Growth Model: Part II

Terry J. Fitzgerald and Edward C. Prescott

October 2002

The Growth Model with Taxes and Government Consumption

There are three tax rates in each period t: τ_{ht} the tax rate on labor income, τ_{kt} the tax rate on capital income, and τ_{ct} the tax rate on consumption. The government consumption share in period t is ν_t (share refers to the total output of the economy). Public school expenditures are not part of public consumption. They are treated as transfers in kind to the household. All services provided to the people by the government are treated as transfers. On the other hand, military expenditures are part of *G*, because they are public goods and there is not a good private substitute.

The households' budget constraints with these taxes is

$$d_{t+1} = \left(1+(1-\tau_{kt})i_t\right)\ \left[d_t + (1-\tau_{ht})w_t h_t - (1+\tau_{ct})c_t + T_t\right].$$

Households' pay the tax on capital income in this setup. The budget constraint for the government is

$$\tau_{ct}c_t + \tau_{kt}(r_t - \delta)k_t + \tau_{ht}w_t h_t = T_t + G_t$$

where $G_t = \nu_t y_t$, and T_t are lump-sum transfers (e.g. public school expenditures). If T_t is negative, $-T_t$ are lump-sum taxes.

The introduction of a public sector requires the following three changes in the equilibrium conditions.

Change 1: Equation (M) becomes, as government purchases are $G_t = \nu_t \; y_t$,

$$G_t + c_t + x_t = F(h_t, k_t, t)$$

or

(M)′
$$c_t + x_t = (1-\nu_t)F(h_t, k_t, t)$$

Change 2: The household equations become

(H1)′
$$\frac{u_1(c_t, 1-h_t)}{u_2(c_t, 1-h_t)} = \frac{(1+\tau_{ct})}{(1-\tau_{ht})w_t}$$

Change 3:

(H2)′
$$\frac{u_1(c_t, 1-h_t)}{u_1(c_{t+1}, 1-h_{t+1})} = \frac{1+(1-\tau_{kt}) \; i_t}{(1+\rho)}.$$

Note that the reason for these changes is that after-tax rather than before-tax prices are used.

Exercise 5: Make the appropriate modifications to the constant growth equilibrium equations (1), (2), and (5) in Part I of these notes. List all the constant growth equations for the model with taxes and population growth.

Equilibrium Paths

Given the parameters $\{\rho, \alpha, \theta, \delta\}$, exogenous paths for $\left\{A_t, N_t, \tau_{ct}, \tau_{ht}, \tau_{kt}, \nu_t\right\}_{t=0,1,2,\ldots}$, and an initial capital stock, competitive equilibrium sequences of prices and quantities can be computed. The competitive equilibrium path, $\{c_t, x_t, d_t, h_t, k_t\}_{t=0}^{\infty}$ and $\{r_t, w_t, i_t\}$, must satisfy conditions (H1)-H(2), F(1)-F(2), (H1)-(B3) and (M).

Welfare Analysis

Ultimately we want to use our model to study the welfare consequences of a change in economic policies. Fiscal policies that change the time path of tax rates and government consumption can be evaluated. Policies that change the path of the productivity parameter $\{A_t\}$ also can be evaluated

Welfare analysis is conducted as follows. Consider two model economies, labeled A and B. The equilibrium path associated with each model economy has a utility U associated with it. These utilities are computed as

$$U^A = \sum_{t=0}^{\infty} \frac{N_t\, u(c_t^A, 1 - h_t^A)}{(1+\rho^A)^t}$$
$$U^B = \sum_{t=0}^{\infty} \frac{N_t\, u(c_t^B, 1 - h_t^B)}{(1+\rho^B)^t},$$

for model economies A and B respectively.

Now ask the question, how much must the equilibrium path of consumption per capita in model A be scaled so that the utility for the scaled economy A is the same as the utility for economy B? Let ϕ denote the scale factor for lifetime consumption. Then mathematically we are looking for the ϕ satisfying

$$U^A(\phi) = \sum_{t=0}^{\infty} \frac{N_t\, u(\phi c_t^A, 1 - h_t^A)}{(1+\rho^A)^t} = U^B.$$

If ϕ is greater than 1, consumption must be scaled up in economy A to produce the same utility as economy B. Thus the equilibrium paths of A must produce a lower utility than the equilibrium of B. The opposite is true if ϕ is less than 1.

Exercise 6: U.S. productivity is 43 percent greater than that of Japan. How much would the Japanese people benefit if it could instantly achieve the U.S. productivity? To answer this question, let the first economy considered be the Japanese economy with the Japanese productivity. The second economy is identical to the Japanese economy except that its productivity is the same as the U.S. productivity. Set the initial capital stock in each economy to the steady state capital stock in the first economy. Values of the other parameters are $\rho = .02$, $\alpha = 1.44$, $\theta = .30$, and $\delta = .07$. The tax rates in both economies are constant over time with $\tau_c = 0.14$, $\tau_h = 0.28$, $\tau_k = 0.50$, and $\nu = 0.9$. Population growth is 0.0 and productivity growth is 2.0 percent in both economies. Find the welfare gains in lifetime consumption equivalents if suddenly Japan became as productive as the United States.

Plot the path of the natural logarithm of total product for the first 30 years. Also plot the growth rate, investment share of output, and the real interest rate. You will need the MacroLab.