



Optimal Capital Taxation Revisited

V. V. Chari

University of Minnesota and Federal Reserve Bank of Minneapolis

Juan Pablo Nicolini

Federal Reserve Bank of Minneapolis,

Universidad Di Tella, and Universidad Autonoma de Barcelona

Pedro Teles

Banco de Portugal, Catolica Lisbon SBE, and CEPR

Working Paper 752

July 2018

DOI: <https://doi.org/10.21034/wp.752>

Keywords: Capital income tax; Long run; Uniform taxation

JEL classification: E60, E61, E62

The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve System, Banco de Portugal, or the European System of Central Banks.

Federal Reserve Bank of Minneapolis • 90 Hennepin Avenue • Minneapolis, MN 55480-0291

<https://www.minneapolisfed.org/research/>

Optimal Capital Taxation Revisited*

V. V. Chari

University of Minnesota and Federal Reserve Bank of Minneapolis

Juan Pablo Nicolini

Federal Reserve Bank of Minneapolis

Universidad Di Tella, Universidad Autonoma de Barcelona

Pedro Teles

Banco de Portugal, Catolica Lisbon SBE, CEPR

July, 2018

Abstract

We revisit the question of how capital should be taxed, arguing that if governments are allowed to use the kinds of tax instruments widely used in practice, for preferences that are standard in the macroeconomic literature, the optimal approach is to never distort capital accumulation. We show that the results in the literature that lead to the presumption that capital ought to be taxed for some time arise because of the initial confiscation of wealth and because the tax system is restricted.

Keywords: capital income tax; long run; uniform taxation

*This is a revised version of a paper that circulated with the title: "More on the Taxation of Capital Income". We thank Isabel Correia, João Guerreiro, Albert Marcet, Ellen McGrattan, Chris Phelan, Catarina Reis, and Ivan Werning for helpful discussions. Chari thanks the NSF for supporting the research in this paper and Teles is grateful for the support of FCT as well as the ADEMU project, "A Dynamic Economic and Monetary Union," funded by the European Union's Horizon 2020 Program under grant agreement 649396. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve System, Banco de Portugal, or the European System of Central Banks. E-mail addresses: varadarajanchari@gmail.com, juanpa@minneapolisfed.org, pteles@ucp.pt.

1 Introduction

How should capital be taxed? How should it be taxed in the long run and along the transition? An influential literature on the optimal Ramsey taxation of capital, as in Chamley (1986) and Judd (1985), compares a tax on labor income with a particular tax on capital income that is capped at some level. The common result is that capital should be taxed at its maximum level initially and for a number of periods, but should not be taxed in the steady state. More recently, Straub and Werning (2015) showed that in that same environment, full taxation of capital can actually last forever.¹ This literature leads to the presumption that capital taxes should be high for some length of time.

In this paper, we take the same Ramsey approach to the optimal taxation of capital in that the tax system is exogenously given, but enlarge the set of instruments to include other taxes widely used in practice in developed economies, such as dividend, consumption, and wealth taxes. We refer to a tax system with this enlarged set of tax instruments as a *rich tax system*. As is well known, many tax policies yield the same distortions, and the theory pins down those distortions in choices. Following the public finance literature, we refer to these distortions as *wedges*.

The main question we address in this paper is, does the Ramsey policy yield intertemporal wedges? If it does, we say future capital is taxed. If it does not, we say that future capital is not taxed. We begin by studying the standard neoclassical growth model with a representative agent. We show that with a rich tax system, capital should not be taxed in the steady state. Along the transition, capital may be taxed or subsidized. We then consider a class of preferences that are standard in the macroeconomics literature and show that with these preferences, future capital should never be taxed, except possibly for one period. We then consider heterogeneous agent economies in which agents differ in their initial wealth. We show that the representative agent results also hold in those economies with heterogeneous agents. Our results differ from those

¹Full taxation of capital income forever is also the optimal outcome in Bassetto and Benhabib (2006), in a different environment. Other relevant literature includes Chari, Christiano, and Kehoe (1994), Atkeson, Chari, and Kehoe (1999), Judd (1999, 2002), Coleman (2000), and Lucas and Stokey (1983).

in the literature described above because of assumptions on the confiscation of initial wealth and on the available tax instruments. We restrict the initial confiscation both directly and indirectly through valuation effects, whereas the literature only restricts direct confiscation. We allow for a rich tax system, whereas the literature considers a restricted tax system.

A central feature of the literature on optimal taxation is that, absent other restrictions, factors in fixed supply should be taxed away completely. This feature implies that in the growth model with a representative agent, the initial capital as well as holdings of government bonds should be taxed, possibly at rates in excess of 100% in order to fund government spending. The Ramsey literature conventionally imposes restrictions on such taxes on initial wealth. With such restrictions on taxes, the Ramsey planner will still be able to affect the value of the initial wealth by moving future taxes. This means that direct confiscation is restricted, but indirect confiscation is not. We restrict both direct and indirect confiscation by imposing the restriction in Armenter (2008) on the value of initial wealth in utility terms, rather than on the taxes themselves.

We show that, for general preferences, capital should not be taxed in the steady state. Along the transition, capital may be taxed or subsidized. For standard preferences in the macroeconomics literature with constant consumption and labor elasticities, future capital should never be taxed.

Once we adopt the standard restriction in the literature that initial taxes are exogenous, but keep a rich tax system, we obtain that capital accumulation in the very first period is distorted and is undistorted thereafter. The reason for that initial distortion is so that initial wealth may be confiscated indirectly through valuation effects. Those distortions will be longer lasting if the tax system is restricted. We impose the restrictions on taxes in the literature described earlier. With taxes on capital and labor only, and with capital taxes restricted to be less than 100%, we recover the results in Chamley (1986), Judd (1985), Bassetto and Benhabib (2006), and Straub and Werning (2015) that capital should be taxed fully for some, possibly infinite, length of time. The key force underlying these results is that such taxation reduces the value of initial wealth and therefore represents an attempt to confiscate initial wealth indirectly, given that the government is not allowed to confiscate that wealth directly.

These alternative time zero restrictions on the Ramsey problem are suggestive of precommitment. We solve for the optimal policy solutions, imposing every period the

same form of one-period commitment, either to wealth in utility terms or to tax rates. When we assume one-period commitment to returns, which amounts to commitment to the next-period wealth in utility terms, the optimal policy solution coincides with the Ramsey solution with full commitment. Instead, with one-period commitment to tax rates, the solution differs from the one with full commitment. The consistency under one assumption and inconsistency under the other rationalize our preferred treatment of the initial confiscation.

We briefly analyze an economy with heterogeneous agents and show that the representative agent results hold in such economies. In the heterogeneous agent economy, as in Werning (2007), there is no reason to impose any restrictions on the initial confiscation. The planner can confiscate directly and indirectly, even if the planner will not necessarily do it for distribution reasons. Given that direct confiscation is allowed for, there is no reason to confiscate indirectly because of the costly distortions on capital accumulation. The solution has the same features as the one in the representative agent economy with a restriction on initial wealth in utility terms. This is an additional justification for the restriction we impose on both direct and indirect confiscation.

Finally, we relate our results to those on uniform commodity taxation (Atkinson and Stiglitz, (1972)). Standard preferences used in the macroeconomic literature are - and homothetic in consumption and labor. With these preferences, the growth model can be recast as a model in which constant returns to scale technologies are used by competitive firms to produce one final consumption composite good and one labor aggregates. In this recast economy, we show that the Diamond and Mirrlees (1971) production efficiency theorem can be extended to obtain that the optimal approach is to not distort the use of intermediate goods. These intermediate goods consist of consumption, labor, and capital at each date in the original economy. This result implies that in the original economy, future capital should never be taxed.

2 A representative agent economy

The model is the deterministic neoclassical growth model with taxes. The preferences of a representative household are defined over consumption c_t and labor n_t ,

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t), \quad (1)$$

satisfying the usual properties.

The production technology is described by

$$c_t + g_t + k_{t+1} - (1 - \delta) k_t \leq F(n_t, k_t) \quad (2)$$

where k_t is capital, g_t is exogenous government consumption, δ is the depreciation rate, and F is constant returns to scale.

We now describe a competitive equilibrium with taxes in which the government finances public consumption and initial debt with (possibly) time-varying proportional taxes. We allow for a rich tax system that includes taxes on consumption τ_t^c , labor income τ_t^n , capital income τ_t^k , dividends τ_t^d , a tax on initial wealth, l_0 , and a nonnegative lump-sum transfer T_0 in period zero.

Capital accumulation is conducted by firms. Given that the technology is constant returns to scale, we assume without loss of generality that the economy has a representative firm. The households trade shares of the firm and receives dividends. In Appendix 1 we describe an alternative, more widely used decentralization in which the households own the capital stock and firms rent capital from the households. The two decentralizations are equivalent, but it is easier to relate the taxes in the decentralization described here to the ones in existing tax systems. We now describe the the firms' problem and the households' problem and define a competitive equilibrium.

Firms The representative firm produces and invests in order to maximize the present value of dividends net of taxes, $\sum_{t=0}^{\infty} q_t (1 - \tau_t^d) d_t$, where q_t is the price of one unit of the good produced in period t in units of the good in period zero and τ_t^d are dividend taxes. Dividends, d_t , are given by

$$d_t = F(k_t, n_t) - w_t n_t - \tau_t^k [F(k_t, n_t) - w_t n_t - \delta k_t] - [k_{t+1} - (1 - \delta) k_t], \quad (3)$$

where w_t is the pretax wage rate and τ_t^k is the tax rate on capital income net of depreciation.

Note that in this way of setting up the competitive equilibrium, dividends are net payments to claimants of the firm. These payments could be interpreted either as payments on debt or as payments to equity holders. To clarify this interpretation, consider an all-equity firm. In this case, our notion of dividends consists of cash dividends plus stock buybacks less issues of new equity. In particular, under this

interpretation, taxes on capital gains associated with stock buybacks are assumed to be levied on accrual and at the same rate as cash dividends. Note also that dividends could be negative if returns to capital are smaller than investment. In this case, a positive tax on dividends would represent a subsidy to the firm. (In a steady state of the competitive equilibrium, it is possible to show that dividends are always positive).

Let the interest rate between periods t and $t + 1$ be defined by

$$\frac{q_t}{q_{t+1}} \equiv 1 + r_t, \text{ with } q_0 = 1. \quad (4)$$

The first-order conditions of the firm's problem are

$$F_{n,t} = w_t \quad (5)$$

$$1 + r_t = \frac{(1 - \tau_{t+1}^d) [1 + (1 - \tau_{t+1}^k) (F_{k,t+1} - \delta)]}{1 - \tau_t^d} \quad (6)$$

where $F_{n,t}$ and $F_{k,t}$ denote the marginal products of capital and labor in period t .

Substituting for d_t from (3) and using (4) – (6), it is easy to show that the present discounted value of dividends is given by

$$\sum_{t=0}^{\infty} q_t (1 - \tau_t^d) d_t = (1 - \tau_0^d) [1 + (1 - \tau_0^k) (F_{k,0} - \delta)] k_0. \quad (7)$$

Households The flow of funds constraint in period t for the households is given by

$$\frac{1}{1 + r_t} b_{t+1} + p_t s_{t+1} = b_t + p_t s_t + (1 - \tau_t^d) d_t s_t + (1 - \tau_t^n) w_t n_t - (1 + \tau_t^c) c_t \quad (8a)$$

for $t \geq 1$, and for period zero,

$$\frac{1}{1 + r_0} b_1 + p_0 s_1 = (1 - l_0) [b_0 + p_0 s_0 + (1 - \tau_0^d) d_0 s_0] + \quad (9)$$

$$(1 - \tau_0^n) w_0 n_0 - (1 + \tau_0^c) c_0 + T_0, \quad (10)$$

where b_{t+1} denotes holdings of government debt that pay one unit of consumption in period $t + 1$, s_{t+1} denotes the households' holdings of the shares of the firm, and p_t is

the price per unit of the firm's shares in units of the good in period t .² Note that the price p_t is the price of shares after dividends have been paid in period t . The initial conditions are given by b_0 and $s_0 = 1$.

The households' problem is to maximize utility (1), subject to (8a), (9), and a no-Ponzi-scheme condition, $\lim_{T \rightarrow \infty} q_{T+1} b_{T+1} \geq 0$.

The first-order conditions of the households' problem include

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{(1 + \tau_t^c)}{(1 - \tau_t^n) w_t}, \quad t \geq 0, \quad (11)$$

$$\frac{u_{c,t}}{(1 + \tau_t^c)} = (1 + r_t) \frac{\beta u_{c,t+1}}{(1 + \tau_{t+1}^c)}, \quad t \geq 0, \quad (12)$$

and

$$1 + r_t = \frac{p_{t+1} + (1 - \tau_{t+1}^d) d_{t+1}}{p_t}, \quad (13)$$

for all t , where $u_{c,t}$ and $u_{n,t}$ denote the marginal utilities of consumption and labor in period t .

The transversality condition implies that the price of the stock equals the present value of future dividends,

$$p_t = \sum_{s=0}^{\infty} \frac{q_{t+1+s}}{q_t} (1 - \tau_{t+1+s}^d) d_{t+1+s}. \quad (14)$$

Using the no-Ponzi scheme condition, the budget constraints of the households, (8a) and (15), can be consolidated into a single budget constraint,

$$\sum_{t=0}^{\infty} q_t [(1 + \tau_t^c) c_t - (1 - \tau_t^n) w_t n_t] \leq (1 - l_0) [b_0 + p_0 s_0 + (1 - \tau_0^d) d_0 s_0] + T_0, \quad (15)$$

Substituting for the price of the stock from (14) for $t = 0$, and using (7) as well as $s_0 = 1$, the budget constraint can be written as

$$\sum_{t=0}^{\infty} q_t [(1 + \tau_t^c) c_t - (1 - \tau_t^n) w_t n_t] \leq W_0 + T_0 \quad (16)$$

²Note that we allow only for a tax on wealth in period zero. It turns out that allowing for taxes on wealth in future periods is equivalent to a consumption tax. Since we allow for consumption taxes, taxes on future wealth are redundant.

where the initial wealth of the households, excluding the lump sum transfer, is given by

$$W_0 \equiv (1 - l_0) [b_0 + (1 - \tau_0^d) [k_0 + (1 - \tau_0^k) (F_{k,0} - \delta) k_0]].$$

A *competitive equilibrium* for this economy consists of a set of allocations $\{c_t, n_t, d_t\}$ and $\{k_{t+1}, b_{t+1}, s_{t+1}\}$, prices $\{q_t, p_t, w_t\}$, and policies $\{\tau_t^c, \tau_t^n, \tau_t^d, \tau_t^k, l_0, T_0\}$, given $\{k_0, b_0, s_0\}$ such that the households maximize utility subject to their constraints, firms maximize value and markets clear in that resource constraints (2) are satisfied and the market for shares clears, $s_t = 1$ for all t .

Note that we have not explicitly specified the government's budget constraint because it is implied by the households' budget constraint and market clearing.

We find it convenient to refer to a subset of the allocations $\{c_t, n_t, k_{t+1}\}_{t=0}^\infty$ as *implementable allocations* if they are part of a competitive equilibrium.

A *Ramsey equilibrium* is the best competitive equilibrium, and the *Ramsey allocation* is the associated implementable allocation.

Given our focus on the extent to which optimal tax systems distort intertemporal decisions, we begin by providing a partial characterization of the distortions introduced by taxes. To obtain this characterization, consider the first-order conditions associated with the equilibrium when lump-sum taxes are available. These are given by

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{1}{F_{n,t}}, \quad (17)$$

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = 1 + F_{k,t+1} - \delta, \quad (18)$$

$$\frac{u_{n,t}}{\beta u_{n,t+1}} = \frac{F_{n,t}}{F_{n,t+1}} [1 + [F_{k,t+1} - \delta]], \quad (19)$$

and the resource constraints (2). We have explicitly characterized the intertemporal labor margin in (19) because we will be interested in understanding when it is optimal to not distort this margin.

In the growth model with distorting taxes, we can combine the first-order conditions of the households and the firms to obtain that taxes introduce wedges in those first-order conditions as follows:

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{(1 + \tau_t^c)}{(1 - \tau_t^n) F_{n,t}}, \quad (20)$$

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = \frac{(1 - \tau_{t+1}^d)(1 + \tau_t^c)}{(1 - \tau_t^d)(1 + \tau_{t+1}^c)} [1 + (1 - \tau_{t+1}^k) [F_{k,t+1} - \delta]], \quad (21)$$

and

$$\frac{u_{n,t}}{\beta u_{n,t+1}} = \frac{(1 - \tau_{t+1}^d)(1 - \tau_t^n)}{(1 - \tau_t^d)(1 - \tau_{t+1}^n)} \frac{F_{n,t}}{F_{n,t+1}} [1 + (1 - \tau_{t+1}^k) [F_{k,t+1} - \delta]]. \quad (22)$$

Notice that a constant dividend tax does not distort any of the marginal conditions. Such a tax of course raises revenues by reducing the value of the firm at the beginning of period zero. In this sense, a constant dividend tax is equivalent to a levy on the initial capital stock. Notice also that a tax on capital income distorts intertemporal decisions in the same way as do time-varying taxes on consumption, dividends, and labor income. Indeed, as shown below, many tax systems can implement the same allocations.

A competitive equilibrium has *no intertemporal distortions in consumption* from period s onward if the first-order conditions (18) and (21) coincide for all $t \geq s$. Similarly, a competitive equilibrium has *no intertemporal distortions in labor* from period s onward if the first-order conditions (19) and (22) coincide for all $t \geq s$. Finally, a competitive equilibrium has *no intertemporal distortions* from period s onward if it has no such distortions for both consumption and labor.

Implementability In order to characterize the Ramsey equilibrium, we begin by characterizing the set of implementable allocations. In order to do so, we substitute prices and taxes from the first-order conditions for the households into the households' budget constraint (16) and use the nonnegativity of the lump sum transfers to obtain

$$\sum_{t=0}^{\infty} \beta^t [u_{c,t} c_t + u_{n,t} n_t] \geq \mathcal{W}_0, \quad (23)$$

where

$$\mathcal{W}_0 = \frac{u_{c,0}}{(1 + \tau_0^c)} W_0. \quad (24)$$

Thus, any implementable allocations, together with initial conditions and period zero policies, must satisfy (23) and the resource constraints (2). We now show that the converse also holds. Specifically, consider an arbitrary allocation that, together with initial conditions and period zero policies, satisfies (23) and (2). We will show that this allocation is implementable. To do so, we construct the remaining elements

of the allocation, prices, and policies and show that all the conditions of a competitive equilibrium are satisfied.

Since multiple tax systems can implement the same allocation, for simplicity we begin by considering the case where $\tau_t^k = 0$ for $t \geq 1$ and $\tau_t^c = \tau_0^c$ for all t . The wage rates w_t are pinned down by (5), and the tax rate on labor τ_t^n is pinned down by (20). Given τ_0^d , the time path of dividend taxes is pinned down by (21), while the time path of consumption prices q_t for $t \geq 1$ is determined by (4) and (6), given $q_0 = 1$. Finally, (14) determines the stock prices p_t , the households' flow of funds determines debt holdings b_{t+1} , and dividends d_t are given by (3). It is immediate that these allocations satisfy all the marginal conditions for households and firms. The lump sum transfers are chosen to satisfy the households' budget constraint. Thus, the so constructed allocation, prices, and policies are a competitive equilibrium.

We summarize this discussion in the following proposition.

Proposition 1: (Characterization of the implementable allocations) Any implementable allocation satisfies the implementability constraint (23) and the resource constraints (2). Furthermore, if a sequence $\{c_t, n_t, k_{t+1}\}$, initial conditions k_0, b_0 and period zero policies $(\tau_0^c, \tau_0^d, \tau_0^k, l_0)$, satisfy (23) and (2), it is implementable.

We emphasize that each implementable allocation can be implemented in numerous ways. For example, consider a tax system that arbitrarily specifies a sequence of taxes on capital income $\tilde{\tau}_t^k$. The other taxes can be constructed using a procedure similar to the one described above. Alternatively, if taxes on dividends are set to zero, time varying taxes on consumption and labor can be chosen to implement the same allocations.

Given that capital taxes here are redundant instruments, what does it mean that capital should not be taxed? In our view, the relevant question is whether it is optimal to have no intertemporal distortions.

Next we consider restrictions on tax rates. One common practice is to impose an upper bound on the capital income tax. One justification for this upper bound is that the tax revenue ought not to exceed the base, so that $\tau_t^k \leq 1$. Such restrictions are imposed in Chamley (1986), Judd (1985), Bassetto and Benhabib (2006), and Straub and Werning (2015). These restrictions do not affect the set of implementable allocations because a rich tax system has alternative taxes.

Note that analogous restrictions on labor and dividend taxes such as $\tau_t^n \leq 1, \tau_t^d \leq 1$ do not restrict the set of implementable allocations. This result follows immediately

from inspecting (20) and (21).

2.1 Ramsey equilibrium

Given Proposition 1, it follows that the Ramsey allocation, together with period zero policies, maximizes utility subject to (23) and (2). Suppose that policies are unrestricted in the sense that any one of the taxes on wealth, dividends, or capital income can be greater than 100%. Then, in a Ramsey equilibrium it is possible to set \mathcal{W}_0 to any arbitrary value. It immediately follows that it is possible to implement the lump-sum tax allocation as the Ramsey equilibrium, and we have the following proposition.

Proposition 2: (No distortions ever) If period zero policies are unrestricted, then the Ramsey equilibrium coincides with the lump-sum tax allocation.

Suppose now that policies and initial conditions are restricted in the sense that the households must be allowed to keep an exogenous value of initial wealth $\bar{\mathcal{W}}$, measured in units of utility. Specifically, we impose the following restriction on the Ramsey problem:

$$\mathcal{W}_0 = \bar{\mathcal{W}},$$

which we refer to as the *wealth restriction in utility terms*. One example of such a restriction is that taxes on wealth, dividends and capital income cannot exceed 100% and that the initial debt, b_0 , is positive. Then, since it is possible to set the tax on wealth equal to 100%, then $\bar{\mathcal{W}}$ is zero.

With this restriction, policies, including initial policies, can be chosen arbitrarily but the households must receive a value of initial wealth in utility terms of $\bar{\mathcal{W}}$ (see Armenter (2008) for an analysis with such a restriction). We show below that this outcome is the equilibrium outcome for an environment with partial commitment.

We now characterize the first order necessary conditions for an interior solution to the Ramsey problem. These are

$$-\frac{u_{c,t}}{u_{n,t}} = \frac{1 + \varphi [1 + \sigma_t^n - \sigma_t^{nc}]}{1 + \varphi [1 - \sigma_t - \sigma_t^{cn}]} \frac{1}{F_{nt}}, t \geq 0 \quad (25)$$

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = \frac{1 + \varphi [1 - \sigma_{t+1} - \sigma_{t+1}^{cn}]}{1 + \varphi [1 - \sigma_t - \sigma_t^{cn}]} [1 + F_{k,t+1} - \delta], t \geq 0 \quad (26)$$

$$\frac{u_{n,t}}{\beta u_{n,t+1}} = \frac{1 + \varphi (1 + \sigma_{t+1}^n - \sigma_{t+1}^{nc})}{1 + \varphi (1 + \sigma_t^n - \sigma_t^{nc})} \frac{F_{n,t} [1 + F_{k,t+1} - \delta]}{F_{n,t+1}}, t \geq 0 \quad (27)$$

together with the constraints. Here,

$$\sigma_t = -\frac{u_{cc,t}C_t}{u_{c,t}}, \sigma_t^n = \frac{u_{nn,t}N_t}{u_{n,t}}, \sigma_t^{nc} = -\frac{u_{nc,t}C_t}{u_{n,t}}, \sigma_t^{cn} = -\frac{u_{cn,t}N_t}{u_{c,t}},$$

and φ is the multiplier of the implementability condition.

Comparing these conditions (25) – (27) with the related conditions with lump-sum taxes (17) – (19) it is clear that the optimal wedges depend on their own and cross elasticities of consumption and labor. If those elasticities are constant, it is optimal to not have intertemporal distortions. Note that in this case, intratemporal wedges are constant and in general positive.

Note that conditions (26) and (27) imply that if elasticities are not constant over time, it is optimal to have intertemporal distortions, but whether it is optimal to effectively tax or subsidize capital accumulation depends on whether elasticities are increasing or decreasing over time.

Note also that if consumption and labor are constant over time, then the relevant elasticities are also constant, so that it is optimal to have no intertemporal distortions. This observation leads to the following proposition.

Proposition 3: (No intertemporal distortions in the steady state) If the Ramsey equilibrium converges to a steady state, it is optimal to have no intertemporal distortions asymptotically.

Consider now preferences that are standard in the macroeconomics literature. These preferences take the form

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma} - 1}{1-\sigma} - \eta n_t^\psi \right]. \quad (28)$$

In this case, the elasticities are constant, so that we have the following proposition.

Proposition 4: (No intertemporal distortions ever) Suppose that preferences are given by (28) and the wealth restriction must be satisfied. Then, the Ramsey solution has no intertemporal distortions for all $t \geq 0$.

Note that the preferences above are separable and homothetic in both consumption and labor. (In Appendix 2 we show that they are the only time-separable preferences with those properties) We use these properties to provide intuition for the results in Section 4 below, where we relate them to results on uniform commodity taxation and production efficiency.

The Ramsey equilibrium characterized in Proposition 3 can be implemented in a variety of ways. In one implementation, the initial wealth tax rate l_0 is chosen to satisfy the wealth constraint, consumption is taxed at a constant rate over time, and all other taxes are set to zero. In an alternative implementation, initial wealth is taxed, labor is taxed at a constant rate, and all other taxes are set to zero.

Consider the particular case in which $\bar{W} = 0$. In this case, the Ramsey policy effectively confiscates all of the households' initial wealth. If this initial wealth is large enough relative to the present value of government expenditures, it is possible to implement the lump-sum tax allocation. If it is not, then taxes are used in all periods to finance the remaining present value of government expenditures. Note that even in this case, Proposition 4 says that it is optimal to not distort intertemporal decisions. That is, in this case one implementation of the Ramsey equilibrium is to tax consumption at a constant positive rate over time and set all taxes, other than the taxes on initial wealth, to zero.

Suppose next that policies are restricted in that the households must keep at least \mathcal{V}_0 initial wealth, but now in units of goods rather than in utility terms. This *wealth restriction in goods units* implies that the constraint faced by the Ramsey planner on the confiscation of initial wealth is

$$\frac{W_0}{1 + \tau_0^c} \geq \bar{\mathcal{V}}.$$

The implementability constraint can then be written as

$$\sum_{t=0}^{\infty} \beta^t [u_{c,t}c_t + u_{n,t}n_t] \geq u_{c,0}\bar{\mathcal{V}}.$$

The problem is the same as before except for the term on the right-hand side. The Ramsey conditions are the same as before in (25), (26), and (27), for all $t \geq 1$. The conditions for period zero are different. The intertemporal condition for consumption between periods zero and one, for example, is now

$$\frac{u_{c,0}}{\beta u_{c,1}} = \frac{1 + \varphi(1 - \sigma_1 + \sigma_1^{cn})}{1 + \varphi\left(1 - \sigma_0 + \sigma_0^{cn} + \frac{\sigma_0\bar{\mathcal{V}}}{c_0}\right)} [1 + F_{k,1} - \delta], \quad (29)$$

and the intratemporal condition at time zero is

$$-\frac{u_{c,0}}{u_{n,0}} = \frac{1 + \varphi \left[1 + \sigma_0^n - \sigma_0^{nc} + \sigma_0^{nc} \frac{\bar{\mathcal{V}}}{c_0} \right]}{1 + \varphi \left[1 - \sigma_0 + \sigma_0^{cn} + \sigma_0 \frac{\bar{\mathcal{V}}}{c_0} \right]} \frac{1}{F_{n,0}}. \quad (30)$$

Since the Ramsey conditions for $t \geq 1$ are unaffected, as before, whether it is optimal to effectively tax or subsidize capital accumulation depends on whether elasticities are increasing or decreasing over time.

With standard macro preferences, since elasticities are constant over time, it is optimal to have no intertemporal distortions from period one onward. Consider now intertemporal distortions in period zero. With standard macro preferences $\sigma_1 = \sigma_0$ and zero cross elasticities, so that if $\bar{\mathcal{V}} > 0$, (29) implies that

$$\frac{u_{c,0}}{\beta u_{c,1}} < 1 + F_{k,1} - \delta.$$

Thus, it is optimal to effectively tax capital accumulation in period zero, or subsidize the consumption good in period zero, relative to consumption in future periods. One intuition for this result is as follows. The households are entitled to an exogenous amount of wealth in period zero. The Ramsey planner finds it optimal to reduce the value of this wealth in utility terms. This value can be reduced by decreasing the marginal utility of period zero consumption. This decrease is achieved by inducing households to increase their period zero consumption relative to consumption in all future periods. We summarize this discussion in the following proposition.

Proposition 5: (No intertemporal distortions after one period) Suppose preferences satisfy (28) and the wealth restriction in goods units must be satisfied. Then the Ramsey solution has no intertemporal distortions for all $t \geq 1$. If $\bar{\mathcal{V}} > 0$, it is optimal to effectively tax capital accumulation from period zero to period one.

In Section 4 below we relate this result to results on uniform commodity taxation and production efficiency.

The Ramsey allocation can be implemented as follows: Set the initial tax rate on wealth to satisfy the wealth restriction; set capital income and consumption taxes to zero in all periods; and set the labor income tax to satisfy (20), (25) and (30). Specifically, set the labor income tax to

$$1 - \tau_0^n = \frac{1 + \varphi \left[1 - \sigma + \frac{\sigma \bar{\mathcal{V}}}{c_0} \right]}{1 + \varphi [1 + \sigma^n]}$$

in period zero and to

$$1 - \tau^n = \frac{1 + \varphi [1 - \sigma]}{1 + \varphi [1 + \sigma^n]}$$

in all future periods. Set the dividend tax to zero in period zero and then to a constant value thereafter. This constant value τ^d must satisfy (29) and (21), so that its value is given by

$$\frac{1 + \varphi (1 - \sigma)}{1 + \varphi \left(1 - \sigma + \frac{\sigma \bar{V}}{c_0}\right)} = 1 - \tau^d.$$

Note that under this implementation, the tax rate on dividends is always less than one and is positive if \bar{V} is positive. An alternative implementation uses the consumption tax rather than the dividend tax. The disadvantage of this implementation is that in order to satisfy (20) and (25), the required tax on labor income might have to be negative to compensate for the effect of the higher consumption tax after period 1. The dividend tax implementation has the advantage that this tax affects only intertemporal decisions, so that the labor income tax can be chosen to satisfy the intratemporal condition. The consumption tax has the disadvantage that it affects inter- and intratemporal decisions. The dividend tax has the disadvantage that, as we remarked earlier, the base on which it is levied could be negative, so that the tax would constitute a subsidy to the firm.

The standard implementation in the literature uses capital and labor income taxes and sets consumption and dividend taxes to zero. A disadvantage of this implementation is that the capital income tax may have to be greater than 100% to implement the Ramsey allocation. Given this disadvantage, the literature typically imposes an additional restriction that the tax rates on capital income cannot exceed some upper limit $\bar{\tau}$. This restriction implies the following additional constraint on the Ramsey problem:

$$\frac{\frac{u_{c,t}}{\beta u_{c,t+1}} - 1}{F_{k,t+1} - \delta} \geq 1 - \bar{\tau}.$$

This restriction may bind for a number of periods as in Chamley (1986) or forever as in Straub and Werning (2015). Straub and Werning (2015) allow the maximum tax rate to be 100% and show that the optimal solution for particularly high levels of initial debt may be to have the capital income tax set at 100% forever. One intuition for the Straub and Werning (2015) finding is that by taxing capital income forever,

real interest rates are zero forever, and that is the way consumption in period zero can be increased the most, reducing the value of the good in the initial period. Given that the initial real rate cannot be below zero, the whole term structure is flattened down to zero.

To see this more clearly, notice that the planner has a strong incentive to make $u_{c,0}$ small so as to reduce the value of initial wealth. We refer to this incentive as the *confiscation motive*. The planner, however, must respect the intertemporal conditions with restricted taxes,

$$\frac{u_{c,t}}{\beta u_{c,t+1}} = 1 + (1 - \tau_{t+1}^k) (F_{k,t+1} - \delta).$$

Given $u_{c,1}$, the confiscation motive provides an incentive to make τ_1^k large to reduce $u_{c,0}$. If the confiscation motive is sufficiently strong, the bound on τ_1^k is met. In this case, the planner has an incentive to make $u_{c,1}$ small to reduce $u_{c,0}$, thereby confiscating initial wealth. Fixing $u_{c,2}$, $u_{c,1}$ in turn can be made small by making τ_2^k large. Again, if the confiscation motive is sufficiently strong, the upper bound will be met. This recursion suggests that the Ramsey solution will have capital taxes be at the upper bound for a length of time, and then zero. If the initial debt is sufficiently large, the confiscation motive is very strong, and the length of time could be infinite as pointed out by Straub and Werning.

With a rich tax system, and with fixed initial policies, the confiscation motive is satisfied in one period by levying a sufficiently high dividend tax in period one, as is apparent from (21). The advantage of the dividend tax is that it effectively allows the tax to apply to a larger base than the capital income tax. This advantage can be seen by inspecting (21) evaluated in period zero with zero consumption taxes and zero dividend taxes in period zero. Notice that if the dividend tax in period one was used fully at 100%, the gross return on capital would be zero. In contrast, full taxation of capital income with $\tau_1^k = 1$ can only reduce the net return to zero, provided $F_{k,1} - \delta \geq 0$.

2.2 Partial commitment equilibria

The notion of a Ramsey equilibrium is developed in an environment in which in period zero the government commits to an infinite sequence of policies. Here we consider an alternative institutional framework in which the government has partial commitment.

We develop a notion of equilibrium for such an environment, referred to as a *partial commitment equilibrium*. In our environment, in any period, governments lack full commitment in the sense that they cannot specify the entire sequence of policies that will be chosen in the future. They do have the ability to constrain the set of policies in the subsequent period. We consider two kinds of constraints.

In the first kind, the government in any period can commit to the one-period returns on assets in utility terms. The government in the following period is free to choose policies as it wishes but must respect the previously committed return constraints. In the second kind, the government in any period can commit to (a subset of) policies in the following period. We show that with the first kind of partial commitment the equilibrium coincides with that under full commitment with constraints on the initial value of wealth. With the second kind of partial commitment, equilibrium outcomes do not coincide with those under full commitment with constraints on initial policies.

Consider the environment with partial commitment on returns. In order to develop our notion of partial commitment in this environment consider the intertemporal Euler equations for bonds and capital from period $t - 1$ to period t :

$$\frac{u_{c,t-1}}{\beta(1+r_{t-1})(1+\tau_{t-1}^c)} = \frac{u_{c,t}}{(1+\tau_t^c)}, \quad (31)$$

$$\frac{u_{c,t-1}(1-\tau_{t-1}^d)}{\beta(1+\tau_{t-1}^c)} = \frac{u_{c,t}(1-\tau_t^d)[1+(1-\tau_t^k)(F_{k,t}-\delta)]}{(1+\tau_t^c)}. \quad (32)$$

Let $\lambda_{1,t}$ denote the right side of (31) and $\lambda_{2,t}$ denote the right side of (32). With partial commitment, the government in period $t - 1$ chooses period $t - 1$ policies as well as $\lambda_{1,t}$ and $\lambda_{2,t}$. The government in period t can choose any policies, but they must have the property that the induced allocations and policies must satisfy the constraints on returns:

$$\lambda_{1,t} = \frac{u_{c,t}}{(1+\tau_t^c)} \quad (33)$$

and

$$\lambda_{2,t} = \frac{u_{c,t}(1-\tau_t^d)[1+(1-\tau_t^k)(F_{k,t}-\delta)]}{(1+\tau_t^c)}. \quad (34)$$

The government in period t chooses period t policies as well as $\lambda_{1,t+1}$ and $\lambda_{2,t+1}$ to constrain policies in period $t + 1$.³ We assume $\lambda_{1,0}$ and $\lambda_{2,0}$ are given.

³We consider the same tax instruments, except that now, in order to treat every period alike, we

In order to understand the nature of partial commitment here, note that the Euler equations for bonds and capital, (31) and (32), will, of course, be satisfied on the equilibrium path. The spirit of this form of partial commitment is that the government must also respect these intertemporal Euler equations off the equilibrium path. The spirit of the assumption that $\lambda_{1,0}$ and $\lambda_{2,0}$ are given is that the economy was operating in previous periods, and the choices made in those previous periods constrain the choices in period zero as well.

Next, we develop a notion of a Markov equilibrium with partial commitment on returns which we call a *nonconfiscatory equilibrium*. It is convenient and without loss of generality to think of the government in period t as choosing allocation, policies, and prices directly in that period. The state of the economy in period t is given by $s_t = \{k_t, b_t, \lambda_{1,t}, \lambda_{2,t}\}$. Let $h_t(s_t)$ denote the policy function in period t that maps the state of the economy into allocation, policies, prices and $\lambda_{1,t+1}, \lambda_{2,t+1}$. The government in period t maximizes welfare, taking as given the continuation value function and the policy functions in period $t + 1$, subject to the marginal conditions of agents, budget constraints, and market clearing conditions in period t . Specifically, the government in period t solves the following problem:

$$v_t(s_t) = \max \{u(c_t, n_t) + \beta v_t(s_{t+1})\}, \quad (35)$$

subject to the period t equilibrium conditions and (33) and (34). Note that the policy function $h_{t+1}(s_{t+1})$ enters the intertemporal Euler equations in these equilibrium conditions. For example, period $t + 1$ policies on consumption, labor, and the consumption tax appear in the households' bond Euler equation,

$$\frac{u_{c,t}}{(1 + \tau_t^c)} = (1 + r_t) \frac{\beta u_c(c_{t+1}(s_{t+1}), n_{t+1}(s_{t+1}))}{(1 + \tau_{t+1}^c(s_{t+1}))},$$

where $c_{t+1}(s_{t+1})$, $n_{t+1}(s_{t+1})$, and $\tau_{t+1}^c(s_{t+1})$ are elements of $h_{t+1}(s_{t+1})$.

A Markov equilibrium with partial commitment on returns, a *non-confiscatory equilibrium*, consists of value functions $v_t(s_t)$ and policy functions $h_t(s_t)$, which solve (35) for all s_t and all t .

Next we show that the Markov equilibrium outcome coincides with the Ramsey

set the initial wealth tax to zero, $l_0 = 0$.

outcome with wealth constraints. Using the same logic as in our characterization result in Proposition 1, it is straightforward to show that the period t equilibrium conditions can be equivalently represented by the resource constraint and by the following period t implementability constraint:

$$\beta\lambda_{1,t+1}b_{t+1} + \beta\lambda_{2,t+1}k_{t+1} = \lambda_{1,t}b_t + \lambda_{2,t}k_t - u_{n,t}n_t - u_{c,t}c_t. \quad (36)$$

Multiplying these constraints by β^t and summing up yields the implementability constraint of the Ramsey problem. Thus, the Ramsey allocation is feasible. Note that future controls do not appear in the objective function, (35), or the constraint set that includes (36). We can then use an argument identical to that in Stokey, Lucas and Prescott (1989)⁴, to show that the functional equation in (35) solves the date zero sequence problem.

We have proved the following proposition.

Proposition 6: (Partial commitment is full commitment) The Markov outcome of an economy with partial commitment in returns coincides with the Ramsey outcome with wealth restriction given by $W_0 = \lambda_{1,0}b_0 + \lambda_{2,0}k_0$.

Kydland and Prescott (1980) propose a method for computing Ramsey outcomes. They show that a Ramsey equilibrium could be characterized recursively starting in period one, with the addition of a state variable. This state variable represents promised marginal utilities which is the analog to $\lambda_{1,t}$ and $\lambda_{2,t}$ in our environment. The government in period zero maximizes discounted utility while being unconstrained by the added state variable. An extensive literature has exploited this recursive formulation to characterize commitment outcomes. We show here that their clever insight can be used to prove that equilibria in environments where policy makers are constrained to not induce regret coincide with equilibria with full commitment and initial wealth constraints.

A partial commitment equilibrium on instruments Consider next an alternative form of partial commitment. In this form, the government in period t chooses a subset of policies, $\{\tau_{t+1}^c, \tau_{t+1}^k, \tau_{t+1}^d\}$ that will be implemented in period $t + 1$. The government in any period t is free to choose the labor income tax, τ_t^n . The spirit of this assumption is that in the literature, as already discussed, this subset of policies is

⁴Theorem 4.3

exogenously fixed in period zero. To that subset of instruments, we extend this spirit to allow for partial commitment in every period. We show that the Markov equilibrium with this form of partial commitment does not in general coincide with the Ramsey outcomes with exogenously specified initial taxes. Together with our results on partial commitment on returns, this result shows that the nature of partial commitment plays a crucial role in determining whether Markov equilibria coincide with commitment equilibria.

Consider the implementability constraint with this form of partial commitment. From (33) and (34), it follows that the implementability constraint can be written as (36), above. Notice here that $\lambda_{1,t+1}$ and $\lambda_{2,t+1}$ depend not only on policies chosen in the current period, $\{\tau_{t+1}^c, \tau_{t+1}^k, \tau_{t+1}^d\}$, but also on allocations and policies that will be chosen in the next period.

A Markov equilibrium is defined analogously to the one above. The state of the economy in period t is given by $s_t = \{k_t, b_t, \tau_t^c, \tau_t^k, \tau_t^d\}$. As before, let $h_t(s_t)$ denote the policy function that maps the state of the economy into allocations, the labor tax rate, prices and the period $t + 1$ taxes, $\{\tau_{t+1}^c, \tau_{t+1}^k, \tau_{t+1}^d\}$. The government in period t solves the problem analogous to the one above.

Note that in a Markov equilibrium, $\lambda_{1,t+1}$, for example, is given by

$$\lambda_{1,t+1} = \frac{u_c(c_{t+1}(s_{t+1}), n_{t+1}(s_{t+1}))}{(1 + \tau_{t+1}^c(s_t))} \quad (37)$$

This equation shows the precise sense in which $\lambda_{1,t+1}$ depends on the policy function that will be followed in the next period. The government in period t takes this future policy function as given in choosing its current optimal policy. Put differently, future controls appear in the constraint set in period t . The arguments in Stokey, Lucas and Prescott (1989) no longer apply.

Lucas and Stokey (1983) provide examples in production economies without capital where the Ramsey outcome is time inconsistent. Chari and Kehoe (1993) characterize Markov equilibria in that environment. Klein, Krusell, and Ríos-Rull (2008) characterize Markov equilibria in environments similar to ours, with partial commitment to instruments. The results in these papers imply that Markov outcomes are in general different from commitment outcomes.

3 Heterogeneous agent model

The results obtained above for the representative agent economy remain under certain conditions in economies with capital-rich and capital-poor agents. In order to show this, consider an economy with an equal measure of two types of agents, 1 and 2. The social welfare function is

$$\theta U^1 + (1 - \theta) U^2$$

with weight $\theta \in [0, 1]$. The individual preferences are assumed to be the standard preferences allowing for possibly different elasticities for the two types of agents,

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma^i} - 1}{1 - \sigma^i} - \eta^i (n_t)^{\psi^i} \right].$$

The resource constraints are

$$c_t^1 + c_t^2 + g_t + k_{t+1} - (1 - \delta) k_t \leq A_t F(n_t^1 + n_t^2, k_t),$$

where $k_t = k_t^1 + k_t^2$.

The taxes are the ones in the rich tax system considered in the representative agent economy that includes taxes on consumption τ_t^c , labor income τ_t^n , capital income τ_t^k , dividends τ_t^d , and a tax on initial wealth, l_0 . Note that we do not allow for the taxes to differ across agents.

With heterogeneous agents, it turns out that we do not need to impose constraints on the initial policies. In particular, for reasons pointed out in Werning (2007), it turns out that without constraints on wealth taxes, it may be optimal for the planner to distort intratemporal decisions.

The implementability conditions can be written as

$$\sum_{t=0}^{\infty} \beta^t [u_{c,t}^1 c_t^1 + u_{n,t}^1 n_t^1] = u_{c,0}^1 (1 - l_0) V_0^1, \quad (38)$$

and

$$\sum_{t=0}^{\infty} \beta^t [u_{c,t}^2 c_t^2 + u_{n,t}^2 n_t^2] = u_{c,0}^2 (1 - l_0) V_0^2, \quad (39)$$

with $V_0^i = [b_0^i + (1 - \tau_0^d) [1 + (1 - \tau_0^k) (F_{k,0} - \delta)] k_0^i]$. Since the taxes must be the same for the two agents, an implementable allocation must also satisfy the following marginal conditions

$$\frac{u_{c,t}^1}{u_{c,t}^2} = \frac{u_{n,t}^1}{u_{n,t}^2}$$

and

$$\frac{u_{c,t}^1}{u_{c,t}^2} = \frac{u_{c,t+1}^1}{u_{c,t+1}^2}.$$

These conditions can be written as

$$u_{c,t}^1 = \gamma u_{c,t}^2 \tag{40}$$

$$u_{n,t}^1 = \gamma u_{n,t}^2, \tag{41}$$

where γ is some endogenous number.⁵

Let φ^1 and φ^2 be the multipliers of the two implementability conditions, (38) and (39). The first-order conditions for $t \geq 1$ imply⁶

$$u_{c,t}^2 \frac{\gamma [\theta + \varphi^1 (1 - \sigma^1)] \frac{\sigma^2}{c_t^2} + [(1 - \theta) + \varphi^2 (1 - \sigma^2)] \frac{\sigma^1}{c_t^1}}{\frac{\sigma^2}{c_t^2} + \frac{\sigma^1}{c_t^1}} = \lambda_t \tag{42}$$

and

$$u_{n,t}^2 \frac{\gamma [\theta + \varphi^1 (1 + \psi^1)] \frac{\psi^2}{n_t^2} + [(1 - \theta) + \varphi^2 (1 + \psi^2)] \frac{\psi^1}{n_t^1}}{\frac{\psi^2}{n_t^2} + \frac{\psi^1}{n_t^1}} = -\lambda_t F_{n,t}, \quad t \geq 1, \tag{43}$$

which together with

$$-\lambda_t + \beta \lambda_{t+1} [f_{k,t+1} + 1 - \delta] = 0$$

imply that, if elasticities are equal, $\sigma^1 = \sigma^2 = \sigma$ and $\psi^1 = \psi^2 = \psi$, future capital should not be taxed from period one on. To see this, notice that, from (40) and (41), c_t^1 must be proportionate to c_t^2 , $c_t^1 = \gamma^{-\frac{1}{\sigma}} c_t^2$, and n_t^1 must also be proportionate to n_t^2 , $n_t^1 = (\gamma)^{\frac{1}{\psi}} n_t^2$. It then follows that the terms multiplying the marginal utilities on the left-hand side of (42) and (43) are time invariant.

⁵See also Greulich, Laczó and Marcat (2016). Werning (2007) also computes optimal taxes with heterogeneous agents taking advantage of this proportionality of marginal utilities.

⁶See Appendix 3 for the derivation.

In period zero, the first-order condition for consumption of type one has an additional term. Using $u_{c,0}^1 = \gamma u_{c,0}^2$, that first order condition is

$$\theta u_{c,0}^1 + \varphi^1 u_{c,0}^1 (1 - \sigma_0^1) + \mu_0 u_{cc,0}^1 - u_{cc,0}^1 (1 - l_0) \left(\varphi^1 V_0^1 + \varphi^2 \frac{V_0^2}{\gamma} \right) = \lambda_0. \quad (44)$$

The first-order condition for an interior solution of l_0 is

$$-u_c^1(0) \left(\varphi^1 V_0^1 + \varphi^2 \frac{V_0^2}{\gamma} \right) = 0$$

Thus, the additional term is zero, so that the first order condition for period zero, (44), has the same form as the ones for $t \geq 1$.

Consider next the additional restriction that the initial wealth tax has to be lower than 100%, $l_0 \leq 1$. If the solution to this problem is interior, the additional term is zero. If the solution has $l_0 = 1$, the additional term is also zero.

The first-order conditions for labor of both types in period zero also have additional terms. The condition for labor of type one in period zero, also using $u_{c,0}^1 = \gamma u_{c,0}^2$, is

$$\theta u_{n,0}^1 + \varphi^1 u_{n,0}^1 (1 + \psi) + \mu_0^n u_{nn,0}^1 - u_{c,0}^1 (1 - l_0) (1 - \tau_0^k) F_{kn,0} \left[\varphi^1 k_0^1 - \varphi^2 \frac{k_0^2}{\gamma} \right] = -\lambda_0 F_{n,0}$$

and similarly for n_0^2 .

The derivative with respect to τ_0^k is $\varphi^1 u_{c,0}^1 (1 - l_0) (F_{k,0} - \delta) k_0^1 + \varphi^2 \frac{u_{c,0}^1}{\gamma} (1 - l_0) (F_{k,0} - \delta) k_0^2$. If there are no restrictions on τ_0^k , the solution is interior, and then $\varphi^1 k_0^1 + \varphi^2 \frac{k_0^2}{\gamma} = 0$. On the other hand, if τ_0^k is restricted to be below 100%, and if the solution is at the corner, $\tau_0^k = 1$, the term in the first order condition for labor in period zero is again zero.

We summarize this discussion in the following proposition.

Proposition 7: (No intertemporal distortions in heterogeneous agent economies) Suppose that preferences for all types of agents are in the class of standard macroeconomic preferences. If all agents have the same preferences, then the Ramsey equilibrium has no intertemporal distortions for all $t \geq 0$.

Note that this proposition holds even if we impose the additional restriction that $l_0 \leq 1$.

This proposition shows that with standard and identical preferences, allowing for

heterogeneity in initial wealth does not overturn the result that, with a rich tax system, future capital should not be taxed.

With heterogeneity and distributional concerns, it may be optimal for the planner to distort intratemporal decisions regardless of whether or not the initial wealth tax is constrained to be below 100%. This result is in striking contrast with the result in the representative agent model. In that model, as stated in Proposition 2, if the initial wealth tax is unconstrained, the outcome coincides with the lump-sum tax allocations and the intratemporal decisions are undistorted.

4 Relation to production efficiency

In this section, we connect our results to the results on production efficiency and uniform taxation. To develop these connections, we set up an alternative economy, which we call an *intermediate goods economy*, that seems different at face value but turns out to be equivalent to the one considered above. In this alternative economy, the representative household consumes a single final good denoted by C and supplies a single final labor input denoted by N . Preferences for the households are given by

$$U(C, N) = \frac{C^{1-\sigma} - \frac{1}{1-\beta}}{1-\sigma} - \eta N^\psi. \quad (45)$$

The economy has three types of firms. The first one is the same as the one described above. We refer to this firm as the *capital accumulation firm*. This firm produces intermediate goods c_t , hires intermediate labor inputs n_t , and accumulates capital according to the technology (2). The second type of firm, referred to as the *consumption firm*, produces the final good C using the intermediate goods c_t according to the constant returns to scale technology given by

$$C = \mathcal{C}(c_0, c_1..) = \left[\sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \quad (46)$$

The third type of firm, referred to as the *labor firm*, produces the intermediate labor inputs using final labor according to the constant returns to scale technology given by

$$N = \mathcal{N}(n_0, n_1, \dots) = \left[\sum_{t=0}^{\infty} \beta^t n_t^\psi \right]^{\frac{1}{\psi}}. \quad (47)$$

In terms of the tax system, we assume that the government can levy a tax on the final consumption good, τ^c , and on final labor denoted by τ^n . In addition, we assume that the government can levy taxes on all intermediate goods, denoted by τ_t^c and τ_t^n . We retain the dividend taxes and the capital income taxes levied on the capital accumulation firm, as well as the initial levy l_0 . We do not impose any taxes on the profits of either the consumption firm or the labor firm because these profits will be zero in equilibrium.

The households' problem is to maximize (45) subject to the budget constraint

$$p(1 + \tau^C)C - w(1 - \tau^N)N \leq (1 - l_0) V_0, \quad (48)$$

where p and w denote the prices of final consumption and labor in units of the consumption good in period zero, and V_0 is the value of initial wealth in units of goods.

The consumption firm's problem is to maximize

$$pC - \sum_{t=0}^{\infty} q_t(1 + \tau_t^c)c_t \quad (49)$$

subject to (46).

The labor firm's problem is to maximize

$$\sum_{t=0}^{\infty} q_t(1 - \tau_t^n)w_t n_t - wN \quad (50)$$

subject to (47).

The capital accumulation firm's problem is the same as above. A competitive equilibrium is defined in the standard fashion.

Next, we show that the competitive equilibria in the two economies coincide in the intermediate goods economy and in the growth economy with distorting taxes. To do so, we first show that we can rewrite the equilibrium in the intermediate goods economy

as an equilibrium in the growth economy with taxes by incorporating the decisions of the consumption firm and the labor firm directly into the households' problem.

Consider the budget constraint of the households. Given that in any competitive equilibrium profits are zero for the consumption and the labor firm, we can substitute $pC = \sum_{t=0}^{\infty} q_t(1 + \tau_t^c)c_t$ from (59) and $wN = \sum_{t=0}^{\infty} q_t(1 - \tau_t^n)w_t n_t$ from (50) into (48) to obtain a budget constraint of the form (16) in the growth economy with taxes. The only difference is the presence of the tax on the final consumption and on final labor, which amounts to rescaling the consumption and labor income taxes in the original economy.

Substituting from (47) and (46) into (45), we see that the households' utility function in the rewritten intermediate goods economy is the same as in the growth economy with taxes.

To establish that the converse holds, note that we can set up the households' problem in the growth economy with taxes as a two-stage problem of first choosing an aggregate value for consumption and labor and then choosing the disaggregated levels of consumption and labor to achieve the desired value for final consumption and labor. Thus, the equilibria in the two economies coincide.

In the intermediate goods economy, the Ramsey problem is to maximize (45) subject to the implementability constraint

$$U_C C + U_N N = \frac{U_C}{(1 + \tau^C)} (1 - l_0) V_0, \quad (51)$$

and the requirement that the allocation is in the *production set* given by (46) and (47) with inequalities and (2).

Suppose now, as before, that the initial wealth in utility terms, given by the right-hand side of (51), must be at least as large as an exogenous value, \bar{W} .

Next we apply the production efficiency theorem of Diamond and Mirrlees (1971) to our intermediate goods economy. This theorem asserts that if pure rents are fully taxed away, any Ramsey allocation must be at the boundary of the production set. In our context, the theorem asserts that if $\bar{W} = 0$, any Ramsey allocation must be at the boundary of the production set. In Appendix 3, we extend this theorem to the case in which \bar{W} is exogenously fixed.

Somewhat loosely speaking, the theorem as extended implies that one way to implement the Ramsey allocation is to tax only final goods and not to tax intermediate

goods. Thus, the theorem implies that it is optimal to not have intertemporal distortions.

More formally, the result that a Ramsey allocation must be at the boundary implies that there exist supporting prices p and w such that the allocation solves

$$\max pC - wN$$

subject to the requirement that the allocation is in the production set.

The first-order conditions for this problem include

$$\frac{\mathcal{C}_{ct}}{\mathcal{C}_{ct+1}} = 1 - \delta + F_{k,t+1} \quad (52)$$

and

$$\frac{\mathcal{N}_{nt}}{\mathcal{N}_{nt+1}} = \frac{F_{n,t}}{F_{n,t+1}} (1 - \delta + F_{k,t+1}). \quad (53)$$

These are conditions of production efficiency. Condition (52) equates the rates at which c_t is transformed into c_{t+1} through the composite C to the rate at which c_t is transformed into c_{t+1} through capital, k_{t+1} . Condition (53) is the analog for labor in consecutive periods. Notice that, since $\beta^t u_c(t) = U_c \mathcal{C}_{ct}$ and $\beta^t u_n(t) = U_N \mathcal{N}_{nt}$, it follows that $\frac{\mathcal{C}_{ct}}{\mathcal{C}_{ct+1}} = \frac{u_{c,t}}{\beta u_{c,t+1}}$ and $\frac{\mathcal{N}_{nt}}{\mathcal{N}_{nt+1}} = \frac{u_{n,t}}{\beta u_{n,t+1}}$. Thus, conditions (52) and (53) imply that it is optimal to have no intertemporal distortions. They also imply that intratemporal distortions are constant. We summarize this discussion in the following proposition, which is the equivalent to Proposition 4 in the intermediate goods economy.

Proposition 4': (No intertemporal distortions ever) Suppose that preferences and technologies are given by (45), (46), and (47) and the wealth restriction must be satisfied. Then, the Ramsey solution has no intertemporal distortions for all $t \geq 0$.

Consider next the case in which the wealth restriction is imposed in units of goods rather than in utility terms. Here, production efficiency is no longer optimal. Nevertheless, it is straightforward to show that an analog of Proposition 5 holds in the intermediate goods economy in the sense that it is not optimal to distort intermediate goods decisions from period one onward. It is optimal to distort intermediate goods in period zero relative to all other intermediate goods in order to partially tax rents.

The intermediate goods economy helps clarify circumstances in which it is optimal to not have intertemporal distortions. We have shown that it is optimal to not have

such distortions when the underlying economy can be represented as a constant returns to scale economy in the production of intermediate goods. In this sense we have shown an equivalence between the principle that intermediate goods taxation is undesirable and intertemporal distortions should not be introduced.

In the process of doing so, we have shown that the celebrated result of Atkinson and Stiglitz (1972), that uniform commodity taxation is optimal when preferences are homothetic and separable, follows from the production efficiency result of Diamond and Mirrlees (1971). Thus, our result of no intertemporal distortions is very closely connected to the uniform commodity taxation result.

Production efficiency in an economy without capital How different are the results above from the ones in an economy without capital as in Lucas and Stokey (1983)? In that production economy, uniform taxation is optimal for standard macro preferences. How does that relate to production efficiency? Can we map that economy into an intermediate goods economy as we did in the growth model?

In Lucas and Stokey (1983), instead of the intertemporal technology (2), the technology is static and given by

$$c_t + g_t \leq A_t n_t. \quad (54)$$

The map between the original economy and the intermediate good economy is the same except that instead of the capital accumulation firms, we have now *production firms* that use technology (54). The conditions for production efficiency in the intermediate goods economy are now

$$\frac{\mathcal{C}_{ct}}{\mathcal{C}_{ct+1}} = \frac{\mathcal{N}_{nt}}{\mathcal{N}_{nt+1}} \frac{A_{t+1}}{A_t}, \quad (55)$$

instead of (52) and (53).

Because \mathcal{C}_{ct} and \mathcal{N}_{nt} are proportionate to $u_{c,t}$ and $u_{n,t}$, condition (55) implies that intratemporal distortions are constant over time. There are no implications for intertemporal wedges because there are no such wedges.

4.1 Production efficiency in a heterogeneous agent economy

Consider now developing an intermediate goods economy that is equivalent to our heterogeneous agent economy. Here we think of the intermediate goods economy as

producing two distinct types of final consumption goods denoted by C^i for $i = 1, 2$. For simplicity, we assume the economy utilizes one common type of final labor, denoted by N . The preferences for households of type i are given by

$$U(C^i, N^i) = \frac{(C^i)^{1-\sigma^i} - \frac{1}{1-\beta}}{1-\sigma^i} - \eta(N^i)^\psi, \quad (56)$$

where N^i denotes the amount of the common final labor supplied by type i . The technologies for the capital accumulation and the labor firm are the same as before. Each consumption good is produced by its own constant returns to scale technology given by

$$C^i = \mathcal{C}^i(c_0^i, c_{1..}^i) = \left[\sum_{t=0}^{\infty} \beta^t (c_t^i)^{1-\sigma^i} \right]^{\frac{1}{1-\sigma^i}} \quad (57)$$

for $i = 1, 2$.

The tax system is the same except that we require that the tax rate on final consumption goods must be the same for the two types. We do so to show the equivalence between the intermediate goods economy and the heterogeneous agent economy with type-independent taxation.

Households of type i maximizes (56) subject to the budget constraint

$$p^i(1 + \tau^C)C^i - w(1 - \tau^N)N^i \leq (1 - l_0) V_0^i, \quad (58)$$

where p^i denotes the price of the final consumption good of type i in units of the consumption good in period zero.

The consumption firm of type i maximizes

$$p^i C^i - \sum_{t=0}^{\infty} q_t (1 + \tau_t^c) c_t^i \quad (59)$$

subject to (57).

The other firms solve the same problems as in the representative agent economy. A competitive equilibrium is defined in the standard fashion.

Next, we show that the competitive equilibria in the intermediate goods economy and the heterogeneous agent economy with type-independent taxes coincide. A key step in this proof is that the tax rates on the final consumption goods are restricted

to be the same. Recall that in the representative agent model, we used zero profits for consumption goods producers and replaced the pre-tax value of consumption in the households' budget constraint in the intermediate goods economy with the value of the intermediate goods. Following the same procedure, we can use (??) and write (58) as

$$(1 + \tau^C) \sum_{t=0}^{\infty} q_t (1 + \tau_t^c) c_t^i - w(1 - \tau^N) N^i \leq (1 - l_0) V_0^i$$

Notice that this budget constraint coincides with the budget constraints in the heterogeneous agent economy except for rescaling. Notice that if the tax rates on the two consumption goods were allowed to be different in the intermediate goods economy, the budget constraints would not coincide and the competitive equilibria would be different.

The rest of the argument that the equilibria coincide is the same as in the representative agent economy. Clearly, if we were to set up an intermediate goods economy that coincided with the heterogeneous agent economy with type-dependent taxes, we could allow the tax rates on the two final consumption goods to be different in the intermediate goods economy.

Consider now the Ramsey problem in the intermediate goods economy with the same tax rate on both consumption goods. This requirement imposes an additional restriction on the Ramsey problem. Straightforward algebra shows that this constraint can be written as

$$\frac{U_{C^i}^i C_{c_t}^i}{-U_{N^i}^i N_{n_t}^i} = \frac{U_{C^j}^j C_{c_t}^j}{-U_{N^j}^j N_{n_t}^j}, t \geq 0. \quad (60)$$

The Ramsey problem is now to maximize utility subject to the implementability constraint, the requirement that the allocation is in the production set and (60). The Ramsey problem in the intermediate goods economy with type-dependent taxes simply drops (60).

We now have the analog of Proposition 7.

Proposition 7': (No intertemporal distortions in heterogenous agent economies) Suppose that preferences for all types of agents are in the class of standard macroeconomic preferences. If all agents have the same preferences, then the Ramsey equilibrium has no intertemporal distortions for all $t \geq 0$.

5 Concluding remarks

Should capital be taxed in the steady state and along the transition? Once we abstract from the initial confiscation of capital, what matters are consumption and labor elasticities. In the steady state they are constant, so capital taxes should be zero in the steady state. For standard macro preferences, those elasticities are always constant, so that capital should never be taxed. These results also hold with heterogeneous agents.

Future capital should not be taxed for standard preferences, because taxing such capital imposes different taxes on different consumption goods and on labor in different periods, when instead uniform consumption and labor taxation are optimal.

Optimal uniform taxation is about production efficiency. We relate the results to the principles of production efficiency in Diamond and Mirrlees (1971).

Indirect confiscation of the initial capital through valuation effects could justify taxing future capital, but that justification lasts for one single period. This is indeed the case if a rich set of taxes is used, that includes taxes used in practice such as dividend taxes or consumption taxes. In any case, direct confiscation of wealth is always preferable to indirect confiscation.

References

- [1] Abel, Andrew B., 2007, “Optimal Capital Income Taxation,” Working Paper 13354, National Bureau of Economic Research.
- [2] Armenter, Roc, 2008, “A Note on Incomplete Factor Taxation,” *Journal of Public Economics* 92 (10-11), 2275–2281.
- [3] Atkeson, Andrew, V. V Chari, and Patrick J Kehoe, 1999, “Taxing Capital Income: A Bad Idea,” *Federal Reserve Bank of Minneapolis Quarterly Review* 23(3), 3–18.
- [4] Atkinson, Anthony B and Joseph E Stiglitz, 1972, “The Structure of Indirect Taxation and Economic Efficiency,” *Journal of Public Economics* 1 (1), 97–119.
- [5] Bassetto, Marco, and Jess Benhabib, 2006, “Redistribution, Taxes, and the Median Voter,” *Review of Economic Dynamics* 9(2), 211-223.

- [6] Chari, V.V., J. Juan P. Nicolini and Pedro Teles, 2016, "More on the Taxation of Capital Income", Mimeo.
- [7] Chamley, Christophe, 1986, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives," *Econometrica* 54 (3), 607–622.
- [8] Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe, 1994, "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy* 102 (4), 617–652.
- [9] Chari, V.V. and Patrick J. Kehoe, 1993, "Sustainable Plans and Debt," *Journal of Economic Theory* 61(2), 230–261.
- [10] Coleman, Wilbur John II, 2000, "Welfare and Optimum Dynamic Taxation of Consumption and Income," *Journal of Public Economics* 76 (1), 1–39.
- [11] Diamond, Peter A. and James A. Mirrlees, 1971, "Optimal Taxation and Public Production I: Production Efficiency," *American Economic Review* 61 (1), 8–27.
- [12] Greulichy, Katharina, Sarolta Laczó and Albert Marcet, 2016, "Pareto-Improving Optimal Capital and Labor Taxes," mimeo, Barcelona GSE.
- [13] Judd, Kenneth L., 1985, "Redistributive Taxation in a Simple Perfect Foresight Model," *Journal of Public Economics* 28 (1), 59–83.
- [14] Judd, Kenneth L., 1999, "Optimal Taxation and Spending in General Competitive Growth Models," *Journal of Public Economics* 71 (1), 1–26.
- [15] Judd, Kenneth L., 2002, "Capital-Income Taxation with Imperfect Competition," *American Economic Review Papers and Proceedings* 92 (2), 417–421.
- [16] Klein, Paul, Per Krusell and José-Victor Rios-Rull, 2008, "Time Consistent Public Policy," *Review of Economic Studies* 75, 789–808.
- [17] Kydland, Finn E. and Edward C. Prescott, 1980, "Dynamic Optimal Taxation, Rational Expectations and Optimal Control," *Journal of Economic Dynamics and Control* 2 , 79–91.
- [18] Lucas, Robert E., Jr. and Nancy L. Stokey, 1983, "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics* 12(1), 55–93.

- [19] Straub, Ludwig and Iván Werning, 2015, “Positive Long Run Capital Taxation: Chamley-Judd Revisited”, mimeo, MIT.
- [20] Werning, Iván, 2007, “Optimal Fiscal Policy with Redistribution,” Quarterly Journal of Economics 122 (3), 925-967.

6 Appendix 1: An alternative decentralization

An economy that would be equivalent to the one we just studied but closer to the standard setup used in the literature would have households accumulating capital and renting it out to firms.

The households own the capital stock and rents it to a representative firm every period at rate u_t . The capital income tax τ_t^k is paid by the representative firm and the dividend tax, τ_t^d , is a tax on capital income net of gross investment paid by the households. This dividend tax more closely resembles the tax proposed by Abel (2007) as a way of collecting lump sum revenue from the taxation of the initial capital stock.

The flow of funds for the households can then be written as

$$\begin{aligned} \frac{1}{1+r_{t+1}} b_{t+1} + k_{t+1} &= b_t + (1-\delta)k_t + u_t k_t - \tau_t^d [u_t k_t - (k_{t+1} - (1-\delta)k_t)] \\ &\quad + (1-\tau_t^n) w_t n_t - (1+\tau_t^c) c_t \end{aligned}$$

for $t \geq 1$, which can be rewritten as

$$\begin{aligned} \frac{1}{1+r_{t+1}} b_{t+1} + (1-\tau_t^d) k_{t+1} &= b_t + (1-\tau_t^d) (1-\delta) k_t + (1-\tau_t^d) u_t k_t \\ &\quad + (1-\tau_t^n) w_t n_t - (1+\tau_t^c) c_t. \end{aligned}$$

In the initial period, the constraint is

$$\begin{aligned} \frac{1}{1+r_1} b_1 + (1-\tau_0^d) k_1 &= (1-l_0) [b_0 + (1-\tau_0^d) (1-\delta) k_0 + (1-\tau_0^d) u_0 k_0] \\ &\quad + (1-\tau_0^n) w_0 n_0 - (1+\tau_0^c) c_0. \end{aligned}$$

The marginal conditions are (11), (12), and

$$1 + r_{t+1} = \frac{(1 - \tau_{t+1}^d) [u_{t+1} + 1 - \delta]}{1 - \tau_t^d}. \quad (61)$$

The representative firm maximizes profits

$$\Pi_t = F(k_t, n_t) - w_t n_t - u_t k_t - \tau_t^k (u_t - \delta) k_t.$$

The capital income tax τ_t^k is a tax on sales minus wages and depreciation of capital, which is the model counterpart to a profit tax.

The price of the good must equal marginal cost,

$$1 = \frac{w_t}{F_n(t)} = \frac{u_t + \tau_t^k (u_t - \delta)}{F_k(t)}. \quad (62)$$

These marginal conditions can be written as (20), (21), and (22).

The households' budget constraint can be written as

$$\begin{aligned} \sum_{t=0}^{\infty} q_t [(1 + \tau_t^c) c_t - (1 - \tau_t^n) w_t n_t] = \\ (1 - l_0) [b_0 + (1 - \tau_0^d) (1 - \delta) k_0 + (1 - \tau_0^d) u_0 k_0] \end{aligned} \quad (63)$$

where $q_t = \frac{1}{(1+r_1)\dots(1+r_t)}$ for $t \geq 1$, with $q_0 = 1$. This uses the no-Ponzi-scheme condition $\lim_{T \rightarrow \infty} q_{T+1} b_{T+1} \geq 0$.

The marginal conditions of the households and firms can be used to write the budget constraint as an implementability condition, which will be written as

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t + u_n(t) n_t] = \frac{(1 - l_0)}{1 + \tau_0^c} \left[b_0 + (1 - \tau_0^d) k_0 + \frac{(1 - \tau_0^d) (F_k(0) - \delta)}{1 + \tau_0^k} k_0 \right].$$

The initial confiscation is restricted regardless of the taxes that are used to obtain it. We define \mathcal{W}_0 to be the exogenous level of initial wealth that the households can keep, measured in units of utility at time 0,

$$u_c(0) \frac{(1 - l_0)}{1 + \tau_0^c} \left[b_0 + (1 - \tau_0^d) k_0 + \frac{(1 - \tau_0^d) (F_k(0) - \delta)}{1 + \tau_0^k} k_0 \right] = \mathcal{W}_0. \quad (64)$$

The implementability condition can then be written as

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t + u_n(t) n_t] = \mathcal{W}_0. \quad (65)$$

The implementability condition (65) and the resource constraints (2) are the only equilibrium restrictions on the sequences of consumption, labor, and capital. The taxes have natural restrictions that the tax revenue does not exceed the base, so that $\tau_t^k \leq 1$ for all t . These restrictions will not be binding in this set up.

The other equilibrium conditions, other than (65) and (2), are satisfied by other variables. The variable τ_t^n is determined by

$$-\frac{u_c(t)}{u_n(t)} = \frac{(1 + \tau_t^c)}{(1 - \tau_t^n) w_t}, \quad (66)$$

The variable r_{t+1} is determined by

$$\frac{u_c(t)}{1 + \tau_t^c} = (1 + r_{t+1}) \frac{\beta u_c(t+1)}{1 + \tau_{t+1}^c}, \quad (67)$$

The variable w_t is determined by

$$1 = \frac{w_t}{F_n(t)}, \quad (68)$$

The variable u_t is determined by

$$\frac{w_t}{F_n(t)} = \frac{u_t + \tau_t^k (u_t - \delta)}{F_k(t)}, \quad (69)$$

and either τ_{t+1}^c given τ_t^c or τ_{t+1}^d given τ_t^d is determined by

$$u_c(t) = \beta u_c(t+1) \frac{(1 + \tau_t^c) (1 - \tau_{t+1}^d)}{(1 + \tau_{t+1}^c) (1 - \tau_t^d)} [u_{t+1} + (1 - \delta)]. \quad (70)$$

The constraint (64) will be satisfied with l_0 .

This implementation does not use τ_t^k and τ_t^d for all t as well as τ_0^c or else τ_t^k and τ_t^c for all t and τ_0^d . They are redundant instruments. It follows that the restrictions $\tau_t^k \leq 1$ will not bind. This also means that the capital tax can always be set equal to

zero.

7 Appendix 2: Time separable homothetic preferences

Time-separable preferences that are also separable between consumption and labor must satisfy

$$\frac{\beta U_c(c_{t+1})}{U_c(c_t)} = \frac{\beta U_c(\lambda c_{t+1})}{U_c(\lambda c_t)}.$$

for all $\lambda > 0$. Differentiating with respect to λ , we have

$$U_c(c_{t+1})c_t U_{cc}(\lambda c_t) = U_{cc}(\lambda c_{t+1})c_{t+1}U_c(c_t)$$

Set $\lambda = 1$. It follows that

$$\frac{c_t U_{cc}(c_t)}{U_c(c_t)} = k$$

is independent of c_t . Standard result (see Pratt) is that $U = \frac{c^{1-\sigma}}{1-\sigma}$.

$$x f'(x) = k f(x)$$

Let $g(x) = \log f(x)$

$$g'(x) = \frac{f'(x)}{f(x)}$$

Rewrite differential equation as

$$g'(x) = \frac{k}{x}$$

so that,

$$g(x) = k \log x + C.$$

It follows that

$$\log f(x) = k \log x + C$$

and therefore

$$f(x) = Cx^k.$$

8 Appendix 3: Ramsey solution in the heterogeneous agent economy

The social welfare function is

$$\theta U^1 + (1 - \theta) U^2.$$

The preferences of each agent are

$$U^i = \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma} - 1}{1-\sigma} - \eta (n_t^i)^{1+\psi} \right], \quad i = 1, 2. \quad (71)$$

The implementability conditions are

$$\sum_{t=0}^{\infty} \beta^t [u_{c,t}^1 c_t^1 + u_{n,t}^1 n_t^1] = u_{c,0}^1 (1 - l_0) V_0^1 \quad (72)$$

and

$$\sum_{t=0}^{\infty} \beta^t [u_{c,t}^2 c_t^2 + u_{n,t}^2 n_t^2] = u_{c,0}^2 (1 - l_0) V_0^2, \quad (73)$$

with $V_0^i = b_0^i + k_0^i + (1 - \tau_0^k) (F_{k,0} - \delta) k_0^i$, together with

$$u_{c,t}^1 = \gamma u_{c,t}^2 \quad (74)$$

$$u_{n,t}^1 = \gamma u_{n,t}^2, \quad (75)$$

where γ is a choice variable and

$$c_t^1 + c_t^2 + g_t + k_{t+1} - (1 - \delta) k_t \leq F(n_t^1 + n_t^2, k_t).$$

The marginal conditions of the Ramsey problem are

$$\begin{aligned} \theta u_{c,t}^1 + \varphi^1 u_{c,t}^1 (1 - \sigma) + \mu_t^c u_{cc,t}^1 &= \lambda_t, \quad t \geq 1 \\ (1 - \theta) u_{c,t}^2 + \varphi^2 u_{c,t}^2 (1 - \sigma) - \mu_t^c \gamma u_{cc,t}^2 &= \lambda_t, \quad t \geq 1 \\ \theta u_{n,t}^1 + \varphi^1 u_{n,t}^1 (1 + \psi) + \mu_t^n u_{nn,t}^1 &= -\lambda_t F_{n,t}, \quad t \geq 1 \end{aligned} \quad (76)$$

$$(1 - \theta) u_{n,t}^2 + \varphi^2 u_{n,t}^2 (1 + \psi) - \mu_t^n u_{nn,t}^2 = -\lambda_t F_{n,t}, \quad t \geq 1. \quad (77)$$

The marginal conditions for c_t^1 and c_t^2 can be used to write

$$\gamma u_{cc,t}^2 [\theta u_{c,t}^1 + \varphi^1 u_{c,t}^1 (1 - \sigma)] + u_{cc,t}^1 [(1 - \theta) u_{c,t}^2 + \varphi^2 u_{c,t}^2 (1 - \sigma)] = \lambda_t \gamma u_{cc,t}^2 + \lambda_t u_{cc,t}^1, \quad t \geq 1 \quad (78)$$

which can be rewritten as

$$\gamma \sigma \frac{u_{c,t}^2}{c_t^2} [\theta u_{c,t}^1 + \varphi^1 u_{c,t}^1 (1 - \sigma)] + \sigma \frac{u_{c,t}^1}{c_t^1} [(1 - \theta) u_{c,t}^2 + \varphi^2 u_{c,t}^2 (1 - \sigma)] = \lambda_t \gamma \sigma \frac{u_{c,t}^2}{c_t^2} + \lambda_t \sigma \frac{u_{c,t}^1}{c_t^1}, \quad t \geq 1 \quad (79)$$

Using

$$u_{c,t}^1 = \gamma u_{c,t}^2, \quad (80)$$

it follows that

$$u_{c,t}^2 \frac{\gamma [\theta + \varphi^1 (1 - \sigma)] \frac{\sigma}{c_t^2} + [(1 - \theta) + \varphi^2 (1 - \sigma)] \frac{\sigma}{c_t^1}}{\frac{\sigma}{c_t^2} + \frac{\sigma}{c_t^1}} = \lambda_t, \quad t \geq 1. \quad (81)$$

With these preferences

$$u_{c,t}^1 = \gamma u_{c,t}^2 \quad (82)$$

implies

$$c_t^1 = \gamma^{-\frac{1}{\sigma}} c_t^2, \quad (83)$$

which, in turn, implies

$$u_{c,t}^2 \frac{\gamma [\theta + \varphi^1 (1 - \sigma)] \sigma \gamma^{-\frac{1}{\sigma}} + [(1 - \theta) + \varphi^2 (1 - \sigma)] \sigma}{\sigma \gamma^{-\frac{1}{\sigma}} + \sigma} = \lambda_t, \quad t \geq 1. \quad (84)$$

For labor, using the marginal conditions for n_t^1 and n_t^2 ,

$$u_{n,t}^2 \frac{\gamma [\theta + \varphi^1 (1 + \psi)] \frac{\psi}{n_t^2} + [(1 - \theta) + \varphi^2 (1 + \psi)] \frac{\psi}{n_t^1}}{\frac{\psi}{n_t^2} + \frac{\psi}{n_t^1}} = -\lambda_t F_{n,t}, \quad t \geq 1. \quad (85)$$

Since

$$u_{n,t}^1 = \gamma u_{n,t}^2, \quad (86)$$

we have

$$n_t^1 = (\gamma)^{\frac{1}{\psi}} n_t^2, \quad (87)$$

and therefore

$$u_{n,t}^2 \frac{\gamma [\theta + \varphi^1 (1 + \psi)] \psi (\gamma)^{\frac{1}{\psi}} + [(1 - \theta) + \varphi^2 (1 + \psi)] \psi}{\psi (\gamma)^{\frac{1}{\psi}} + \psi} = -\lambda_t F_{n,t}, \quad t \geq 1. \quad (88)$$

This implies that the intratemporal wedge is constant and that the intertemporal wedge for labor is also zero.

For period zero, the condition for c_0^1 is

$$\theta u_{c,0}^1 + \varphi^1 u_{c,0}^1 (1 - \sigma) + \mu_0^c u_{cc,0}^1 - u_{cc,0}^1 (1 - l_0) \left(\varphi^1 V_0^1 + \varphi^2 \frac{V_0^2}{\gamma} \right) = \lambda_0.$$

This is obtained by replacing $\frac{u_{c,0}^1}{\gamma}$ for $u_{c,0}^2$ in the implementability conditions. The derivative with respect to l_0 is $\varphi^1 u_{c,0}^1 V_0^1 + \varphi^2 \frac{u_{c,0}^1}{\gamma} V_0^2$ which can be rewritten as $u_{c,0}^1 \left[\varphi^1 V_0^1 + \varphi^2 \frac{V_0^2}{\gamma} \right]$. At the optimum, either this is zero or $l_0 = 1$. Either way, the last term in the first order condition is zero.

There are also terms for labor of both types in period zero. For n_0^1 , we have

$$\theta u_{n,0}^1 + \varphi^1 u_{n,0}^1 (1 + \psi) + \mu_0^n u_{nn,0}^1 - u_{c,0}^1 (1 - l_0) (1 - \tau_0^k) F_{kn,0} \left[\varphi^1 k_0^1 - \varphi^2 \frac{k_0^2}{\gamma} \right] = -\lambda_0 F_{n,0} \quad (89)$$

and similarly for n_0^2 .

The derivative with respect to τ_0^k is $\left[\varphi^1 k_0^1 + \varphi^2 \frac{1}{\gamma} k_0^2 \right] u_{c,0}^1 (1 - l_0) (F_{k,0} - \delta)$. If the solution is interior, then

$$u_{c,0}^1 (1 - l_0) (F_{k,0} - \delta) \left[\varphi^1 k_0^1 + \varphi^2 \frac{k_0^2}{\gamma} \right] = 0, \text{ so that } \varphi^1 k_0^1 + \varphi^2 \frac{k_0^2}{\gamma} = 0.$$

Otherwise, if the solution is at the corner, then $\tau_0^k = 1$, and the time zero term in the first-order condition is again zero.