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On the Needed Quantity of Government Debt

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ABSTRACT

People are having longer retirement periods, and population growth is slowing and has even stopped in some countries. In this paper we determined the implications of these changes for the needed amount of government debt. The needed debt is near zero if there are high tax rates and the transfer share of gross national income (GNI) is high. But, with such a system there are huge dead-weight losses as the result of the high tax rate on labor income. With a savings system, a large government debt to annual GNI ratio is needed, as large as 5 times GNI, and welfare is as much as 24 percent higher in terms of lifetime consumption equivalents than the tax-and-transfer system.

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Section 1: Introduction

We need to change our way of thinking when it comes to government debt. The reason that we have to change our thinking is that population demographics have changed dramatically. There are more retired workers because people are living longer lives and because population growth has stopped in most of the advanced industrial countries. With these demographics, large government debt is a feature of the retirement financing system that maximizes the *lifetime welfare of all including our grandchildren*, namely the saving system. The alternative system where the government taxes the workers' labor income and/or consumption to finance the consumption of retirees has little or no government debt. However, *welfare for all,* including our grandchildren, is much lower for this system, which has no government debt.

Unlike the pure consumption loan model of Samuelson (1958), taxing income of workers and making lump sum transfers to them when old is not equivalent to there being large quantities of *explicit* government debt which people buy during their working life and sell during their retirement life. The reason is that people value their non-market time. As a result, taxing labor income and/or consumption, lowers labor supply. The welfare of a saving-for-retirement system is much higher than the welfare of the tax-and-transfer system.

We begin with the Samuelson pure consumption loan model and show that a pay-as-you-go retirement system and a savings system are equivalent. This follows from Ricardian equivalence as the tax is lump sum and there are no redistributions.[1] In this

[1] There is an interesting but minor issue associated with staring the savings system. These issues are addressed in Prescott and Rios-Rull (2005), who develop an equilibrium concept for these environments with the property that the generation that sets up the system can not do better than future generations in the same situation. If the initial generation did better, then all would choose to be the initial generation.

world people receive a large endowment of the consumption good when young and a small endowment when old. They prefer smooth consumption over their lifetime to having consumption concentrated when young. Under the pay-as-you-go system, there is no government debt. Under the saving system, there is a large amount of government debt. We show that Ricardian equivalence holds and that someone born into either system enjoys the same level of welfare.

We then modify the economy in an important way. Rather than having the endowment when young be the consumption good, the endowment is productive time. This time can be used in the market to produce the consumption good or used for valued non-market activities. We label non-market activities leisure as this is conventional in macroeconomics even though some of these non-market activities are non-market work. As has been established, the time allocation decision between market and non-market activities responds to incentives. There is a dead-weight loss associated with taxing labor income of young people to finance lump sum transfers to old.

With people valuing leisure as well as consumption when young and consumption when old, the welfare of someone born into a world with a savings-for-retirement system is much higher than the welfare of someone born into a world with a tax-and-transfer system. The saving system has large government debt and the transfer system has.

Some naively think that government debt is a burden on the young. This is not the case since the welfare of the young and the government debt are both large in the efficient saving-for-retirement system. Indeed, in the inferior tax-and-transfer system, government debt is zero. We go on to show that switching from a pay-as-you-go system to a saving system makes everyone better off and there are *no costs* of making the switch.

All that needs to be done is to stop taxing labor income. In the transition period there would be transfers to old financed by a large deficit. Subsequently the stock of government debt would be at its needed level.

The economies discussed so far have no reproducible productive assets; that is there is no capital. In fact, there are large quantities of reproducible productive assets held by households, of the order of 3.5 times annual gross national income (GNI). We introduce production using both capital and labor, where labor is time allocated to market production. We calibrate this model economy to the behavior of the advanced industrial countries including Australia, France, Japan, and the United States. This means that the model economies mimic the behavior of these actual economies on key aggregate dimensions given their policies.

We find that with a pure saving-for-retirement system the ratio of government debt to annual GNI can be as high as 5 under plausible demographic assumptions. For the tax-and-transfer retirement system, the government *owns* some productive assets and there is no government debt.

Currently there are large implicit government liabilities. In the United States, the Medicare and Social Security *implicit* liabilities are many times GNI. The Congressional Budget Office estimates these liabilities to be about 4 times GNI. These implicit liabilities are not government debt. They are called *responsibilities* by the U.S. Treasury and *mandated expenditures* by the Congressional Budget Office.

We emphasize that financing retirement is not a problem of insurance. Getting older each year is a predictable event. In the case of the United States, the Survivor and Disability Insurance System and the welfare system are insurance, not retirement programs. Financing retirement and providing insurance are fundamentally different activities and are best separated. In the case of annuities, of course, there is insurance within a cohort against living too long, as annuities are insurance against outliving savings. Annuities are provided by the private sector as are many other forms of insurance. Therefore we abstract from uncertain lifetimes and as a result there is no need for annuities. All our conclusions hold with uncertain lifetimes provided there are annuities.

Section 2: Illustrative Examples

2.1. The Pure Consumption Loan Model

People are endowed with 11 units of the consumption good when young and 1 unit when old. An equal number of people are born every period. A person born in period t, has preferences ordered by

(2.1) $$\log c_{1,t} + \log c_{2,t+1}$$

where c_1 is consumption when young and c_2 is consumption when old. The second subscript subscripts indicate when the consumption occurs.

The period t society resource constraint is

(2.2) $$c_{1,t} + c_{2,t} \leq 12.$$

All variables are per person; therefore, $c_{1,t}$ is consumption per young person at date t.

With no government, $c_{1,t} = 11$ and $c_{2,t} = 1$ for all t. A feasible allocation that is much better *for all* is $c_{1,t} = 6$ and $c_{2,t} = 6$ for all t. This can be supported as a competitive equilibrium with either of two very different government policies. The first has a government debt of 5. The young buy this debt from the old and sell it when they are old. Given the nature of preferences, the equilibrium interest rate is zero and so government interest rate payments are 0 and there is no deficit. If the endowment grew 50 percent per period,[2] the interest would be positive and there would be the need for deficit equal to one-quarter to total output. The deficit pays for the interest on the government debt. Currently the U.S. number for explicit government debt to annual GDP ratio is 0.3.

The second system taxes the endowment at rate $\tau = \frac{5}{11}$ and transfers the proceeds in equal shares to the old. The equilibrium allocation is the same as in the saving system with government debt. This allocation maximizes utility per generation. Therefore, both systems maximize welfare in this sense but have starkly different amounts of government debt.

2.2. A Three Period Consumption Loan Model

We define the debt facing a person entering the workforce as the present value of taxes used to finance transfers minus the present value of transfers that the individual will receive. The following example shows that in the transfer-and-tax system new entrants to the work force have debt, but in the saving-for-retirement system new entrants are debt free. This may seem surprising given that with the tax-and-transfer system there is no government debt while with the saving-for-retirement system there is a large quantity of government debt. But, the tax-and-transfer system, not the saving system, imposes a debt burden on future generations. This is shown by the following example.

The individual's utility function is

[2] Given a period corresponds to 20 years, this growth rate is in line with the U.S. historical experience over the last 140 years (see McGrattan and Prescott 2003) .

(2.3) $$\log c_{1,t} + \frac{1}{(1+\rho)} \log c_{2,t+1} + \frac{1}{(1+\rho)^2} \log c_{3,t+2}$$

where c_1 is consumption when young, c_2 is consumption when middle aged, c_3 is consumption when old, and the discount rate ρ is positive. The young and the middle aged are endowed with 15 units of the consumption good. The old receive no endowment.

The first government policy is to tax the young and middle aged to provide for the old. This policy will have no government debt. The young and middle aged will be taxed at a rate of 1/3 on their endowment. The government transfers the revenue lump-sum to the old. The equilibrium for this policy is $c_1 = c_2 = c_3 = 10$. The interest rate is $i = \rho$, and there is no government debt.

Under this tax-and-transfer system, each cohort has promises of transfers from the government when old. Each cohort also must pay taxes when young and when middle aged. The present value of the taxes or transfers is specified in the table below. Given the interest rate is positive for this model economy, the present value of taxes exceeds the present value of benefits. This means the young are born with debt even though there is no government debt for this policy.

When cohort is	Present value of tax or benefit
Old	$10/(1+i)^2$
Middle Aged	$-5/(1+i)$
Young	-5

The equilibrium for the saving-for-retirement system has the same consumption allocation and interest rate. Government debt held by the old (before receiving interest) is $10/(1+i)$. The pre-interest government debt held by middle age people is 5. Total government debt therefore is $10/(1+i)+5$. The new entrants into the labor force have no debt with this savings-for-retirement system yet there is a large amount of government debt. Government debt that is held by residents of a country cannot be though of as a burden on grandchildren.

2.3. Introducing Valued Non Market Time

When Samuelson wrote his classic paper in 1958, the Lucas and Rapping (1969) paper introducing labor supply into macroeconomics had not been written. We now introduce labor supply into the Samuelson pure consumption loan as follows. Rather than the individuals being endowed with stocks of the consumption good, they are endowed with time that they allocate between market production and non-market activities.

(2.4) $$\log c_{1,t} + \log c_{2,t+1} - v_1 l_{1,t} - v_2 l_{2,t+1}.$$

The v parameters are the disutility of labor supplied to the market, l_t and l_{t+1}. Behind this utility is the Rogerson (1988) and Hansen (1985) labor indivisibility in the life cycle framework (see Prescott, Rogerson, and Wellenius, 2006 for endogenizing the indivisibility). The values of the labor disutility parameters are $v_1 = 0.1$ and $v_2 = 1$. There is great disutility of working when old. This is why people retire in our economy.

The technology is such that one unit of time produces one unit of the consumption good, which implies the equilibrium wages are $w_t = w = 1$ for all t. In this section we will only consider policies for which consumption when young equals consumption when old. Given the preferences, this implies that the equilibrium interest rate must be zero.

The budget constraint of a household born at date t is

(2.5) $$c_{1,t} + c_{2,t+1} = (1-\tau)wl_{1,t} + (1-\tau)wl_{2,t+1} + b\,.$$

Here τ is the tax rate on labor income, w is the wage rate, and b retirement benefits paid by the government. The government's budget constraints are

(2.6) $$b = \tau\, l_t w\,.$$

If there are no transfers and no government debt, the equilibrium values of consumption and labor supply are

(2.7) $$c_1 = 10 \text{ and } c_2 = 1 \text{ and } l_1 = 10 \text{ and } l_2 = 1\,.$$

This allocation can be improved upon. The following two systems are better for all:

The tax-and-transfer system

If transfers and taxes are such that consumptions of young and old are equal, the values of the policy parameters must be $\tau = 0.5$ and $b = 3$. The equilibrium values of the variables are $l_1 = 5$ and $l_2 = 0$ and $c_1 = c_2 = 2.5$. There is no government debt with this system. This system is better than no government, but can be improved upon.

The government debt system

Government debt is selected so that the consumptions of the young and old are equated. The equilibrium allocation is $l_1 = 10$ and $l_2 = 0$ and $c_1 = c_2 = 5$. Government debt is 5 and the interest rate is zero. The government debt to output ratio is 0.50.

The tax-and-transfer system has lower consumption but more non-market time, something which is valued by the household. The questions remain: which system is better and how much better? To answer this question, we determine by what percent the two consumptions must be increased with the tax-and-transfer system to compensate individuals for being born into world with that system rather than being born into a world with a savings-for-retirement system. The percentage by which consumptions in the tax-and-transfer system must be increased for the individual to be indifferent between being born into that world and being born into one with the savings system is 21 percent. Thus the dead-weight loss of a tax-and-transfer system to finance retirement versus a saving-for-retirement system is 21 percent in lifetime consumption equivalents.

This establishes that the government debt is not per se a burden on the young as a young person would prefer to be born in the world with high government debt rather than in the world with no government debt.

2.4. The Model with Capital Accumulation

In the economies considered so far there are no reproducible productive assets namely capital. Saving in the form of reproducible productive assets reduces the amount of government debt needed to support the steady state allocation that maximizes the utility of people. We show this in this section.

We modify the economy to have durable capital goods that enhance production possibilities. The production function is Cobb-Douglas with

$$C_t + X_t \le A K_t^{1/3} L_t^{2/3} \tag{2.7}$$

The capital letters denote aggregate variables with C denoting consumption, X denoting investment, K denoting both capital services and capital stock as one unit of capital produces one unit of capital services, and L denoting labor services. This choice of this

Cobb-Douglas was dictated by the constancy of capital income share near 1/3 over time and across countries. The capital depreciation rate is 100 percent so

(2.8) $$K_{t+1} = X_t .$$

Given a period is 25 years, this is a reasonable depreciation rate.

The capital-output ratio is big in this economy being 1/3. Given a period is 25 years, the annualized capital income ratio is 25 times larger or 8.33, which is almost two times what the ratio is for economies throughout the world. In the following sections, a calibrated model is used to draw quantitative inference as to the welfare consequences of different policies, where the capital output ratio is in line with the real world. But this example suffices to make the point that less government debt is needed in the pure savings system if there is capital accumulation. We make the convenient normalization $A = (2/3)^{-2/3}$ and assume $v_1 = 1/3$. The disutility of working when old, v_2, is large enough that the old do not work, so an equilibrium condition is $l_2 = 0$.

The equilibrium allocation is $c_1 = c_2 = X = K = 1$ and $l_1 = 3$. Aggregate output is 3 and capital income share is $1/3$. Thus the amount of government debt is zero. This is because the labor income of young is 2, their consumption 1, and consequently their savings 1. As investment is 1, the amount of government debt needed to support this allocation is zero. For this numerical example, savings in the form of capital eliminates the need for any government debt.

The examples of this section establish the need to use economic theory and measurement to determine the consequences of different debt, transfer, and tax policies to determine the amount of needed government debt relative to gross national income. We now use both theory and measurement to draw some inference as to the needed amount of government debt.

Section 3: Model Economy Used for Policy Evaluation

This section develops a model that we use to predict the consequences of two retirement policies under alternative demographic assumptions. In all cases we find the balanced growth path with the property that the return on capital is 5 percent before taxes which determines the size of transfers and government debt given the tax rates on labor and capital income.[3] In section 4, the model is used to evaluate retirement policies under different demographic assumptions.

This model includes capital accumulation and extends the lifetime of each agent to N years with the first N_w years being the working life and the last N_r years being the retirement life.[4] The inputs to production are capital services and labor services as in section 2.4. The assets held by people are capital and government debt.

3.1 The Model Economy

Policies

The policy parameters are $(\tau_h, \tau_k, \{D_t, \psi_t\}_{t=0}^{\infty})$ where τ_h is the tax rate on labor income, τ_k is the tax rate on capital income net of depreciation, $\{D_t, \psi_t\}$ is the paths of government debt $\{D_t\}$ and transfers to all people $\{\psi_t\}$. Policy is such that the steady

[3] No equilibrium exists with dynamic inefficiency of Diamond (1965) if borrowing and lending contracts are permitted by infinitely lived organizational entities. There will be an equilibrium for our policies because the policy pegged interest rate exceeds the growth rate of the economy. Abel, Mankiw, Summers, and Zeckhauser (1989) empirically show that there is not dynamic inefficiency in the case for the United States.

[4] In this model, we take the retirement age as given. See Rogerson and Wellenius (2007) for endogenizing the retirement decision.

state after-tax interest rate, *i*, is 4 percent. With this restriction and the two tax rates, the paths of government debt and aggregate transfers are determined.

Within this model economy, we consider two policies for financing retirement. The first is a pay-as-you-go tax-and-transfer system with a tax on labor income and net capital income. The capital tax is paid by the owner of capital, the stand-in bank. The labor income tax is paid by the workers. The transfers are made lump sum each period to all people in the economy in the same amount. Having a transfer to only retired agents would change the accounting in the economy, but not the equilibrium allocation. The second policy is a savings policy for which the government does not tax the labor income of the workers. People save for retirement by holding assets at the stand-in bank, which owns the capital stock and any government debt.

People

A cohort is born every year with cohort size growing at rate η. We normalize the measure of the initial population to 100. The consumption of those younger than working age is implicitly included in their parents' consumption. People begin working at age 20 and are endowed with one unit of time each period of their working life, which is N_w years and then have a retirement life, which is N_r years. Their lifetime is $N+20$ years where $N = N_w + N_r$. In each year of their working life, people divide their time endowment between market work and leisure.

The lifetime utility of a person that enters the workforce in period $t=1$ is

(3.1) $$\sum_{t=1}^{N_w} \beta^t [(1-\alpha)\log c_t + \alpha \log(1-h_t)] + \sum_{t=N_w+1}^{N} \beta^t [(1-\alpha)\log c_t + \alpha \log(1)].$$

People value consumption and leisure in each period. Since each person is endowed with one unit of time each period, the amount of leisure is simply $(1-h_t)$ where h_t is the time allocated to the market. Retired people allocate no time to the market. The parameter α is the leisure share parameter. The parameter $\beta \leq 1$ is the rate at which people discount future utility. The first part of the utility function is the utility during the working lifetime, and the second part is the utility during retirement. Macroeconomic observations dictate this choice of utility for the intertemporal leisure elasticity of substitution. These observations include business cycle facts, prosperities and depressions, and labor supply differences across the major advanced industrial countries.[5]

The lifetime budget constraint of an individual entering the workforce at the beginning of year 1 is

(3.2) $$\sum_{t=1}^{N_w} \frac{1}{(1+i)^t}\left(c_t-(1-\tau_h)w_t h_t\right)-\sum_{t=N}^{N} \frac{\psi_t}{(1+i)^t}=0 .$$

The present value of cash flows is zero.

Each year a worker chooses how much time to allocate to the market, how much to consume, and how much to save. Retired workers do not work and have no labor supply decision, just a consumption decision.

Define the agents that enter the workforce in period 1 as cohort 1. The period budget constraints for the cohort 1 person are

(3.3) $$a_{t+1}=a_t(1+i_{t-1})+(1-\tau_h)w_t h_t+\psi_t-c_t ,$$

where a_t are beginning of period t assets. Initial assets of cohort 1 are $a_1=0$. The individual's allocation of assets between tangible capital and government debt is not

[5] See Prescott (2006)

determined. The sum, however, is determined as is the aggregate holdings of these two assets. At the beginning of the period, interest is paid on the level of assets held at the end of last period. This interest rate, i_{t-1}, is determined in period *t-1* and paid in period *t*. The tax rate on labor income is τ_h, and the consumers' after-tax labor income in period *t* is $(1-\tau_h)w_t h_t$.

Stand-in Bank

To handle the indeterminacy of individual portfolio composition, we introduce a stand-in bank. This bank has liabilities to individuals in the form of deposits or assets, A_t, and owns both the capital stock, K_t, and the government debt, D_t. Therefore, $A_t = K_t + D_t$. The bank rents the capital stock to the stand-in firm for use in production at the rate of r_t. After production, the firm returns the capital to the bank less depreciation. The bank chooses how much to invest in capital stock and pays interest to depositors.

The capital stock owned by the bank evolves according to:

(3.4) $$K_{t+1} = (1-\delta)K_t + X_t$$

Where X_t denotes investment and $0 < \delta < 1$ is the depreciation rate. The technology of the stand-in bank displays constant returns to scale so profits are zero, and we need not worry about ownership of the bank. Therefore the interest rate paid on assets is exactly equal to the after tax net income from renting the capital. This means

(3.5) $$i_{t-1} = (1-\tau_k)(r_t - \delta)$$

must be satisfied, where r_t is the year *t* rental price of capital and τ_k is the tax rate on net capital income.

Stand-in Firm

The firm rents capital services from the bank and labor services from the households for use in producing output that can be used for consumption or investment. The production function of the stand-in firm is

(3.6) $$Y_t = K_t^{\theta}(Z_t H_t)^{1-\theta}$$

The parameter Z_t grows at rate γ and the parameter θ is the capital share parameter. The equilibrium wage rate is equal to the marginal product of labor, and the rental rate of capital is equal to the marginal product of capital. The production technology of the stand-in firm displays constant returns to scale; therefore, we need not worry about the ownership of the firm.

Government

The government in the model economy receives revenue from a tax on labor income and net capital income.

(3.7) $$\text{Revenue}_t = \tau_k(r_t - \delta)K_t + \tau_h w_t H_t$$

The government spends this revenue on a lump-sum transfer to all individuals and interest payments on government debt. Let $\Psi_t = N\psi_t$ denote aggregate transfers. The law of motion for the amount of government debt is:

(3.8) $$D_{t+1} = \Psi_t + i_{t-1}D_t - \text{Revenue}_t$$

Aggregates

As can be seen from the above, the only things that the government does are tax, make transfers, and pay interest on the government debt. In the case when the government debt is negative, the government has an interest bearing deposit at the stand-in bank. Any public consumption is treated as a transfer in kind to households.

Output is used only for consumption by individuals and investment by the bank. Therefore, $Y_t = C_t + X_t$. Aggregate consumption at date t is the sum of the consumption of all individuals alive at time t

$$C_t = \sum_{b=t-N}^{t} (1+\eta)^b N^t c_t \tag{3.9}$$

where the superscript b denotes the cohort, or year of birth and N^t is the size of cohort t. Aggregate assets, A_t , are determined in the same way. In the case at aggregate hours H_t , the summation begins at $b=t - N_w$.

3.2 Model Parameter values

We consider balanced growth or steady state equilibria. For such equilibria (Y_t , K_t ,C_t ,X_t ,A_t ,D_t) grow at rate g, (w_t, c_t, ψ_t, a_t) grow at rate γ , (H_t , N_t , N^b) grow at rate η, and (h_t, r_t, i_t) are constant. Along the balanced growth path, the government holds assets or issues debt in order to keep the interest rate fixed at i.

Before we can derive any quantitative policy implications using the model, we must calibrate the model. We calibrate this model economy to the behavior of the advanced industrialized countries. This means that the model economy mimics the behavior of these actual economies on a number of key aggregate dimensions. There are six parameters we need to calibrate, (γ, θ, δ, Z, β, α). The first four are technology parameters and the last two are preference parameters. In the calibration we need valued for the three demographic parameters (N_w, N_r, η) and the two tax rates The demographic parameters and tax rates will vary across examples

We use U.S. data to tie down the parameters. We normalize the value of aggregate output to one. We use data on the U.S. economy where the marginal effective tax rate on labor income is 0.40.[6] We set $\tau_k = 0.20$. The tax rate on corporate capital is higher than this, but the tax rates on other capital including owner occupied housing and consumer durables is lower than this figure. We include depreciation on consumer durables in depreciation and impute services to consumer durables in much the same way national account impute services to owner occupied housing. This inclusion increases the capital cost share parameter.

The preference and technology parameters ($\gamma, \theta, \delta, \beta$) can be calibrated using the following observations: (i) the growth rate of output is 2 percent per year; (ii) the annual capital to output ratio is 3.5; (iii) capital cost share is 0.35; (iv) depreciation of the capital stock relative to output is 0.175; and the before tax return on capital is 5 percent. With the tax rate on net capital income of 20 percent, the after-tax return on capital and household interest rate are 4 percent.

In order to calibrate the preference parameter α and the technology parameter Z, we need to use the demographic parameters, the normalization of the initial population to 100, and the observation that the fraction of productive time of working age people allocated to the market given the tax rate of labor of 40 percent is 0.25. The demographic parameters are calibrated using observations from the U.S. The average working life is 45 years. The average adult lifetime is 85 years. Last, the population growth rate is 1 percent per year.

[6] See Prescott (2002).

We will deal only with policies that result in the after tax return on capital being 4 percent as this has been the after tax return on non-corporate capital in the United States since 1929 the first year for which National Income Accounts for the United States exists. This is the approximate return realized by household previously to 1929 on important savings instruments. For more details see McGrattan and Prescott (2003).

The motivation for the selection of a 2 percent growth rate of output is that this has been the trend growth rate in the United States over the last 135. The accounts for the model economy are reported in Table 1. The calibrated parameters are listed in Table 2. Appendix A explains the algorithm for solving for the balanced growth path of the economy.

3.3 Demographic Assumptions

The two main demographic assumptions that influence the optimal amount of government debt are the length of the retirement period and the population growth rate. Holding an individual's working life fixed, a longer retirement period increases the amount of assets a person of a given age holds. We explore various demographic assumptions. Population growth rates are now low and even negative in some advanced industrial countries. As asset positions differ with age, changing the age distribution will have consequences for the needed amount of government debt.

Section 4: Policy Evaluations under various Demographic Assumptions

When comparing alternate retirement policies, we focus on the welfare criterion of lifetime consumption equivalents of someone entering the workforce. We emphasize that ours is a steady state analysis. We examine the level of government debt needed for each of the retirement policies in our calibrated model.

The two main findings that emerge from the policy evaluations are that the optimal amount of government debt depends on the demographics of the economy and that the move from a tax-and-transfer retirement policy to a savings policy is welfare improving. A summary of the results is presented in Tables 3 and 4.

4.1 Current United States

The first case we examine roughly captures the current U.S. economy. This is the benchmark case we used for calibration. The population growth rate is 1 percent per year, and the growth rate of technology is 2 percent per year. The lifespan is 85 years. The actual retirement policy in the U.S. economy is somewhere between a full tax-and-transfer system and a complete savings system. Here we simply compare a system with the U.S. marginal effective tax rate on labor income with a savings system.

In the tax-and-transfer system, labor income is taxed at 40 percent, and net capital income is taxed at 20 percent. The equilibrium government debt to output ratio in the model economy is 0. The government capital to output ratio is 0.62. The government owns 18 percent of the productive assets in the economy. This is about double U.S. governments holdings of capital.

If the government policy changed such that the only tax levied was on net capital income, the ratio of government debt to output would rise to 1.30 and government holding of capital would be zero.. In addition, aggregate output would increase from 1.00 to 1.43. Because of the increase in the after-tax wage faced by the worker, labor allocated to the market increases. In the tax-and-transfer system, a person entering the workforce today allocates 25 percent of his time endowment to the market. In the savings system, a worker allocates 36 percent of his time to the market. However, the reduction in leisure is more than offset by increases in consumption with the savings system.

In order to evaluate these two policies, we consider the welfare of a person entering the workforce today. By entering the economy with the savings system, his welfare is 9.2 percent higher in terms of lifetime consumption equivalents. This means that in order for him to be indifferent between the tax-and-transfer system and the savings system, his consumption in the tax-and-transfer economy would need to be increased by 9.2 percent in every period of his life. To summarize, the economy with the savings system has a significantly larger government debt to GNP ratio, but it provides higher welfare for an agent entering the workforce.

4.2 Future United States

Next, we examine is a possible future United States economy. This economy has zero population growth and people have a longer lifespan at 95 years. Technology growth is 2 percent per year. This economy has a tax-and-transfer system with a tax rate on labor income of 40 percent and net capital income of 20 percent. Aggregate output is 0.907, and the government debt to output ratio is 1.7. Individuals allocate 28 percent of their time endowment to the market.

If the retirement policy changed to a saving-for-retirement system, there are no transfers from the government. The labor income tax is 7.5 as tax revenues are needed to help finance to interest on the government debt. With a switch to this policy, aggregate output would increase to 1.21, and the government debt to output ratio would increase to 4.5. Individuals would now allocate 38 percent of their time endowment to the market.

Aggregate output increases but market work also increases, so what is the effect on the welfare of a person entering the workforce? In terms of lifetime consumption equivalents, the welfare of a person entering the workforce is 5.5 percent higher in the economy with a saving system than in the economy with a tax-and-transfer system.

4.3 Old United States

The third case we examine is motivated by the historical U.S. economy. In the past, the population growth rate was 1 percent per year, and the growth rate of technology was 2 percent per year. The average lifespan was 75 years. A saving-for-retirement system would have had a tax rate on net capital income of 20 percent and no tax on labor income. Under the savings system, there is no government debt. The government owns 44 percent of the capital stock. Under a tax-and-transfer system, the government owns even more of the capital stock. Consider a tax-and-transfer system with a 40 percent tax rate on labor income and a 20 percent tax rate on net capital income. If there had been a tax-and-transfer system like this in place, the government would have owned 66 percent of the capital stock. Therefore, in an economy where the population is growing and there is a short retirement period, the tax-and-transfer system requires that the government own two-thirds of the productive assets in the economy.

4.4 Current Japan

The fourth example is motivated by the current Japanese economy. The growth rate of the population is zero. The growth rate of technology is 2 percent per year, and the average lifespan is 85 years. The tax-and-transfer system taxes labor income at 40 percent and net capital income at 20 percent. The government debt to output ratio is 0.0, and the government capital to output ratio is 0.11. The government owns 3 percent of the capital stock. Aggregate output is 0.943 and individuals allocate 26 percent of their time endowment to the market.

In an economy with a saving-for-retirement system, there is no tax on labor income. The government debt to output ratio in this economy is 2.15. Aggregate output increases to 1.34 and time allocated to the market increases to 36.7 percent. In terms of lifetime consumption equivalents, the welfare of a person entering the workforce is 7.8 percent higher in the economy with the saving system than in the economy with a tax-and-transfer system. This means that a person's consumption in the tax-and-transfer economy would need to be increased by 7.8 percent in every period in order for him to be indifferent between entering the workforce in the two economies. So the move from a tax-and-transfer system to a saving system increases the welfare of a person entering the workforce and increases the government debt to output ratio.

4.5 Current France

The last economy we examine is motivated by the current French economy. This economy has zero population growth, retirement at age 60, and a lifespan of 85 years. Technology growth is 2 percent per year. In the tax-and-transfer system, the tax rate on labor income is 60 percent, and the tax rate on net capital income is 20 percent. Aggregate output is 0.672, and the government has no debt. The government capital to output ratio is 0.59 which means the government owns 11 percent of the productive assets in the economy.

With a saving-for-retirement system, the government levies a 5 percent labor income tax and a 20 percent tax on net capital income. As a result, output nearly doubles to 1.24, and the government debt to output ratio increases to 3.42. Worker's time allocated to the market increases from 0.21 to 0.38. This system significantly reduces the deadweight loss due to the labor income tax. As a result, the welfare gain of switching from the tax-and-transfer system to the savings system is 23 percent in terms of lifetime consumption equivalents. For economies with high labor income tax rates and long retirement periods, the switch from a tax-and-transfer system to a savings system generates a large welfare gain and needs a sizable amount of government debt.

Section 5: Deficit Implications

As shown in Tables 3 and 4, a savings system has a larger budget deficit than a tax-and-transfer system for all four examples. The large deficit means government debt increases, but it does not increase relative to output. Along the balanced growth path, the budget deficit to output ratio remains constant as does the government debt to output ratio. In a savings system, the government taxes net capital income and makes interest payments and small transfers. From Table 3, the current U.S. a savings system generates capital tax revenue equal to 3.5 percent of output. The government makes lump-sum transfers back to the people equal to 2.2 percent of output. That leaves 1.3 percent of output for paying the interest on the government debt. The total interest payments on this debt are equal to 5.2 percent of output. The deficit each period stays constant at 3.9 percent of output.

The current U.S. system is neither all tax-and-transfer nor all saving-for-retirement. There is a significant amount of private saving for retirement. The current U.S. deficit is approximately 2 percent of output. So moving to a welfare improving saving-for-retirement system would increase the government budget deficit.

Section 6: Conclusion

With the welfare improving saving-for-retirement system, the needed amount of explicit government debt is big if the fraction of retirees is large. With a tax-and-transfer system, government debt is small, but the present value of promised transfers is large. In addition, with such a system there are huge dead-weight losses as the result of the high tax rate on labor income.

With the current demographics in the United States, moving from a tax-and-transfer system to a saving-for-retirement system would increase the government debt to output ratio to 1.3 and would result in a welfare gain of more than 9 percent in terms of lifetime consumption equivalents. With plausible future demographics for the United States with no population growth and longer retirement periods, the government debt to output ratio would increase to 4.5.

The gains of a switch to a saving-for-retirement system are even large in Western Europe where effective labor income tax rates are significantly higher than in the United States. The welfare gains are as large as 24 percent in terms of lifetime consumption equivalents. As the needed government debt is 3.4 times GNI this means that the Maastricht Treaty would have to be revised to permit more than 0.6 times GNI of debt.

For all of the plausible demographic assumptions, moving from a tax-and-transfer system to a saving-for-retirement system increases government debt and is welfare improving. Government debt is not a burden on our grandchildren. Our grandchildren will be better off in a world with a saving-for-retirement system and sizable government debt.

Table 1

The Accounts with Output Normalized to 1

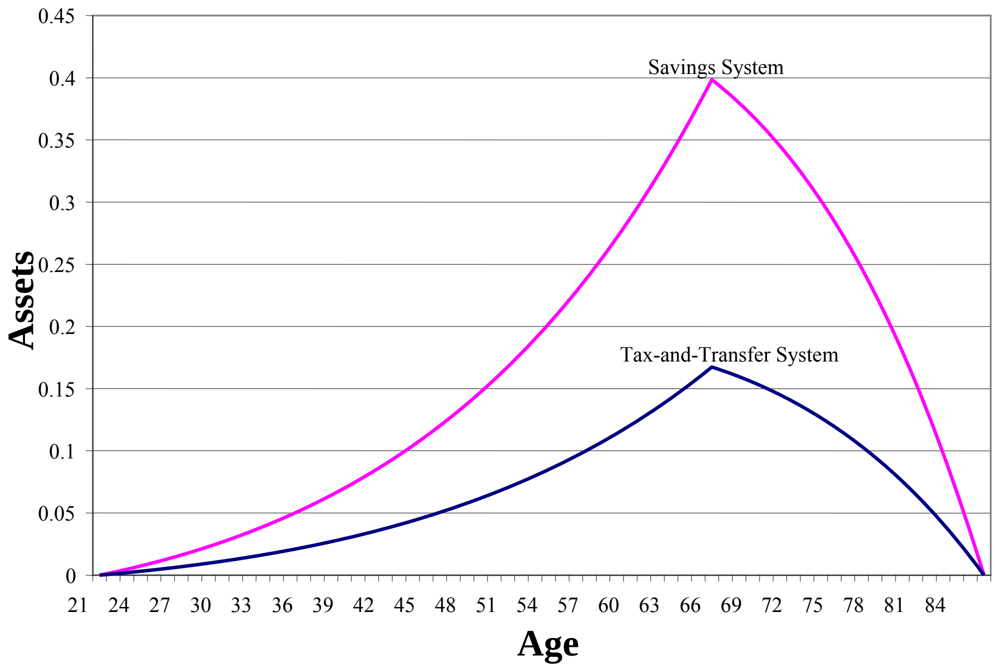
GNP	1.000
Consumption	0.719
Investment	0.281
GNP at Cost	
Depreciation	0.175
Compensation	0.650
Profits	0.175
Profit tax	0.035
After tax profits	0.140
Aggregate Inputs	
K	3.500
H	18.95
Individual Variables	
Labor supply of workers	0.25
Consumption	0.0719

Table 2

Calibrated Parameter Values

Parameters	Values	
γ	0.0200	Average growth in per capita consumption
θ	0.3500	Capital cost share
δ	0.0500	Depreciation and capital stock
Z	0.0269	Y normalized to 1 in base year.
α	0.6822	Fraction of time workers allocate to market when the labor tax is 0.40
β	0.9808	Fact that consumption of each individual grows by factor $(1+\gamma)$
N_w	45	
N	85	
η	0.01	
Population	100	

Lifetime Asset Profile for Current U.S. Demographics



Cross Sectional Asset Holdings for Current U.S. Demographics

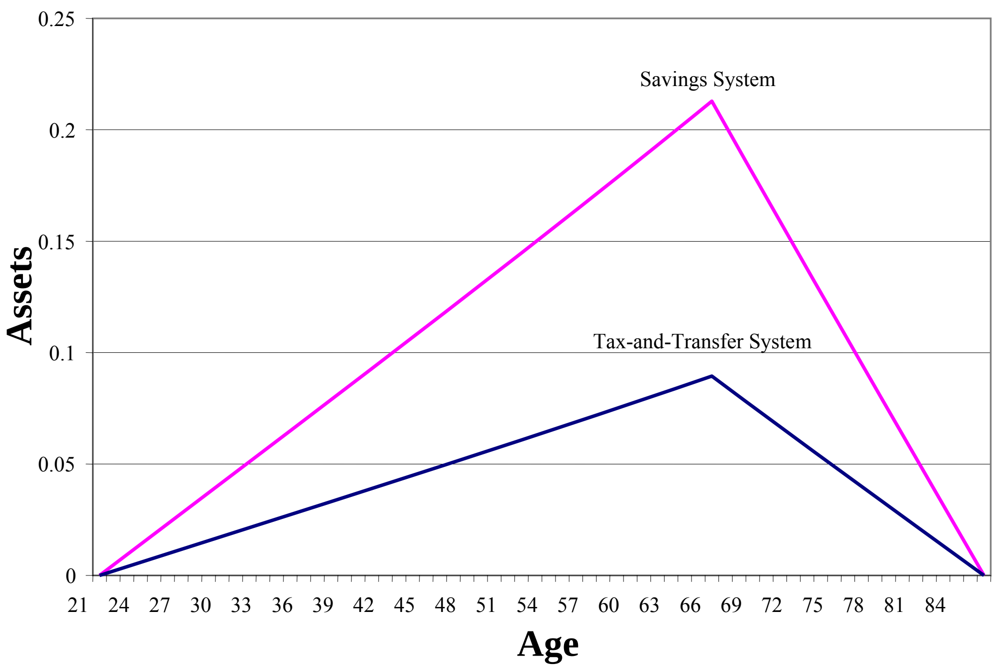


Table 3

	Current U.S. η=0.01, retire at 65, die at 85		**Future U.S.** η=0.00, retire at 65, die at 95	
	Tax-and-Transfer	*Savings*	*Tax-and-Transfer*	*Savings*
Output	1.000	1.429	0.907	1.210
Government (relative to output)				
Receipts:				
Tax Revenue	0.295	0.035	0.295	0.084
Interest Received	0.031	0.000	0.000	0.000
Expenditures:				
Transfers	0.301	0.022	0.261	-0.006
Interest Paid	0.000	0.052	0.068	0.180
Deficit	-0.025	0.039	0.034	0.090
Household Balance Sheet (relative to output):				
Total Assets	2.877	4.795	5.191	8.003
Total Capital	2.877	3.500	3.500	3.500
Total Government Debt	0.000	1.295	1.691	4.503
Government Balance Sheet (relative to output):				
Total Capital	0.623	0.000	0.000	0.000
Total Debt Outstanding	0.000	1.295	1.691	4.503
Individual Consumption	0.00719	0.01027	0.00684	0.00914
Transfer/Consumption	0.418	0.031	0.346	-0.008
Individual Labor Supply	0.250	0.357	0.286	0.382
Labor income tax rate	0.400	0.000	0.400	0.075
Welfare Gain in L.C.E		9.18%		5.51%

Table 4

	Current Japan η=0.00, retire at 65, die at 85		**Current France** η=0.00, retire at 60, die at 85	
	Tax-and-Transfer	*Savings*	*Tax-and-Transfer*	*Savings*
Output	0.943	1.340	0.672	1.242
Government (relative to output)				
Receipts:				
Tax Revenue	0.295	0.035	0.425	0.068
Interest Received	0.006	0.000	0.029	0.000
Expenditures:				
Transfers	0.297	-0.008	0.437	-0.001
Interest Paid	0.000	0.086	0.000	0.137
Deficit	-0.004	0.043	-0.017	0.068
Household Balance Sheet (relative to output):				
Total Assets	3.390	5.650	2.914	6.920
Total Capital	3.390	3.500	2.914	3.500
Total Government Debt	0.000	2.150	0.000	3.420
Government Balance Sheet (relative to output):				
Total Capital	0.110	0.000	0.586	0.000
Total Debt Outstanding	0.000	2.150	0.000	3.420
Individual Consumption	0.00711	0.01012	0.00507	0.00938
Transfer/Consumption	0.394	-0.011	0.578	-0.001
Individual Labor Supply	0.258	0.367	0.207	0.382
Labor income tax rate	0.400	0.000	0.600	0.050
Welfare Gain in L.C.E		7.83%		23.38%

Appendix A:

The following specifies the algorithm use to find the equilibrium of the model described in Section 2.1. We are looking for an equilibrium where (Y_t ,K_t ,C_t ,X_t ,A_t ,D_t) grow at rate g, (w_t, c_t, ψ_t, a_t) grow at rate γ, (H_t, N_t , N^b) grow at rate η, and (h_t, r_t, i_t) are constant. Along the balanced growth path, the government holds assets or issues debt in order to keep the interest rate fixed at i. Given the parametric set of economies, we are looking for balanced growth path of this economy given the interest rate, the tax rate on net capital income and the labor income tax rate.

The first step is to solve for the rental price of capital, r, using the capital tax rate and the depreciation rate along with the bank's no profit condition, $i = (1-\tau_k)(r-\delta)$. The second step is to solve for the capital labor ratio using the profit maximizing condition of the stand-in firm with respect to capital, $r = \theta \frac{Y}{K}$. Then solve for the capital to labor capital ratio using the production function, $\frac{Y}{K} = Z\left(\frac{H}{K}\right)^{1-\theta}$. Then solve for the real wage, w, using the stand-in firm's other profit maximizing condition, $w = (1-\theta)Z\left(\frac{K}{H}\right)^{\theta}$. The next step is to solve for per capita consumption, c, individual labor supplied, h, aggregate consumption and aggregate labor supply. This requires solving four equations in four unknowns.

$C = \sum_{i=-N}^{1}(1+\eta)^i \mathrm{N}^1 c$: the definition of aggregate consumption

$H = \sum_{i=-Nw}^{1}(1+\eta)^i \mathrm{N}^1 h$: the definition of aggregate labor supply

$C+(\gamma+\delta)K = ZK^{\theta}H^{1-\theta}$: the aggregate resource constraint

$\frac{\alpha c}{(1-h)} = (1-\tau_h)w$: the household's intratemporal marginal condition

where N^1 is the size of cohort 1.

After solving for c and h, solve for ψ using the lifetime budget constraint of the household. $\sum_{t=1}^{N_w} \frac{1}{(1+i)^t}\left(c_t-(1-\tau_h)w_t h_t\right) - \sum_{t=N}^{N} \frac{\psi_t}{(1+i)^t} = 0$

The next step is to solve for aggregate assets in the economy in period t=1, $A_1 = \sum_{b=1-N}^{1} a_1^b$

First, calculate the lifetime asset profile of an individual born in period 1 using

$a_{t+1} = a_t(1+i_{t-1}) + (1-\tau_h)w_t h_t + \psi_t - c_t$ and $a_1^1 = 0$.

Since the economy is on the balanced growth path and we know the vector of

$\{a_t^1\}_{t=1}^{N}$, we can solve for the vector of $\{a_1^b\}_{b=-N}^{1}$

using the following relationship: $a_t^{-b} = \frac{a_b^t}{(1+\gamma)^b}$

Therefore the total assets in the economy in period 1: $A_1 = \sum_{j=1}^{N} \frac{a_j^1}{(1+\eta)^j(1+\gamma)^j} N^1$

The final step solves for government debt, *D*. First, using the bank's balance sheet condition, *A* = *K* + *D*, and aggregate assets, find *D*. A corresponding step involves looking at the government's budget constraint. Solve for total tax revenue and total transfers as follows: $\text{Revenue} = \tau_k(r-\delta)K + \tau_h wH$

Transfers, $\Psi = \sum_{i=-N}^{0}(1+\eta)^i N^1 \psi$. Then find government debt using the government's budget constraint, $\text{Revenue} = \Psi + (i-\gamma)D$.

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