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Some Explorations Into Optimal Cyclical Monetary Policy

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ABSTRACT

We consider the nature of optimal cyclical monetary policy in three different stochastic models with various shocks. The first is a pure liquidity effect model, the second is a cost of changing prices model, and the third is an optimal seignorage model. In each case we solve for the optimal monetary policy and describe how money growth and interest rates respond to shocks under the optimal policy. The shocks we consider are money demand shocks, productivity shocks, and government consumption shocks. All of the models have the feature that the Friedman rule of setting the nominal interest rate to zero is not optimal. Optimal policies are always time inconsistent even though lump sum taxation is allowed. At least in some instances we find that optimal policy dictates responses of money growth and interest rates which run counter to conventional wisdom.

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1. Introduction

The purpose of this paper is to conduct a preliminary exploration into the nature of optimal cyclical monetary policy. We solve for the optimal monetary policy in three different stochastic models and describe how money and interest rates respond to a variety of shocks under the optimal policy. The goal is to gain some intuition into how monetary policy should respond to different types of shocks in the context of some simple models. At least in some instances we find that optimal policy dictates responses of money growth and interest rates which run counter to conventional wisdom. For example, we find that the optimal cyclical response of the money growth rate need not be countercyclical and may depend on the nature of the shock that affects output.

We view the research undertaken in this paper as the first step toward formulating and analyzing models which, hopefully, will lead to reasonable prescriptions for the conduct of cyclical monetary policy. It may also lead to useful insights into business fluctuations. As Sims [1996] has observed (p. 118), "...most of the observed variation in monetary policy instruments - interest rates and monetary aggregates - cannot be treated as exogenously generated by random shifts in policy...because monetary contraction is rarely a spontaneous policy decision, the apparently eloquent fact that monetary contractions are followed by recession is hard to interpret."

The exercises we undertake in this paper are closely related to those undertaken by Taylor [1993]. In that book Taylor writes down a system of equations describing an economy. Many of these equations are interpreted as describing the optimizing behavior of households or firms; they could be thought of as the stochastic Euler equations arising from a dynamic stochastic optimization problem. However, Taylor does not indicate

explicitly the objective functions and constraints of households or firms which presumably lead to these equations. One consequence of this is that there is no clear basis for attributing any particular objective function for a policy maker. For example, a particular Euler equation containing past values as well as expectations of future values might be justified by appealing to dynamic adjustment costs. However, these adjustment costs are not considered in the policy maker's objective.

In contrast to Taylor's approach, our exercises are undertaken in models with a representative household so that we not only make clear what the objective functions and constraints are which lead to particular Euler equations but we also have a natural objective function for the policy maker. This is not to say that our models are free of ad hoc features. But, whatever ad hoc features we put into our models they are explicitly embedded in either objective functions or constraints, the Euler equations are consistent with the postulated objective functions and constraints for households, and the policy maker's objective function is the natural one of maximizing the welfare of the representative household. Therefore if we postulate some adjustment costs for the household these adjustment costs will also appear in the policy maker's objective function.

We hasten to add that we do not intend the above remarks as criticisms of Taylor's [1993] work. His work is far more ambitious in scope than ours and it will be no easy task to extend our analysis of extremely simple models to much more complex ones which contain many more of the essential features of real economies.

We believe that an essential ingredient of any model which attempts to study the nature of optimal monetary policy is that in the corresponding deterministic version of the model, the Friedman rule, interpreted as

setting the nominal interest rate to zero, should not be optimal. Otherwise the problem of optimal monetary policy is trivial. We consider three different stochastic models with the feature that the Friedman rule is not optimal. All three are of the standard cash-in-advance (CIA) type monetary models. In each case our interest is in understanding how money growth and interest rates respond to a variety of different shocks under the optimal policy.

In the literature one can distinguish among (at least) three different ways in which money affects the economy. First, money affects asset prices in financial markets through liquidity type effects. Second, money affects goods markets due to the fact that goods prices are posted in nominal terms and are costly to change. Third, money is a source of seignorage revenue to the government which may be useful when other forms of raising revenues involve costs. Each of our three models is designed to focus on one of the above features.

Our first model is a pure liquidity effect model. Here the household faces some convex costs of going to the financial market to adjust its portfolio of money and interest bearing securities. We interpret this as an analytically tractable general equilibrium version of a Baumol-Tobin transaction cost model of money demand. In any model with a Baumol-Tobin style transaction cost for converting securities into money an injection of money into the financial market (whether anticipated or not) must lead initially to a drop in the nominal interest rate. The reason is that households face costs of converting securities into money and vice versa and, hence, a drop in the nominal interest rate is required to induce households to go to the financial market and pick up the money injection. Thus, both anticipated as well as unanticipated money injections will

produce a liquidity effect type response of the nominal interest rate which, furthermore, will be persistent. We assume the transaction costs to be convex in order to maintain the convenience of working with a representative household model. Because of these costs the Friedman rule will not be optimal. We consider how money growth and interest rates respond to money demand shocks, productivity shocks, and government consumption shocks under the optimal policy.

Our second model is a model of costly price changes, very similar to the model in Rotemberg [1994]. Here the representative producer faces costs of changing nominal prices. A permanent increase in the money supply will, therefore, lead to a gradual rise in the price level to its new steady state value. During this time output will initially rise above its steady state value and then gradually return to its initial steady state value. Again, due to the costs of price change it will not be optimal to implement the Friedman rule by deflating the money supply. We do not permit the government to pay interest on currency in circulation. Thus, it will not be possible to implement the Friedman rule by paying interest on money financed by lump sum taxes thus keeping the money supply and the price level constant and avoiding the costs of price change. We consider how money growth and interest rates respond to productivity shocks and government consumption shocks under the optimal policy.

Our third model is one of optimal seignorage. Here while the government has access to lump sum taxes there is some loss in revenues received by the government relative to the taxes levied. One can think of this simply as there being a leaky bucket which transports taxes paid by households to tax revenues collected by the government. Consequently, it will be optimal to finance some portion of government consumption by seignorage. Again the

Friedman rule will not be optimal. We describe how money growth and interest rates respond to money demand, productivity, and government consumption shocks under the optimal policy.

The rest of this paper is organised as follows. In sections 2, 3 and 4 we describe the liquidity effect model, the cost of price change model, and the optimal seignorage model, respectively. In section 5 we describe the cyclical characteristics of the optimal policies for each of the three models. Section 6 summarizes and suggests some directions for future work.

2. A Pure Liquidity Effect Model

The representative household maximizes

$$(2.1a) \quad E_0 \sum_{t \geq 0} \beta^t \{U(c_{1t}, c_{2t}, \theta_t) - \phi(z_t/m_t)\}$$

subject to the following constraints:

$$(2.1b) \quad z_t/p_t \equiv (m_t + b_t + T_t)/p_t - b_{t+1}/[(1+R_t)p_t] \geq c_{1t},$$

$$(2.1c) \quad (m_t + b_t + T_t)/p_t - b_{t+1}/[(1+R_t)p_t] - c_{1t} + y_t - c_{2t} - m_{t+1}/p_t \geq 0.$$

In the above problem c_{1t} and c_{2t} are the household's purchases of cash goods and credit goods in period t , respectively, and θ_t is a random shock to the relative desirability of cash goods and credit goods (a proxy for a money demand shock). The household consists of a seller and a shopper. The seller receives a random endowment of y_t units of goods in period t . The shopper starts period t with m_t units of money and b_t units of nominal bonds and proceeds to the financial market. There he receives nominal transfers of T_t and engages in financial market transactions which leave him with z_t

units of money which he takes to the cash goods market. The function ϕ in the household's preferences reflects a cost (in terms of disutility) of undertaking financial market transactions in order to change money balances. We assume that $\phi(1) = \phi'(1) = 0$ and that ϕ is convex. Thus, the household could avoid these costs if the shopper chose to proceed directly to the cash goods market with its starting money balances m_t . However, attempting to change the amount of cash to be taken to the cash goods market requires financial market transactions which impose some costs. Our intent is to capture a Baumol-Tobin type transaction demand for money in a general equilibrium setting in a way that is tractable and preserves the convenience of the representative household model.

To complete the description, p_t and R_t denote the price level and the nominal interest rate, respectively. Constraint (2.1b) is the CIA constraint and constraint (2.1c) is the budget constraint.

Let $\beta^t \lambda_t$ and $\beta^t \mu_t$ be the nonnegative multipliers associated with the constraints (2.1b) and (2.1c), respectively. The first order necessary conditions (FONCs) for the problem (2.1) are as follows.

$$(2.2a) \quad u_{1t} - (\lambda_t + \mu_t) = 0,$$

$$(2.2b) \quad u_{2t} - \mu_t = 0,$$

$$(2.2c) \quad -\mu_t + \beta E_t(\lambda_{t+1} + \mu_{t+1}) p_t / p_{t+1} - \beta E_t(p_t / m_{t+1}) (1 - z_{t+1} / m_{t+1}) \phi'(z_{t+1} / m_{t+1}) = 0.$$

$$(2.2d) \quad -(\lambda_t + \mu_t) / (1 + R_t) + (p_t / m_t) \phi'(z_t / m_t) / (1 + R_t) + \beta E_t(\lambda_{t+1} + \mu_{t+1}) p_t / p_{t+1} - \beta E_t(p_t / m_{t+1}) \phi'(z_{t+1} / m_{t+1}) = 0,$$

In equilibrium we must have the following.

$$(2.3a) \quad z_t = m_{t+1},$$

$$(2.3b) \quad c_{1t} + c_{2t} = y_t.$$

Note that (2.3a) follows from the government budget constraint and (2.3b) is the resource constraint.

Using (2.1b) and (2.3a) we can write

$$(2.4) \quad p_t/p_{t+1} = (p_t/m_{t+1})(m_{t+1}/m_{t+2})(m_{t+2}/p_{t+1}) = c_{1,t+1}x_{t+1}^{-1}/c_{1t},$$

where x_{t+1} is the gross money growth rate in period $t+1$.

We can use (2.3a) and (2.4) to combine (2.2a), (2.2b) and (2.2c) into the following equation.

$$(2.5) \quad u_{2t} = \beta E_t \{ u_{1,t+1} c_{1,t+1} x_{t+1}^{-1} - (1-x_{t+1}) \phi'(x_{t+1}) \} / c_{1t}.$$

Policy maker's problem

Now we can state the problem of the policy maker. Note that for the policy maker $\phi(z_t/m_t) = \phi(m_{t+1}/m_t) = \phi(x_t)$. Therefore, the policy maker's problem is to

$$\text{maximize } E_0 \sum_{t \geq 0} \beta^t \{ U(c_{1t}, c_{2t}, \theta_t) - \phi(x_t) \}$$

subject to (2.3b) and (2.5) by choosing stochastic processes for $\{c_{1t}, c_{2t}, x_t\}$.

Note the following aspects of the solution to this model.

First, while the usual Fisher relation between the real, nominal and the inflation rates does hold across steady states, it does not hold along

the dynamic path. That is, $1+R_t \neq u_{1t}/[\beta E_t\{u_{1,t+1}p_t/p_{t+1}\}]$. This can be seen from the FONCs (2.2 a,d).

Second, the solution to this problem will clearly be time inconsistent. To see this note that the constraint (2.5) which arises from household behavior is forward looking and involves expectations of future values. However, when the future arrives those variables are predetermined and the policy maker can ignore whatever expectations households may have had about those variables in the past. Hence, if the policy maker is allowed to reoptimize at some future date t he will choose a different solution for date t than the one he chose at date zero.¹

3. A Cost of Changing Nominal Prices Model

There is a continuum of goods as well as households both indexed by $i \in [0,1]$. Each household consists of a consumer, a worker, and a seller. Seller i is a monopolist in the market for good i . Let c_{it} , n_t and p_{it} be the amount of good i consumed by the household in period t , amount of labor supplied by the household in period t , and the nominal price of good i in period t , respectively. A household's preferences are given by the following.

$$(3.1a) \quad E_0 \sum_{t \geq 0} \beta^t \{U(c_t, 1-n_t) - \phi(p_{it}/p_{i,t-1})\}$$

¹To see a concrete illustration of time inconsistency note that the initial money growth rate x_0 appears only in the objective function of the policy maker and nowhere else. Since $\phi'(1) = 0$, it is clearly optimal to set $x_0 = 1$. However, the solution for future money growth rates will, in general, differ from unity since they appear in the constraint (2.5). Therefore, if the planner is allowed to reoptimize at some future date t then he would want to set x_t to unity which will be different from the optimal choice of x_t from the point of view of date zero.

where

$$(3.1b) \quad c_t = [\int_0^1 c_{it}^\rho di]^{1/\rho}.$$

The function ϕ represents a cost (in terms of disutility) which seller i faces in his price setting. We assume that $\phi(1) = \phi'(1) = 0$ and that ϕ is convex.

The production technology is simple; one unit of labor produces θ_t units of any good where θ_t is random.

We now describe the problems faced by the consumer-worker and the monopolist seller separately.

The consumer-worker's problem

The consumer-worker maximizes

$$(3.2a) \quad E_0 \sum_{t \geq 0} \beta^t U(c_t, 1-n_t)$$

(where c_t is given by 3.1b) subject to the following constraints

$$(3.2b) \quad (m_t + b_t + T_t)/p_t - b_{t+1}/[(1+R_t)p_t] \geq \int_0^1 p_{it} c_{it} di/p_t,$$

$$(3.2c) \quad (m_t + b_t + T_t)/p_t - b_{t+1}/[(1+R_t)p_t] - \int_0^1 p_{it} c_{it} di/p_t + w_t n_t/p_t + \pi_t/p_t - m_{t+1}/p_t \geq 0.$$

Constraint (3.2b) is the CIA constraint and constraint (3.2c) is the budget constraint. In constraints (3.2), m_t and b_t are the money and nominal bonds held by a typical household at the beginning of period t , T_t are the nominal transfers received by the household in period t , R_t , w_t and p_t are the nominal interest rate, nominal wage and the price level (to be defined

shortly) in period t , and π_t are the nominal monopoly profits made by the seller and passed back to the household. Note that the left side of (3.2b) is the amount of real money balances held by the household after transactions in the financial market.

Let $\beta^t \lambda_t$ and $\beta^t \mu_t$ be the nonnegative multipliers associated with constraints (3.2b) and (3.2c), respectively. The FONCs for the above problem are as follows.

$$(3.3a) \quad u_{1t} (c_t/c_{it})^{1-\rho} - (\lambda_t + \mu_t) p_{it}/p_t = 0,$$

$$(3.3b) \quad -u_{2t} + \mu_t w_t/p_t = 0,$$

$$(3.3c) \quad -\mu_t + \beta E_t (\lambda_{t+1} + \mu_{t+1}) p_t/p_{t+1} = 0,$$

$$(3.3d) \quad -(\lambda_t + \mu_t)/(1+R_t) + \beta E_t (\lambda_{t+1} + \mu_{t+1}) p_t/p_{t+1} = 0.$$

We assume that the government consumes the amount g_{it} of good i in period t . The government budget constraint is as follows.

$$(3.4) \quad \int_0^1 p_{it} g_{it} di + T_t = m_{t+1} - m_t + b_{t+1}/(1+R_t) - b_t.$$

Combining the government budget constraint (3.4) with the CIA constraint (3.2b) at equality we have

$$(3.5) \quad m_{t+1} = \int_0^1 p_{it} g_{it} di + \int_0^1 p_{it} c_{it} di.$$

Further, the FONC (3.3a) implies

$$(3.6) \quad (c_{it}/c_{jt})^{1-\rho} = (p_{jt}/p_{it}).$$

It is convenient to define the price level p_t as follows.

$$(3.7) \quad p_t = \left[\int_0^1 p_{it}^{-\rho/(1-\rho)} di \right]^{-(1-\rho)/\rho}.$$

We now assume that government consumption of various goods is given as follows.

$$(3.8) \quad g_{it} = g_t (p_{it}/p_t)^{-1/(1-\rho)},$$

where g_t is random.

Denoting aggregate demand for good i in period t from households and government by z_{it} and using (3.5)-(3.8) we can derive the following demand function for z_{it} .

$$(3.9) \quad z_{it} \equiv c_{it} + g_{it} = (m_{t+1}/p_t) (p_{it}/p_t)^{-1/(1-\rho)}.$$

We are now in a position to state the problem of a typical monopolist seller selling good i .

The monopolist seller's problem

The monopolist seller of good i maximizes

$$(3.10a) \quad E_0 \sum_{t \geq 0} \beta^t \{ \beta (\lambda_{t+1} + \mu_{t+1}) (p_{it} z_{it}^{-w_t} n_{it}) / p_{t+1} - \phi(p_{it}/p_{i,t-1}) \}$$

subject to:

$$(3.10b) \quad z_{it} = \theta_t n_{it} = (m_{t+1}/p_t) (p_{it}/p_t)^{-1/(1-\rho)}.$$

Note that $(p_{it}z_{it}-w_t n_{it})$ are the seller's period t nominal profits. Since these dollars can only be spent on goods at $(t+1)$ they are worth $(\lambda_{t+1}+\mu_{t+1})/p_{t+1}$ in terms of utils at $(t+1)$ and, hence, need to be discounted by β to convert them into utils in period t . In view of (3.3c) we could also replace $\beta(\lambda_{t+1}+\mu_{t+1})/p_{t+1}$ by μ_t/p_t which is the multiplier on nominal profits in constraint (3.2c).

After substituting out for z_{it} and n_{it} from (3.10b) into (3.10a) we can write the Euler equation with respect to p_{it} for the resulting problem as follows.

$$(3.11) \quad \beta E_t [(\lambda_{t+1}+\mu_{t+1})/p_{t+1}] (m_{t+1}/p_t) (p_{it}/p_t)^{-1/(1-\rho)} [w_t/(\theta_t p_{it}) - \rho]/(1-\rho) - \phi'(p_{it}/p_{i,t-1})/p_{i,t-1} + \beta E_t \phi'(p_{i,t+1}/p_{it}) p_{i,t+1}/(p_{it})^2 = 0.$$

We now assume that all the sellers start with identical initial conditions, that is, $p_{i,-1} = p_{-1}$ for all i . This will imply that $p_{it} = p_t$ and $c_{it} = c_t$, $g_{it} = g_t$, $z_{it} = z_t = c_t + g_t = m_{t+1}/p_t$ for all i . It follows that we can write

$$(3.12) \quad p_t/p_{t+1} = (p_t/m_{t+1})(m_{t+1}/m_{t+2})(m_{t+2}/p_{t+1}) = z_{t+1} x_{t+1}^{-1}/z_t,$$

where x_{t+1} is the gross money growth rate in period $t+1$.

Further, condition (3.3a) simplifies to

$$(3.3a)' \quad u_{it} - (\lambda_t + \mu_t) = 0.$$

Using (3.3b), (3.3c), (3.1a)' and (3.12) we can write

$$(3.13) \quad w_t/p_t = u_{2t}z_t/[\beta E_t u_{1,t+1}z_{t+1}x_{t+1}^{-1}].$$

Multiplying (3.11) by p_t and then substituting from (3.12), (3.3a)' and (3.13) into (3.11) we can rewrite (3.11) as follows.

$$(3.14) \quad [(u_{2t}z_t/\theta_t) - \rho\beta E_t\{u_{1,t+1}z_{t+1}x_{t+1}^{-1}\}]/(1-\rho) - (z_{t-1}x_t/z_t)\phi'(z_{t-1}x_t/z_t) + \beta E_t(z_t x_{t+1}/z_{t+1})\phi'(z_t x_{t+1}/z_{t+1}) = 0.$$

The market clearing condition in the goods market is as follows.

$$(3.15) \quad z_t = c_t + g_t = \theta_t n_t.$$

The policy maker's problem

Now we are in a position to describe the policy maker's problem. The policy maker's objective is to maximize the welfare of the representative household. This leads to the following objective function for the policy maker.

$$(3.16) \quad E_0 \sum_{t \geq 0} \beta^t [u(c_t, 1-n_t) - \phi(p_t/p_{t-1})] = E_0 \sum_{t \geq 0} \beta^t [u(c_t, 1-n_t) - \phi(z_{t-1}x_t/z_t)].$$

The policy maker maximizes the objective function on the right side of the equality in (3.16) by choosing stochastic processes for $\{x_t, c_t, n_t\}$ subject to (3.14) and (3.15) and taking c_{-1} as a given initial condition.

Note the following aspects of the solution to this problem.

First, it can be shown that the deterministic steady state value of x ,

the gross money growth rate, is between β and unity. Thus, the solution is characterized by price deflation but the nominal interest rate will be positive. The reason is as follows. In the absence of monopolistic price setting and price adjustment costs the optimal policy is to deflate the money supply at the rate β thereby setting the nominal interest rate to zero and eliminating the inflation tax distortion in labor supply. If labor supply were inelastic so that there would have been no inflation tax distortion but there is monopolistic price setting with price adjustment costs then the optimal policy would be to maintain a constant money supply thereby eliminating the costs of price adjustment. With both features present the optimal policy is somewhere in between.

Second, the solution to this problem will clearly be time inconsistent. The reason is that the constraint (3.14) which arises from household behavior is forward looking and involves expectations of future values. However, when the future arrives those variables are predetermined and the policy maker can ignore whatever expectations households may have had about those variables in the past. Hence, if the policy maker is allowed to reoptimize at some future date t he will choose a different solution for date t than the one he chose at date zero.

4. An Optimal Seignorage Model

The representative household maximizes

$$(4.1a) \quad E_0 \sum_{t \geq 0} \beta^t u(c_{1t}, c_{2t}, \theta_t)$$

subject to the following CIA and budget constraints.

$$(4.1b) \quad (m_t + b_t)/p_t - b_{t+1}/[(1+R_t)p_t] + g_t + \phi(\tau_t) - \tau_t \geq c_{1t},$$

$$(4.1c) \quad (m_t + b_t)/p_t - b_{t+1}/[(1+R_t)p_t] - \tau_t - c_{1t} + y_t - c_{2t} - m_{t+1}/p_t \geq 0,$$

where c_{1t} and c_{2t} are the amounts of cash goods and credit goods consumed, θ_t is a random preference shock to the relative desirability of cash goods versus credit goods (a proxy for a money demand shock), τ_t are real lump sum taxes paid by the household, $(g_t + \phi(\tau_t))$ is total government consumption of which the first term is exogenous and the second term represents costs of collecting taxes, and y_t is the total endowment of goods. Note that we are modeling total government consumption as arising purely from credit good purchases. This is reflected in (4.1b) in which the term $(b_{t+1}/[(1+R_t)p_t] - g_t - \phi(\tau_t))$ represents the market value of new debt issued by the government in the financial market. The government later issues additional debt with market value $(g_t + \phi(\tau_t))$ in the credit goods market to finance its purchases so that the total debt issued adds up to $b_{t+1}/[(1+R_t)p_t]$ in terms of market value.²

Letting $\beta^t \lambda_t$ and $\beta^t \mu_t$ be the nonnegative multipliers associated with the constraints (4.1b) and (4.1c), respectively, we can derive the following usual FONCs for the above problem.

$$(4.2a) \quad u_{1t} - (\lambda_t + \mu_t) = 0,$$

$$(4.2b) \quad u_{2t} - \mu_t = 0,$$

$$(4.2c) \quad -\mu_t + \beta E_t(\lambda_{t+1} + \mu_{t+1})p_t/p_{t+1} = 0,$$

²Even though the constraints (4.1b,c) are written as though only nominal riskless bonds are issued, later we only impose the present value budget constraint on the government. This amounts to assuming complete contingent markets for nominally denominated securities together with spot markets in goods.

$$(4.2d) \quad -(\lambda_t + \mu_t)/(1+R_t) + \beta E_t(\lambda_{t+1} + \mu_{t+1})p_t/p_{t+1} = 0.$$

The government's budget constraint is as follows.

$$(4.3) \quad g_t + \phi(\tau_t) = \tau_t + (m_{t+1} - m_t)/p_t + b_{t+1}/[(1+R_t)p_t] - b_t/p_t.$$

We assume that $\phi(0) = \phi'(0) = 0$ and that ϕ is convex. Note that $[\tau_t - \phi(\tau_t)]$ are net tax revenues received by the government. Thus, the function ϕ represents the leakage in the tax collection system.

In equilibrium we have

$$(4.4a) \quad m_{t+1}/p_t = c_{1t},$$

$$(4.4b) \quad y_t = c_{1t} + c_{2t} + g_t + \phi(\tau_t).$$

Using (4.4a) we can write

$$(4.5) \quad p_t/p_{t+1} = c_{1,t+1} x_{t+1}^{-1}/c_{1t},$$

where x_{t+1} is the gross money growth rate in period $t+1$.

We can combine (4.2a)-(4.2c) along with (4.5) to obtain the following.

$$(4.6) \quad u_{2t} = \beta E_t u_{1,t+1} c_{1,t+1} x_{t+1}^{-1}/c_{1t}.$$

We can also use (4.2a) and (4.2d) to write the government budget constraint in the following present value form.

$$(4.7) \quad E_0 \sum_{t \geq 0} \beta^t u_{1t} [\tau_t - \phi(\tau_t) + (m_{t+1} - m_t)/p_t - g_t] \geq u_{1,0} b_0/p_0.$$

Note that we can write

$$(4.8) \quad (m_{t+1} - m_t)/p_t = (m_{t+1}/p_t)(1-x_t^{-1}) = c_{1t}(1-x_t^{-1})$$

and

$$(4.9) \quad b_0/p_0 = (b_0/m_0)(m_0/m_1)(m_1/p_0) = (b_0/m_0)x_0^{-1}c_{1,0}.$$

Plugging (4.8) and (4.9) into (4.7) we have the following present value form of the government budget constraint.

$$(4.10) \quad E_0 \sum_{t \geq 0} \beta^t u_{1t} [\tau_t - \phi(\tau_t) + c_{1t}(1-x_t^{-1}) - g_t] \geq u_{1,0}(b_0/m_0)x_0^{-1}c_{1,0}.$$

We can simplify the above government budget constraint further by using (4.6) as follows. Note that

$$(4.11) \quad \begin{aligned} E_0 \sum_{t \geq 0} \beta^t u_{1t} c_{1t} x_t^{-1} &= u_{10} c_{1,0} x_0^{-1} + E_0 \sum_{t \geq 1} \beta^t u_{1t} c_{1t} x_t^{-1} \\ &= u_{10} c_{1,0} x_0^{-1} + E_0 \sum_{t \geq 1} \beta^t E_{t-1} \{u_{1t} c_{1t} x_t^{-1}\} \\ &= u_{10} c_{2,0} x_0^{-1} + E_0 \sum_{t \geq 1} \beta^t u_{2,t-1} c_{1,t-1} / \beta \\ &= u_{10} c_{1,0} x_0^{-1} + E_0 \sum_{t \geq 0} \beta^t u_{2t} c_{1t}. \end{aligned}$$

Substituting (4.11) in (4.10) and simplifying we have the following form of the government budget constraint.

$$(4.12) \quad E_0 \sum_{t \geq 0} \beta^t [u_{1t} \{\tau_t - \phi(\tau_t) + c_{1t} - g_t\} - u_{2t} c_{1t}] \geq u_{1,0} [(b_0 + m_0)/m_0] x_0^{-1} c_{1,0}.$$

The policy maker's problem

Now we can state the policy maker's problem. The policy maker's objective is to

$$\text{maximize } E_0 \sum_{t \geq 0} \beta^t u(c_{1t}, c_{2t}, \theta_t)$$

subject to (4.4b) and (4.12) by choosing x_0 and stochastic processes for $\{c_{1t}, c_{2t}, \tau_t\}$ and taking b_0/m_0 as an initial condition.

Note the following obvious aspects of the solution to this problem.

First, from (4.12), if $(b_0 + m_0) > 0$ then it is optimal to set $x_0 = \infty$. If $(b_0 + m_0) < 0$ so that the government is a creditor then it is optimal to set x_0 to a sufficiently low value so that the shadow value on the constraint (4.12) is zero. The interest earnings on the government's initial credit are used to deflate the money supply in order to support the Friedman optimum of a zero nominal interest rate without any taxes. If $(b_0 + m_0) = 0$ then x_0 is indeterminate.³

Second, from equations (4.2) it is easy to see that $1 + R_t = (\lambda_t + \mu_t) / \mu_t = u_{1t} / u_{2t}$. Hence, the solutions for c_{1t} and c_{2t} determine the solution for the nominal interest rate.

Third, the solution for the money growth rate process $\{x_t\}$ is indeterminate. Any stochastic process for $\{x_t\}$ which satisfies (4.6) given the solutions for $\{c_{1t}, c_{2t}\}$ for the policy maker's problem will support the optimum. In general one can write the solution for the money growth rate process as follows

³In the quantitative analysis of this model, described in section 5, this case is assumed to prevail.

$$(4.13) \quad x_{t+1} = \beta u_{1,t+1} c_{1,t+1} / [u_{2t} c_{1t} + \varepsilon_{t+1}], \quad t \geq 0,$$

where $\{\varepsilon_{t+1}\}$ is any stochastic process which satisfies $E_t \varepsilon_{t+1} = 0$.

Fourth, note that the solutions for (c_{1t}, c_{2t}, τ_t) and, hence, R_t , depend only on the date t realizations (θ_t, y_t, g_t) . This can be seen by writing down the FONCs for the policy maker's problem.

Fifth, the solution is clearly time inconsistent. This is manifested in two ways. First, even if $(b_0 + m_0)$ is zero, it will not, in general, be the case that $(b_t + m_t)$ will be zero for $t > 0$. Therefore, the solution for x_t calculated at zero will not be optimal when date t arrives and the policy maker is allowed to reoptimize. Second, the value of the multiplier on the government budget constraint (4.12) will depend on the date zero realizations (θ_0, y_0, g_0) . The value of the multiplier will be different at some future date t if the planner is allowed to reoptimize. Therefore, the solution for c_{1t} and c_{2t} calculated at date zero will not be optimal when date t arrives if the planner is allowed to reoptimize.

5. Cyclical Features of Optimal Policies

5.1 The liquidity effect model

We first describe the model specification.

The model period is taken to be one quarter.

The utility discount factor β is chosen to generate a quarterly real interest rate of 0.5 percent. That is, $\beta = 1/(1+0.005)$. The utility function is chosen to be of the following constant elasticity of substitution (CES) type.

$$(5.1) \quad u(c_1, c_2, \theta) = [\theta c_1^\rho + (1-\theta)c_2^\rho]^{1/\rho}, \quad \rho = 0.85.$$

The random variables θ_t and y_t are each assumed to follow independent first order autoregressions with a common serial correlation coefficient of 0.85 and mean values of 0.46 and 1.0, respectively. The value of ρ and the mean value of θ are chosen to generate a quarterly velocity of money equal to 5 and an interest rate semielasticity of money demand equal to 5, where we use the monetary base as our empirical measure of money.

The standard deviation of the innovations in y is specified as 0.009. This value is chosen to be consistent with a value of 0.017 for the standard deviation of output relative to trend, as reported by Kydland and Prescott [1990] for quarterly U.S. data during 1954-89. The standard deviation of the innovation in θ is chosen to be 0.0007.⁴

The cost function $\phi(z/m)$ is specified as follows.

$$(5.2) \quad \phi(z/m) = \gamma(z/m-1)^2/2, \quad \gamma = 5.$$

The above parameter values lead to a steady state optimal money growth rate (and inflation rate) of about -0.25 percent (annual) and a steady state optimal nominal interest rate of about 1.77 percent (annual). Using these numbers we can calculate the steady state value of the cost ϕ in units of consumption goods by dividing the value of ϕ from (5.2) by the marginal utility of credit goods. This yields a cost of about 1.8×10^{-6} . It's clear that extremely small costs are sufficient to generate a significantly

⁴This value is chosen, rather arbitrarily, in order to produce a 50 basis point response in the nominal interest rate due to a one standard deviation innovation in θ when the money growth rate follows an exogenous autoregressive process - an AR(1) with positive serial correlation.

positive steady state optimal nominal interest rate.

The optimal monetary policy for this model is given by the following decision rule for the money growth rate.

$$(5.3) \quad x_t = \text{constant} - 1.41(c_{1,t-1}/y_{t-1}) - 0.001y_t - 0.019\theta_t - 0.15y_{t-1} + 4.12\theta_{t-1}.$$

Before we describe the features of the optimal cyclical monetary policy we describe how the model behaves under a simple exogenous money growth rate rule. We assume that the money growth rate follows a first order autoregressive process with positive serial correlation. The mean, serial correlation coefficient and the innovation standard deviation of the money growth rate process are chosen to be -0.1 percent/quarter, 0.85 and 0.0007, respectively.⁵ Figure 1 depicts the responses to a one standard deviation innovation in the money growth rate of inverse velocity (c_1/y), the nominal interest rate (R), the expected inflation rate (π^e), and the ex post inflation rate (π).⁶ The liquidity effect on R is quite apparent; moreover, unlike other liquidity effect models, the effect here is persistent. The presence of the Baumol-Tobin style transaction costs in financial markets leads to a persistent liquidity effect when the money growth rate process is persistent.

Figures 2 and 3 describe the responses to one standard deviation

⁵The empirical value for the innovation standard deviation of the money growth rate process is about 0.0046. We chose the lower value 0.0007 in order to keep the interest rate response nonnegative.

⁶All of the impulse responses described here and subsequently are in levels and not in terms of deviations from steady state values. The ex post inflation rate at date t (π_t) is defined as: $\pi_t = (p_t/p_{t-1}) - 1$, and the expected inflation rate at date t (π_t^e) is defined as: $\pi_t^e = E_t \pi_{t+1}$.

innovations in θ (money demand shock) and y (productivity/output shock).

Now we describe the features of the optimal cyclical monetary policy.

Figure 4 depicts the responses to a one standard deviation innovation in productivity/output and Figure 5 depicts the responses to a one standard deviation innovation in the money demand shock (θ), i.e., the shock to the demand for cash goods relative to credit goods. As can be seen, in both cases, optimal policy calls for an initial expansion in the money growth rate. Presumably, the reason is that in both cases the household desires to purchase more cash goods and the expansion in the money growth rate accommodates this increased demand for cash goods. A comparison of Figures 2 and 3 with Figures 4 and 5 shows that the optimal monetary policy calls for a relatively persistent dynamic response of the money growth rate to either shock. The money growth rate first rises above its steady state level and then falls below it before gradually returning to its steady state level. A higher value of γ , implying higher adjustment costs, has the expected effects. It raises the steady state value of π towards zero and diminishes the response of inflation to either shock.

5.2 *The cost of price change model*

We first describe the model specification.

The model period is taken to be one quarter.

The utility discount factor β is chosen to generate a quarterly real interest rate of 0.5 percent. That is, $\beta = 1/(1+0.005)$. The utility function is chosen to be of the following log-linear type.

$$(5.4) \quad u(c, 1-n) = \alpha \log(c) + (1-\alpha) \log(1-n), \quad \alpha = 0.41.$$

The parameter ρ (see 3.1b) which determines the elasticity of substitution between any two goods among the continuum of goods consumed is chosen to be 0.83. In the absence of costs of changing prices this parameter determines the markup of price over marginal cost; specifically the markup equals $(1-\rho)/\rho$ ⁷. Our parametrization implies a markup of 20 percent.

The random variable g_t (government consumption) is assumed to follow a first order autoregression with a serial correlation coefficient of 0.85 and a mean of 0.1. The standard deviation of the innovation in g is specified as 0.0002. This value is chosen to be consistent with the standard deviation of government consumption (relative to trend) of 0.02, as reported by Kydland and Prescott [1990] for quarterly U.S. data during 1954-89. The random variable θ_t (labor productivity) is also assumed to follow a first order autoregression with a serial correlation coefficient of 0.85 and mean of 1.0. The standard deviation of the innovation in θ is specified as 0.0046. This value is chosen to be consistent with the standard deviation of labor productivity (relative to trend) of 0.0088 as reported by Kydland and Prescott [1990] for quarterly U.S. data during 1954-89.

The cost function $\phi(z)$ is specified as follows.

$$(5.5) \quad \phi(z) = \gamma(z-1)^2/2, \quad \gamma = 17.$$

The above specifications yield an optimal steady state nominal interest rate of 0.96 percent (annual) and an optimal steady state money growth rate (and inflation rate) of -1.03 percent (annual).

⁷This can be seen from the FONC (3.11) for the monopolist's optimization problem (3.10). In the absence of price adjustment costs (3.11) simplifies to: $p_{it}/(w_t/\theta_t) = 1/\rho$, where w_t/θ_t is the marginal cost.

The steady state share of working time in total time (n) equals $1/3$. Our parameterization also implies a steady state value of 0.016 percent of output for the cost of changing prices in units of consumption goods which works out to about \$1 billion for the U.S. It is clear that extremely small costs of changing prices can lead to somewhat positive nominal interest rates as being optimal.

The optimal monetary policy for this model is given by the following decision rule for the money growth rate.

$$(5.6) \quad x_t = \text{constant} - 3.04z_{t-1} + 2.58z_{t-2}x_{t-1}/z_{t-1} + 0.77\theta_t + 0.41\theta_{t-1} + 1.55g_t + 0.30g_{t-1},$$

where $z_t = c_t + g_t$.

Before we describe the features of the optimal cyclical monetary policy we describe how the model behaves under a simple exogenous money growth rate rule. We assume that the money growth rate follows a first order autoregressive process with positive serial correlation. The mean, serial correlation coefficient and the innovation standard deviation of the money growth rate process are chosen to be -0.25 percent/quarter, 0.85 and 0.0046, respectively. Figure 6 depicts the responses to a one standard deviation innovation in the money growth rate of consumption (c), labor input (n), the nominal interest rate (R), the expected inflation rate (π^e), and the ex post inflation rate (π). It can be seen that due to the cost of nominal price adjustment a monetary innovation is expansionary; consumption and labor input initially increase, and the response of prices is attenuated. Interestingly, the response of consumption, labor input, and, hence, output, is hump shaped. This feature of the response does depend on the serial

correlation in money growth with a high enough serial correlation giving rise to such a hump shaped response and a zero or low serial correlation giving rise to a response that peaks in the impact period and then decays monotonically to the steady state. This feature has been previously noted by Rotemberg [1994, pp.10-11] in his sticky price model.

Figures 7 and 8 depict the responses to one standard deviation innovations in productivity and government consumption, respectively, under the same exogenous money growth rate rule. In Figure 7 note that the response of labor to the innovation in productivity is negative. This is due to the presence of government consumption in the model. If government consumption were zero then due to the assumed log linear utility function the opposing wealth and substitution effects on labor would have implied a zero response of labor to the productivity innovation. The presence of government consumption enhances the wealth effect and leads to a negative response of labor. However, the output response is still positive as can be seen from the response of consumption. In Figure 8, it can be seen that the responses to a government consumption innovation are as expected. Due to the negative wealth effect on leisure and consumption, labor input and, hence, output increase, whereas consumption is crowded out.

Now we describe the features of the optimal cyclical monetary policy.

Figure 9 depicts the responses to a one standard deviation innovation in productivity. As can be seen the optimal policy calls for an expansion in the money growth rate. Presumably, the reason is that without such an expansion nominal prices would have to fall (since output rises - see the response of consumption) thus imposing costs of lowering prices. The expansion in money growth leads to an attenuated response of prices and saves on price adjustment costs. Interestingly, this response of the money

growth rate exacerbates consumption variability. It is conceivable that the optimal response of the money growth rate to an innovation in productivity may become countercyclical if consumers are sufficiently risk averse, i.e., dislike consumption variability sufficiently.

Figure 10 depicts the responses to a one standard deviation innovation in government consumption. Again, as can be seen, the optimal policy calls for an expansion in the money growth rate. Presumably, the reason is that the innovation in government consumption is expansionary; it leads to an increase in labor input and output and, hence, to a potential decrease in prices. The monetary expansion helps to save on the adjustment costs of lowering prices. Since the innovation in government consumption crowds out consumption the monetary expansion also serves to smooth the impact on consumption.

The nature of the response of the money growth rate in Figures 9 and 10 bears some comment. In both cases the response exacerbates output variability. In one case (productivity innovation) it also exacerbates consumption variability whereas in the other case (government consumption innovation) it serves to smooth consumption. These findings suggest that optimal monetary policy need not necessarily be countercyclical. As noted earlier it is conceivable that the optimal response of the money growth rate to an innovation in productivity may become countercyclical if consumers are sufficiently risk averse; however, it is likely that the optimal response of the money growth rate to an innovation in government consumption will continue to be procyclical. This suggests that the nature of the optimal cyclical response of the money growth rate may also depend on the nature of the shock that affects output. These observations appear counter to the conventional wisdom regarding countercyclical optimal monetary policy.

5.3 The optimal seignorage model

We first describe the model specification.

The model period is taken to be one quarter.

The utility discount factor β is chosen to generate a quarterly real interest rate of 0.5 percent. That is, $\beta = 1/(1+0.005)$. The utility function is chosen to be of the following CES type.

$$(5.7) \quad u(c_1, c_2, \theta) = [\theta c_1^\rho + (1-\theta)c_2^\rho]^{1/\rho}, \quad \rho = 0.85.$$

The random variables θ_t , y_t , g_t are each assumed to follow independent first order autoregressions with a common serial correlation coefficient of 0.85 and mean values of 0.46, 1.0 and 0.2, respectively. The specifications of ρ and the mean value of θ are chosen to generate a quarterly velocity of money equal to 5 and an interest rate semielasticity of money demand equal to 5, where we use the monetary base as our empirical measure of money.

The standard deviations of the innovations in y and g are specified as 0.009 and 0.002, respectively. These values are chosen to be consistent with the standard deviations of output and government consumption (relative to their respective trend values) of 0.017 and 0.02, respectively, as reported by Kydland and Prescott [1990] for quarterly U.S. data during 1954-89. The standard deviation of the innovation in θ is specified as 0.0019.

The cost function $\phi(\tau)$ is specified as follows.

$$(5.8) \quad \phi(\tau) = \gamma\tau^2/2, \quad \gamma = 0.37.$$

The above specifications yield an optimal steady state nominal interest rate of 6 percent (annual). This implies an optimal steady state inflation rate and money growth rate of 4 percent (annual) and a seignorage of 0.2 percent of output. Further, the optimal steady state tax τ equals 0.205 and the steady state cost ϕ equals 0.8 percent of output or about \$50 billion dollars in terms of current dollars. In order to judge the empirical plausibility of this cost number note that the annual budget of the Internal Revenue Service has averaged about \$8 billion/year during 1994-97. To this number must be added the value of resources (including time) spent by households and businesses in making and keeping records and preparing and filing tax returns. According to a study by Slemrod and Sorum [1984] for the U.S. these costs may be around 5 percent of total tax revenue or about 1 percent of total output, thus amounting to about \$60 billion. Hence, the total cost associated with the tax system may be about \$68 billion. In light of this fact the model generated number of \$50 billion seems very reasonable. Indeed, the optimal steady state inflation rate may be somewhat higher than 4 percent (annual). It is interesting that empirically reasonable costs of tax collection can imply that observed levels of the inflation rate and the seignorage/GNP ratio in the U.S. may be nearly optimal.

We now describe the responses of the nominal interest rate, the money growth rate and taxes to innovations in θ , y and g . The following table summarizes our findings.

Table 1

	effect on R_t (annual rate)	effect on x_{t+1} (annual rate)	effect on τ_t
one standard deviation innovation in θ_t	0.01	-0.016	0.0004
one standard deviation innovation in y_t	-0.005	-0.0124	-0.00027
one standard deviation innovation in g_t	0.0009	0.0024	0.00004

Note that, as explained in section 2, while the effect on the contemporaneous nominal interest rate is determined the effect on the contemporaneous money growth rate (x_t) is indeterminate. However, the effect on the next period money growth rate (x_{t+1}) is determinate. Further, the effects of innovations in period t die out over time in absolute value at the geometric rate ρ^t where ρ is the common serial correlation coefficient of θ , y and g . This is because, as explained in section 2, the solution has the feature that the solution variables depend only on the current values of the shocks.

The innovations in the money demand shock as well as output increase current real balances (equivalently, consumption of cash goods) and, hence, the base for current and future seignorage tax revenues. Table 1 suggests that this is compensated by a decrease in the tax rate on real balances as evidenced by the fall in the future money growth rate. Table 1 also suggests that an innovation in government consumption is financed partly by a rise in seignorage tax revenues, as evidenced by the rise in the future money growth rate, and partly by a rise in lump sum taxes.

As can be seen in Table 1, a one standard deviation innovation to money

demand (θ) in period t raises the nominal interest rate by 1 percentage point (annual) and lowers the money growth rate next period by 1.6 percentage points (annual). A one standard deviation innovation to output in period t lowers the nominal interest rate by 50 basis points (annual) and lowers the money growth rate by 1.24 percentage points (annual) contemporaneously. A one standard deviation innovation in government consumption raises the nominal interest rate by about 9 basis points and raises the money growth rate by 0.24 percentage points contemporaneously.

6. Summary and Future Work

In this paper we considered three simple models of optimal cyclical monetary policy. The first was a pure liquidity effect model, the second was a cost of changing nominal prices model, and the third was an optimal seignorage model. In each case we solved for the optimal cyclical monetary policy and described how money growth and the nominal interest rate respond to various shocks under the optimal policy. The shocks we considered were money demand shocks, productivity shocks, and government consumption shocks.

All of the models have the feature that the Friedman rule of setting the nominal interest rate to zero is not optimal. Optimal policies are always time inconsistent even though lump sum taxation is allowed. The qualitative as well as the quantitative properties of optimal policies seem intuitively reasonable. At least in some instances we found that optimal policy dictates responses of money growth and interest rates which run counter to conventional wisdom. For example, we found that the optimal cyclical response of the money growth rate need not be countercyclical and may depend on the nature of the shock that affects output.

In future work we plan to pursue several different avenues. One avenue

is to combine the different models into one and include capital in order to study the properties of optimal cyclical policies when several empirically relevant features are simultaneously taken into account. We also plan to study multi-country versions of such models and incorporate exchange rate policies as well. Hopefully, this line of research will lead to reasonable prescriptions for the conduct of cyclical monetary policy. It may also lead to useful insights into business fluctuations. As the quotation from Sims [1996, p.118] noted in the introduction suggests, most of the observed variation in interest rates and monetary aggregates is endogenous. In view of this it may be interesting to study the business cycle properties of the models as well.

Another avenue to explore is the nature of time consistent policies in these models in contrast with optimal policies with commitment which, as we noted earlier, are time inconsistent.

Yet another avenue to explore is the problem of signal extraction. Policy makers (and possibly households as well) may not observe the exogenous shocks directly and may either not observe enough endogenous variables or observe endogenous variables only with some error so that they cannot recover the shocks perfectly. In this case, policy makers have to set policy on the basis of (a possibly limited number of) observed endogenous variables taking the signal extraction problem into account. Another way in which the problem of signal extraction become relevant is when there are both temporary and persistent components to exogenous shocks which are not observed separately. Solving the signal extraction problem requires knowledge of the relative contributions of different shocks to the variations in endogenous variables. In this way, the study of optimal policies makes contact with the real business cycle literature which

attempts to determine (among other things) the contribution of different shocks to business cycles.

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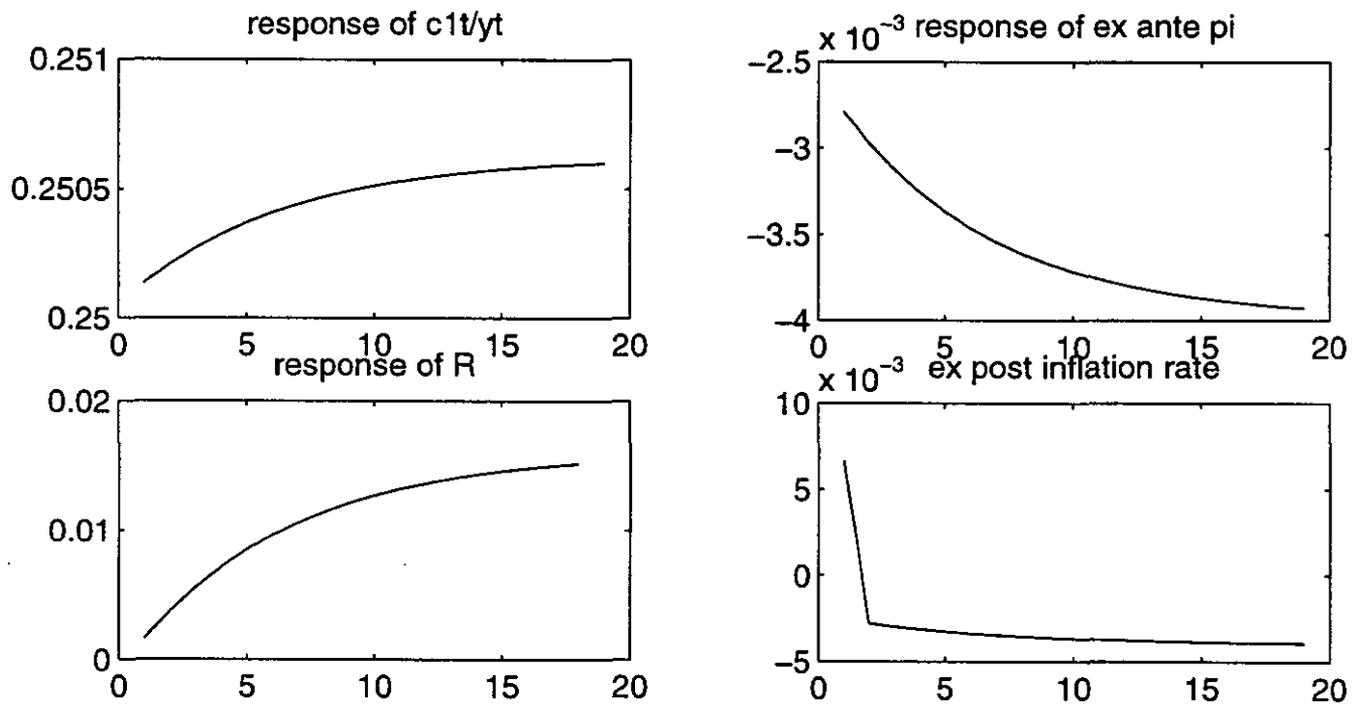


Figure 1

Responses to a Money Growth Innovation
 (Exogenous Money Growth Process—AR(1) Positively Correlated)

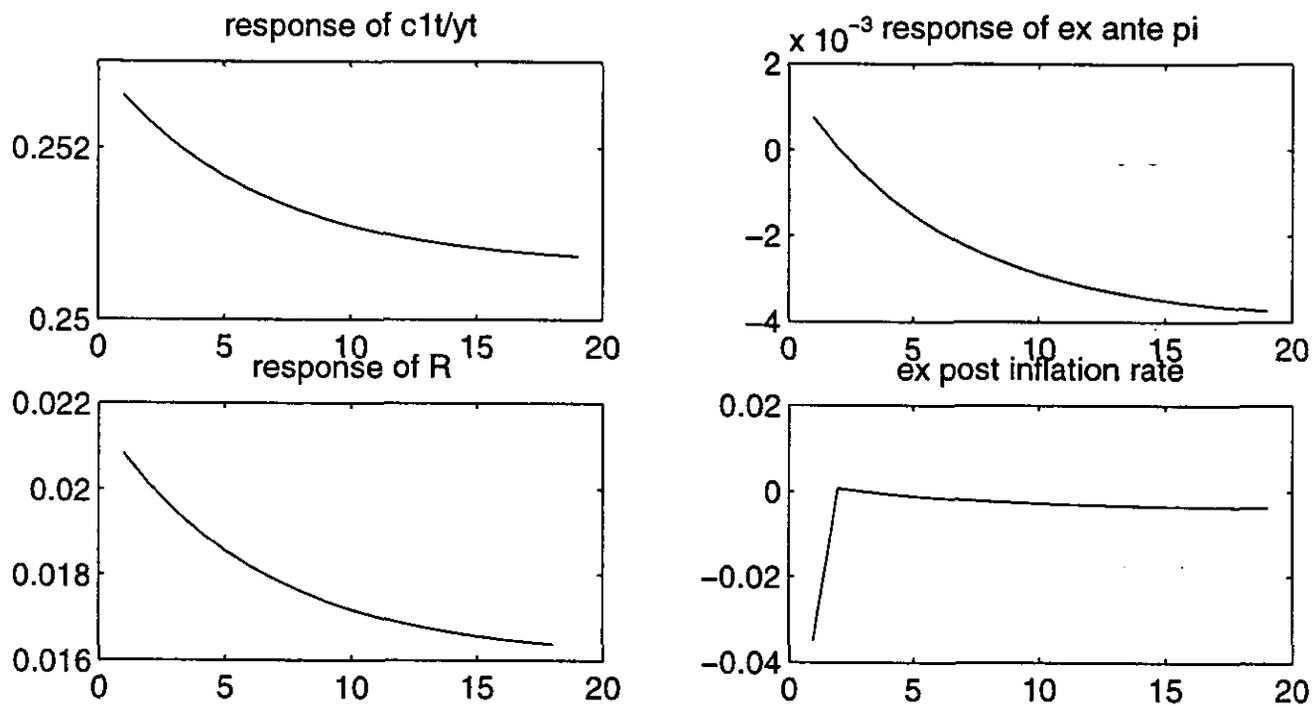


Figure 2

Responses to a Money Demand Innovation
 (Money Demand Process—AR(1) Positively Autocorrelated)

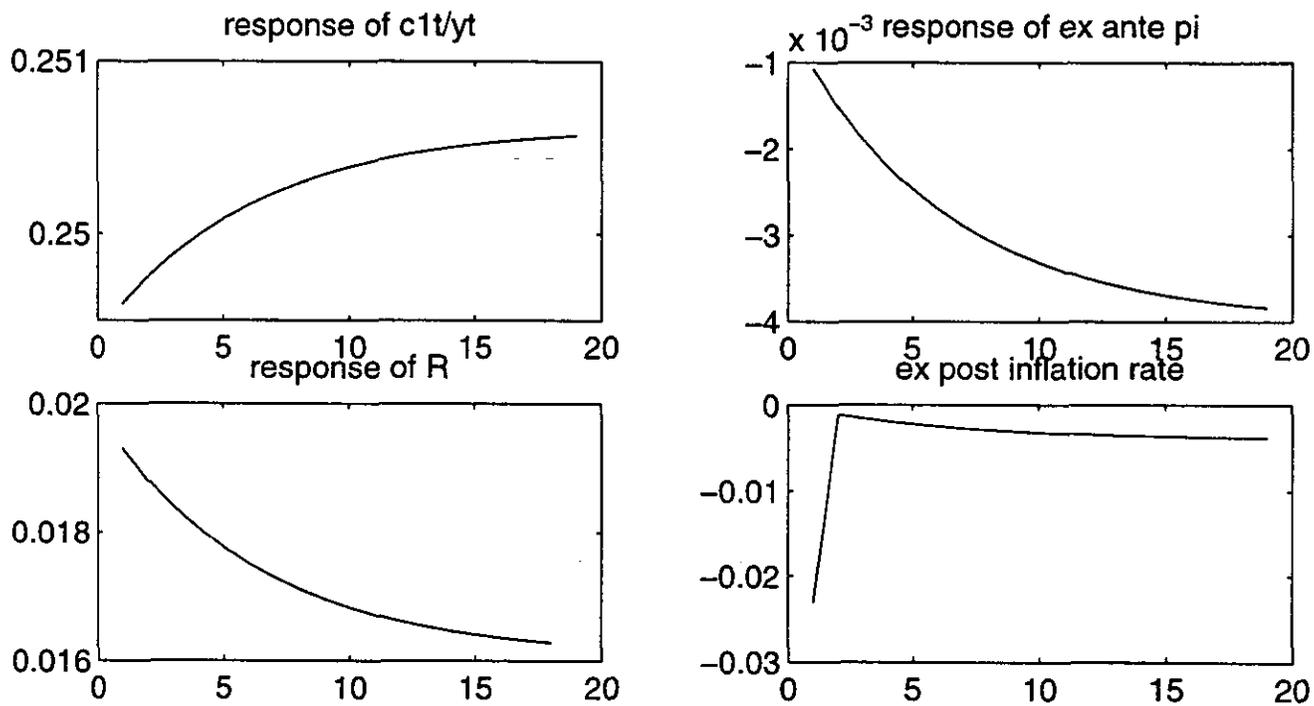


Figure 3

Responses to a Productivity Innovation
 (Productivity Process—AR(1) Positively Correlated)

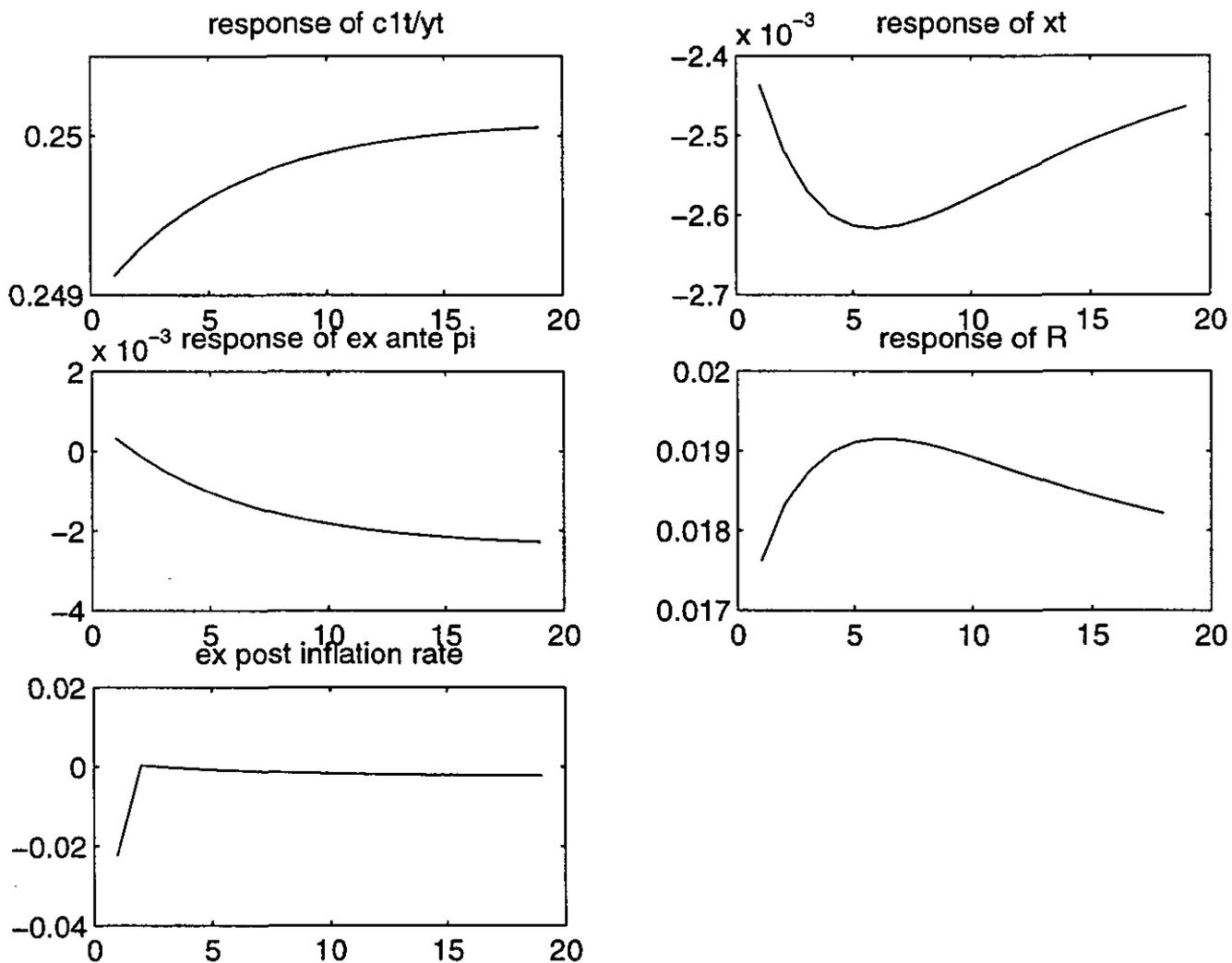


Figure 4

Responses to a Productivity Innovation
(Under Optimal Policy)

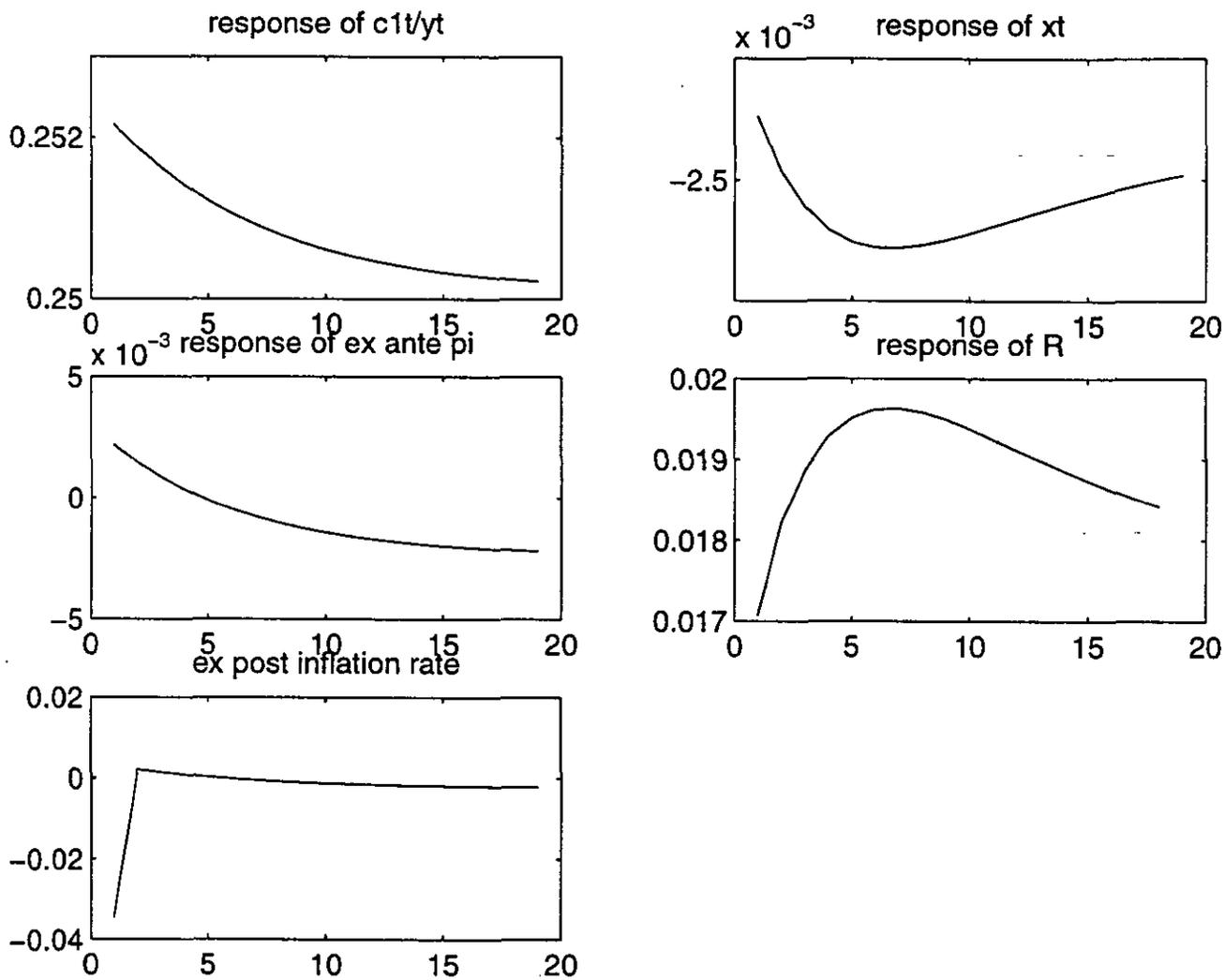


Figure 5

Responses to a Money Demand Innovation
(Under Optimal Monetary Policy)

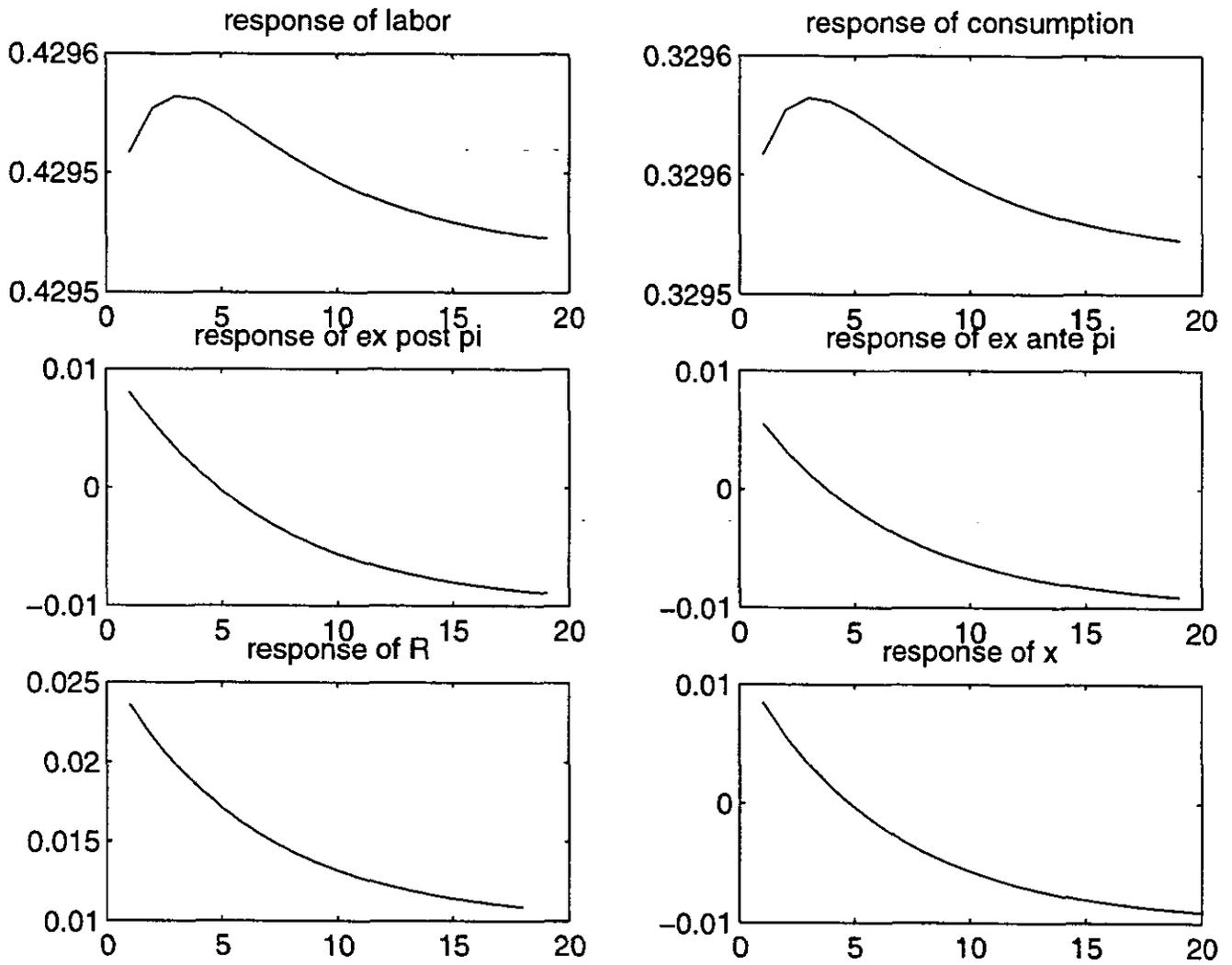


Figure 6

Responses to a Money Growth Innovation
 (Exogenous Money Growth Process—AR(1) Positively Correlated)

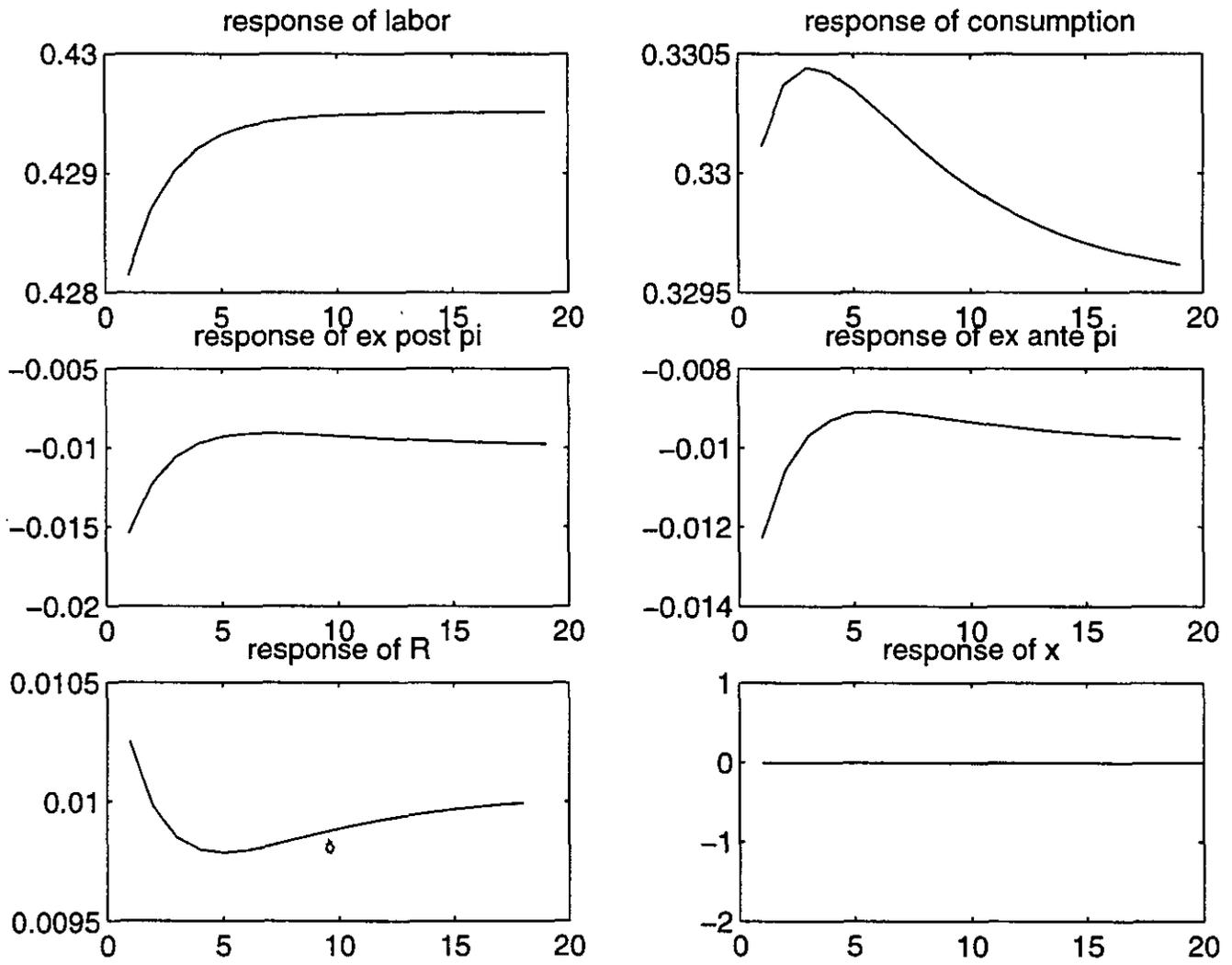


Figure 7

Responses to a Productivity Innovation
 (Productivity Process—AR(1) Positively Correlated)

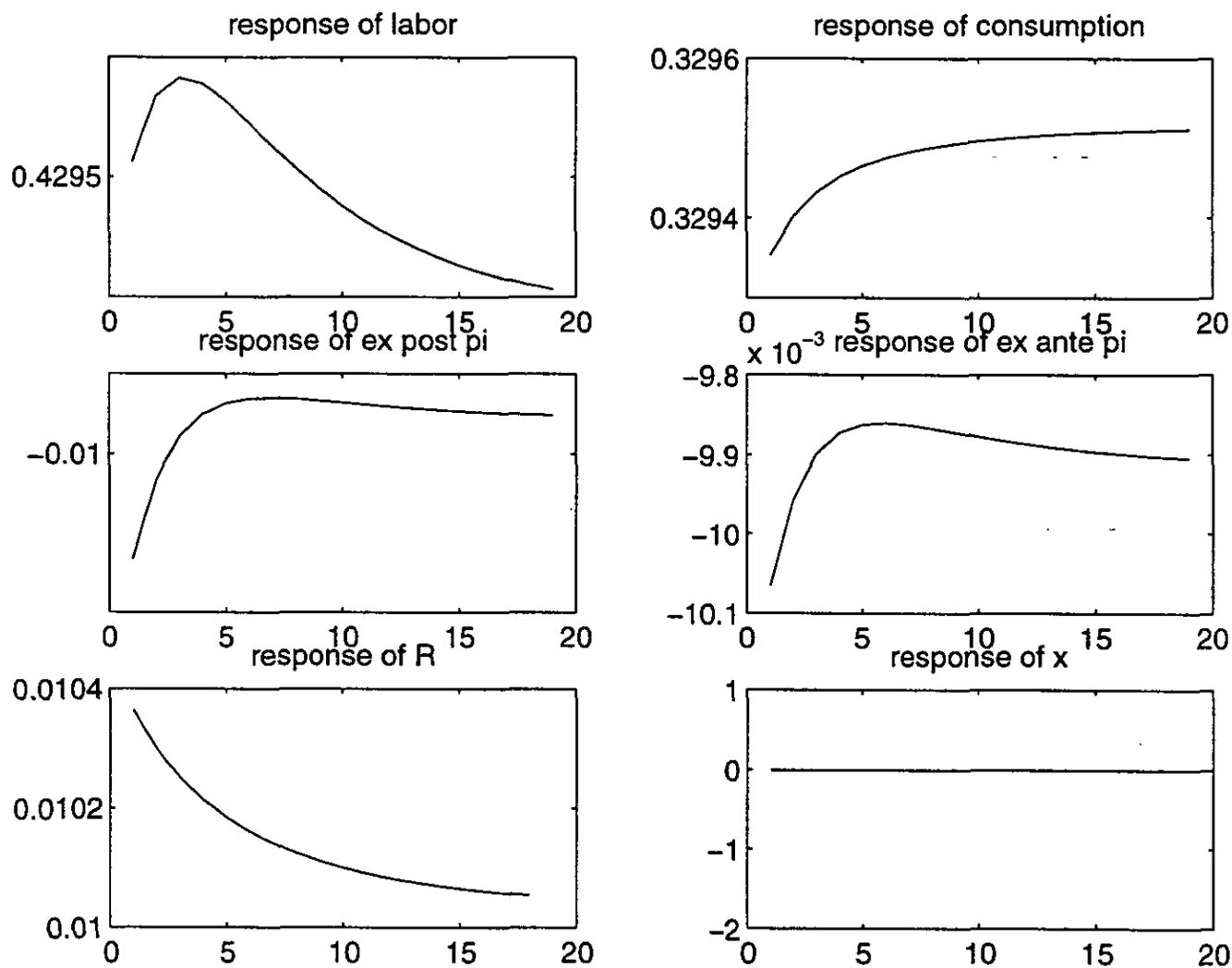


Figure 8

Responses to a Government Purchases Innovation
 (Government Purchases Process—AR(1) Positively Correlated)

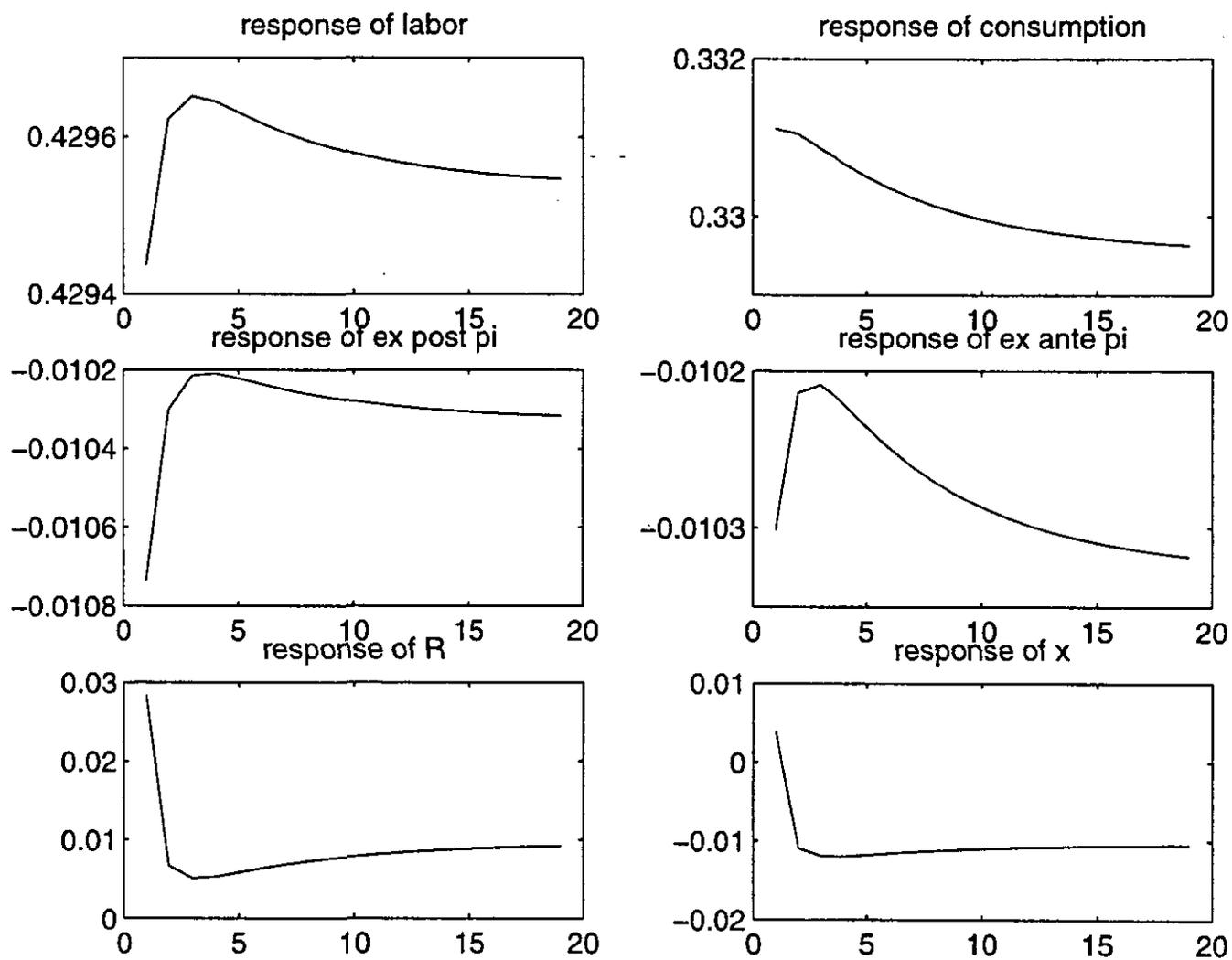


Figure 9

Responses to a Productivity Innovation
 (Under Optimal Monetary Policy)

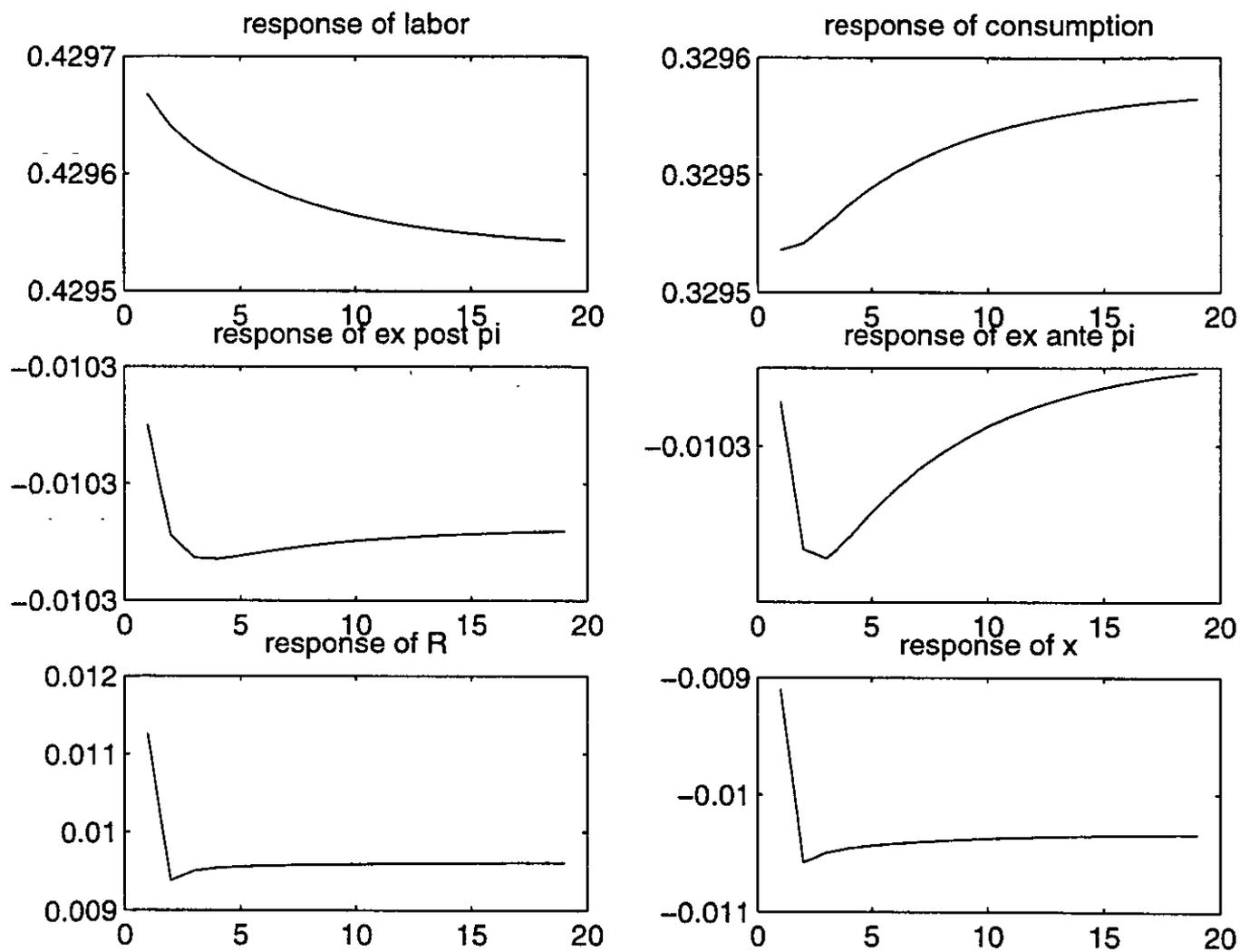


Figure 10

Responses to a Government Purchases Innovation
 (Under Optimal Monetary Policy)