

The Competitive Provision of Fiat Money

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Herein, it is demonstrated that the competitive provision of fiat money is generically either inefficient or infeasible.

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## The Competitive Provision of Fiat Money

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In discussions of fiat money, it is often assumed that fiat money is provided by the "government," that fiat money is imposed from outside the model being used. One can well suppose, however, that there is "inside" fiat money, fiat money being provided by a banking system or other issuers of liabilities. This paper considers the competitive provision of inside fiat money. We conclude that the competitive provision of fiat money is generically inefficient or infeasible, even in the best of circumstances. There is, then, good reason why the provision of fiat money should not be left to the private market. The assumption that the provision of fiat money is a government task is a good one.

By "best of circumstances" we mean that all technologies exhibit constant returns to scale. There is costless full information regarding the privately produced money, so there are no returns in information costs from reducing the number of producers. There are no costs to inhibiting counterfeiting, so there are no returns to scale in enforcement. There are no direct costs of producing fiat money, so no returns to scale in this technology. Finally, the technology of servicing fiat money exhibits constant returns to scale.

Our model of fiat money is a modified version of Samuelson's (7) pure consumption-loans model (p.c.l.m.). This model was chosen for several reasons. In the first place, it is a venerable and well-known coherent model of fiat money. Secondly, the model is tractable and easily allows us to address the issues raised by the competitive provision of fiat money. For a more detailed defense of the p.c.l.m. as a model of fiat money, see Bryant (1) and Wallace (8).

Most of our assumptions are standard ones in the p.c.l.m. Time is discrete and divided into periods with no last period. Each period a two-period-lived generation is born, indexed by its date of birth. Each generation has the

same number of identical members,  $N$ , with preferences and endowments common to all individuals of all generations. Each individual is born with one unit of the single transferable but nonstorable consumption good, and that is her only endowment for life. There are no production technologies. Lastly, the common utility function has as domain the individual's first- and second-period consumption of the consumption good. The utility function has the following properties: it is (a) strictly concave, (b) two-smooth, (c) strictly increasing in both arguments, (d) with infinite marginal utility for consumption in a period with zero consumption, (e) with consumption in both periods strictly noninferior (normal goods). This very simple specification of the p.c.l.m. is chosen for convenience only.

While most of the assumptions are standard ones for the p.c.l.m., there are some important differences. These differences stem from the fact that the p.c.l.m. assumes that fiat money has always existed, while we examine the provision of it. To examine the provision of fiat money, we assume a first period, rather than an infinite past as in the p.c.l.m. We also assume a one-time, first-period cost to the servicing of fiat money, instead of costless servicing of fiat money as in the p.c.l.m. However, as it is a first-period cost, the model behaves like the standard p.c.l.m. after the first period.

The new assumption of the cost of providing fiat money requires more explanation. We assume that this cost is incurred in the first period, while money changes hands in the second period. This assumption turns out to be innocuous, as the cost arising in the second period has little effect on the implications of the model, as is made precise later. We assume no ongoing costs to the maintenance of fiat money, only the one-time setup cost. However, ongoing maintenance costs can easily be added without substantially altering the conclusions, which also is made precise later.

The one-time cost turns out to be crucial for the competitive provision of fiat money. It is worth noting that this cost implies that there is a real nondepreciating "durable good" in the economy, the fact of having incurred the setup cost. The fiat money is not just a fictional durable asset. The necessity of a one-time setup cost is not unique to the competitive provision of fiat money, but was also found essential for the collusive provision of fiat money in Bryant (3). This suggests a more general hypothesis that the existence of fiat money requires that it represents a real nondepreciating asset.

The very nature of fiat money dictates the peculiar form that the one-time setup cost should take. Unlike other goods, fiat money does not come in any natural units. Therefore, it seems unreasonable to model costs as a function of a "nominal" quantity of money. Rather, we must model cost as a function of real money balances, of the "purchasing power" of the fiat money produced. Our cost can be viewed as a one-time setup cost to service a given real volume of transactions per period for all periods.

The cost of providing fiat money being a function of real balances is an important difference between money and other goods. In particular, price provides no mechanism whereby supply by the first generation is equated to demand for money by the second generation. The price of the good money influences the cost of producing it. To allow the first-generation suppliers to compete for "consumers" we introduce the possibility of rebates. Rebates loosen the link between real costs of supplying money and the real amount of money purchased. We can also assume that money suppliers compete by incurring excess setup costs, but direct payments are more efficient, and they are marginally neater to model.

The hypothesis that the competitive provision of valued fiat money is either inefficient or infeasible is not new. For a discussion of this proposition see Klein (6). Here we clarify and demonstrate the correctness of this hypothesis in a coherent model of fiat money, the p.c.l.m.

Proposition I

The substance of this paper flows from a single proposition concerning monetary equilibria. First we present and prove the proposition, and then discuss its implications. Let  $C > 0$  be the goods cost per unit of goods value of fiat money purchased.

Proposition I:

(a) If  $C > 1$ , then

- (1) Without rebates there is no monetary equilibrium.
- (2) With nonnegative rebates there is no monetary equilibrium.
- (3) With a full system of rebates there is a stationary monetary equilibrium with negative rebates and "superefficient" provision of fiat money.

(b) If  $C = 1$ , there is a stationary monetary equilibrium with efficiently supplied money and rebates equal to zero if they are allowed.

(c) If  $C < 1$ ,

- (1) Without rebates there is no monetary equilibrium.
- (2) With rebates there is a stationary monetary equilibrium with positive rebates and inefficient provision of fiat money.

Proof: We prove Proposition I first considering the model with rebates, then the model without rebates. In the proof "superefficient" is defined.

In the proof we use the following notation. The individual's utility function is  $U(e_1, e_2)$  where  $e_1$  and  $e_2$  are first- and second-period consumption. The first-period decision variable of a generation one individual is  $K_1$ , the amount of goods he spends in a setup costs for fiat money.  $M_2$  is the nominal amount of money he produces, somehow denominated.  $R_2$  are the rebates he pays as a percentage of money produced.  $P_2 M_2$  is the consumption goods value of the money he produces. The corresponding quantities for generation two are  $K_2$ ,  $M_2 - M_3$ ,

$R_3$ , and  $P_3(M_2 - M_3)$ , and so on for future generations. Each individual's money has its own price, but we do not lose any generality in assuming that money is denominated so that the prices are equal. As individuals are the same, we have dropped subscripts referring to the individual.

The problem of the representative individual of the first generation in a monetary equilibrium can be written as:

$$\begin{aligned} & \max_{K_1, M_2} U[1 - K_1, P_2(1 - R_2)M_2] \\ & \text{s.t. } P_2 M_2 \leq \frac{K_1}{C}. \end{aligned}$$

In equilibrium,  $R$  is determined to equate supply and demand of real balances, or  $NK_1 = NCP_2M_2$ .

The implied first-order necessary condition is:

$$(1) \quad U_1[1 - K_1, P_2(1 - R_2)M_2] = \frac{1 - R_2}{C} U_2[1 - K_1, P_2(1 - R_2)M_2]$$

or substituting in the equilibrium condition:

$$(2) \quad CU_1[1 - CP_2M_2, P_2(1 - R_2)M_2] = (1 - R_2)U_2[1 - C_2P_2M_2, P_2(1 - R_2)M_2].$$

Now let us turn to the problem of the second generation. It can be written as:

$$\begin{aligned} & \max_{M_2, K_2, M_3} U[1 - K_2 - P_2(1 - R_2)M_2, P_3(1 - R_3)M_3] \\ & \text{s.t. } P_3M_3 - P_2M_2 \leq \frac{K_2}{C}. \end{aligned}$$

The equilibrium condition is  $NCP_3M_3 = NK_2 + NK_1$ . Assuming we are in a monetary equilibrium, that  $M_2, M_3 > 0$ , one first-order necessary condition is

$$(3) \quad U_1[1 - K_2 - P_2(1 - R_2)M_2, P_3(1 - R_3)M_3] = U_2[1 - K_2 - P_2(1 - R_2)M_2, P_3(1 - R_3)M_3].$$

The first-order condition on  $K_2$  then becomes

$$(4) \quad -U_1[1-K_2-P_2(1-R_2)M_2, P_3(1-R_3)M_3] \\ + \frac{1-R_2}{C} U_2[1-K_2-P_2(1-R_2)M_2, P_3(1-R_3)M_3] \leq 0, = \text{if } K_2 > 0.$$

Now, let us consider the possibility of a stationary monetary equilibrium-- $K_j = 0$ ,  $R_j = R_2$ ,  $M_j = M_2$ ,  $P_j = P_2$  for  $j = 2, \dots$ --at  $1-R_2 = C$ . By observation, both (2) and (3) are solved simultaneously as they become identical equations. Also, by observation (4) holds at equality with  $K_2 = 0$  as it reduces to (2) or (3). As the problem of future generations just replicates the problem of generation two, we conclude that there is a stationary monetary equilibrium with  $1 - R = C$  (the zero profit condition).

The effective amount of fiat money produced is  $N(1-R_2)P_2M_2$ . This is produced at cost  $NCP_2M_2$ . If  $NCP_2M_2/N(1-R_2)P_2M_2$  is

$$\left. \begin{array}{l} \text{greater than} \\ \text{equal to} \\ \text{less than} \end{array} \right\} C, \text{ we say the provision of money is } \left. \begin{array}{l} \text{inefficient} \\ \text{efficient} \\ \text{superefficient} \end{array} \right\}.$$

Proposition I parts (a)-(3), (b), and (1)-(2) are immediate.

Now, let us suppose that rebates are not possible. In any monetary equilibrium (3) and (4) must hold with  $R_2 = R_3 = 0$ . But this can occur only if  $C \geq 1$ . Moreover, from (4), if  $C > 1$ , then  $K_2 = 0$ , implying  $P_3M_3 = P_2M_2$ . (1) and (3) then become

$$(5) \quad U_1(1-K_1, P_2M_2) = \frac{1}{C} U_2[1-K_1, P_2M_2]$$

$$(6) \quad U_1(1-P_2M_2, P_2M_2) = U_2[1-P_2M_2, P_2M_2].$$

For this to be feasible  $K_1 \geq CP_2M_2 > P_2M_2$ . From strict noninferiority we know that

$$\frac{\partial}{\partial e_1} \left[ \frac{U_1(e_1, e_2)}{U_2(e_1, e_2)} \right] < 0.$$

But this implies that  $\frac{1}{C} > 1$ , contradiction. Therefore, a monetary equilibrium is possible only if  $C = 1$ . The remaining parts of Proposition I--(a)-(1), (a)-(2), (c)-(1)--follow immediately.

#### Comments on Proposition I

We have five comments on Proposition I.

The proof of Proposition I does not depend upon the assumption that  $C > 0$ . In particular, if  $C = 0$ , then  $1 - R = 0$  and the effective amount of fiat money outstanding is zero.

Proposition I does not rule out efficient nonstationary monetary equilibria for  $C < 1$ . However, any such monetary equilibrium involves the payment of positive rebates, and, therefore, is inefficient. Proposition (I) part (c)-(2) can be modified to read "If  $C < 1$ , with rebates there is no efficient monetary equilibrium."

Proposition I allows for negative rebates. However, negative rebates make no sense. We have already seen that negative rebates allow the economy to produce an "effective" amount of fiat money at cost less than  $C$  per real unit. In other words, the setup cost of a unit of real transactions is less than  $C$ , which contradicts our assumptions on costs. Therefore, Proposition I part (a) can be modified to "If  $C > 1$ , then no monetary equilibrium is feasible." In this circumstance there must be nonprice rationing of real balances.

Fourthly, in the introduction it is claimed that the addition of on-going maintenance cost has little effect on the results. Now we can make this precise. Suppose there are maintenance costs of  $C^* < C$  in every odd period after the first period. Then Proposition I is the same with " $C - C^*$ " replacing " $C$ ."

Lastly, in the introduction it is also claimed that the setup costs occurring in the second period, when the money is exchanged, has little effect on the results. To be precise, the only change is to Proposition I part (b), which becomes:

(b) If  $C = 1$ ,

- (1) Without rebates there are infinitely many stationary monetary equilibria with efficiently supplied money; real balances are indeterminate.
- (2) With rebates there is one stationary monetary equilibrium with efficiently supplied money and no rebates.

The problem for the competitive provision of fiat money is rents, positive or negative, generated in the "production" of real balances. Our conclusion is that the competitive provision of fiat money is inefficient or infeasible except in the serendipitous case that the added cost of initiating fiat money just equals its value.

## References

\*This paper was partially motivated by comments from Robert Lucas and Edward Prescott on Bryant and Wallace (4) and (5). However, the views expressed herein are solely those of the author and do not necessarily represent those of Lucas or Prescott or of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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