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Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics

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ABSTRACT

We solve equilibrium models of lumpy investment wherein establishments face persistent shocks to common and plant-specific productivity. Nonconvex adjustment costs lead plants to pursue generalized (S, s) rules with respect to capital; thus, their investments are lumpy. In partial equilibrium, this yields substantial skewness and kurtosis in aggregate investment, though, with differences in plant-level productivity, these nonlinearities are far less pronounced. Moreover, nonconvex costs, like quadratic adjustment costs, increase the persistence of aggregate investment, yielding a better match with the data.

In general equilibrium, aggregate nonlinearities disappear, and investment rates are very persistent, regardless of adjustment costs. While the aggregate implications of lumpy investment change substantially in equilibrium, the inclusion of fixed costs or idiosyncratic shocks makes the average distribution of plant investment rates largely invariant to market-clearing movements in real wages and interest rates. Nonetheless, we find that understanding the *dynamics* of plant-level investment requires general equilibrium analysis.

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1 Introduction

In recent years, the mechanics of changes in the distribution of capital across establishments have been emphasized in studies of aggregate investment. An influential body of research suggests that there are important nonlinearities in aggregate investment originating from the establishment level. In particular, nonconvex costs of capital adjustment lead establishments to adjust capital infrequently in the form of lumpy investments. As explained by Caballero and Engel (1999), a large aggregate shock in such a setting may lead to a substantial increase in the number of establishments undertaking capital adjustment. This, in turn, implies a time-varying elasticity of aggregate investment demand with respect to shocks, and such nonlinearities help explain the data.

The substantial heterogeneity that characterizes (S, s) models of capital adjustment has largely dissuaded researchers from undertaking general equilibrium analysis.[1] However, in Khan and Thomas (2003), we solved a stochastic dynamic general equilibrium model where nontrivial heterogeneity in production arose from nonconvex adjustment costs that caused plants to adopt optimal (S, s) decision rules with respect to capital. We found that the aggregate nonlinearities predicted by previous partial equilibrium studies were present in our model economy when real wages and interest rates were held fixed, but disappeared in general equilibrium. An important assumption in this earlier analysis was that differences in capital were the sole source of heterogeneity across plants. In abstracting from persistent differences in plant-specific productivity, the theory could not usefully address a richer set of establishment-level facts that have been recently documented (see Cooper and Haltiwanger (2002)). In this paper, we extend the analysis, allowing plants to differ both in their capital stocks and in their total factor productivity. We also allow plants to undertake low levels of investment without incurring adjustment costs. The result is, to the best of our knowledge, the first model to match the available data on the average distribution of establishment-level investment rates.

We find that the introduction of *additional heterogeneity reduces the aggregate nonlinearities that exist in partial equilibrium.* This result that idiosyncratic shocks reduce

[1] Examples of partial equilibrium (S, s) models include Caballero and Engel (1999), Caballero, Engel and Haltiwanger (1995), Cooper, Haltiwanger and Power (1999) and Cooper and Haltiwanger (2002).

the aggregate effects of (S, s) policies is not new; it was first established in a model of irreversibilities by Bertola and Caballero (1994). However, we find that the additional risk reduces not only the changes in the number of establishments undertaking capital adjustment, but also the extent of adjustment by each such establishment. As a result, our analysis suggests that plant-specific productivity shocks may actually reverse the amplification of aggregate investment that is commonly associated with partial equilibrium lumpy investment models. In particular, the rise in aggregate investment demand following a positive aggregate productivity shock in the lumpy investment model may be less than the corresponding rise in a standard model without adjustment costs.[2]

One long-standing challenge for the empirical investment literature has been explaining the persistence of aggregate investment rates. As described in Caballero (1999), this motivated the ad-hoc introduction of distributed lags in early empirical investment equations. Subsequent explicit q-theoretic models introduced persistence by assuming convex capital adjustment costs. However, the lagged investment rate was found to be significant in model specification tests that included it as an additional regressor, reflecting the q-model's inability to explain the serial correlation of investment rates (Chirinko (1993)). Moreover, absent ad-hoc lagged regressors, estimates of the model's adjustment cost parameter are widely viewed as implausibly large, as they imply very slow adjustment speeds (Chirinko (1993), Cooper and Ejarque (2001)).

Our second central result is that, *in partial equilibrium, aggregate investment rates are less volatile and far more persistent in the presence of nonconvex adjustment costs*, irrespective of idiosyncratic productivity shocks. By delaying capital adjustment for some establishments, these costs deliver gradual changes in aggregate investment. Thus, partial equilibrium models may tend to emphasize these costs because they increase the persistence of aggregate investment rates in such settings, bringing them closer to the data.

General equilibrium analysis remains essential in any evaluation of the aggregate implications of nonconvexities. Changes in real wages and interest rates imply dramatic reductions in the volatility of aggregate investment and large increases in its persistence sufficient to match the serial correlation in the data. Perhaps most important is the result that, *in general equilibrium, the persistence and skewness of aggregate investment rates are*

[2] As carefully explained by Caballero (1999), models with investment irreversibilities, such as Bertola and Caballero (1994) and Veracierto (2002), do not generate lumpy plant investment nor the corresponding amplification of aggregate investment demand.

essentially unaffected by nonconvex capital adjustment costs. As a result, lumpy investment does not lead to aggregate nonlinearities, a finding that is entirely robust to the inclusion of persistent differences in plant-level productivity.

By contrast, *equilibrium has relatively little impact on the average cross-sectional distribution of plant investment rates when there are either nonconvex capital adjustment costs or large idiosyncratic productivity differences.* In such settings, there is a permanent source of heterogeneity and a nontrivial distribution of investment rates. Moreover, given relatively small fluctuations in aggregate productivity, much of a plant's investment, on average, derives from reallocation of the investment good across plants driven by differences in their individual states. While equilibrium movements in real wages and interest rates dampen fluctuations in aggregate investment, they have little impact on such reallocation. As a result, the average cross-sectional distribution for the stochastic economy under both partial and general equilibrium closely resembles the distribution in the deterministic steady state. Nonetheless, the distribution of plant investment rates does change over time with the aggregate state, and the magnitude of these changes is very sensitive to relative prices. Thus, an understanding of the dynamics of plant-level investment would seem to require equilibrium analysis.

While idiosyncratic shocks are important in explaining plant-level investment, we find that the role of nonconvexities changes substantially in their presence. Nonconvex adjustment costs cease to be important in generating the plant-level investment spikes that are the hallmark of lumpy investment. In fact, their primary role shifts to one of *reducing* investment spikes, while they have a secondary role in yielding the stark asymmetry in the occurrence of positive versus negative spikes observed in the data.

Most of our analysis assumes that plants face adjustment costs whenever they invest. As a result of this assumption, plants invest infrequently, and inaction is too prevalent relative to the establishment-level data. To resolve this discrepancy, we extend the model, allowing plants to undertake low levels of investment exempt from adjustment costs. To the best of our knowledge, this extended model is the first to match the average distribution of investment rates in the data. Nonetheless, our extension has no effect on aggregate dynamics; we show that these are quantitatively indistinguishable from the basic lumpy investment model.

2 Model

In our model economy, there are both fixed costs of capital adjustment and persistent differences in plant-specific productivity, which together lead to substantial heterogeneity in production. In this section, we describe the economy beginning with production units, then follow with households and equilibrium. Next, using a simple implication of equilibrium, we characterize the capital adjustment decisions of production units as a two-sided generalized (S, s) policy. This decision rule for investment is what distinguishes the model from the stochastic neoclassical growth model.

2.1 Production and capital adjustment

We assume a large number of production units. Each establishment produces its output using predetermined capital stock k and labor n, via an increasing and concave production function, F:

$$y = z\varepsilon F(k, n).$$

Here, z reflects stochastic total factor productivity common across plants, while ε is plant-specific productivity. For convenience, we assume that z follows a Markov chain, $z \in \{z_1, \ldots, z_{N_z}\}$, where

$$\Pr\left(z' = z_j \mid z = z_i\right) \equiv \pi_{ij} \geq 0,$$

and $\sum_{j=1}^{N_z} \pi_{ij} = 1$ for each $i = 1, \ldots, N_z$. Similarly, we assume that $\varepsilon \in \{\varepsilon_1, \ldots, \varepsilon_{N_\varepsilon}\}$, where

$$\Pr\left(\varepsilon' = \varepsilon_l \mid \varepsilon = \varepsilon_k\right) \equiv \pi^{\varepsilon}_{kl} \geq 0,$$

and $\sum_{l=1}^{N_\varepsilon} \pi^{\varepsilon}_{kl} = 1$ for each $k = 1, \ldots, N_\varepsilon$.

In each period, a plant is defined by its predetermined stock of capital, k, its idiosyncratic productivity level, ε, and its current cost of capital adjustment, $\xi \geq 0$, denominated in units of labor. Given the current aggregate state of the economy, it decides its current level of employment, n, production occurs, and its workers are paid. After production, the plant determines whether to pay its fixed cost and undertake an active capital adjustment. It may alternatively avoid the cost by setting investment to 0 and passively allowing its capital to depreciate. We summarize the salient features of this choice below, denoting the plant's investment by i and the depreciation rate by δ, and measuring the adjustment cost in units of output using the real wage rate, ω.[3]

$i \neq 0$	fixed cost $= \omega\xi$	$\gamma k' = (1-\delta)\,k + i$
$i = 0$	fixed cost $= 0$	$\gamma k' = (1-\delta)\,k$

For the plant, capital adjustment involves a nonconvexity, since the cost ξ is independent of the scale of adjustment. At the same time, we assume that ξ varies across plants and over time for any given plant. Each period, every plant draws a cost from the time-invariant distribution $G : [0, \overline{\xi}] \rightarrow [0, 1]$. As a result, given its end-of-period stock of capital, a plant's current adjustment cost has no implication for its future adjustment. Thus, it is sufficient to describe differences across plants by their idiosyncratic productivity, ε, and capital, k. We summarize the distribution of plants over (ε, k), where $\varepsilon \in \mathcal{E} \equiv \{\varepsilon_1, \ldots, \varepsilon_{N_e}\}$ and $k \in \mathcal{K} \subseteq \mathbf{R}_+$, using the Borel probability measure μ defined on the $\sigma-$algebra generated by the open subsets of the product space $\mathcal{S} = \mathcal{E} \times \mathcal{K}$. The aggregate state of the economy is then described by (z, μ), and the distribution of plants evolves over time according to a mapping, Γ, from the current aggregate state, $\mu' = \Gamma(z, \mu)$. We will define this mapping below.

Let $v^1(\varepsilon_k, k, \xi; z_i, \mu)$ represent the expected discounted value of a plant entering the period with (ε_k, k) and drawing an adjustment cost ξ, when the aggregate state of the economy is (z_i, μ). We state the dynamic optimization problem for the typical plant using a functional equation defined by (1) and (2). First we define the beginning of period expected value of a plant, prior to the realization of its fixed cost draw, but after the determination of $(\varepsilon_k, k; z_i, \mu)$:

$$v^0(\varepsilon_k, k; z_i, \mu) \equiv \int_0^{\overline{\xi}} v^1(\varepsilon_k, k, \xi; z_i, \mu)\, G(d\xi)\,. \tag{1}$$

Assume that $d_j(z_i, \mu)$ is the discount factor applied by plants to their next-period expected value if aggregate productivity at that time is z_j and current productivity is z_i. (Except where necessary for clarity, we suppress the indices for current aggregate and plant productivity below.) The plant's profit maximization problem, which takes as given the evolution of the plant distribution, $\mu' = \Gamma(z, \mu)$, is then described by the following

[3]Throughout the paper, primes indicate one-period-ahead values, and all variables measured in units of output are deflated by the level of labor-augmenting technological progress, which grows at the rate $\gamma - 1$.

functional equation:

$$v^1(\varepsilon, k, \xi; z, \mu) = \max_n \Bigg[z\varepsilon F(k,n) - \omega(z,\mu)\, n + (1-\delta)\, k \tag{2}$$

$$+ \max \Bigg\{ -\xi\omega(z,\mu) + \max_{k'} \left(-\gamma k' + \sum_{j=1}^{N_z} \pi_{ij} d_j(z,\mu) \sum_{l=1}^{N_e} \pi^{\varepsilon}_{kl} v^0\left(\varepsilon_l, k'; z_j, \mu'\right) \right),$$

$$- (1-\delta)\, k + \sum_{j=1}^{N_z} \pi_{ij} d_j(z,\mu) \sum_{l=1}^{N_e} \pi^{\varepsilon}_{kl} v^0 \left(\varepsilon_l, \frac{(1-\delta)}{\gamma} k; z_j, \mu' \right) \Bigg\} \Bigg].$$

Given (ε, k, ξ) and the equilibrium wage rate $\omega(z,\mu)$, the plant chooses current employment n. Next it selects whether to adjust capital, the value of which is represented by the first term in the internal binary maximum choice above, or avoid its current fixed cost by setting investment to 0. Rather than subtracting investment from current profits, we adopt an equivalent but notationally more convenient approach in (2); there, the value of nondepreciated capital augments current profits, and the plant is seen to repurchase its entire capital stock each period. Since adjustment costs do not affect the choice of current employment, we denote the common employment selected by all type (ε, k) plants using $N(\varepsilon, k; z, \mu)$. Further, let $K(\varepsilon, k, \xi; z, \mu)$ represent the choice of capital for the next period by plants of type (ε, k) with adjustment cost ξ.

2.2 Households

The economy is populated by a unit measure of identical households. Household wealth is held as one-period shares in plants, which we denote using the measure λ.[4] They determine their current consumption, c, hours worked, n^h, as well as the number of new shares, $\lambda'(\varepsilon', k')$, to purchase at price $\rho_1(\varepsilon', k'; z, \mu)$. Households receive prices $\rho_0(\varepsilon, k; z, \mu)$ for their current shares and real wage $\omega(z,\mu)$ for their labor effort. Their lifetime expected

[4] Households also have access to a complete set of state-contingent claims. However, as there is no heterogeneity across households, these assets are in zero net supply in equilibrium. Thus, for brevity, we do not explicitly model them.

utility maximization problem is listed below.

$$W(\lambda; z, \mu) = \max_{c, n^h, \lambda'} \left[U\left(c, 1 - n^h\right) + \beta \sum_{j=1}^{N_z} \pi_{ij} W\left(\lambda'; z_j, \mu'\right) \right] \tag{3}$$

subject to

$$c + \int_{\mathcal{S}} \rho_1\left(\varepsilon', k'; z, \mu\right) \lambda'\left(d\left[\varepsilon' \times k'\right]\right) \leq \omega(z, \mu) n^h + \int_{\mathcal{S}} \rho_0(\varepsilon, k; z, \mu) \lambda(d[\varepsilon \times k]).$$

Let $C(\lambda; z, \mu)$ describe the household choice of current consumption, $N^h(\lambda; z, \mu)$ the current allocation of time to working, and $\Lambda(\varepsilon', k', \lambda; z, \mu)$ the quantity of shares purchased in plants that begin the next period with productivity ε' and k' units of capital.

2.3 Recursive equilibrium

A *recursive competitive equilibrium* is a set of functions

$$\left(\omega, (d_j)_{j=1}^{N_z}, \rho_0, \rho_1, v^1, N, K, W, C, N^h, \Lambda\right)$$

such that plants and households maximize their expected values, and the markets for assets, labor and output clear:

1. v^1 satisfies (1) - (2), and (N, K) are the associated policy functions for plants.
2. W satisfies (3), and $\left(C, N^h, \Lambda\right)$ are the associated policy functions for households.
3. $\Lambda(\varepsilon_l, k', \mu; z, \mu) = \mu'(\varepsilon_l, k')$.
4. $N^h(\mu; z, \mu) = \int_{\mathcal{S}} \left(N(\varepsilon, k; z, \mu) + \int_0^{\bar{\xi}} \xi \mathcal{J}\left(\frac{(1-\delta)}{\gamma} k - K(\varepsilon, k, \xi; z, \mu)\right) G(d\xi) \right) \mu(d[\varepsilon \times k])$, where $\mathcal{J}(x) = 0$ if $x = 0$; $\mathcal{J}(x) = 1$ if $x \neq 0$.
5. $C(\mu; z, \mu) = \int_{\mathcal{S}} \left(z\varepsilon F(k, N(\varepsilon, k; z, \mu)) - \int_0^{\bar{\xi}} [\gamma K(\varepsilon, k, \xi; z, \mu) - (1-\delta) k] G(d\xi) \right) \mu(d[\varepsilon \times k])$.
6. $\mu'(\varepsilon_l, B) = \int_{\{(\varepsilon_k, k, \xi) \mid K(\varepsilon_k, k, \xi; z, \mu) \in B\}} \pi_{kl}^{\varepsilon} G(d\xi) \mu(d[\varepsilon_k \times k])$ defines Γ.

2.4 (S, s) decision rules

Using C and N, as given by (4) and (5), to describe the market-clearing values of consumption and hours worked by the household, it is straightforward to show that equilibrium requires $\omega(z,\mu) = \frac{D_2 U(C,1-N)}{D_1 U(C,1-N)}$ and that $d_j(z,\mu) = \frac{\beta D_1 U\left(C'_j, 1-N'_j\right)}{D_1 U(C,1-N)}$. We may then compute equilibrium by solving a single Bellman equation that combines the plant-level profit maximization problem with the equilibrium implications of household utility maximization. Let p denote the price plants use to value current output, where

$$p(z,\mu) = D_1 U(C, 1-N), \tag{4}$$

$$\omega(z,\mu) = \frac{D_2 U(C, 1-N)}{p(z,\mu)}. \tag{5}$$

A reformulation of (2) then yields an equivalent description of a plant's dynamic problem. Suppressing the arguments of the price functions,

$$\begin{aligned} V^1(\varepsilon, k, \xi; z, \mu) = \max_n \Bigg(& [z\varepsilon F(k,n) - \omega n + (1-\delta)k] p \\ + \max \Bigg\{ & -\xi\omega p + \max_{k'} \left(-\gamma k' p + \beta \sum_{j=1}^{N_z} \pi_{ij} \sum_{l=1}^{N_e} \pi^{\varepsilon}_{kl} V^0\left(\varepsilon_l, k'; z_j, \mu'\right) \right), \\ & -(1-\delta)kp + \beta \sum_{j=1}^{N_z} \pi_{ij} \sum_{l=1}^{N_e} \pi^{\varepsilon}_{kl} V^0\left(\varepsilon_l, \frac{(1-\delta)}{\gamma} k; z_j, \mu'\right) \Bigg\} \Bigg), \end{aligned} \tag{6}$$

where

$$V^0(\varepsilon, k; z, \mu) \equiv \int_0^{\overline{\xi}} V^1(\varepsilon, k, \xi; z, \mu)\, G(d\xi). \tag{7}$$

Equations (6) and (7) will be the basis of our numerical solution of the economy. This solution exploits several results that we now derive. First, note that plants choose labor $n = N(\varepsilon, k; z, \mu)$ to solve

$$z\varepsilon D_2 F(k,n) = \omega(z,\mu).$$

Next, we examine the capital choice of establishments undertaking active adjustment decisions. Define the gross value of undertaking adjustment as that arising in the first term of the internal binary maximum within (6):

$$E(\varepsilon, z, \mu) \equiv \max_{k'} \left(-\gamma k' p(z,\mu) + \beta \sum_{j=1}^{N_z} \pi_{ij} \sum_{l=1}^{N_e} \pi^{\varepsilon}_{kl} V^0\left(\varepsilon_l, k'; z_j, \mu'\right) \right). \tag{8}$$

Note that the target capital stock solving this maximization problem is independent of both k and ξ, but not ε, given persistence in plant-specific productivity. As a result, all plants sharing the same current productivity ε that actively adjust their capital stock choose a common *target* level of capital for the next period, $k' = k^*(\varepsilon, z, \mu)$, which solves the right-hand side of (8). This independence of target capital from current capital implies that the gross value of adjustment, $E(\varepsilon, z, \mu)$, is itself independent of current capital.

Referring again to the functional equation in (6), it is now clear that a plant will absorb its fixed cost and adjust if the net value of achieving the target capital, $E(\varepsilon, z, \mu) - \xi\omega p$, is at least as great as its continuation value under nonadjustment (line three). It follows immediately that a plant of type (ε, k) will undertake active capital adjustment if its fixed adjustment cost, ξ, lies at or below some (ε, k)-specific threshold value. Let $\widehat{\xi}(\varepsilon, k; z, \mu)$ describe the level of ξ that leaves a type (ε, k) plant indifferent between active capital adjustment and inaction (simply allowing its capital to depreciate):

$$\begin{aligned} & -p(z,\mu)\,\widehat{\xi}(\varepsilon,k;z,\mu)\,\omega(z,\mu) + E(\varepsilon,z,\mu) \\ = & -p(z,\mu)(1-\delta)k + \beta\sum_{j=1}^{N_z}\pi_{ij}\sum_{l=1}^{N_e}\pi_{kl}^{\varepsilon}V^0\left(\varepsilon_l, \frac{(1-\delta)}{\gamma}k; z_j, \mu'\right). \end{aligned} \tag{9}$$

Next, define $\xi^T(\varepsilon, k; z, \mu) \equiv \min\left\{\overline{\xi}, \max\left\{0, \widehat{\xi}(\varepsilon, k; z, \mu)\right\}\right\}$, so that $0 \le \xi^T(\varepsilon, k; z, \mu) \le \overline{\xi}$. Plants with adjustment costs at or below $\xi^T(\varepsilon, k; z, \mu)$ will adjust their capital stock.

Using the target capitals and threshold adjustment costs identified above, the plant-level decision rule for capital may be conveniently summarized; any establishment identified by the plant-level state vector $(\varepsilon, k, \xi; z, \mu)$ will begin the subsequent period with a capital stock given by

$$k' = K(\varepsilon, k, \xi; z, \mu) = \begin{cases} k^*(\varepsilon, z, \mu) & \text{if} \quad \xi \le \xi^T(\varepsilon, k; z, \mu), \\ \frac{(1-\delta)k}{\gamma} & \text{if} \quad \xi > \xi^T(\varepsilon, k; z, \mu). \end{cases} \tag{10}$$

Based on (10), we now explicitly define the evolution of the plant distribution, $\mu' = \Gamma(z, \mu)$. For all $(\varepsilon_l, k) \in \mathcal{S}$,

$$\begin{aligned} \mu'(\varepsilon_l, k) \;=\; & \sum_{k=1}^{N_\varepsilon}\pi_{kl}^{\varepsilon}\left[\Big(1 - \mathcal{J}(k - k^*(\varepsilon_k, z, \mu))\Big)\int_{\mathcal{S}} G\left(\xi^T(\varepsilon_k, k; z, \mu)\right)\mu(d[\varepsilon_k \times k]) \right. \\ & + \left.\left[1 - G\left(\xi^T\left(\varepsilon_k, \frac{\gamma}{1-\delta}k; z, \mu\right)\right)\right]\mu\left(\varepsilon_k, \frac{\gamma}{1-\delta}k\right)\right]. \end{aligned} \tag{11}$$

It then follows that the market-clearing levels of consumption and hours required to determine p and ω using (4) and (5) are given by

$$
\begin{aligned}
C &= \int_{\mathcal{S}} \Big(z\varepsilon F\left(k, N\left(\varepsilon, k; z, \mu\right)\right) - G\left(\xi^T\left(\varepsilon, k; z, \mu\right)\right) \Big[\gamma k^*\left(\varepsilon, z, \mu\right) \\
&\quad - \left(1-\delta\right) k\Big]\Big) \mu\left(d\left[\varepsilon \times k\right]\right) && (12) \\
N &= \int_{\mathcal{S}} \left[N\left(\varepsilon, k; z, \mu\right) + \int_0^{\xi^T(\varepsilon,k;z,\mu)} \xi G\left(d\xi\right) \right] \mu\left(d\left[\varepsilon \times k\right]\right). && (13)
\end{aligned}
$$

3 Model solution

We evaluate the plant-level and aggregate implications of nonconvex capital adjustment costs using several numerical experiments across which we vary the stochastic process for idiosyncratic shocks to plants' total factor productivity and the parameterization of capital adjustment costs. All other production parameters, as well as preferences, are held constant throughout. Each experiment is based on a 5000-period model simulation, and the same random draw of aggregate productivity is used in each. In the next section, we discuss functional forms and parameter values for technology and preferences that are identical across models. In section 3.2, we explain the choice of idiosyncratic shocks, and, in section 3.3, we specify the distribution of capital adjustment costs.

3.1 Common parameters

Across all our model economies, we assume that the representative household's period utility is the result of indivisible labor (Hansen (1985), Rogerson (1988)): $u(c, L) = \log c + \varphi L$, and the establishment-level production function takes a Cobb-Douglas form, $z\varepsilon F(k, N) = z\varepsilon k^{\theta} N^{\nu}$. We fix the length of a period to correspond to one year, allowing us to use evidence on establishment-level investment in the parameterization of the adjustment cost distribution below. Model parameters are selected to ensure agreement with observed long-run values for key postwar U.S. aggregates in a version of the model without capital adjustment costs described in the appendix. However, the aggregate first moments in all model economies are extremely similar.

As proven in lemma 2 of the appendix, macroeconomic aggregates are insensitive to the presence of idiosyncratic productivity differences in the models we study that do not involve capital adjustment costs, (one with plant-level productivity shocks and one without). We use this pair of *standard* models to derive parameter values for technology and preferences that are consistent with empirical counterparts. Next, we apply the same values to the lumpy investment models. The mean growth rate of technological progress is chosen to imply a 1.6 percent average annual growth rate of real per capita output, and the discount factor, β, is then set to imply an average real interest rate of 4 percent. Given the rate of technological progress, the depreciation rate, δ, is selected to match an average investment-to-capital ratio of 10 percent, corresponding to the average value for the private capital stock between 1954 and 2002 in the U.S. Fixed Asset Tables. Labor's share is then set to 0.64 as in Prescott (1986); given this value, capital's share of output is determined by targeting an average capital-to-output ratio of 2.353 as in the data. Finally, the parameter governing the preference for leisure, φ, is taken to imply an average of one-third of available time spent in market work. Table 1 summarizes the resulting parameter values.

We determine the stochastic process for total factor productivity using the Crucini residual approach described in King and Rebelo (1999). A continuous shock version of the standard model, assuming $\log z' = \rho_z \log z + \varepsilon_z'$ with $\varepsilon_z' \sim N\left(0, \sigma_{\varepsilon_z}^2\right)$, is solved using an approximating system of stochastic linear difference equations, given an arbitrary initial value of ρ_z. This linear method isolates a decision rule for output of the form $Y = \pi_z(\rho_z)\,\psi(z) + \pi_k(\rho_z)\,k$, where the coefficients associated with z and k are functions of ρ_z. Rearranging this solution, data on GDP and capital are then used to infer an implied set of values for the technology shock series. Maintaining the assumption that these realizations are generated by a first-order autoregressive process, the persistence and variance of this implied series yield new estimates of $\left(\rho_z, \sigma_{\varepsilon_z}^2\right)$, and the process is repeated until these estimates converge. The resulting values for the persistence and variance of the technology shock process are not uncommon; $\rho_z = 0.8254$ and $\sigma_{\varepsilon_z} = 0.0124$. Next, we discretize this productivity process using a grid of 5 possible shock realizations; $N_z = 5$.

3.2 Plant-specific shocks

Given the parameter selection above, we consider two distinct stochastic processes for idiosyncratic productivity. These identify our *full* and *common productivity* models. The *full* models, with and without fixed costs of capital adjustment, have persistent idiosyncratic shocks. We introduce these using the estimated persistence and variability from

Cooper and Haltiwanger (2002). In particular, the idiosyncratic component of a plant's total factor productivity is assumed to follow a log-normal process $\log \varepsilon' = \rho_\varepsilon \log \varepsilon + \eta_\varepsilon$ where $\rho_\varepsilon = 0.53$, and the standard deviation of the white noise innovation η_ε is 0.0785. This implies that idiosyncratic shocks have a standard deviation relative to the aggregate shock of $\frac{8}{3}$, as in Cooper and Haltiwanger (2002). As in that paper, we use an 11-value discretization of this log-normal process; $N_\varepsilon = 11$. The *common productivity* models eliminate differences in plants' total factor productivity, setting $\sigma_{\eta_\varepsilon} = 0$. We use these models as controls to isolate the effect of persistent differences in plant-specific productivity for the role of nonconvex costs in investment dynamics.

3.3 Capital adjustment costs

The parameters above fully specify the *standard* models without capital adjustment costs. All that remains now is to determine the distribution of adjustment costs that distinguish the *lumpy investment* models. We assume that these costs are uniformly distributed, with cumulative distribution function $G(\xi) = \xi/\overline{\xi}$. We then select $\overline{\xi}$ so that the *full lumpy investment* model matches the fraction of plants experiencing positive investment spikes reported by Cooper and Haltiwanger (2002).

Constructing their own plant capital series using data on both retirements and investment from the Longitudinal Research Database, Cooper and Haltiwanger (2002) provide a detailed set of time-averaged moments on plants' investment rates, which are summarized in table 2. They define any plant with an investment rate (ratio of investment to capital) less than 1 percent in absolute value as inactive. Positive investment rates are those exceeding 1 percent, while negative investment rates are those falling below -0.01. Finally, they define positive spikes as positive investment rates exceeding 0.2, and negative spikes as observations of $\frac{i}{k} < -0.2$. As seen in table 9 (panel B, row 1), the selection of $\overline{\xi} = 0.011$ implies that, on average, roughly 18.6 percent of establishments invest more than 20 percent of their existing stock of capital in our full model. Note that this upper bound for the fixed costs also implies a very close match to the average fraction of establishments experiencing a negative investment spike, which is 1.4 percent in both model and data.

The cost of matching the empirical observations on positive and negative spikes in our basic model of lumpy investment is that it requires plant-level investments to be, on average, quite infrequent. The fraction of inactive observations is markedly larger in the model than apparent in the data, 77.8 percent versus 8.1 percent. This is a standard shortcoming of quantitative models of lumpy investment; see Cooper and Haltiwanger (2002). Table 9 suggests that idiosyncratic shocks and fixed costs are in themselves insufficient to reproduce the average distribution of plant investment rates in the data. One possible explanation is that fixed costs do not apply to investments when they are sufficiently minor relative to a plant's existing capital. In section 5, we develop an extension to the model along these lines. We find that this resolves the inconsistencies between model and data without altering aggregate results.

3.4 Forecasting rules

Solving the standard models is fairly straightforward, even in the presence of persistent plant-level shocks. Despite a distribution of plants over capital and productivities, the endogenous aggregate state vector is fully described by total capital and a time-invariant distribution of plants' shares of the aggregate capital stock as a function of their idiosyncratic productivity level (as shown in the appendix). Given the invariance in the distribution of relative capital, the aggregate state vector contains only two time-varying elements, total capital and aggregate productivity, and standard methods may be used to solve the model. The one novelty in our approach is that we apply a nonlinear solution method using piecewise polynomial cubic spline interpolation of the planner's value function. This method, which to our knowledge is not often used in macroeconomics, is described briefly in Khan and Thomas (2003) and, in more detail, in Thomas (2004). In partial equilibrium, the same nonlinear approach is applied to solving plants' value functions for the lumpy investment models. The distribution of adjustment costs implies that value functions are smoother objects than decision rules, and the splines are robust interpolants for such discrete choice problems.

General equilibrium solution of the lumpy investment models requires the determination of market-clearing real wages and interest rates which, in turn, depend on agents' expectations of future wages and interest rates. We adapt the solution method described in Khan and Thomas (2003) to allow for a two-dimensional distribution of plants over capital and idiosyncratic productivity. The upper bound on the distribution of capital adjustment costs implies that all plants adjust in finite time and the economy has, in this sense, finite memory. Thus, at each productivity, the distribution of plants over capital may be described using a finite vector of capital levels and the associated number of plants holding each such level.

While not high-dimensional, our aggregate state vector is still large. In the common productivity model with lumpy investment, it involves 31 variables. The nonlinear solution method predicated by our focus on aggregate nonlinearities makes this numerically intractable, so we use selected moments as a proxy for the distribution in the aggregate state vector, following the method of Krussel and Smith (1997). Specifically, we solve for equilibrium under the assumption that plants and households use only these moments in forming expectations of future wages and interest rates. This allows us to tractably approximate rational expectations equilibrium and evaluate the aggregate business cycle implications arising from nonconvex costs of capital adjustment at the plant level.

Table 3 presents agents' forecasting rules for the common productivity model. In determining their current decisions, agents forecast the future proxy state, m_1', assumed to be the first moment of the distribution of plants over capital, using the mean of the current distribution, m_1 (and current aggregate productivity). Similarly, when solving for agents' value functions, we have them assume that the value of current output, p, is a log-linear function of this mean.[5] Note that adjusted R-squares are very high, and standard errors are small; almost all the true variation in the mean of the distribution, and in the relative price of output, may be explained using these simple forecasting rules.

In the full lumpy investment model, there is a two-dimensional distribution of plants over capital and idiosyncratic productivity. Here, the 11-point discretization of the persistent plant productivity process implies an aggregate state vector with 551 variables. Nonetheless, we find that the solution method described above is robust to this additional source of heterogeneity. The equilibrium forecasting rules are presented in table 4. Note that there is no loss of accuracy in the forecasting rules with the introduction of persistent differences in plant-specific productivity, though we continue to use only the unconditional mean of the distribution of capital as a proxy for the aggregate endogenous state. This suggests that our general equilibrium solution method may be applied to a broad class of models currently studied in partial equilibrium.

[5] Our solution algorithm iterates between an inner loop and an outer loop, as in Krusell and Smith (1997). In the inner loop, agents' value functions are solved based upon a given set of forecasting rules. Given these value functions, the economy is simulated in the outer loop, where p is endogenously determined in each date. Next, the resulting simulation data are used to update the forecasting rules for the inner loop.

4 Results

As indicated above, our results are based upon comparisons of four models differentiated by their capital adjustment costs and idiosyncratic productivity processes. We review these models here. First, as we are interested in assessing the effects of plant-level nonconvexities, we compare results for *standard* equilibrium business cycle models with corresponding results for models where plants are subject to nonconvex capital adjustment costs; we label the latter group *lumpy (investment)* models. Second, we explore the effect of introducing persistent changes in plant-specific productivity in both standard and lumpy models. We do this by contrasting the results for *full* models, where such changes exist, with those for *common productivity* models, where there are no differences in total factor productivity across plants. A central focus of this exploration is the impact of general equilibrium changes in prices on both aggregate and plant-level investment dynamics. Thus, all four models are solved both in *partial equilibrium*, by which we mean that real wages and interest rates are held constant at their steady state values, and in *general equilibrium*. We begin with a study of the aggregate implications of lumpy investment, with and without plant-specific variation in total factor productivity, under partial, then general, equilibrium.

4.1 Aggregate investment in partial equilibrium

The empirical investment literature has focused on changes in investment rates - that is, movements in the ratio of investment to capital. Across a broad variety of empirical studies, capital adjustment costs have been found to be important in matching the persistence of investment rates (Caballero (1999)). Finally, almost all of the analysis of nonconvex capital adjustment costs has been done in partial equilibrium. Here, we explore the aggregate effects of lumpy investment on investment rates in partial equilibrium versions of both the full and common productivity models.

4.1.1 Persistence

Table 5 reports the first four moments of aggregate investment rates for the standard and lumpy models, in both full and common productivity variants, under partial equilibrium. Beginning with the standard models, where there are no nonconvex costs of capital adjustment, note that aggregate investment rates are negatively autocorrelated and very volatile. In partial equilibrium, and without capital adjustment costs, investment responds immediately to changes in aggregate productivity. Thus, while productivity may be persistent, investment is not. (Capital stocks are of course persistent, since they track productivity with a one-period lag.)

Our first result is that, *in partial equilibrium, capital adjustment costs not only reduce the volatility of aggregate investment rates, but also increase their persistence.* The reason for this increased persistence is straightforward. Fixed costs of capital adjustment induce inaction among plants with relatively high current costs or capital close to their target value. Thus, in the aggregate, investment initially responds less to a change in aggregate productivity than in the standard model without adjustment costs. However, aggregate productivity changes are very persistent and, as a result, in subsequent periods many of those initially inactive plants undertake capital adjustments. Thus, in partial equilibrium, investment is both less variable and more persistent with capital adjustment costs. A similar result holds for models with convex adjustment cost; such costs induce all plants to undertake concurrent but gradual capital adjustment. In our lumpy investment models, by contrast, aggregate investment is more gradual because nonconvex costs give rise to an extensive margin, which in turn implies that only a fraction of plants adjust each period.

4.1.2 Nonlinearities

The lumpy investment models exhibit considerable skewness and excess kurtosis in partial equilibrium aggregate investment rates, a feature not shared by the corresponding standard models. It is this central and well-known feature of lumpy investment that has motivated much interest in its empirical usefulness.[6] Interestingly, when comparing the lumpy investment models in panels A and B of table 5, we see that there is much less skewness and excess kurtosis in the distribution of aggregate investment rates in the full model. *In partial equilibrium, plant-level productivity shocks sharply reduce the skewness and kurtosis in aggregate investment rates.* This is our second result.

To explain both the skewness of investment rates and why it is reduced by the presence of plant-specific productivity shocks in the full model, we study the response of plants to a

[6] See, for example, Caballero and Engel (1999), Caballero, Engel and Haltiwanger (1995) and Cooper, Haltiwanger and Power (1999).

5 percent rise in aggregate total factor productivity versus a 5 percent fall. Consider first the common productivity model, which is characterized by a one-dimensional distribution of plants over capital. The first column of figure 1 shows a typical period, aggregate productivity having been at its mean level for 19 periods. In the top panel, we show the distribution of plants over capital; there, the highest value with positive mass is the target capital adopted by all adjusting plants absent any changes in aggregate productivity, which is just over 1.41. The dashed curve shows adjustment rates as a function of capital. Here, we see a rising adjustment hazard, as plants with capital further from the target are willing to suffer larger costs and thus have a higher probability of capital adjustment. The lowest capital level held by any plant is 0.64, and such plants adjust with full probability. The lower panel of the column shows the actual number of plants that adjust to the target capital stock from each existing level. The total adjusting each period is 0.22.

The second column of figure 1 illustrates the partial equilibrium response to a rise in aggregate total factor productivity. Since changes in aggregate productivity are expected to persist, plants' target capital stock rises to 1.88, increasing the gap between actual and target capital for each type of plant. With plants of each type now willing to pay larger fixed costs, adjustment rates increase sharply, and the total number of adjusting plants jumps to 0.78. This rise in the extensive margin, total plants adjusting capital, reinforces the rise in the intensive margin, the average investment undertaken by each adjusting plant. As a result, aggregate capital rises by far more than it would in the absence of an increase in adjustment rates.

By contrast, the final column of figure 1 reveals that an equivalent fall in aggregate productivity leads to a sharp decrease in adjustment rates. The fall reduces plants' target capital stock for next period to 1.05, which is lower than the capital stock actually held by more than a fifth of plants. As a result, the fraction of plants for which adjustment is sufficiently valuable to offset the associated fixed costs declines markedly. This fall is most pronounced near the middle of the distribution, where current capital, once adjusted for depreciation and exogenous technological progress, is closest to the target capital stock for next period. As a result, the adjustment hazard takes on a U shape over the mass of plants and, overall, the number of adjusting plants falls from its average level of 0.22 to a low of 0.07.

We have seen that adjustment rates rise in response to a positive productivity shock, but fall in the face of a negative productivity shock. As illustrated in figure 2A, this asymmetry reinforces the rise in aggregate capital when productivity increases and dampens the fall associated with a reduction in productivity.[7] This is the key nonlinearity of the lumpy investment model that generates skewed investment rates. The graph also shows that this asymmetry is dampened for the full model, where plants face not only common, but also idiosyncratic, changes to their total factor productivity.

The second and third panels of figure 2 compare the common productivity and full lumpy investment models to the standard model without capital adjustment costs. For the latter, changes in aggregate capital are unaffected by idiosyncratic shocks. From figure 2B, we see that the percentage increase in aggregate capital demand in the common productivity lumpy investment model actually exceeds that of the standard model. In contrast, the full lumpy investment model exhibits a lesser rise relative to the standard model, as seen in figure 2C. Thus, large and persistent idiosyncratic shocks actually reverse the amplification possible under lumpy investment. Nonetheless, in contrast to the standard model, both lumpy investment models continue to exhibit an asymmetric response in capital to positive versus negative shocks. In the common productivity lumpy investment model, the percentage rise in total capital is more than five times larger than the subsequent percentage fall. For the full lumpy investment model, the asymmetry is halved.

In an effort to understand the response of aggregate capital for the full model with lumpy investment under partial equilibrium in figure 2, we now turn to examine plant level adjustment for this model. The top panel of figure 3 illustrates the stationary distribution of plants over capital and idiosyncratic productivity in our full lumpy investment model. The presence of large plant-level differences in total factor productivity implies considerably greater dispersion in capital than in the common productivity model. Mean reversion in idiosyncratic productivity delivers a distribution that is concentrated around the mean level of productivity. Nonetheless, persistence in this productivity process leads plants with higher productivity levels to have, on average, higher capital stocks. In the lower panel, we see that adjustment rates (in the region of positive mass) are U shaped. As target capital stocks rise with plant productivity, the lowest adjustment rate for any given productivity level, that associated with a (depreciation-adjusted) current capital closest to the target for the next period, is increasing in plant productivity, as is the threshold value of capital below which adjustment rates are one.

[7] Of course, as was seen in figure 1, the distribution of adjustment over plant types shifts with aggregate shocks, which changes the average investment per adjusting plant.

In response to the rise in aggregate total factor productivity examined in figure 1 for the common productivity model, the adjustment hazards associated with each productivity in the full model shift leftward (into a higher capital range). As the target capital stock associated with each idiosyncratic productivity level rises, most plant are willing to accept higher adjustment costs. The top panel of figure 4 shows the total adjustors from each plant type after the rise in aggregate productivity. Relative to the stationary state, there is increased adjustment among plants with both high and low capital stocks. The lower panel of figure 4 shows the total adjusting from each plant type after a fall in aggregate productivity. In this case, target capital stocks are reduced at each idiosyncratic productivity. As the gap between actual and target capital now becomes largest for plants with relatively high capital stocks, most adjustment is concentrated among such plants. Clearly, the asymmetry discussed above in the context of the common productivity model is still present. However, it is less acute. A rise in common aggregate productivity increases total adjustors from its average value of 0.22 to 0.58, while a fall reduces adjusters to 0.21, only slightly below the stationary state level.

One reason for the dampened asymmetry under idiosyncratic shocks is simply that they lead to greater dispersion in the distribution of plants over capital than exists in the common productivity model. In figure 1, we saw that the distribution of plants in the common productivity model was monotonically rising in capital. This implied that leftward versus rightward shifts in the adjustment hazard had very different effects on the overall number adjusting. In the full model, by contrast, the distribution of plants over capital has less concentration at the highest levels of capital; the most common levels of capital lie below them. This immediately implies less asymmetry in adjustment.[8]

There is, however, a second reason for dampened asymmetry, one involving adjustments in the intensive margin. At each level of idiosyncratic productivity, there are lesser shifts in the adjustment hazards of the full model, relative to those in the common productivity model, in response to changes in aggregate productivity. These reduced shifts correspond to smaller changes in the target capitals selected by adjusting plants. For example, in response to the positive aggregate shock examined above, the average rise in target capital, weighted by the number of plants at each idiosyncratic shock level, is only 16.67 percent in the full model, while it is 33.33 percent in the common productivity model. Given the persistence

[8] This dampening of changes in extensive-margin adjustment is similar to the result of Bertola and Caballero (1994).

of the idiosyncratic shock, plants with lower productivity levels increase their target capital by less than do those with higher productivity. At the same time, the possibility of large idiosyncratic shocks in future periods that may offset the current rise in aggregate productivity reduces even high productivity plants' willingness to increase capital. Thus, it is not only extensive margin changes, but also those at the intensive margin, that are reduced by the inclusion of large plant-specific idiosyncratic shocks, thereby reducing the skewness in the distribution of aggregate investment rates that otherwise characterizes models of lumpy investment under partial equilibrium (above in table 5).

4.2 General equilibrium

In general equilibrium, the aggregate differences between the lumpy investment models and the standard models are largely eliminated. Table 6 shows that the standard deviation of aggregate investment rates is identical across the standard and lumpy investment models, whether or not there are idiosyncratic variations in plant productivity. Moreover, there are virtually no differences in the persistence of aggregate investment rates, which are far higher than their partial equilibrium counterparts, and very close to the data.[9] Persistence in aggregate investment rates is an immediate result of consumption smoothing by the representative household in general equilibrium. The omission of this channel in partial equilibrium places an emphasis on capital adjustment costs to generate some of this persistence that is otherwise lost.

General equilibrium also eliminates most of the differences in skewness and excess kurtosis across models. Moreover, comparing any one model to its partial equilibrium counterpart in table 5, we see that *equilibrium dramatically reduces the skewness and excess kurtosis in the distribution of aggregate investment rates*. This is our third result. As discussed above, the skewness exhibited by lumpy investment models in partial equilibrium arises because changes in aggregate productivity are followed by large movements in target capital that cause sharp, concurrent changes in the fraction of plants undertaking capital adjustment. When we impose market-clearing, however, such aggregate investment spikes would imply large movements in consumption. This consumption volatility is sharply restrained by procyclical real interest rates, which dampen the changes in target capital arising from aggregate shocks.

[9] The first-order autocorrelation of the aggregate investment rate is 0.7068 in the data.

For example, the rise in aggregate productivity that caused a 16.67 percent average increase in target capital in the partial equilibrium full model of lumpy investment now induces only a 1.51 percent increase. This is a standard result of households' preference for smooth consumption profiles, as familiar from the optimal growth model. As real interest rates rise with an increase in aggregate productivity, plants' incentive to increase capital is mitigated. Thus, the adjustment hazards move far less in general equilibrium. Large shifts in hazards, which interact with the underlying distribution of plants, are a prerequisite for significant variation in the number of adjusting plants. In the absence of such large shifts, the fraction of adjustors changes relatively little with aggregate shocks. Consequently, there is little variation in extensive margin adjustment, precluding aggregate nonlinearities.

Tables 7 and 8 confirm this finding. While partial equilibrium suggests that there are pronounced differences in the variability of output and investment when either lumpy model is compared to its standard counterpart, these differences disappear in general equilibrium. Examining the variabilities and contemporaneous correlations of output, investment share, employment and capital, we see that the aggregate business cycle is essentially unaffected by lumpy investment and by idiosyncratic shocks to plants.[10]

4.3 Plant-level investment

Tables 9 and 10 examine investment dynamics at the plant level in both the basic and the common productivity models. Using these tables, we will focus on three particular aspects of plant investment in this section: persistence, the effects of equilibrium and the role of nonconvex costs. We will also examine how each of these aspects is affected by the presence of large idiosyncratic shocks to productivity.

4.3.1 Persistence in the standard models

One striking feature of tables 9 and 10 is that, in most cases, there is a negative autocorrelation in plant investment rates. In fact, across these tables, the only case of persistent plant investment is that in table 9 corresponding to the standard common productivity model in general equilibrium.

[10] Here, we report moments for investment's share of output rather than investment, since investment is at times negative in the partial equilibrium simulation. We do not report the moments for consumption's share, as they are immediate from $\frac{C}{Y} = 1 - \frac{I}{Y}$.

Consider first the standard model under common productivity, where there is a representative firm and no difference between plant and aggregate investment. In general equilibrium, capital adjusts gradually to changes in aggregate productivity, due to equilibrium movements in wages and interest rates; thus, investment is persistent. In partial equilibrium, by contrast, capital adjustment is completed immediately following a change in aggregate productivity, and, as a result, we see no persistence in investment.

Continuing to examine the standard model, we next consider the effect of idiosyncratic productivity differences on establishment-level investment. In the standard full model, plants' decision rules for capital are independent of their existing stocks, as proven in lemma 1 of the appendix. Holding the aggregate state constant, and absent adjustment costs, capital at the plant tracks idiosyncratic productivity with a one-period lag; a change in plant productivity this period causes an immediate and complete adjustment in capital for the next period. As a result, while plants' capital stocks inherit the persistence of the idiosyncratic shock process, their investments lack persistence. This tends to generate negative autocorrelation in plant investment rates in the full model, where plants experience large and mildly persistent movements in their productivities. Moreover, the partial equilibrium dynamics of the common productivity model, discussed in the paragraph above, imply that changes in aggregate productivity only reinforce this tendency. Thus, investment rates are negatively autocorrelated in the full standard model in partial equilibrium, as seen in the first row of table 10A.

As we have already noted, general equilibrium introduces gradual changes in the total capital stock of the common productivity standard model. The same holds for the full standard model, since lemma 2 of the appendix implies that its dynamics are fully recoverable using a representative firm approach. However, comparing the first row of tables 9A and 10A, we see that changes in the equilibrium aggregate state fail to have significant impact on the persistence, or indeed the average distribution, of plant-level investment rates. The same is true for both the common productivity and full lumpy investment models. We will return to this issue in section 4.3.3.

4.3.2 Persistence in the lumpy investment models

In the lumpy investment models, fixed costs of capital adjustment lead to a large number of inactive plants on average, as seen in both rows of tables 9B and 10B. In partial equilibrium, this inaction makes adjustments in the total capital stock more gradual, and thereby increases the persistence of aggregate investment rates, as we discussed in section 4.1.1. However, when we examine the common productivity models, we see that this is not the case at the establishment level. First, recall from equation (10) that the target capital stock for any plant is independent of its current capital. Thus, active changes at the plant are not gradual, leading investment to lack persistence. Moreover, in the absence of idiosyncratic shocks, an active adjustment by the typical plant in any given period is generally followed by one or more periods of zero investment, given rising adjustment hazards. This also tends to generate a negative autocorrelation in plant investment rates, and we see a sharp difference relative to the persistent investment undertaken by the representative plant in the corresponding equilibrium standard model.

In the presence of large idiosyncratic shocks, the effect of nonconvex costs on plant-level investment persistence is reversed; that is, the full lumpy investment model exhibits more persistence in investment rates (a less negative autocorrelation) than does the corresponding standard model. As was the case with the full standard model, the plant-specific productivity shocks cause a negative autocorrelation in plant investment. However, this is mitigated by nonconvex adjustment costs for two reasons. First, following a shock to its productivity, an adjusting plant is cautious in selecting the size of its capital adjustment in an effort to avoid readjusting, and hence paying another fixed cost, in the near future when its productivity may change again. Moreover, the resulting reduction in the distances between target capitals associated with differing plant-specific productivity levels implies that fewer plants find it worthwhile to undertake an active adjustment in response to such a shock. Thus, in the full model, we see substantially more inaction and a less negative autocorrelation in investment rates when adjustment costs are present. Overall, plant-level investment becomes less volatile.

4.3.3 Effects of equilibrium

As noted above, market-clearing changes in real wages and interest rates lead to sharp changes in plant investment behavior in the common productivity standard model. However, when we compare row 2 of tables 9B and 10B, this does not appear true for the common productivity lumpy investment model. Much of plant-level investment there represents a reallocation of the investment good from nonadjusting to adjusting plants. As such reallocation has no implication for aggregate investment, it is unaffected by equilibrium movements in real wages and interest rates. We also find little effect of general equilibrium in the results for both the full standard and full lumpy investment models. The average fraction of plants exhibiting inaction is largely unaffected, as are the average fractions exhibiting spikes and positive and negative investment rates, and the negative autocorrelation in investment rates remains.

This brings us to our fourth result. *In the presence of either nonconvex capital adjustment costs or large idiosyncratic productivity differences, equilibrium has relatively little impact on the average cross-sectional distribution of plant investment rates.* Both nonconvex costs and idiosyncratic shocks lead to a nontrivial distribution of plants over individual states. Each plant responds to its capital stock and its current productivity and/or fixed cost, so investment differs across plants, and the investment of any given plant relative to others' changes over time. In each period, there is a reallocation of investment across plants that does not affect total investment demand and, hence, is not affected by changes in the relative price of consumption. Moreover, given that the calibrated aggregate shock to total factor productivity has relatively low variance, much of an individual plant's investment, on average, results from such reallocation. Thus, irrespective of equilibrium price movements, the average cross-sectional distribution for the stochastic economy closely resembles that of the deterministic steady state.[11] This suggests that model-based estimation of capital adjustment costs, such as Cooper and Haltiwanger (2002), may not be very sensitive to equilibrium analysis if such average moments are used.

There is, however, a caveat to our finding. The distribution of plant investment rates changes over time with the aggregate state, and such changes can be very sensitive to movements in real interest rates. For example, consider the common productivity lumpy investment model. There, the average fraction of inactive plants is roughly 0.78 in both partial and general equilibrium. However, the standard deviation of this fraction is 0.12 when real wages and interest rates are held fixed at their steady state values, while it is 0.01 in equilibrium. Similarly, while the mean fraction of plants exhibiting positive spikes is the same, the standard deviation of this fraction is 0.12 under partial equilibrium versus 0.01 in general equilibrium, and the standard deviation of the size of positive spikes in partial

[11] By contrast, the average distribution of investment rates in the standard common productivity model merely represents the time-averaged observations of a single representative plant's investment across dates. There, equilibrium price determination is essential.

equilibrium is five times that with market-clearing changes in relative prices.[12] Analogous results hold with regard to the remaining cross-sectional moments of table 9 versus 10, both for this model and for those with idiosyncratic productivity differences. Based on this, we conclude that equilibrium analysis is essential in understanding the *dynamics* of plant-level investment.

4.3.4 Role of nonconvex costs

Examining the lumpy investment models in panel B of table 9, we find that idiosyncratic shocks allow a better fit to the data, in that they imply both negative investment rates and negative spikes. However, comparing each row of panel B to its standard model counterpart in panel A reveals another important aspect of these shocks. Their presence substantially alters the role of nonconvex adjustment costs in shaping investment at the plant.

Notice the changes in the plant investment moments that occur in moving from the standard model to the lumpy investment model under common productivity, and compare these to the changes that occur in moving from the full standard to the full lumpy model. We have already discussed how large idiosyncratic shocks change the effect of nonconvex costs for investment persistence at the plant; in the absence of these shocks, nonconvex costs reduce persistence, while this is reversed in their presence. Perhaps more importantly, in the common productivity models of table 9, we see that nonconvex costs lead to the defining features of lumpy investment: positive spikes and inaction. However, comparison of the two standard models in panel A reveals that the idiosyncratic shocks on their own substantially raise the plant observations of both positive and negative spikes. In fact, for the full standard model, fixed costs are no longer necessary to generate investment spikes; they are already overstated relative to the data. Instead, in the full models, the primary role of the adjustment costs now seems to be to induce inaction, *reduce* spikes, and increase the asymmetry between the average fractions of plants exhibiting positive versus negative spikes. In this sense, nonconvex costs have a quite different effect upon plant-level investment when we assume large and persistent differences in plant-level total factor productivity.

[12] This higher variability in partial equilibrium is caused by large changes in target capital that, in turn, cause big swings in adjustment rates, as was seen in the example of section 4.1.2.

5 Extended model

Thus far we have examined the interaction of idiosyncratic productivity differences and nonconvex adjustment costs under the assumption that all nonzero plant-level investments incur fixed costs. Given that assumption, in order to match the average occurrence of positive and negative spike episodes in the plant-level data, we found it necessary to substantially exaggerate inaction. In this section, we work to correct this problem by extending the model to allow some low-level capital adjustments that are exempt from fixed costs.

In this *extended lumpy* model, we assume that plants choosing investment rates satisfying $a \leq \frac{i}{k} \leq b$, where $a \leq 0 \leq b$, do not incur any adjustment costs. Note that this includes our previous lumpy investment model as a special case when $a = b = 0$. However, when $a < 0 < b$, a plant not paying its adjustment cost can still undertake some active increase or reduction in its capital. In this case, unlike the model examined above, investment at the plant is almost never 0; thus, the frequency of inactive observations may be reduced.

After production, a plant with current capital k and adjustment cost draw ξ can either pay its fixed cost ($\omega\xi$ in units of current output) and undertake an *unconstrained* investment to reach any chosen $k' \in \mathcal{K}$, or it can avoid the cost by selecting a *constrained* investment, $i \in [ak,\ bk]$. Note that the constrained investment choice set directly implies a set of possible values for k'. Let $\Lambda(k) \subseteq \mathcal{K}$ represent the set of capital stocks available to a constrained investor with current capital k:

$$\Lambda(k) = \Big[\frac{1-\delta+a}{\gamma}k,\ \frac{1-\delta+b}{\gamma}k\Big].$$

To facilitate our description of the plant's problem, we define the gross continuation value associated with any future capital stock, k', as

$$q\left(\varepsilon, k'; z, \mu'\right) \equiv -\gamma k' + \sum_{j=1}^{N_z} \pi_{ij} d_j\left(z, \mu\right) \sum_{l=1}^{N_e} \pi^{\varepsilon}_{kl} v^0\left(\varepsilon_l, k'; z_j, \mu'\right). \tag{14}$$

As before, let $v^1(\varepsilon_k, k, \xi; z_i, \mu)$ represent the expected discounted value of a plant entering the period with (ε_k, k) and drawing an adjustment cost ξ when the aggregate state of the economy is (z_i, μ), where $v^0(\varepsilon_k, k; z_i, \mu)$ is the expectation over the adjustment cost defined in (1). Taking as given the evolution of the plant distribution, $\mu' = \Gamma(z, \mu)$, the plant solves the following dynamic optimization problem:

$$v^1(\varepsilon,k,\xi;z,\mu)=\max_n\Bigg[z\varepsilon F(k,n)-\omega(z,\mu)n+(1-\delta)k \tag{15}$$
$$+\max\left\{-\xi\omega(z,\mu)+\max_{k'\in\mathcal{K}}q\left(\varepsilon,k';z,\mu'\right),\ \max_{k'\in\Lambda(k)}q\left(\varepsilon,k';z,\mu'\right)\right\}\Bigg].$$

Given the equilibrium wage rate $\omega(z,\mu)$, a plant of type (ε,k,ξ) first chooses its current employment n. This choice remains independent of ξ as in our previous model; thus, we continue to denote the common employment selected by all type (ε,k) plants as $N(\varepsilon,k;z,\mu)$. Next, the plant decides upon either an unconstrained or a constrained choice of its capital stock for next period. The unconstrained choice, in the first term of the binary maximum above, requires payment of the fixed labor cost of capital adjustment. However, if $k'\in\Lambda(k)$ is selected, the second term in the binary maximum applies, and this cost is avoided.

As before, let $K(\varepsilon,k,\xi;z,\mu)$ represent the capital decision rule for plants of type (ε,k) with adjustment cost ξ. A *recursive competitive equilibrium* is then a set of functions,

$$\left(\omega,(d_j)_{j=1}^{N_z},\rho_0,\rho_1,v^1,N,K,W,C,N^h,\Lambda\right),$$

such that plants and households maximize their expected values, and the markets for assets, labor and output clear:

1. v^1 satisfies (1) and (14) - (15), and (N,K) are the associated policy functions for plants.

2. W satisfies (3), and $\left(C,N^h,\Lambda\right)$ are the associated policy functions for households.

3. $\Lambda(\varepsilon_l,k',\mu;z,\mu)=\mu'(\varepsilon_l,k')$.

4. $N^h(\mu;z,\mu)=\int_{\mathcal{S}}\left(N(\varepsilon,k;z,\mu)+\int_0^{\overline{\xi}}\xi\mathcal{J}\left(\frac{K(\varepsilon,k,\xi;z,\mu)-\frac{(1-\delta)}{\gamma}k}{k}\right)G(d\xi)\right)\mu(d[\varepsilon\times k])$, where $\mathcal{J}(x)=0$ if $x\in\left[\frac{a}{\gamma},\frac{b}{\gamma}\right]$; $\mathcal{J}(x)=1$ otherwise.

5. $C(\mu;z,\mu)=\int_{\mathcal{S}}\left(z\varepsilon F(k,N(\varepsilon,k;z,\mu))-\int_0^{\overline{\xi}}[\gamma K(\varepsilon,k,\xi;z,\mu)-(1-\delta)k]G(d\xi)\right)\mu(d[\varepsilon\times k])$.

6. $\mu'(\varepsilon_l,B)=\int_{\{(\varepsilon_k,k,\xi)\,|\,K(\varepsilon_k,k,\xi;z,\mu)\in B\}}\pi_{kl}^{\varepsilon}G(d\xi)\,\mu(d[\varepsilon_k\times k])$ defines Γ.

5.1 Characterizing the extended model

We follow our previous method in reformulating the plant's dynamic problem. Recall that $p(z,\mu) = D_1 U(C, 1-N)$ and $\omega(z,\mu) = \frac{D_2 U(C,1-N)}{p(z,\mu)}$. Suppressing the arguments of these price functions,

$$V^1(\varepsilon, k, \xi; z, \mu) = \max_n \Bigg([z\varepsilon F(k,n) - \omega n + (1-\delta)k] p \qquad (16)$$
$$+ \max \left\{ -\xi \omega p + \max_{k' \in \mathcal{K}} Q(\varepsilon, k'; z, \mu'), \max_{k' \in \Lambda(k)} Q(\varepsilon, k'; z, \mu') \right\} \Bigg),$$

where

$$Q(\varepsilon, k'; z, \mu') \equiv -\gamma k' p + \beta \sum_{j=1}^{N_z} \pi_{ij} \sum_{l=1}^{N_e} \pi_{kl}^{\varepsilon} V^0(\varepsilon_l, k'; z_j, \mu') \qquad (17)$$

and

$$V^0(\varepsilon, k; z, \mu) \equiv \int_0^{\overline{\xi}} V^1(\varepsilon, k, \xi; z, \mu) G(d\xi). \qquad (18)$$

Equations (16) - (18) are the basis of our numerical solution of the extended model economy.

Note that, as before, plants choose labor $n = N(\varepsilon, k; z, \mu)$ to solve $z\varepsilon D_2 F(k,n) = \omega(z,\mu)$. In examining the capital choice made by a type (ε, k, ξ) plant, we define the gross value associated with the unconstrained capital choice, $E(\varepsilon, z, \mu)$, and the value of the constrained choice, $E^C(\varepsilon, k, z, \mu)$, as follow:

$$E(\varepsilon, z, \mu) \equiv \max_{k' \in \mathcal{K}} Q(\varepsilon, k'; z, \mu') \qquad (19)$$
$$E^C(\varepsilon, k, z, \mu) \equiv \max_{k' \in \Lambda(k)} Q(\varepsilon, k'; z, \mu'). \qquad (20)$$

As in our previous model, the solution to the unconstrained problem in (19) depends upon ε, but does not depend upon k or ξ. Thus, defining the capital that solves this problem as the plant's *target capital*, we again have the result that all plants sharing the same current productivity ε and paying their fixed costs will adjust to a common target capital for the next period, $k' = k^*(\varepsilon, z, \mu)$. Plants that do not pay adjustment costs, instead undertaking constrained capital adjustments solving (20), will choose future capital that may depend on their current capital, $k' = k^C(\varepsilon, k, z, \mu)$. (The exception occurs for plants with $k^*(\varepsilon, z, \mu) \in \Lambda(k)$; for such plants, the constraint in (20) does not bind, and the target capital may be achieved without an adjustment cost.)

Examining (16), we see that a plant will absorb its fixed cost to undertake an unconstrained capital adjustment if the net value of achieving the target capital, $E(\varepsilon, z, \mu) - \xi\omega p$, is at least as great as its continuation value under constrained adjustment, $E^C(\varepsilon, k, z, \mu)$. Let $\widehat{\xi}(\varepsilon, k; z, \mu)$ describe the fixed cost that leaves a type (ε, k) plant indifferent between these options:

$$-p(z,\mu)\,\widehat{\xi}(\varepsilon, k; z, \mu)\,\omega(z,\mu) + E(\varepsilon, z, \mu) = E^C(\varepsilon, k, z, \mu). \tag{21}$$

Next define $\xi^T(\varepsilon, k; z, \mu) \equiv \min\left\{\overline{\xi}, \max\left\{0, \widehat{\xi}(\varepsilon, k; z, \mu)\right\}\right\}$, so that $0 \le \xi^T(\varepsilon, k; z, \mu) \le \overline{\xi}$. Any plant with an adjustment cost at or below its type-specific threshold, $\xi^T(\varepsilon, k; z, \mu)$, will pay the fixed cost and adjust to its target capital.

Using the constrained and unconstrained choices of future capital, alongside the threshold adjustment costs, the plant-level decision rule for capital is as follows. Any establishment identified by the plant-level state vector $(\varepsilon, k, \xi; z, \mu)$ will begin the subsequent period with capital given by

$$k' = K(\varepsilon, k, \xi; z, \mu) = \begin{cases} k^*(\varepsilon, z, \mu) & \text{if} \quad \xi \le \xi^T(\varepsilon, k; z, \mu), \\ k^C(\varepsilon, k, z, \mu) & \text{if} \quad \xi > \xi^T(\varepsilon, k; z, \mu). \end{cases} \tag{22}$$

Based on (22), we now explicitly define the evolution of the plant distribution, $\mu' = \Gamma(z, \mu)$. This law of motion is somewhat involved because we have to account for those plants that can reach their unconstrained target capital stock without paying fixed costs. For all $\left(\varepsilon_l, \widehat{k}\right) \in \mathcal{S}$, define the indicator function $\overline{\mathcal{J}}(x) = 1$ for $x = 0$; $\overline{\mathcal{J}}(x) = 0$ for $x \neq 0$, and we have

$$\begin{aligned} \mu'\left(\varepsilon_l, \widehat{k}\right) = \sum_{k=1}^{N_\varepsilon} \pi_{kl}^{\varepsilon} \Bigg[& \overline{\mathcal{J}}\left(\widehat{k} - k^*(\varepsilon_k, z, \mu)\right)\Bigg(\int_{\mathcal{S}} G\left(\xi^T(\varepsilon_k, k; z, \mu)\right)\mu(\varepsilon_k, dk) \\ & + \int_{\left[\frac{\gamma}{1-\delta+b}k^*(\varepsilon_k,z,\mu),\ \frac{\gamma}{1-\delta+a}k^*(\varepsilon_k,z,\mu)\right]\cap\mathcal{K}} \mu(\varepsilon_k, dk)\Bigg) \\ & + \int_{\left[0,\ \frac{\gamma}{1-\delta+b}k^*(\varepsilon_k,z,\mu)\right)\cap\mathcal{K}} \left[1 - G\left(\xi^T(\varepsilon_k, k; z, \mu)\right)\right]\overline{\mathcal{J}}\left(\widehat{k} - k^C(\varepsilon_k, k, z, \mu)\right)\mu(\varepsilon_k, dk) \\ & + \int_{\left(\frac{\gamma}{1-\delta+a}k^*(\varepsilon_k,z,\mu),\ \infty\right)\cap\mathcal{K}} \left[1 - G\left(\xi^T(\varepsilon_k, k; z, \mu)\right)\right]\overline{\mathcal{J}}\left(\widehat{k} - k^C(\varepsilon_k, k, z, \mu)\right)\mu(\varepsilon_k, dk)\Bigg]. \end{aligned} \tag{23}$$

The first two lines in equation (23) apply only when $\widehat{k} = k^* (\varepsilon_k, z, \mu)$, for each given ε_k, $k = 1, \ldots, N_\varepsilon$. The first line captures plants that pay fixed costs to adjust to this target. The second line reflects all plants (ε_k, k) that achieve this target without paying fixed costs, because $k^* (\varepsilon_k, z, \mu) \in \Lambda (k)$. The third and fourth lines of the equation apply when $\widehat{k}$ is not the target capital stock for the given idiosyncratic shock value. The set of plants in the third line are those that have drawn adjustment costs above their threshold, $\xi^T (\varepsilon_k, k; z, \mu)$, and face a binding upper constraint on their capital choice, as $k^* (\varepsilon_k, z, \mu) > \frac{\gamma}{1-\delta+b} k$. Of these plants, those with $\widehat{k} = k^C (\varepsilon, k, z, \mu)$ adjust to $\widehat{k}$. The fourth line represents plants not paying adjustment costs that have current capital too high to allow them to reach the unconstrained target; they adopt $\widehat{k}$ if $\widehat{k} = k^C (\varepsilon, k, z, \mu)$.

Finally, the market-clearing level of consumption is now given by

$$C = \int_{\mathcal{S}} \Big(z\varepsilon F (k, N (\varepsilon, k; z, \mu)) - G \left(\xi^T (\varepsilon, k; z, \mu)\right) \Big[\gamma k^* (\varepsilon, z, \mu) - (1 - \delta) k\Big] \\ - \Big[1 - G \left(\xi^T (\varepsilon, k; z, \mu)\right)\Big] \Big[\gamma k^C (\varepsilon, k, z, \mu) - (1 - \delta) k\Big] \Big) \mu (d [\varepsilon \times k]) . \tag{24}$$

This equation, alongside that determining total hours worked in (13), defines the equilibrium output price and wage in equations (4) and (5).

5.2 Calibration and model solution

Our goal in extending the lumpy investment model is to provide a better match with the microeconomic data on establishment-level investment. Recall from table 9 that the full lumpy model (panel B, row 1) was more successful than its common productivity counterpart in that it produced some plant-level observations of negative investment and negative spikes. However, it still dramatically overpredicted the extent of inaction, with inactive investments representing more than three-quarters of plant-year observations. (By contrast, the data exhibit such low investment rates only 8 percent of the time.) Consequently, the model had far too few observations of active positive and negative investment.

The extended full lumpy model maintains all parameter values of the original full model other than those involving the capital adjustment costs. Here we depart from existing quantitative (S, s) investment studies (for example, Cooper and Haltiwanger (2002), Thomas (2002) and Khan and Thomas (2003)) by assuming that plants do not face capital adjustment costs when they undertake nonzero investments that are sufficiently small relative to their existing capital stocks. To implement this, we assume symmetric bounds for the cost-exempted investment rates; $-a = b$. Next, we select the value of b, alongside the upper support on adjustment costs, $\overline{\xi}$, to best match three moments from the plant-level investment data: the average fractions of plants exhibiting inaction, positive investment spikes and negative investment spikes. This leads to a choice of $-a = b = 0.015$ and $\overline{\xi} = 0.00975$.

We solve the extended model using broadly the same numerical method that we used in solving the original equilibrium lumpy investment models. However, because plants that do not pay their fixed costs now typically invest to future capitals that depend upon both their current stock and their current productivity, the size of the distribution in the aggregate state vector is dramatically increased.[13] In equilibrium, this object involves a support with 2250 values of capital across the 11 idiosyncratic shock levels. Nonetheless, when we solve this model in general equilibrium following the approach discussed in section 3.4, no forecasting coefficient changes by more than 0.002 relative to those reported in table 4 for the original full lumpy model. Furthermore, the adjusted R-squares and standard errors in the forecasting regressions are either unchanged or marginally improved. These similarities suggest that reducing the incidence of nonconvex adjustment costs has little effect on the aggregate economy, as will be confirmed in the results below.

5.3 Results

As our motive for developing this extension was to improve the lumpy investment model's predictions for average plant-level investment rates, we begin by discussing the plant results under partial and general equilibrium in table 11. The most notable feature of the table is that the distance between model and data is now largely eliminated. The average fraction of plants exhibiting inaction, at 0.048, is just 3 percentage points below its empirical value of 0.081, while it is 73 percentage points below its exaggerated counterpart from the original full lumpy model.

Consider a plant with current capital sufficiently far from its target capital that it cannot reach this target without incurring a fixed adjustment cost. If it chooses not to pay its fixed cost, it can nonetheless undertake an adjustment of up to 1.5 percent of its current stock toward the target. When this plant undertakes such a constrained investment, it is

[13] By contrast, in the original model where all nonzero investments incurred fixed costs, the future capital of any plant not paying its fixed cost was simply $\frac{1-\delta}{\gamma}$ times its current stock, regardless of its productivity.

inactive only if the bounds on its constrained adjustment choice do not bind, and the investment rate that achieves its target is below 1 percent in absolute value. By contrast, in the original lumpy model, any plant not paying its fixed cost was necessarily inactive.

Indeed, when we compare the plant-level moments of the full lumpy model of table 9 with those of its extended counterpart in table 11, we see that the majority of plants that were previously inactive are now engaged in positive investment. Such plants are partly offsetting the effects of depreciation in periods when they choose not to engage in large investments that would attain their target but incur a fixed cost. As a result, the average fraction of plant-year observations that have positive investment rates, at 0.72, is now close to its empirical counterpart. At the same time, plants can now also undertake small negative adjustments while avoiding their fixed costs. Moreover, the ability to undertake small positive investments exempt from adjustment costs in the future reduces their reluctance to disinvest after a fall in productivity. Consequently, the observation of negative investment rates has also risen substantially, and now exceeds the data by 13 percentage points.

Aside from its better ability to explain the average establishment-level moments, the extended model changes little in our main findings about idiosyncratic shocks. In section 4.3.3, we saw that market-clearing movements in real wages and interest rates have little effect on either the average distribution of plant-level investment rates or their persistence. The second and third rows of table 11 reveal that this is still very much the case. Moreover, the role of nonconvex adjustment costs under idiosyncratic shocks is unaltered in the extended model. Comparing table 11 to the full standard model in panel A of table 9, we see that adjustment costs continue to reduce investment spikes and drive an asymmetry between positive and negative rates.

The extended model does not alter our findings about aggregate investment dynamics; its aggregate moments are largely indistinguishable from those of the original full lumpy model. Examining row 2 of table 12, note that the extended lumpy model continues to exhibit more persistence, lower volatility, and more skewness and kurtosis in its partial equilibrium aggregate investment rates than does the full standard model without adjustment costs (table 5A, row 1). As before, market-clearing changes in real wages and interest rate induce a sharp rise in persistence and a sharp reduction in volatility and nonlinearities. Indeed, the extended model's general equilibrium results for aggregate investment rates match those in the rows of table 6 very closely. Finally, in table 13, we see that, under both partial and general equilibrium, the business cycle behavior of output, investment's share, employment and capital are all unchanged relative to their lumpy investment counterparts in rows 2 and 4 of table 7.

6 Concluding Remarks

We have studied partial and general equilibrium models of lumpy investment with and without persistent differences in plants' total factor productivity. In partial equilibrium, we found that lumpy investment caused increased persistence and nonlinearities in aggregate investment, although nonlinearities were reduced in the presence of persistent idiosyncratic shocks. Across all models, investment persistence rose substantially with the inclusion of general equilibrium changes in relative prices, and this persistence was quantitatively unaffected by the presence of either capital adjustment costs or idiosyncratic productivity differences. Finally, our equilibrium models of lumpy investment exhibited little aggregate nonlinearity relative to the corresponding models without adjustment costs.

Examining investment at the plant, we found that the lumpy investment model succeeded in matching the average distribution of investment rates in the establishment data only when it was extended to allow for both persistent idiosyncratic productivity shocks and some low-level investment rates not subject to adjustment costs. Across models, we found that this average cross-sectional distribution was relatively unchanged by equilibrium if either fixed adjustment costs or plant-specific productivities were present. However, irrespective of these idiosyncratic variables, we saw that market-clearing changes in real wages and interest rates had important consequences for the higher moments of the plant investment distribution. Most notably, they reduced variability in the fractions of plants undertaking large capital adjustments, as well as the size of these investments, thus eliminating the potentially large distributional changes associated with aggregate nonlinearities. Finally, when present, idiosyncratic productivity shocks appeared to play a leading role in explaining investment at the plant, yielding a diminished role for fixed adjustment costs, particularly with regard to investment spikes.

In concluding, it may be useful to reiterate why the heterogeneity caused by idiosyncratic shocks or nonconvex adjustment costs makes the average distribution of plant investment rates so insensitive to equilibrium changes in real interest rates, while such movements qualitatively change the behavior of aggregate investment. Changes in interest rates dampen movements in aggregate investment demand and deliver a smooth path for household consumption. However, consumption is almost entirely unaffected by the reallocation of capital from one plant to another at a point in time in response to idiosyncratic variables. Indeed, when plants' output is perfectly substitutable, as it is in all of the models examined here, this reallocation of resources across plants is optimal from the perspective of households.

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Appendix: Idiosyncratic shocks in the standard model

In this appendix, we derive several analytical results for the full standard model characterized by persistent plant-specific total factor productivity shocks and no nonconvex costs of capital adjustment. In lemma 1, under the assumption of Cobb-Douglas production, we establish that the plant decision rule for next period's capital stock may be expressed as the product of two functions whose arguments are the current plant-specific productivity term and the aggregate state, respectively. Thus, in the absence of capital adjustment costs, a plant's decision rule for future capital is independent of its current capital. Moreover, this decision rule is separable in plant-level and aggregate variables.

It is then immediate that, given any initial distribution of plants, future distributions involve only N_ε time-varying values of capital with positive mass. The separability of plants' capital stock decision rules into a plant-specific and an aggregate component implies that the shares of the aggregate capital stock across plant types are time-invariant. In other words, the distribution of capital across plants, once normalized, satisfies a time-invariance property. This time-invariance property implies that in any period the entire distribution of capital, and thus production, may be described using a time-invariant share distribution and the aggregate capital stock, as established in lemma 2. As a result, the aggregate capital stock is sufficient to fully characterize variation in the endogenous state vector of the full version of the standard model, just as under common productivity. Moreover, it follows that all aggregate dynamics of the full model may be recovered using a representative firm approach, although for brevity we omit the details of this analysis.

We begin our analysis of the standard model by describing the problem of a plant. In the absence of capital adjustment costs, the value of any plant of type (ε_k, k) will solve the following functional equation:

$$\begin{aligned} v^1(\varepsilon_k, k; z_i, \mu) \;=\; & \max_{n,k'} \Bigg(z_i \varepsilon_k F(k,n) - \omega(z_i,\mu) n - \gamma k' + (1-\delta) k \Bigg) \\ & + \sum_{j=1}^{N_z} \pi_{ij}^z d_j(z_i,\mu) \sum_{l=1}^{N_\varepsilon} \pi_{kl} v^1\left(\varepsilon_l, k'; z_j, \mu'\right), \end{aligned} \tag{25}$$

subject to $\mu' = \Gamma(z_i,\mu)$. Let $N(\varepsilon_k, k; z_i, \mu)$ describe the plant's employment choice and $K(\varepsilon_k, k; z_i, \mu)$ its decision rule for next period's capital stock. The description of households in section 2.2 of the text is unchanged.

A *recursive competitive equilibrium* is a set of functions

$$\left(\omega, (d_j)_{j=1}^{N_z}, \rho_1, \rho_0, v^1, N, K, W, C, N^h, \Lambda, \Gamma\right)$$

such that plants and households maximize their expected values and the markets for assets, labor and output clear:

1. v^1 satisfies (25) and (N, K) are the associated policy functions for plants.
2. W satisfies (3) and $\left(C, N^h, \Lambda\right)$ are the associated policy functions for households.
3. $\Lambda\left(\varepsilon_l, k', \mu; z, \mu\right) = \mu'\left(\varepsilon_l, k'\right)$.
4. $N^h\left(\mu; z, \mu\right) = \int_{\mathcal{S}} N\left(\varepsilon, k; z, \mu\right) \mu(d\left[\varepsilon \times k\right])$.
5. $C\left(\mu; z, \mu\right) = \int_{\mathcal{S}} \Big(z\varepsilon F\left(k, N\left(\varepsilon, k; z, \mu\right)\right) - \gamma K\left(\varepsilon, k, \xi; z, \mu\right) + (1-\delta) k\Big) \mu(d\left[\varepsilon \times k\right])$.
6. $\mu'\left(\varepsilon_l, B\right) = \int_{\{(\varepsilon_k, k) \,|\, K(\varepsilon_k, k; z, \mu) \in B\}} \pi_{kl}^{\varepsilon} \mu\left(d\left[\varepsilon_k \times k\right]\right)$ defines Γ.

A Plant's capital decision rule

Let $\alpha \in (0,1)$ represent capital's share of production and $\nu \in (0,1)$ be labor's share, where $\alpha + \nu < 1$. The choice of employment, n, solves $\max_n \left(sk^{\alpha} n^{\nu} - \omega n\right)$, where $s = z\varepsilon$ and ω is the real wage. This yields the employment decision rule $n = \left(\frac{\nu s k^{\alpha}}{\omega}\right)^{\frac{1}{1-\nu}}$, allowing us to express production as $y = s^{\frac{1}{1-\nu}} k^{\frac{\alpha}{1-\nu}} \left(\frac{\nu}{\omega}\right)^{\frac{\nu}{1-\nu}}$. Production net of labor costs is then given by the following:

$$y - \omega n = (1-\nu)\, s^{\frac{1}{1-\nu}} k^{\frac{\alpha}{1-\nu}} \left(\frac{\nu}{\omega}\right)^{\frac{\nu}{1-\nu}}. \tag{26}$$

Substituting (26) into (25), we remove the static employment decision:

$$\begin{aligned} v^1\left(\varepsilon_k, k; z_i, \mu\right) &= \max_{k'} \Bigg((1-\nu) \left[z_i \varepsilon_k\right]^{\frac{1}{1-\nu}} k^{\frac{\alpha}{1-\nu}} \left(\frac{\nu}{\omega\left(z_i, \mu\right)}\right)^{\frac{\nu}{1-\nu}} \\ &\quad -\gamma k' + (1-\delta) k \Bigg) + \beta \sum_{j=1}^{N_z} \pi_{ij}^z d_j\left(z_i, \mu\right) \sum_{l=1}^{N_\varepsilon} \pi_{kl} v^1\left(\varepsilon_l, k'; z_j, \mu'\right). \end{aligned} \tag{27}$$

The first-order condition is

$$-\gamma + \beta \sum_{j=1}^{N_z} \pi_{ij}^z d_j\left(z_i, \mu\right) \sum_{l=1}^{N_\varepsilon} \pi_{kl}^{\varepsilon} D_2 v^1\left(\varepsilon_l, k'; z_j, \mu'\right) = 0.$$

Combining this with the Benveniste-Scheinkman condition below,

$$D_2 v^1 (\varepsilon_k, k; z_i, \mu) = \left(\alpha \left[z_i \varepsilon_k\right]^{\frac{1}{1-\nu}} k^{\frac{\alpha}{1-\nu}-1} \left(\frac{\nu}{\omega(z_i, \mu)}\right)^{\frac{\nu}{1-\nu}} + (1-\delta)\right),$$

we have a stochastic Euler equation for capital:

$$\gamma = \sum_{j=1}^{N_z} \pi_{ij}^z d_j (z_i, \mu) \sum_{l=1}^{N_\varepsilon} \pi_{kl}^\varepsilon \left(\alpha \left[z_j \varepsilon_l\right]^{\frac{1}{1-\nu}} (k')^{\frac{\alpha}{1-\nu}-1} \left(\frac{\nu}{\omega(z_j, \mu')}\right)^{\frac{\nu}{1-\nu}} + (1-\delta)\right). \tag{28}$$

Define the following terms:

$$L_0 (\varepsilon_k) = \left(\sum_{l=1}^{N_\varepsilon} \pi_{kl}^\varepsilon (\varepsilon_l)^{\frac{1}{1-\nu}}\right)^{\frac{1-\nu}{1-(\alpha+\nu)}} \tag{29}$$

$$L_1 (z_i, \mu) = \left(\frac{\gamma - \sum_{j=1}^{N_z} \pi_{ij}^z d_j (z_i, \mu) \left(\alpha z_j^{\frac{1}{1-\nu}} \left(\frac{\nu}{\omega(z_j, \mu')}\right)^{\frac{\nu}{1-\nu}}\right)}{\sum_{j=1}^{N_z} \pi_{ij}^z d_j (z_i, \mu) \left(1-\delta\right)}\right)^{\frac{1-\nu}{\alpha+\nu-1}}. \tag{30}$$

Simplification of (28) and use of the definitions in equations (29) - (30) proves the following.

Lemma 1 *The capital decision rule for a plant, $K(\varepsilon_l, k; z_i, \mu)$, is independent of k and takes the form $L_0(\varepsilon_l) L_1(z_i, \mu)$.*

B Aggregation

The result that plants' future capital stocks are independent of their current capital stocks is the central mechanism behind our aggregation result. This result is not shared by the lumpy investment model because of the inaction arising from its fixed adjustment costs.

We next exploit the result that the ratio of capital across any two plants depends only on their lagged productivity levels to describe how the dynamics of this economy may be solved as a standard optimal growth model, with the aggregate state vector effectively reduced to simply the aggregate capital stock and exogenous productivity.

Let $H = (h_1, \ldots, h_{N_\varepsilon})^T$ be the vector representing the time-invariant distribution of idiosyncratic shock values solving

$$H = \begin{vmatrix} \pi^{\varepsilon}_{1,1} & \pi^{\varepsilon}_{1,2} & \cdots & \pi^{\varepsilon}_{1,N_\varepsilon} \\ \pi^{\varepsilon}_{2,1} & \pi^{\varepsilon}_{2,2} & \cdots & \pi^{\varepsilon}_{2,N_\varepsilon} \\ \vdots & \vdots & & \vdots \\ \pi^{\varepsilon}_{N_\varepsilon,1} & \pi^{\varepsilon}_{N_\varepsilon,2} & \cdots & \pi^{\varepsilon}_{N_\varepsilon,N_\varepsilon} \end{vmatrix} H.$$

Since lemma 1 proves that capital decision rules are independent of current capital, it follows that all plants with the same current idiosyncratic shock value, ε_l, will choose the same capital stock for next period, $k_l = L_0(\varepsilon_l) L_1(z_i, \mu)$, $l = 1, \ldots, N_\varepsilon$. Thus, there will be N_ε capitals stock values with positive mass next period, and h_l plants, all currently having the idiosyncratic shock value ε_l, will begin the next period with k_l. Define the mean of this distribution of capital $K' = \sum_{l=1}^{N_\varepsilon} h_l k'_l$. Using lemma 1, we know

$$K' = \sum_{l=1}^{N_\varepsilon} h_l L_0(\varepsilon_l) L_1(z_i, \mu). \tag{31}$$

Toward establishing a time-invariant *relative* distribution of plants over capital, it is useful to define the following share terms:

$$\chi_m \equiv \frac{L_0(\varepsilon_m)}{\sum_{l=1}^{N_\varepsilon} h_l L_0(\varepsilon_l)}, \quad m = 1, \ldots, N_\varepsilon. \tag{32}$$

Define the vector of these share terms as $\chi \equiv \left(\chi_1, \ldots, \chi_{N_\varepsilon}\right)$.

While all plants with the same current idiosyncratic shock value will choose a common capital stock for next period, their subsequent idiosyncratic productivities will differ. Let $\widetilde{H}$ describe the two-dimensional distribution of plants over ε_{t-1} and ε_t. An element of this $N_\varepsilon \times N_\varepsilon$ matrix, $\widetilde{h}_{lm}$, represents the number of plants that had $\varepsilon_{t-1} = \varepsilon_l$ and $\varepsilon_t = \varepsilon_m$:

$$\widetilde{h}_{l,m} = \pi_{l,m} h_l, \qquad \text{for } l = 1, \ldots, N_\varepsilon \text{ and } m = 1, \ldots, N_\varepsilon. \tag{33}$$

In any period $t+1$, where $t \geq 0$, the distribution of plants is then completely characterized by $\widetilde{H}$ and χ together with the aggregate capital stock, K_{t+1}. This establishes lemma 2 below.

Lemma 2 *Let K be the aggregate capital stock, and define $k_l \equiv \chi_l K$, $l = 1, \ldots, N_\varepsilon$. For each ε_m, $m = 1, \ldots, N_\varepsilon$, $\mu(\varepsilon_m, k_l) = \widetilde{h}_{l,m} \geq 0$, and elsewhere $\mu = 0$.*

Thus, the distribution of plants over both idiosyncratic productivity levels and capital stocks has N_ε^2 elements in all. More importantly, this distribution is completely characterized by two time-invariant objects, $\widetilde{H}$ and χ, and the aggregate capital stock. It follows, then, that the aggregate state vector of the full standard model has only two time-varying elements, aggregate capital and exogenous aggregate productivity.

Figure 1: Adjustment responses in common productivity lumpy model

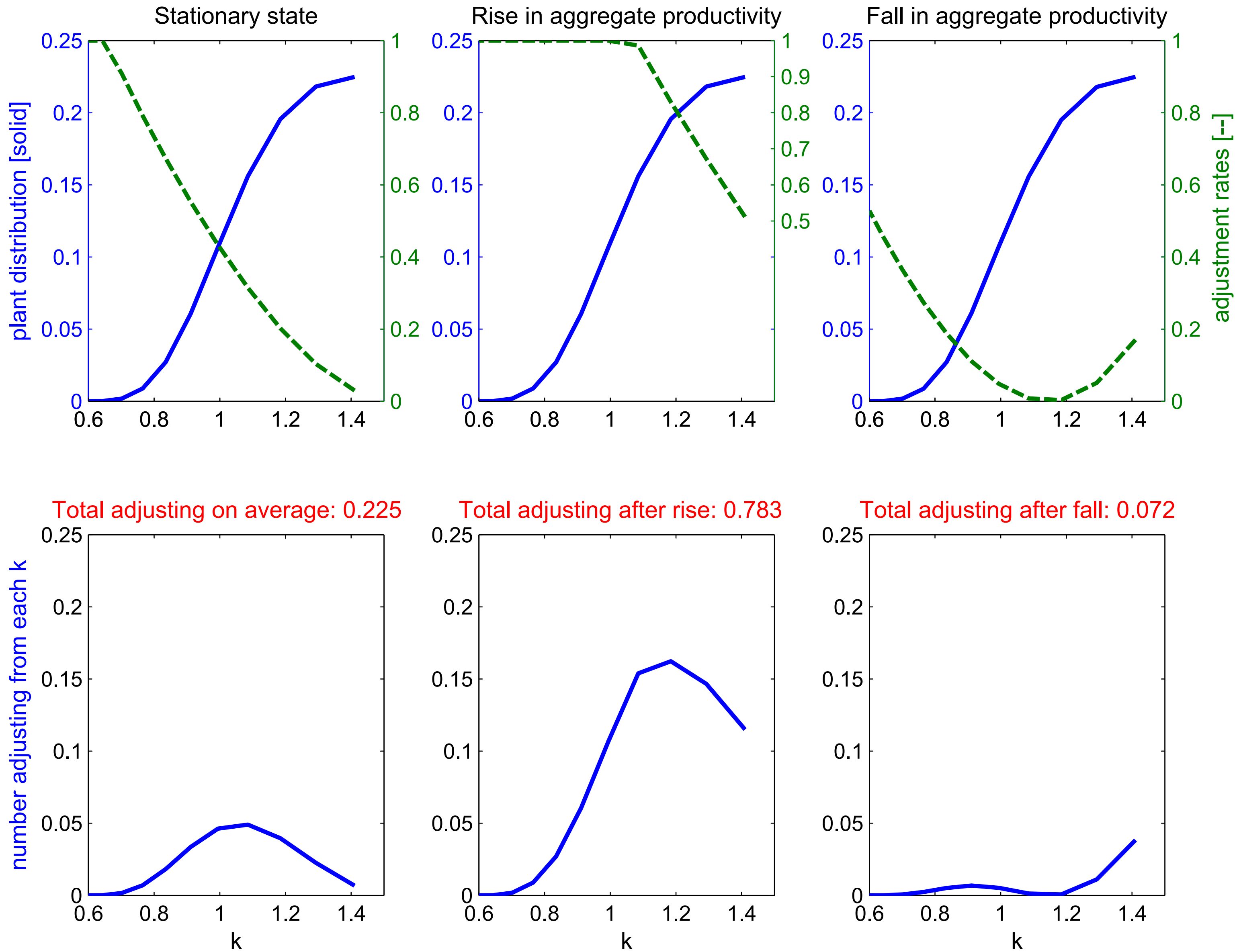

Figure 2: Asymmetries and amplification

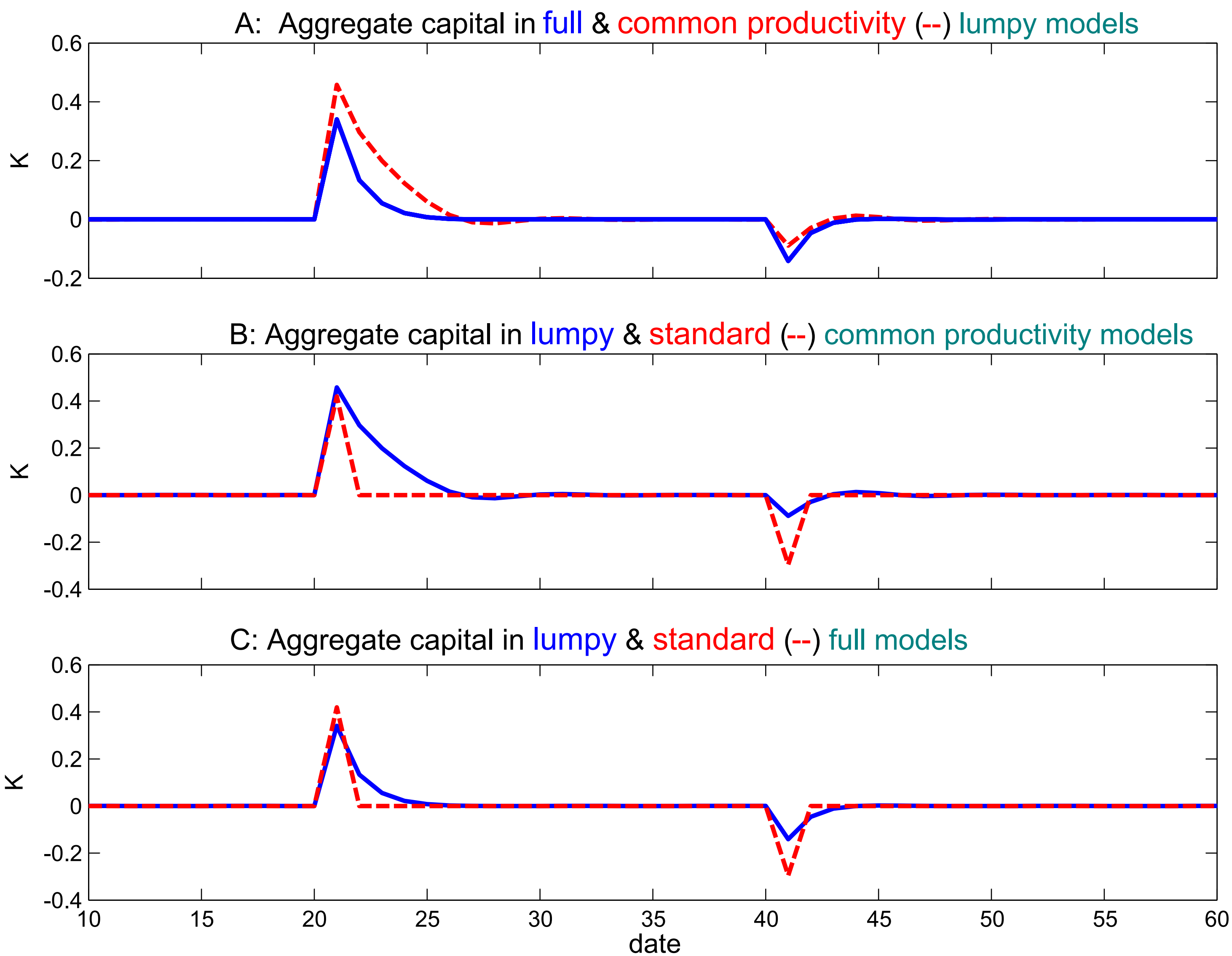

Figure 3: Stationary state distribution and hazard in full lumpy model

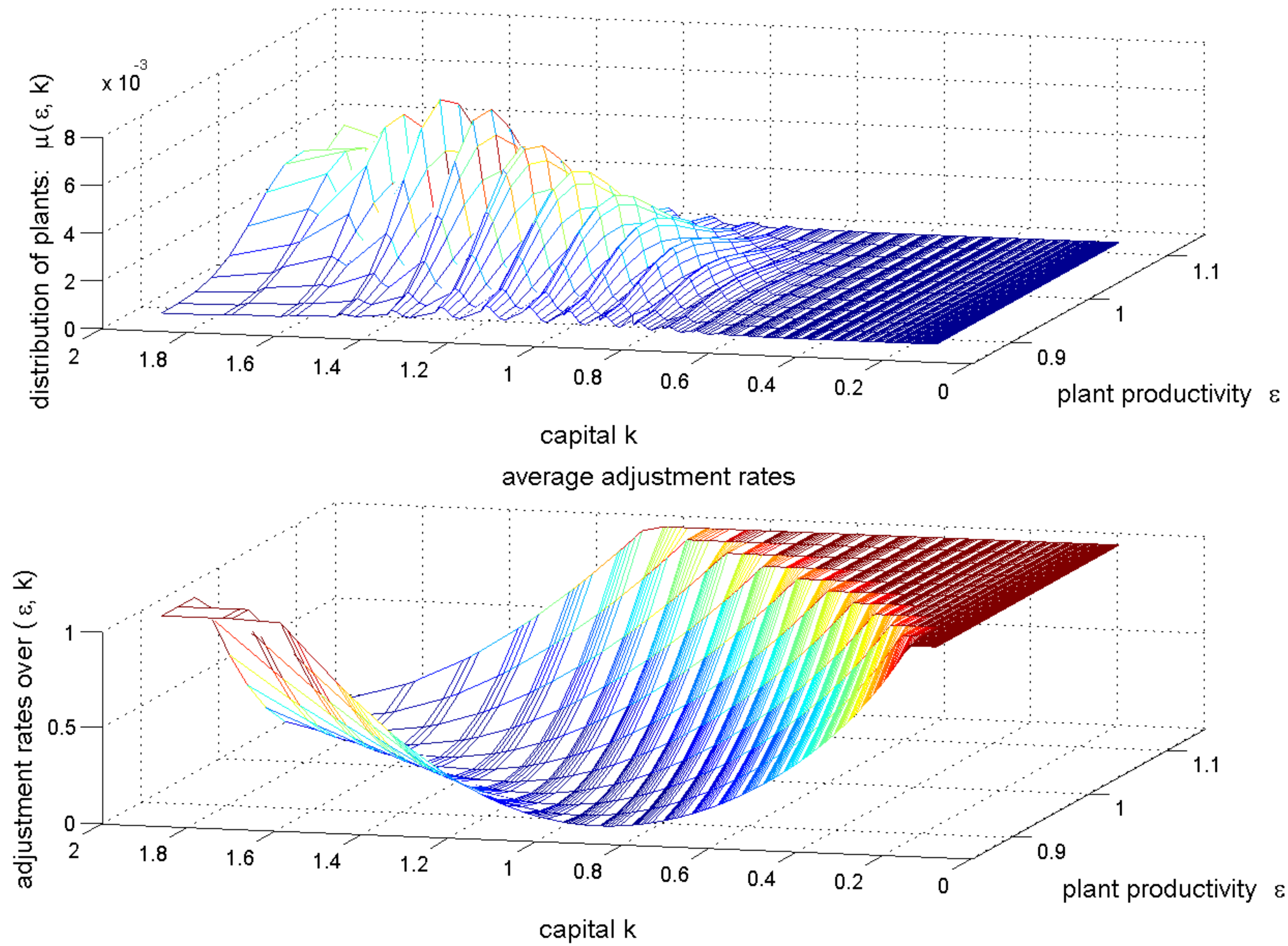

Figure 4: Adjustment responses to aggregate shocks in the full model

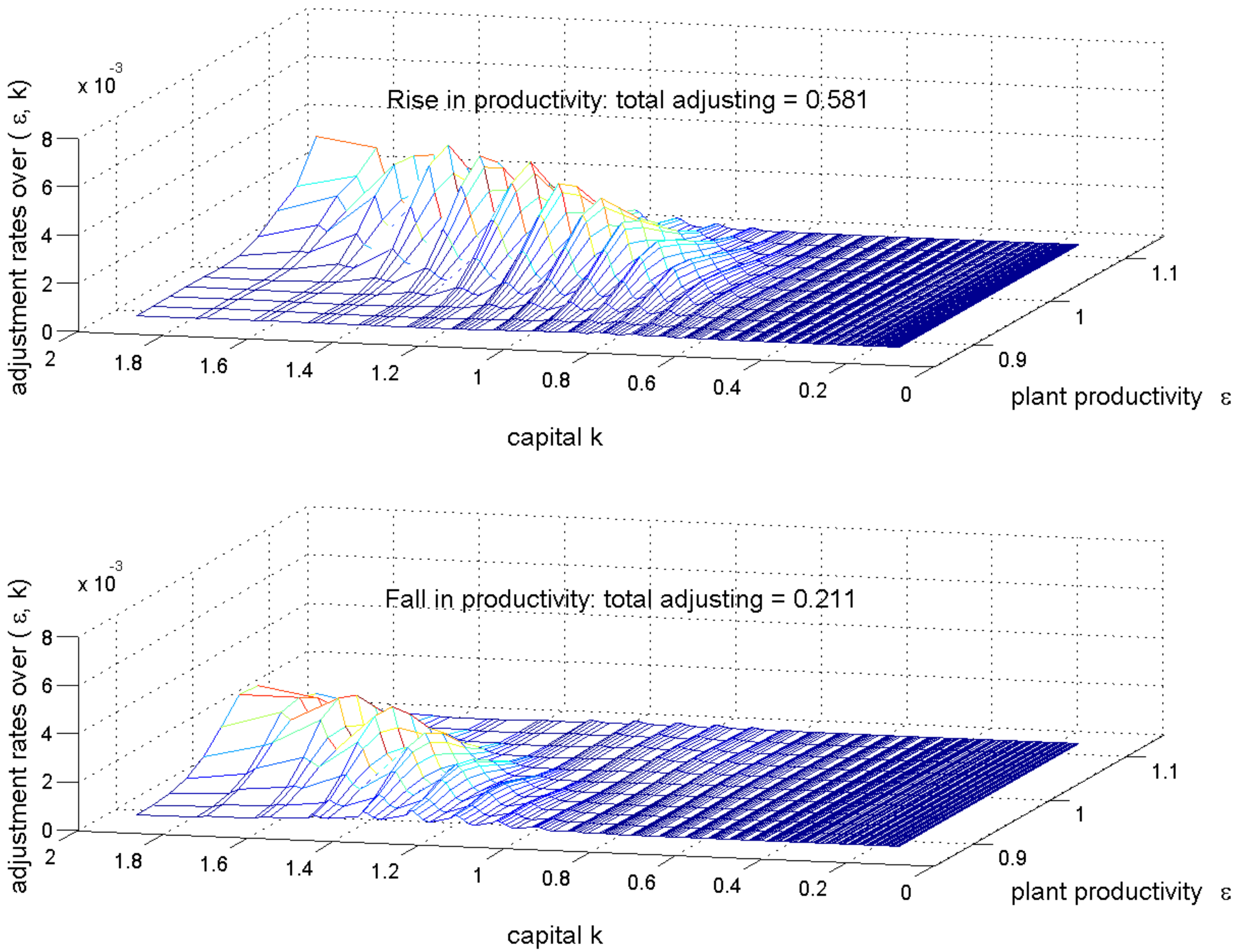

Table 1. Common parameter choices

γ	β	δ	α	ν	A	ρ_z	σ
1.016	0. 977	0.069	0.256	0.640	2.400	0.8254	0.0124

Table 2. Plant-level investment rate data

Inaction	Active Positive	Active Negative	Positive Spike	Negative Spike
0.081	0.800	0.104	0.180	0.014

NOTE. – Moments based on the Longitudinal Research Database derived by Cooper and Haltiwanger (2002). Plant-level investment-to-capital ratio, i/k, moments are as follow. Inaction: fraction of plant-year obs. with $|i/k| < 0.01$; Active Positive: fraction of obs. with $i/k \geq 0.01$; Active Negative: fraction of obs. with $i/k \leq - 0.01$; Positive Spike: fraction of obs. with $i/k > 0.20$; Negative Spike: fraction of obs. with $i/k < - 0.20$.

Table 3. Forecasting rules in *common productivity* lumpy investment model

z_1 (302 obs.)	β_1	β_2	S.E.	R^2
m_1'	0.019	0.797	0.24 e-3	0.99990
p	0.981	-0.396	0.15 e-3	0.99985
z_2 (1158 obs.)	**β_1**	**β_2**	**S.E.**	**R^2**
m_1'	0.029	0.798	0.25 e-3	0.99989
p	0.967	-0.391	0.15 e-3	0.99985
z_3 (1894 obs.)	**β_1**	**β_2**	**S.E.**	**R^2**
m_1'	0.040	0.798	0.25 e-3	0.99990
p	0.953	-0.387	0.13 e-3	0.99988
z_4 (1300 obs.)	**β_1**	**β_2**	**S.E.**	**R^2**
m_1'	0.051	0.797	0.20 e-3	0.99993
p	0.938	-0.383	0.11 e-3	0.99990
z_5 (346 obs.)	**β_1**	**β_2**	**S.E.**	**R^2**
m_1'	0.063	0.795	0.19 e-3	0.99994
p	0.924	-0.380	0.10 e-3	0.99993

NOTE. – Forecasting rules are conditional on current productivity, z_i. Each regression takes the form $\log(y) = \beta_1 + \beta_2 \log(m_1)$, where $y = m_1'$ or p.

Table 4. Forecasting rules in *full* lumpy investment model

z_1 (302 obs)	β_1	β_2	S.E.	R^2
m_1'	0.021	0.796	0.22 e-3	0.99992
p	0.976	-0.396	0.07 e-3	0.99997

z_2 (1158 obs)	β_1	β_2	S.E.	R^2
m_1'	0.032	0.797	0.23 e-3	0.99991
p	0.962	-0.391	0.07 e-3	0.99997

z_3 (1894 obs)	β_1	β_2	S.E.	R^2
m_1'	0.043	0.797	0.22 e-3	0.99992
p	0.948	-0.387	0.06 e-3	0.99997

z_4 (1300 obs)	β_1	β_2	S.E.	R^2
m_1'	0.054	0.796	0.18 e-3	0.99994
p	0.933	-0.383	0.05 e-3	0.99998

z_5 (346 obs)	β_1	β_2	S.E.	R^2
m_1'	0.066	0.794	0.17 e-3	0.99995
p	0.919	-0.380	0.04 e-3	0.99999

NOTE. – Forecasting rules are conditional on current productivity, z_i. Each regression takes the form $\log (y) = \beta_1 + \beta_2 \log (m_1)$, where $y = m_1'$ or p.

Table 5. Distribution of aggregate investment rates in partial equilibrium

	Persistence	Standard Deviation	Skewness	Kurtosis
A. full models (large idiosyncratic shocks)				
Standard	- 0.092	0.105	0.541	2.099
Lumpy Investment	0.223	0.066	1.314	4.527
B. common productivity models				
Standard	- 0.092	0.105	0.541	2.099
Lumpy Investment	0.227	0.072	2.324	8.643

Table 6. Distribution of aggregate investment rates in general equilibrium

	Persistence	Standard Deviation	Skewness	Kurtosis
A. full models (large idiosyncratic shocks)				
Standard	0.627	0.008	0.086	0.316
Lumpy Investment	0.636	0.008	0.098	0.297
B. common productivity models				
Standard	0.627	0.008	0.086	0.316
Lumpy Investment	0.638	0.008	0.103	0.293

Table 7. Aggregate moments in the *full* models

	Z	Y	I/Y	N	K
A. Standard Deviations Relative to Output					
Standard: General Equilibrium	0.587	1.950	0.537	0.670	0.498
Lumpy Investment: General Equilibrium	0.590	1.940	0.531	0.663	0.499
Standard: Partial Equilibrium	*0.137*	*8.338*	*2.768*	*1.000*	*1.093*
Lumpy Investment: Partial Equilibrium	*0.174*	*6.571*	*2.003*	*1.000*	*1.020*
B. Contemporaneous Correlations with Output					
Standard: General Equilibrium	1.000	1.000	0.957	0.958	0.020
Lumpy Investment: General Equilibrium	1.000	1.000	0.959	0.959	0.019
Standard: Partial Equilibrium	*0.707*	*1.000*	*-0.218*	*1.000*	*0.938*
Lumpy Investment: Partial Equilibrium	*0.718*	*1.000*	*0.019*	*1.000*	*0.888*

NOTE. – For Y, we report the percentage standard deviation; for I/Y, we report the standard deviation relative to Y, and for Z, N, and K, we report percentage standard deviations relative to Y.

Table 8. Aggregate moments in the *common productivity* models

	Z	Y	I/Y	N	K
A. Standard Deviations Relative to Output					
Standard: General Equilibrium	0.587	1.950	0.537	0.670	0.498
Lumpy Investment: General Equilibrium	0.590	1.939	0.533	0.665	0.502
Standard: Partial Equilibrium	*0.137*	*8.338*	*2.748*	*1.000*	*1.093*
Lumpy Investment: Partial Equilibrium	*0.168*	*6.804*	*2.117*	*1.000*	*1.063*
B. Contemporaneous Correlations with Output					
Standard: General Equilibrium	1.000	1.000	0.957	0.958	0.021
Lumpy Investment: General Equilibrium	1.000	1.000	0.960	0.960	0.019
Standard: Partial Equilibrium	*0.707*	*1.000*	*-0.218*	*1.000*	*0.938*
Lumpy Investment Partial Equilibrium	*0.687*	*1.000*	*-0.029*	*1.000*	*0.897*

NOTE. – For Y, we report the percentage standard deviation; for I/Y, we report the standard deviation relative to Y, and for Z, N, and K, we report percentage standard deviations relative to Y.

Table 9. General equilibrium effects of idiosyncratic shocks on plant-level investment

	Inaction	Positive Spike	Negative Spike	Positive Invest.	Negative Invest.	Invest. Persistence
LRD Data	0.081	0.180	0.014	0.800	0.104	0.007
	A. Standard models					
Full	0.004	0.390	0.141	0.585	0.411	-0.234
Common productivity	0.000	0.000	0.000	1.000	0.000	0.627
	B. Lumpy Investment models					
Full	0.777	0.186	0.014	0.199	0.024	-0.156
Common productivity	0.776	0.198	0.000	0.224	0.000	-0.221

NOTE. – LRD data are reproduced from Cooper and Haltiwanger (2002). Inaction: fraction of plant observations with $|i/k|<0.01$; Positive Spike: fraction of obs. with $i/k > 0.20$; Negative Spike: fraction of obs. with $i/k < -0.20$; Positive Invest.: fraction of obs. with $i/k \geq 0.01$; Negative Invest.: fraction of obs. with $i/k \leq -0.01$; Invest. Persistence: first-order autocorrelation of plant-level investment rates.

Table 10. Partial equilibrium effects of idiosyncratic shocks on plant-level investment

	Inaction	Positive Spike	Negative Spike	Positive Invest.	Negative Invest.	Invest. Persistence
	A. Standard models					
Full	*0.000*	*0.398*	*0.144*	*0.598*	*0.402*	*- 0.216*
Common productivity	*0.000*	*0.145*	*0.009*	*0.856*	*0.144*	*- 0.092*
	B. Lumpy Investment models					
Full	*0.764*	*0.188*	*0.024*	*0.201*	*0.036*	*- 0.145*
Common productivity	*0.782*	*0.194*	*0.000*	*0.216*	*0.002*	*-0.192*

NOTE. –Inaction: fraction of plant obs. with $|i/k| < 0.01$; Positive Spike: fraction of obs. with $i/k > 0.20$; Negative Spike: fraction of obs. with $i/k < -0.20$; Positive Invest.: fraction of obs. with $i/k \geq 0.01$; Negative Invest.: fraction of obs. with $i/k \leq -0.01$; Invest. Persistence: first-order autocorrelation of plant-level investment rates.

Table 11. Plant-level investment in *extended full lumpy* model

	Inaction	Positive Spike	Negative Spike	Positive Invest.	Negative Invest.	Invest. Persistence
LRD Data	0.081	0.180	0.014	0.800	0.104	0.007
General Equilibrium	0.048	0.180	0.015	0.720	0.232	-0.148
Partial Equilibrium	*0.052*	*0.184*	*0.025*	*0.688*	*0.260*	*-0.137*

NOTE. –Extended full lumpy model has an upper support on adjustment costs of 0.00975, and zero adjustment costs for investment rates satisfying $|i/k| < 0.015$. All other parameters are identical to those for the full lumpy model in previous tables.

Table 12. Distribution of aggregate investment rates in *extended full lumpy* model

	Persistence	Standard Deviation	Skewness	Kurtosis
General Equilibrium	0.634	0.008	0.096	0.294
Partial Equilibrium	*0.207*	*0.067*	*1.265*	*4.338*

Table 13. Aggregate moments in *extended full lumpy* model

	Z	Y	I/Y	N	K
A. Standard Deviations Relative to Output					
General Equilibrium	0.589	1.943	0.533	0.665	0.500
Partial Equilibrium	*0.171*	*6.676*	*2.043*	*1.000*	*1.026*
B. Contemporaneous Correlations with Output					
General Equilibrium	1.000	1.000	0.959	0.959	0.020
Partial Equilibrium	*0.718*	*1.000*	*0.005*	*1.000*	*0.893*

NOTE. – For Y, we report the percentage standard deviation; for I/Y, we report the standard deviation relative to Y, and for Z, N, and K, we report percentage standard deviations relative to Y.