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## **How Industries Migrate When Agglomeration Economies Are Important\***

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### **ABSTRACT**

The Economics of QWERTY suggests that historical accidents can trap economies in inefficient equilibria. This paper suggests that such accidents do not have the force that proponents claim. The paper presents a mechanism that may unravel a locational advantage caused by an historical accident. In the model, there are agglomeration benefits from concentrating industry in a particular location because it enables a large variety of local suppliers to emerge. Firms differ by the extent to which they purchase from local suppliers. Low-tier firms purchase little; high-tier firms purchase more. When the industry migrates, the lowest-tier products move first.

\*The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

## 1. Introduction

The possibility of multiple equilibria under increasing returns has received much attention recently. Work by Arthur (1989), Krugman (1991a), Farrell and Saloner (1985), and David (1985) emphasizes that in such situations equilibrium outcomes may be determined by accidents of history. For example, Krugman (1991a) discusses several sources of increasing returns from concentrating the production of a particular industry in a particular location. With these agglomeration economies, there may be multiple equilibria that differ according to where the industry concentrates. Krugman traces the current concentration of the carpet industry in Dalton, Georgia, to an historical accident in 1895 in which a teenaged girl named Catherine Evans made a tufted bedspread for a gift. To take another example, David (1985) argues that there are network externalities to the widespread adoption of a standard keyboard. David attributes the emergence of the QWERTY keyboard standard to an historical accident. This example is so frequently cited in the literature that Krugman calls this literature "The Economies of QWERTY."

A recurring theme of this literature is that the equilibrium selected by the historical accident may turn out to be an inefficient equilibrium. For example, in the case of the QWERTY keyboard, it is argued that there exists an alternative keyboard with a superior layout of the keys. However, according to the argument, people continue to use the QWERTY keyboard because everyone else uses it.<sup>1</sup>

Can historical accidents have such force? If for some reason Salt Lake City becomes a more natural place to produce carpets than Dalton, Georgia, is it possible that the industry will remain in Georgia because of a bedspread made 100 years ago? This paper suggests that historical accidents may have less force than what the previous literature suggests. This paper develops a theory of how economic agents initially situated in an inferior location or standard might migrate to the superior location or standard. The departure from the previous literature is to consider a model where economic agents differ in the weight they place on being *near* other agents. Some agents value agglomeration benefits a great deal; others value them little.

In the model, there are agglomeration benefits that arise from concentrating industry in a particular location because concentration enables a large variety of local specialized suppliers to emerge at the location. The model follows the recent literature (e.g., Abdel-Rahman (1988), Krugman (1991b)) and uses the Spence (1976) and Dixit and Stiglitz (1977) model of monopolistic competition to model the emergence of local specialized suppliers. In the previous literature, e.g., Krugman (1991b), there is a single type of manufactured final good. In this paper, there is a

continuum of types of final goods that vary in their production requirement for the output of local specialized suppliers. Low-tier products have a negligible requirement for such output. High-tier products have a substantial requirement. In Section 2, I argue that this continuum of product types can be interpreted as a continuum of quality types. High quality goods are more sophisticated and tend to have a greater requirement for the output of local specialized suppliers than low quality goods.

In the model, there are two locations, the North and South. The South has a natural advantage for production in the industry, due, perhaps, to a superior climate. Because of some historical accident, the industry is initially in the North. The paper addresses two questions. Does the industry eventually migrate to the South? If so, how does the industry migrate?

If a migration takes place, this is how it goes. Producers of the lowest quality products; i.e., the producers that place little weight on access to local suppliers, immediately move South to take advantage of the better climate. As this bottom end of the quality spectrum builds up in the South, a network of local suppliers begins to emerge to cater to the local production in the South. The emergence of this supplier network in the South attracts products formerly produced in the North with a medium requirement for access to local suppliers. As the range of products produced in the South expands, the supplier base in the South grows. But as the supplier base grows, the South becomes more attractive to higher-end products, and the range of products produced in the South expands even more. Ultimately, through this sequence of events, the entire industry migrates to the South, despite the initial pull of agglomeration economies in the North.

If the natural advantage of the South is very small, it is possible in this model for part of the industry to get stuck in the North. The goods with the lowest production requirements for the output of specialized suppliers will always move South. However, since these products make little use of specialized suppliers, the supplier network that emerges in the South to cater to these products may be too small to attract the next tier of production. Hence, the migration path I have described above might get cut off in the middle.

While it is possible for part of the industry to get stuck in the North, this is less likely to happen in my model with a continuum of product types than in the standard model with a single product type. I compare my model with a version of the standard model with the same average level of agglomeration economies. When agglomeration economies matter, the condition that determines when the industry necessarily migrates to the South is substantially weaker in my model than in the standard model. The comparison of the two models is most dramatic in the special case where

intermediate inputs are completely nontradeable across locations. In this case, in the standard model, there always exists a *bad* equilibrium with all production in the North. (If everyone else is in the North, a producer will not unilaterally move to the South because it will not be able to obtain intermediate inputs). In contrast, in this same case in the continuum model, for a wide range of parameters this *bad* equilibrium can be unraveled; i.e., for these parameters, the industry necessarily moves to the South in the long run.

This paper is similar in spirit to Rauch (1993).<sup>2</sup> He focuses on the role industrial park developers play in coordinating a migration. In his model, the developer is a *large* agent who takes actions that can effect what happens to the entire economy. In my model, all agents are *small*; i.e., no agent's action has an effect on the economy as a whole. If the possibility of a developer were introduced into my model, the industry would be even less likely to get stuck in the North.

The question of *how* the industry migrates is as important as the question of *whether* the industry migrates. I show that along the migration path, each new cohort of entrants to the South pulls up the average intensity of demand for the output of specialized suppliers compared to the demand of preexisting firms. The paper then provides a new theory for why a developing region will begin producing unsophisticated, low quality goods and then gradually over time expand its product set to include sophisticated, high quality goods. This pattern is a common feature of economic development and industrialization. In Section 2, I discuss the case of the migration of the cotton textile industry from New England to the South. The South began producing coarse, low quality goods, but eventually the production of even the highest quality goods shifted to the South. An analogous pattern has been observed in developing economies in Asia.<sup>3</sup> The usual explanation for this phenomenon is that the production of high-end products requires workers with high skills and, as an economy develops, its skill base increases, as in Stokey (1991). There also may be learning-by-doing where the experience of producing a particular product on the quality ladder may confer benefits for producing the next product on the ladder, as in Stokey (1988). I do not want to discount the importance of these explanations. Rather, I want to emphasize that the forces I highlight in this paper can also give rise to this kind of phenomena and may play a contributing role.

The rest of the paper is organized as follows. Section 2 briefly discusses the migration of the cotton textile industry. The purpose of this section is to relate features of the model to some real-world counterparts. Sections 3 and 4 describe and analyze a static version of the model. The static model is easier to explain than the dynamic model, and much of what is learned about the static

model applies to the dynamic model. Section 5 puts the static model in an overlapping generations framework and makes the model dynamic. Section 6 concludes.

## 2. The Cotton Textile Example

One example of an industry where the forces I highlight in this paper may be important is in the cotton textile industry. The American cotton textile industry was heavily concentrated in New England in the nineteenth century. Over the period 1880 to 1930, most of this industry migrated to the Piedmont region of the South. One source of puzzlement to economic historians is why the migration was so drawn out, given obvious advantages of the South in labor costs and access to raw cotton.<sup>4</sup> In the face of clear-cut advantages of the South, why were people still building new textile mills in New England in 1900?

One factor that is not emphasized in the literature is the role agglomeration economies in New England may have played in slowing the migration.<sup>5</sup> The remarkable tendency of this industry to be geographically concentrated, whether we look across centuries or across continents, suggests that agglomeration economies may be important in this industry. For example, in the nineteenth century, the vast majority of this industry was within 50 miles of Providence, Rhode Island; now the vast majority is in North and South Carolina. The scale effect that allows for the emergence of specialized suppliers is a plausible source of agglomeration benefits in the cotton textile industry. In this industry, there are a large number of stages of production, e.g., spinning, weaving, knitting, dyeing, bleaching, printing, the manufacture of textile equipment, and so forth. Often times these stages are undertaken by specialized firms, e.g., bleacheries, dyehouses, print houses, specialized equipment manufacturers, and so forth.<sup>6</sup> The advantages of being located near these specialized suppliers include saving on transportation costs and facilitating coordination of a complicated production process. This advantage is captured in the model with a transportation cost parameter. Moreover, there are advantages to being located near different kinds of specialized suppliers. Sometimes a mill might want to have its woven cloth printed and other times dyed. In the first case, it is useful to be near a printing house and in the second case a dyehouse. This advantage is captured in the model with a preference-for-variety parameter. There existed a vast network of specialized suppliers in New England in the nineteenth century. In contrast, in the South in 1890 there were no bleacheries, dyehouses, or print houses.<sup>7</sup> The existence of this network in the North, and the absence of this network in the South, may help explain why people were still building new textile mills in New England in 1900.

One of the well-known facts about the migration of the cotton textile industry is that the migration proceeded in stages. The production of the lowest quality products shifted to the South first, and over time, successively higher quality products were produced in the South.<sup>8</sup> Not so well appreciated is the fact that the low quality products that moved South first were products for which access to specialized suppliers was not that important. The first product made in the South was coarse cloth that did not go through finishing stages such as dyeing or printing and that was used for clothing for poor farm workers. Over the course of time, the products produced in the South expanded to include higher quality products that required the various finishing stages of bleaching, dyeing, and printing, and so forth. A network of specialized suppliers gradually emerged in the South to undertake these activities.

The connection between the quality of a good and the importance of access to specialized suppliers is something that holds beyond the cotton textile industry. High quality products tend to be unstandardized and undergo rapid change compared with low quality products. For example, high quality clothing tends to be fashion-oriented. Access to local, specialized suppliers is important for the production of unstandardized goods because of the flexibility it affords in a changing environment.<sup>9</sup> For example, in the garment industry in New York, if a producer of a high-fashion dress suddenly has a need for a large red button, it can be readily procured from a local supplier. Analogously, if a manufacturer of high-tech equipment in Silicon Valley suddenly has a need for a specialized chip or a specialized piece of software, it can be readily procured from a local supplier.

### 3. The Static Model

The first part of this section describes a static version of the model. The second part defines equilibrium in the static model.

#### A. Description of the Model

There is a continuum of final-good product types indexed by  $\beta$  on the interval  $[0, \bar{\beta}]$ . The product types differ in how important intermediate inputs are in the production process. In particular, to produce one unit of  $\beta$ -type product requires  $\beta$  units of a composite intermediate input.

There are two possible locations for production, the North and the South. A key feature of the model is that the prices of the composite intermediate input in general differ across the two locations, because these prices depend upon the scale of production at the two locations. Let  $v_N$  be the price of one unit of the composite in the North, and let  $v_S$  be the price in the South.

A second key feature of the model is that the South has a natural advantage for production

in this industry. Assume some other inputs such as labor or capital are required to produce one unit of a  $\beta$ -type product in addition to the  $\beta$  units of the composite intermediate already mentioned. Assume that the cost of these other inputs is  $\alpha_N$  dollars in the North and  $\alpha_S$  dollars in the South. The costs,  $\alpha_N$  and  $\alpha_S$ , of these other inputs are held fixed in the analysis. Assume that  $\alpha_N > \alpha_S$ , so that these other inputs cost more in the North than the South. This difference could arise if the weather is less favorable in the North than the South requiring more units of labor in the North to produce one unit of the final good. Alternatively, it could be due to higher wages in the North.

The discussion above implies that the total cost to produce one unit of a  $\beta$ -type product in the North is  $c_N(\beta) = \beta v_N + \alpha_N$ , the cost of  $\beta$  units of the composite input plus the  $\alpha_N$  dollars for the additional inputs. Analogously,  $c_S(\beta) = \beta v_S + \alpha_S$ .

To keep the analysis simple, I assume that there is an inelastic demand for a measure  $\bar{Q}$  units of each final-good  $\beta$  in the economy. The transportation cost to ship final goods between the two locations is zero. Hence in equilibrium, the price of a final-good  $\beta$  is independent of the location of production of the good.

It remains to describe the composite intermediate input sector. The intermediate input discussed above is a composite of a continuum of differentiated inputs indexed by  $s \in [0, \infty)$ . Let  $m(s)$  denote the quantity of differentiated input  $s$  employs. The production function for the composite intermediate is

$$(1) \quad M = \left[ \int_0^\infty m(s)^{\frac{1}{\mu}} ds \right]^\mu.$$

This production function has a constant elasticity of substitution equal to  $\frac{\mu}{\mu-1}$ . If  $\mu$  were equal to one, the specialized products would be perfect substitutes, and final-good producers would have no preference for variety. Assume that  $\mu > 1$ , which implies that the elasticity is less than infinity and greater than one.<sup>10</sup> The bigger is  $\mu$ , the stronger the preference for variety.

To produce a particular specialized-input  $s$ , a specialized factory must be set up in either the North or the South. The cost of setting up this factory in either location is  $\theta$  dollars.<sup>11</sup> The marginal cost to produce one more unit of a specialized input is constant at 1 dollar. Hence, to produce  $m(s)$  units of input,  $s$  costs  $\theta + m(s)$  dollars.

Specialized intermediate inputs can be shipped between the two locations at a cost. These costs are of the *iceberg* variety. For any given amount of an input shipped, a fraction  $\tau \in (0, 1]$  of the input is lost in the process. This transportation cost setup is analogous to Krugman (1991b).

In the model, all agents are small. The final-good producers all behave competitively. The

producer of a particular specialized-input  $s$  has a monopoly over this input. However, there is free entry into the production of specialized inputs, so the equilibrium concept is that of monopolistic competition. The location decisions of final-good producers and of specialized-input producers are all made simultaneously.

It is easy to reinterpret this model as a model of two alternative technology standards. The specialized-inputs  $s$  could be reinterpreted as products, such as software, that complement the technology standard. The parameter  $\beta$  in this reinterpretation indicates how much weight a particular consumer places on being able to obtain these complementary products. For example, a low- $\beta$  consumer may write his or her own software, and therefore, may not place much weight on the variety of software available for a technology. The parameter  $\tau$  determines how well the software written for one standard can be used for the other technology standard. If  $\tau = 1$ , the software written for one standard is useless for the other standard. If  $\tau = 0$ , the software for the two standards is interchangeable. It should be noted that this is a model of *unsponsored* technologies. Using the terminology of Arthur (1989, p. 117), “Sponsored technologies are proprietary and capable of being priced and strategically manipulated; unsponsored technologies are generic and not open to manipulation and pricing.” In other words, there is no analog of Microsoft controlling one technology and Apple controlling the other technology.

## B. Definition of Equilibrium

To define an equilibrium some additional notation is needed. Let  $Q_N(\beta)$  and  $Q_S(\beta)$  be the quantity of final-good  $\beta$  produced in the North and South, respectively. Let  $n_N$  and  $n_S$  denote the number (or, formally, the measure) of different specialized inputs produced at each location. Let  $p_N(s)$  be the *delivered* price of one unit of specialized-input  $s$  in the North. The price depends upon the location because of transportation costs. If a particular specialized input  $s$  is not available in the North at any price, then  $p_N(s) = \infty$ . Analogously, I define  $p_S(s)$  to be the price of specialized-input  $s$  in the South.

The price  $v_N$  for a unit of the composite intermediate in the North is determined by calculating the minimum cost way to construct one unit of the composite, given the production function (1) and given the price  $p_N(s)$  for each specialized-input  $s$  in the North. The price  $v_S$  is calculated in the analogous way.

An equilibrium is an allocation that satisfies the following conditions. First, final-good producers choose to locate in the least-cost location. Formally, if  $Q_N(\beta) > 0$  (so that product  $\beta$  is

produced in the North), then  $c_N(\beta) \leq c_S(\beta)$ . Analogously, if  $Q_S(\beta) > 0$ , then  $c_S(\beta) \leq c_N(\beta)$ . Second, producers of specialized inputs choose their locations and their prices to maximize profit. Third, there is free entry, so producers of specialized inputs must earn zero profit. Fourth, the total output of product  $\beta$  must equal the amount of the inelastic demand for the product in the economy,  $Q_N(\beta) + Q_S(\beta) = \bar{Q}$ .

#### 4. Equilibrium in the Static Model

Analysis of equilibrium is facilitated by making a few observations. As I will discuss below, the prices  $v_N$  and  $v_S$  of the composite at the two locations depend upon the demand for the composite at the two locations. If the demands are exactly the same at the two locations, then the prices are the same,  $v_N = v_S$ . If demand is higher in the North, then the price is lower in the North,  $v_N < v_S$ , and so on.

It is clear that there is always an equilibrium in which all final-good producers and all specialized-input suppliers locate in the South. To see this, note that if all production is in the South, then the composite is cheaper in the South,  $v_S < v_N$ . Since the cost of the other inputs is also lower in the South,  $\alpha_S < \alpha_N$ , the total cost of production in the South is lower for each  $\beta$ -type product, i.e.,  $\alpha_S + \beta v_S < \alpha_N + \beta v_N$ . Therefore, all final-good producers prefer to locate South. Suppliers of specialized inputs also prefer to locate in the South to avoid the transportation cost  $\tau$ . Hence, it is an equilibrium for all production to locate in the South.

The key question considered in this section is whether or not there exists an equilibrium with production in the North. If there is such an equilibrium, it must be the case that more than half of the demand for the composite intermediate is in the North. In this case, the composite is cheaper in the North,  $v_N < v_S$ . This will offset to some degree the cost advantage of the South for other inputs,  $\alpha_S < \alpha_N$ . The weight placed on the lower composite cost in the North will depend upon the  $\beta$  type of the product. A product with  $\beta = 0$  places no weight on the lower price of the composite in the North, so such a product will always locate in the South. It is clear that if there is an equilibrium with any production in the North, there will be some cutoff level of  $\hat{\beta}$  such that (1) all  $\beta < \hat{\beta}$  strictly prefer to locate in the South, (2) all  $\beta > \hat{\beta}$  strictly prefer to locate in the North, and (3) the cutoff product  $\hat{\beta}$  is indifferent between the two locations.

This discussion suggests a two-step procedure for determining whether or not an equilibrium exists with production in the North. The first step is to take as given that  $\beta < \hat{\beta}$  locate South and  $\beta > \hat{\beta}$  locate North and to determine the composite prices at each location consistent with

equilibrium of monopolistic competition in the specialized input sector. The second step is to determine the set of  $\hat{\beta}$  values that are consistent with the equilibrium location decisions of final-good producers.

### A. Step 1: Taking the Cutoff $\hat{\beta}$ as Given

Now take as given that for final goods with  $\beta \leq \hat{\beta}$ , all production is in the South ( $Q_S = \bar{Q}$ ,  $Q_N = 0$ ) and for final goods with  $\beta > \hat{\beta}$ , all production is in the North ( $Q_N = \bar{Q}$ ,  $Q_S = 0$ ). This subsection solves for the equilibrium of monopolistic competition in the intermediate input market. The treatment will be brief, since this analysis is standard in the literature.

Each specialized-input producer will have a monopoly over its particular variety. Given the form of the production function, each specialized-input supplier faces a constant elasticity of demand equal to  $\frac{\mu}{\mu-1}$ . Hence, each producer will set a price equal to a constant markup  $\mu$  over marginal cost. Suppose a particular intermediate input  $s'$  is produced by a specialized-input supplier located in the North. The marginal cost of delivering one unit of the input in the North is one dollar. Hence,  $p_N(s') = \mu$ . The marginal cost to deliver one unit in the South is higher because of the transportation cost. For every unit shipped, a fraction  $(1 - \tau)$  survives the trip. Hence, the marginal cost to deliver one unit to the South is  $\frac{1}{1-\tau}$  dollars, so the price is  $p_S(s') = \frac{1}{(1-\tau)}\mu$ .<sup>12</sup> By analogous reasoning, if a specialized input  $s''$  is produced in the South, the delivered price of the good in the South is  $p_S(s'') = \mu$ , and the delivered price of the good in the North is  $p_N(s'') = \frac{1}{(1-\tau)}\mu$ .

Now take as given that there are a measure  $n_N$  different specialized inputs produced in the North and  $n_S$  different specialized inputs produced in the South. I will derive two equilibrium conditions that  $n_N$  and  $n_S$  must solve. The total number of different specialized inputs produced in the economy is  $n_N + n_S$ . Given the preference for variety built into the production function (1), each final-good producer will employ all the different specialized inputs in the production of the composite intermediate input. The relative usage of specialized inputs at a particular location will, of course, depend upon the relative price of the specialized inputs at the location. Consider a final-good producer located in the North. Such a producer faces a price of  $\mu$  for specialized inputs produced in the North and a price of  $\frac{1}{(1-\tau)}\mu$  for specialized inputs produced in the South. Let  $m_{NN}$  and  $m_{NS}$  solve the following problem:

$$\min_{m'_{NN}, m'_{NS}} \mu n_N m'_{NN} + \frac{1}{(1-\tau)} \mu n_S m'_{NS}$$

subject to

$$1 = \left[ n_N m_{NN}^{\frac{1}{\mu}} + n_S m_{NS}^{\frac{1}{\mu}} \right]^\mu.$$

That is,  $m_{NN}$  and  $m_{NS}$  are the input levels of specialized inputs originating, in the North and the South, respectively, that minimize the cost of constructing one unit of the composite commodity in the North. Let  $v_N$  be the value of the minimized cost to construct one unit of the composite material input in the North. It is straightforward to show that this minimum cost is

$$(2) \quad v_N = \left( n_N \cdot \mu^{-\frac{1}{\mu-1}} + n_S \cdot \left( \frac{\mu}{(1-\tau)} \right)^{-\frac{1}{\mu-1}} \right)^{-(\mu-1)}.$$

The minimum cost  $v_N$  decreases in  $n_N$  and  $n_S$  because of the preference for variety. Analogous notation can be defined for the problem of constructing one unit of the composite intermediate in the South in the cost-minimizing way.

It is straightforward to calculate the demand for delivered units of the composite intermediate at the two locations. These equal

$$(3) \quad \begin{aligned} M_N &= \int_{\hat{\beta}}^{\bar{\beta}} \beta \bar{Q} d\beta = \bar{Q} \left[ \frac{\bar{\beta}^2}{2} - \frac{\hat{\beta}^2}{2} \right] \\ M_S &= \int_0^{\hat{\beta}} \beta \bar{Q} d\beta = \bar{Q} \left[ \frac{\hat{\beta}^2}{2} \right]. \end{aligned}$$

To see this, recall there is an inelastic demand in the economy of  $\bar{Q}$  for each  $\beta$ -type product. Furthermore, to produce one unit of  $\beta$ -type product, there is a fixed requirement of  $\beta$  units of the composite intermediate. Hence, the demand for the composite intermediate arising from production of final-good  $\beta$  is  $\beta \bar{Q}$ . The total demand  $M_N$  for the composite in the North is obtained by integrating over the demand of the product types that locate there, i.e.,  $\beta$  from  $\hat{\beta}$  to  $\bar{\beta}$ . Analogously, total demand in the South is obtained by integrating over the demand of the product types that locate there,  $\beta$  from 0 to  $\hat{\beta}$ .

Now consider the problem of a specialized-input supplier. The optimal prices, taking entry as given, have been discussed above. So it remains to determine the entry condition. Let  $x_N$  denote the quantity of sales of a specialized-input supplier that locates in the North,

$$x_N = m_{NN} M_N + \frac{1}{(1-\tau)} m_{SN} M_S.$$

The first and second terms are the sales to final-good producers located, in the North and the South, respectively. Note that sales to the latter include the portion of the good that dissipates in

transportation costs. The profit of a specialized-input supplier in the North is

$$\pi_N = \mu x_N - x_N - \theta.$$

The first term is revenue (the price is  $\mu$ , and the quantity sold is  $x_N$ ). The second term subtracts variable cost, and the third term subtracts the fixed cost. In an analogous way, one can calculate the quantity of sales  $x_S$  and the profit  $\pi_S$  of a specialized-input supplier that locates in the South.

Taking the cutoff  $\hat{\beta}$  as given, and taking as given the variety of products  $n_N$  and  $n_S$  produced at each location, we can calculate the variables  $v_N$  and  $v_S$ ,  $x_N$  and  $x_S$ ,  $\pi_N$  and  $\pi_S$ . In an equilibrium of monopolistic competition, it must be that if there is production of specialized inputs at a particular location, then with free entry, profit at that location must be zero. If there is no production at a location, then profit at the location must be nonpositive. Formally,

$$(4) \quad \begin{aligned} \pi_j &= 0, \text{ if } n_j > 0, j \in \{N, S\} \\ &\leq 0, \text{ if } n_j = 0. \end{aligned}$$

Straightforward calculations can be used to prove the following lemma.

**Lemma 1.** For each cutoff  $\hat{\beta}$ , there exist unique equilibrium levels of  $n_N^e(\hat{\beta})$  and  $n_S^e(\hat{\beta})$  which solve the zero-profit conditions (4).

Figure 1 shows how the variety levels  $n_N^e$  and  $n_S^e$  vary with  $\hat{\beta}$  in the monopolistic competition equilibrium. Assuming  $\tau < 1$ , there exists a point  $\hat{\beta}^- > 0$ , such that if  $\hat{\beta} < \hat{\beta}^-$ , no specialized-input suppliers locate in the South,  $n_S = 0$ . In this range of  $\hat{\beta}$ , somewhat paradoxically the variety of producers in the North  $n_N$  actually increases as  $\hat{\beta}$  increases, and production is shifted to the South. This is an artifact of the twin assumptions of iceberg transportation costs and inelastic demand for delivered units of the composite. As production is shifted to the South for  $\beta$  in this range, the total production of the composite in the economy increases because more of the composite dissipates in transportation costs. This increase in total production enables  $n_N$  to increase in this range. For  $\hat{\beta}$  above  $\hat{\beta}^-$ , there is sufficient final-good production in the South to support the existence of specialized-input suppliers in the South. As  $\hat{\beta}$  increases in this range, the variety of suppliers  $n_S$  in the South increases, while the variety of suppliers  $n_N$  in the North decreases. When  $\hat{\beta}$  is increased to  $\hat{\beta} = \frac{1}{\sqrt{2}}\bar{\beta}$ , it is straightforward to calculate (using the formulas given in (3)) that total demand for the composite is exactly the same at the two locations,  $M_N = M_S$ . At this point, product variety is exactly the same at the two locations,  $n_N = n_S$ . As  $\hat{\beta}$  is increased even further, eventually a point  $\hat{\beta}^+ < \bar{\beta}$  is reached, such that for all  $\beta > \hat{\beta}$ , all suppliers locate in the South. As mentioned

above, this discussion assumes that  $\tau < 1$ . If  $\tau = 1$ , then transportation of intermediate inputs is impossible. In this case, for any  $\hat{\beta} \in (0, 1)$ , the equilibrium levels of  $n_N^e(\hat{\beta})$  and  $n_S^e(\hat{\beta})$  are both positive.

## B. Step 2: Equilibrium Levels of $\hat{\beta}$

The previous subsection solves for the equilibrium of the economy and takes as given that final goods  $\beta \leq \hat{\beta}$  locate production in the South, while  $\beta > \hat{\beta}$  locate in the North. This subsection determines what levels of  $\hat{\beta}$  are consistent with optimal location decisions of final-good producers.

I begin by introducing some new notation. As discussed in the previous subsection, a given cutoff level  $\hat{\beta}$  determines unique  $n_N(\hat{\beta})$  and  $n_S(\hat{\beta})$  levels which pin down the composite prices  $v_N(\hat{\beta})$  and  $v_S(\hat{\beta})$  at each location. Recall that a  $\beta$ -type product requires  $\beta$  units of the composite intermediate input. Define  $\hat{\Delta}(\hat{\beta})$  to be the difference in the cost between the South and the North of acquiring the composite intermediate inputs required for a  $\hat{\beta}$  good, given that  $\beta$  below  $\hat{\beta}$  locate in the South and  $\beta$  above  $\hat{\beta}$  locate in the North; i.e.,

$$\hat{\Delta}(\hat{\beta}) \equiv \hat{\beta} \left[ v_S(\hat{\beta}) - v_N(\hat{\beta}) \right].$$

This difference in cost will play an important role in the analysis. To see why, suppose that there exists a  $\hat{\beta}$ , such that  $\hat{\Delta}(\hat{\beta}) = \alpha_N - \alpha_S$ . In this case, given that final-good production is distributed according to  $\hat{\beta}$ , the difference in the composite cost between the South and the North for a  $\hat{\beta}$ -type producer exactly offsets the North's disadvantage in the cost of the other inputs. A  $\hat{\beta}$ -type product is then indifferent between the North and the South, while  $\beta > \hat{\beta}$  strictly prefer the North, and  $\beta < \hat{\beta}$  strictly prefer the South. This cutoff is then an equilibrium.

To determine when an equilibrium exists with production in the North, I need to determine how  $\hat{\Delta}(\hat{\beta})$  varies with  $\hat{\beta}$ . The following lemma lists several properties of  $\hat{\Delta}(\hat{\beta})$  that are useful here.

### Lemma 2.

*Case 1.* Suppose either (a)  $\tau < 1$  or (b)  $\tau = 1$  and  $\mu \leq 2$ . Then (1)  $\lim_{\hat{\beta} \rightarrow 0} \hat{\Delta}(\hat{\beta}) = 0$ . (2) There exists a point  $\hat{\beta}^* > 0$ , such that if  $\beta < \hat{\beta}^*$ ,  $\hat{\Delta}(\hat{\beta})$  is strictly increasing, and if  $\beta > \hat{\beta}^*$ ,  $\hat{\Delta}(\hat{\beta})$  is strictly decreasing.

*Case 2.* Suppose  $\tau = 1$  and  $\mu > 2$ . Then  $\lim_{\hat{\beta} \rightarrow 0} \hat{\Delta}(\hat{\beta}) = \infty$ , and  $\hat{\Delta}(\hat{\beta})$  is strictly decreasing over the entire range of  $\hat{\beta}$ .

Lemma 2 distinguishes between two cases. In Case 1, either some transportation is possible ( $\tau < 1$ ), or if it is not, then  $\mu \leq 2$ . In this case,  $\hat{\Delta}(\hat{\beta})$  goes to zero as  $\hat{\beta}$  goes to zero. This is obvious in the case where  $\tau < 1$  because here the difference in the composite price between the two locations

is finite (equal to the cost of transportation), and the weight  $\hat{\beta}$  on this finite difference goes to zero. This is not obvious for the case where transportation is impossible ( $\tau = 1$ ) because the cost of the composite  $v_S(\hat{\beta})$  in the South goes to infinity when  $\hat{\beta}$  goes to zero, in this case. However, for the case of  $\mu \leq 2$ , it turns out that  $\beta v_S(\hat{\beta})$ , the price of the composite weighted by  $\hat{\beta}$ , goes to zero, so  $\hat{\Delta}(\hat{\beta})$  goes to zero.

In Case 1,  $\hat{\Delta}(\hat{\beta})$  is first increasing in  $\hat{\beta}$  up to a point  $\hat{\beta}^*$ , and then it is decreasing. The slope of  $\hat{\Delta}(\hat{\beta})$  is

$$(5) \quad \frac{d\hat{\Delta}}{d\hat{\beta}} = [v_S - v_N] + \hat{\beta} \left[ \frac{dv_S}{d\hat{\beta}} - \frac{dv_N}{d\hat{\beta}} \right].$$

For small  $\hat{\beta}$ , the composite intermediate is more expensive in the South, so the first term is positive. Increasing  $\hat{\beta}$  puts more weight on the composite cost disadvantage of the South, and this tends to increase  $\hat{\Delta}$ . The second term is negative. This term reflects the fact that as  $\hat{\beta}$  is increased and more production is shifted to the South, the composite cost disadvantage in the South decreases. For small  $\hat{\beta}$ , there is little weight on this negative second term, and the net effect of an increase in  $\hat{\beta}$  is positive. For large  $\hat{\beta}$ , the weight on the negative second term is large and the first term is negative. Hence, for large  $\hat{\beta}$ , the slope (5) is negative.

Now consider Case 2. Here transportation is impossible, and  $\mu$  is relatively large. In this case,  $\beta v_S(\hat{\beta})$  goes to infinity as  $\beta$  goes to zero. In this case, the slope (5) is strictly decreasing over the entire range of  $\hat{\beta}$ .

For Case 1, define  $\hat{\Delta}^*$  to equal  $\hat{\Delta}(\hat{\beta}^*)$ . For Case 2, set  $\hat{\Delta}^* = \infty$ . The variable  $\hat{\Delta}^*$  is the maximum value of the cost difference  $\hat{\Delta}(\hat{\beta})$  over all  $\hat{\beta}$ . Whether or not an equilibrium with production in the North exists depends on the relationship between  $\hat{\Delta}^*$  and the difference in the other input costs  $\alpha_N - \alpha_S$  at the two locations.

**Proposition 1.** Assume either  $\tau < 1$  or  $\mu \leq 2$ . If  $\alpha_N - \alpha_S > \hat{\Delta}^*$ , the unique equilibrium is for all production in the industry to be in the South. If  $\alpha_N - \alpha_S < \hat{\Delta}^*$ , there are three equilibria: one with all production in the South and two equilibria with interior values of  $\hat{\beta}$  in which some production is in the North.

The proof of this result is illustrated in Figure 2. This illustrates what the  $\hat{\Delta}(\beta)$  function looks like in Case 1 (i.e., either  $\tau < 1$  or  $\tau = 1$  and  $\mu \leq 2$ ). Consider first the case where the South's natural advantage  $\alpha_N - \alpha_S$  is bigger than  $\hat{\Delta}^*$ . This case is labeled "large  $\alpha_N - \alpha_S$ " in the figure. As can be seen in the figure, the unique equilibrium is at point  $A$  where all production is in the South, i.e.,  $\hat{\beta} = \bar{\beta}$ . For any  $\hat{\beta}$ , the value of the natural advantage exceeds any composite cost advantage of

the North; i.e.,  $\alpha_N - \alpha_S$  is greater than  $\hat{\Delta} = \hat{\beta}[v_S - v_N]$ .

Now consider a second case where the natural advantage of the South is less than  $\hat{\Delta}^*$ . This case is labeled “small  $\alpha_N - \alpha_S$ ” in the figure. Here there are three equilibria. There is an equilibrium at  $A$  where all production is in the South. There are also equilibria at  $B$  and  $C$ . Consider point  $B$ . Here the cutoff is  $\hat{\beta}_B$ , where the natural advantage of the South is exactly equal to  $\hat{\Delta}(\hat{\beta}_B)$ . Type  $\hat{\beta}_B$  is indifferent between the two locations. Since higher  $\beta$  weight the price of intermediate inputs more, product-types  $\beta > \hat{\beta}_B$  strictly prefer the North, and product-types  $\beta < \hat{\beta}_B$  strictly prefer the South, so  $\hat{\beta}_B$  is an equilibrium. By similar reasoning, point  $C$  is also an equilibrium.<sup>13</sup>

### C. Discussion of the Critical Natural Advantage

I conclude this section by discussing when the condition for uniqueness holds. For this purpose, it is useful to define a normalized value of the natural advantage of the South,

$$\text{Normalized Natural Advantage} \equiv \frac{\alpha_N - \alpha_S}{\frac{\bar{\beta}}{2} v_S(\bar{\beta})}.$$

It equals the absolute difference in costs of other inputs  $\alpha_N - \alpha_S$ , divided by expenditure on the composite for the average  $\beta$ -type product (when all production is in the South, i.e., when  $\hat{\beta} = \bar{\beta}$ ). Given this normalization, I restate Proposition 1 as follows. There exists a unique equilibrium in which all production is in the South if and only if

$$(6) \quad \text{Normalized Natural Advantage} > \text{Critical Level},$$

where the critical level is defined by

$$(7) \quad \text{Critical Level} \equiv \frac{\hat{\Delta}^*}{\frac{\bar{\beta}}{2} v_N(0)}.$$

It turns out that the critical level (7) depends only on the transportation cost  $\tau$  and the preference-for-variety parameter  $\mu$  and is independent of all the remaining parameters of the model. Table 1 presents the critical levels for various values of  $\tau$  and  $\mu$ . For each given value of  $\tau$  and  $\mu$ , there exists a unique equilibrium with all production in the South if and only if the normalized natural advantage exceeds the critical value in the table.

Consider first the case where the transportation cost is zero,  $\tau = 0$ . In this case, there are no agglomeration economies, and the critical level is zero. For any arbitrarily small natural advantage of the South, the unique outcome is for the whole industry to be in the South.

When  $\tau$  is greater than zero, the critical value is above zero. In this case, when the natural advantage of the South is sufficiently small, there exist equilibria with production in the North.

The table illustrates that the cutoff increases with the transportation cost  $\tau$  and the preference for variety  $\mu$ .<sup>14</sup> That is, when agglomeration economies become more important, it is more likely that an equilibrium with some production in the North exists.

To place these values in perspective, I consider the following exercise. Consider an alternative version of the model in which there is only a single product type instead of a continuum of product types. I refer to this alternative model as the *single-type model* and the original model as the *continuum model*. Suppose that in the single-type model, the entire distribution is concentrated at  $\frac{\bar{\theta}}{2}$ , the midpoint of the distribution in the continuum model. Suppose that all the other parameters are the same for the two models, e.g.,  $\tau$  and  $\mu$ . The total demand and average demand for intermediate inputs is exactly the same in the two models. The models differ in the distribution of this demand across products.

Analogous to the continuum model, for the single-type model it is possible to determine a critical level of the normalized natural advantage. If the normalized natural advantage is above this level, there is a unique equilibrium with production in the South. If it is below the critical level, then an equilibrium with production in the North exists. It turns out that the critical level for the single-type case depends upon  $\tau$  but is independent of  $\mu$ . The last column of Table 1 presents the critical values for various levels of  $\tau$ .

Suppose first that  $\tau = 1.00$ , so transportation is impossible. For the single-type model, the critical value of the normalized natural advantage is infinity. No matter how big the natural advantage of the South, there is always an equilibrium with production in the North. It is easy to see why. If all production is in the North, there is no production of intermediate inputs in the South. If a final-good producer were to locate in the South, the producer would face a composite price that was infinitely high. This would outweigh any finite advantage of the South. This can be contrasted with the continuum model. For  $\mu \leq 2$  (which holds if the elasticity of substitution is greater than 2), the critical values are finite. For example, consider the case of  $\mu = 1.1$  (an elasticity of substitution of 11). The critical value in this case is .14. This means that if the natural advantage of the South is at least 14 percent of the average expenditures on the composite, the unique equilibrium is for the entire industry to be located in the South. If one were to take the same economy and concentrate the distribution of product types at the center  $\frac{\bar{\theta}}{2}$ , an equilibrium would exist with production in the North for any natural advantage of the South.

Now consider the case where  $\tau$  is less than one but still large; e.g.,  $\tau = .80$ . The critical values in the single-type case are substantially greater than the corresponding values in the continuum

case for the same  $\tau$ . Thus, the range of natural advantage under which an equilibrium exists with production in the North is much smaller in the continuum model than in the single-type case.

Finally, consider the case where  $\tau$  is small but not zero; e.g.,  $\tau = .20$ . With transportation costs close to zero, agglomeration economies are not important. The critical values are small for both the continuum model as well as the single-type model. Note that for such low values of  $\tau$ , the critical value in the single-type model can actually be slightly less than the critical value in the continuum model for some values of  $\mu$ .

## 5. The Dynamic Model

This section makes the analysis dynamic by considering an overlapping-generations model of final-good producers. It assumes that as of period 0, all previous generations of final-good producers have located in the North. This section asks whether or not the entire industry eventually migrates to the South, and if so, how the industry migrates. The section finds that if the natural advantage of the South exceeds the critical level from the static model, then eventually all final-good producers locate in the South. Furthermore, the migration proceeds in a particular way. Each new entering cohort of final-good producers locating in the South pulls up the average intensity of demand for the composite relative to the demand of preexisting producers in the South. Initially, newly entering high- $\beta$  producers may choose to locate in the North, analogous to the people who built new textile mills in New England in 1900. But eventually all new entrants locate in the North.

Suppose that time is discrete,  $t \in \{0, 1, 2, \dots, T\}$ . For technical reasons, I assume that the horizon is finite; i.e.,  $T < \infty$ . However, I assume that  $T$  is so large that the choice of  $T$  has no effect on what happens early on in the economy.

In each period, a new generation of final-good producers enters the economy. Each generation lives for  $h$  periods. In each generation, there are a measure  $\frac{\bar{Q}}{h}$  of final-good producers. In each generation, producers vary by  $\beta$ -type product, and this product type is uniformly distributed between 0 and  $\bar{\beta}$ . Adding up over the  $h$  generations, in any given period there is a total measure of  $\bar{Q} = h \frac{\bar{Q}}{h}$  final-good producers in the economy, the same as in the static model just considered.

The specialized-input sector is the same as in the static model. A specialized-input supplier must pay a fixed cost of  $\theta$  dollars to set up a factory. The factory only lasts for one period. If a specialized-input supplier desires to produce in the following period, the supplier must pay the fixed cost of  $\theta$  dollars again.

The thing that distinguishes this model from the static model is that when a new generation of

final-good producers enters, these producers make a once-and-for-all location decision. If a producer chooses to locate in the North in the first period of his or her life, the producer must remain in the North over all the remaining  $h - 1$  periods of his or her life. Assume that when making the location decision, a new producer uses a discount factor  $\delta \in [0, 1]$  to discount future profits.

Assume that as of the start of period 0, all the  $h - 1$  previous generations that are still alive have all located in the North. I will not try to model how such an initial state ever came about. Perhaps previous to period 0, the North actually had a natural advantage over the South. Perhaps the South did not even exist previous to period 0.

I should note an obvious asymmetry here between the treatment of final-good producers and specialized-input suppliers. On one hand, the location choices of final-good producers have consequences  $h - 1$  periods into the future. On the other hand, the location choices of intermediate-input suppliers are made every period. In a more realistic model, the location choices of intermediate-input suppliers would also have some degree of permanence. I rule this out to make the analysis tractable, but I do not think the analysis would be qualitatively different if it were added in.

The analysis of this dynamic model is similar to the analysis of the static model. In any given period  $t$ , there will be some distribution of final-good production that will determine the distribution of the demand  $M_{N,t}$  and  $M_{S,t}$  for the composite intermediate at the two locations in period  $t$ . Given  $M_{N,t}$  and  $M_{S,t}$ , we can determine the prices  $v_{N,t}$  and  $v_{S,t}$  of the composite from the monopolistic-competition equilibrium of the specialized-input market.

Final-good producers take as given the sequence of future composite prices at the two locations when making their location choice. The discounted cost to a final-good producer of type  $\beta$ , born at time  $t$ , of locating in the North equals the discounted value of the cost of other inputs, plus the discounted cost of  $\beta$  units of the composite in each period,

$$(8) \quad c_{N,t}(\beta) = \left[1 + \delta + \dots + \delta^{h-1}\right] \alpha_N + \beta \left[v_{N,t} + \delta v_{N,t+1} + \dots + \delta^{h-1} v_{N,t+h-1}\right].$$

The discounted cost  $c_{S,t}(\beta)$  of locating in the South is defined in a similar way. Final-good producers choose their location to minimize discounted cost. It is immediate from the form of the discounted cost (8) that in any equilibrium of this economy there is a cutoff rule  $\hat{\beta}_t^e$  in each period  $t$ , such that all new entrants with  $\beta < \hat{\beta}_t^e$  locate in the South, and all new entrants with  $\beta > \hat{\beta}_t^e$  locate in the North. It is straightforward to show that an equilibrium always exists in this economy (the appendix contains a proof). The equilibrium is not always unique, as will be clear in an example below.

The next result provides a characterization of equilibria in this economy. For this characterization, I assume that the natural advantage of the South exceeds the critical level defined in the analysis of the static model; i.e.,

$$\text{Assumption 1. } \alpha_N - \alpha_S > \hat{\Delta}^*.$$

The analysis is complicated, and in order to make some headway, I consider only two special cases. The first case is where generations live two periods, but the discount factor is general; i.e.,

$$\text{Special Case 1: } h = 2 \text{ and } \delta \in [0, 1].$$

The second case is where the number  $h$  of generations is arbitrary, but the discount factor is zero; i.e.,

$$\text{Special Case 2: } h \geq 2 \text{ and } \delta = 0.$$

I suspect the result holds for the intermediate case of general  $h$  and  $\delta$ , but I have not been able to prove it. To state the result, I have to introduce some additional notation. Let  $a_t$  be the average cutoff in period  $t$  of the  $h - 1$  previous generations that are still around at the beginning of period  $t$ ,

$$(9) \quad a_t = \frac{\hat{\beta}_{t-1} + \hat{\beta}_{t-2} + \dots + \hat{\beta}_{t-(h-1)}}{h-1}.$$

**Proposition 2.** Suppose Assumption 1 holds. Assume either Special Case 1 or 2 applies. (1) There exists a  $k > 0$  that is independent of  $T$ , such that in any equilibrium if  $\hat{\beta}_t^e < \bar{\beta}$ , then  $\hat{\beta}_t^e > a_t + k$ . (2) Let  $T > \frac{\bar{\beta}(h-1)}{k}$ . There exists a  $t' < T$ , such that if  $t \geq t'$ , then  $\hat{\beta}_t^e = \bar{\beta}$ .

The result holds when the natural advantage of the South exceeds the critical level. In this case, there exists a constant  $k > 0$ , such that in any period where there is entry in the North (i.e.,  $\hat{\beta}_t^e < \bar{\beta}$ ), the equilibrium cutoff  $\hat{\beta}_t^e$  exceeds the average cutoff of the preexisting generations by the amount of this constant  $k$ . The easiest way to state the result is for the case of  $h = 2$ , because in this case, the average cutoff of the preexisting generations is just the last-period cutoff; i.e.,  $a_t = \hat{\beta}_{t-1}$ . The result implies that for the case of  $h = 2$ , the cutoff  $\hat{\beta}_t^e$  strictly increases over time, in increments of at least  $k$ , until it hits  $\bar{\beta}$ , where it stays for all subsequent periods.

It is useful to illustrate the equilibrium path with a numerical example. Suppose that  $\mu = 1.5$  and  $\tau = .8$ . Suppose that the Normalized Natural Advantage is .81. This exceeds the critical level of .76 for this  $\mu$  and this  $\tau$  (see Table 1), so all production is in the South in the unique equilibrium

of the static model. Suppose that for the dynamic model, final-good producers live four periods ( $h = 4$ ) and that the discount factor is  $\delta = .1$ .

Table 2 illustrates two possible equilibrium paths. The first line in the table reports the levels of the variables in period  $t = -1$ . As mentioned above, I assume that prior to period 0, it is impossible for new entrants to locate in the South, so all new entrants locate in the North. The total demand in the economy for delivered units of the composite intermediate is normalized to one. Of this total demand in any given period, .75 is fixed by the location decisions of the three preexisting generations of entrants. Table 2 provides the distribution of the demand from preexisting generations across the two locations. In period  $t = -1$ , all of this demand is in the North. Table 2 provides the cutoff  $\hat{\beta}_t$  which determines where the new entrants locate. In period  $t = -1$ , the cutoff is  $\hat{\beta}_{-1} = 0$ ; i.e., all  $\beta \in [0, 1]$  locate in the North (they have no other choice). Table 2 reports the measure  $n_{N,t}$  and  $n_{S,t}$  of specialized-input suppliers at each location. These are normalized so that when all demand for the composite is concentrated at one location, the equilibrium variety is 1.000. This is what happens in period  $t = -1$ . Table 2 also reports the distribution of the total output across the two locations. In period  $t = -1$ , all output is in the North.

For this given set of the model's parameters, all production is in the South in the long-run in any equilibrium. In this example, eventually all new entrants locate in the South. Consider the path referred to as the *Slow-Transition Path*. In period 20 and all subsequent periods, the cutoff  $\hat{\beta}_t$  is at the maximum level of one; i.e., all new entrants locate in the South beginning in period 20. For this set of the model's parameters, this is the slowest possible transition; i.e., there does not exist an equilibrium with any entry in the North in period 20 or thereafter. The situation is different in the single-type model with the same parameters, i.e., in the model where average agglomeration economies are the same, but demand is  $\frac{\bar{\beta}}{2}$  for each final good rather than being distributed on the interval  $[0, \bar{\beta}]$ . In this alternative model, there exists an equilibrium in which the industry is permanently stuck in the North; i.e., all producers locate in the North in every period.<sup>15</sup>

Consider period 0 in the Slow-Transition Path. In this period, the equilibrium cutoff is  $\hat{\beta}_0 = .102$ . Hence, about 10 percent of all new entrants locate in the South in the initial period. Since entry represents one fourth of all output, the South's share of period-0 total output is about 2.5 percent. The demand for the composite in the South in period 0 is too small to support any specialized-input suppliers, so all of the composite used in the South in period 0 is imported. The scarcity of specialized suppliers in the South explains why entrants with  $\beta > .102$  chose to locate in the North. This entry in the North is analogous to building new textile mills in New England in

Now consider the subsequent periods in the Slow-Transition Path. Period 1 begins with a small demand for intermediate inputs already in the South equal to .003. The equilibrium cutoff in period 1 is  $\hat{\beta}_1 = .103$ , slightly higher than the previous cutoff. Going into the next period, the preexisting demand for intermediate inputs in the South increases a little, and the cutoff increases a little. This process repeats itself, and production in the South gradually builds up. Specialized-input suppliers begin to emerge in the South. By period 19, the cutoff has grown to  $\hat{\beta}_{19} = .501$ , so about half of the entrants locate in the South and half locate in the North. In this period, 35 percent of final-good output is in the South. By period 19, the South has established an specialized-input industry with a measure .235 of producers compared with .922 in the North. In period 20, the cutoff shifts up to  $\hat{\beta}_{20} = 1$ , and all entrants locate in the South. However, there still is production in the North because of the location decisions of previous entrants. In period 23, the long-run outcome is attained with all production in the South.

In the *Fast-Transition Path*, period 2 is the last period with entry in the North. Given these parameters, this is the fastest transition path; i.e., there does not exist an equilibrium without any entry in the North in period 2. The Fast-Transition Path is qualitatively similar to the Slow-Transition Path in that the cutoff increases over time. The difference is that things move faster.

## 6. Conclusion

This paper develops a theory of how industries migrate when agglomeration economies are important. It determines when an industry will not get stuck at the *wrong* location. It determines the features of the migration path.

One avenue for future research is to use this model to help determine how important agglomeration economies are in an industry. It is sometimes difficult to estimate the importance of agglomeration economies by looking at a particular incidence of agglomeration, e.g., the cotton textile industry in New England in the nineteenth century. By looking at an industry on the move, we might be able to learn some things about agglomeration economies that are not apparent when the industry is sitting still. If agglomeration economies are important, the migration might proceed in certain ways; e.g., certain kinds of products may move first. If agglomeration economies are not important, the migration may proceed in other ways. These different implications may enable one to identify the importance of agglomeration economies.

## Notes

<sup>1</sup>For an alternative view, see Leibowitz and Margolis (1990).

<sup>2</sup>See also Leibowitz and Margolis (1995).

<sup>3</sup>See, for example, Young (1992).

<sup>4</sup>Wright (1981, p. 605) writes, “Why did it take fifty years for the South to triumph? With hourly wage rates 30 to 50 percent below Northern levels as early as 1880, as Lars Sandberg has written, the Southern advantage ‘now seems so obvious that the principal task of economic historians is to explain why it did not happen sooner.’”

<sup>5</sup>Hekman (1980) has emphasized that agglomeration economies may have been an important reason for why the industry initially concentrated in New England.

<sup>6</sup>In the time period under consideration, spinning and weaving would often be done in the same establishment. But the finishing stages of printing, dyeing and bleaching were often undertaken in an establishment different from the one spinning the yarn. The 1890 Census reported that of plain cloth printed in that year, 80 percent was printed in specialized printing houses and only 20 percent in cotton mills that wove cloth. Analogously, specialized dyehouses accounted for 91 percent of the woven fabric that was dyed, and specialized bleacheries accounted for 87 percent of the woven fabric that was bleached (U.S. Bureau of the Census, 1895, p. 182).

<sup>7</sup>See U.S. Bureau of the Census (1895, p. 232).

<sup>8</sup>For example, Table IV of Wright (1981) shows how the average yarn number (an indicator of yarn quality) monotonically increased over the period 1880 to 1920.

<sup>9</sup>See Lichtenberg (1960 p. 58) and Hall (1959, p. 12) for discussions of this point. More recently, there have been many discussions of the importance of what has been called *flexible specialization*, e.g., Piore and Sabel (1984).

<sup>10</sup>This rules out the case of  $\mu < 0$ . It is standard in the literature to ignore this case since it implies inelastic demands which are inconsistent with an equilibrium of monopolistic competition.

<sup>11</sup>An alternative assumption is that the cost of producing intermediate inputs is lower in the South for the same reason that other costs are lower in the South. For example, it could be the ratios of fixed cost and the marginal costs in the South, to their counterparts in the North, are the same as the ratio of  $\alpha_S$  to  $\alpha_N$ . The algebra is more tedious for this case, but the basic results do not change.

<sup>12</sup>Note that there is no arbitrage opportunity here because  $p_S(s') = \frac{1}{(1-\tau)} p_N(s')$ .

<sup>13</sup>Point  $C$  is not a stable equilibrium in the sense of stability invoked in the literature.

<sup>14</sup>For large  $\mu$  and large  $\tau$ , normalized  $\hat{\Delta}^*$  begins to decrease in  $\mu$ .

<sup>15</sup>There also exists an equilibrium in which all producers locate in the South in every period.

<sup>16</sup>The result that in the initial period there is continuing investment in the North is reminiscent of what happens in Chari and Hopenhayn (1991). In that model, individuals continue to invest in old technologies because there are complementarities between what the current entering generation does today and what previous generations did before. The same is true here.

## Appendix

This appendix begins by proving the claim in the text that an equilibrium exists. It then presents a proof of Proposition 2.

**Proposition 3.** An equilibrium exists.

*Proof.* Let  $\hat{B}$  denote a vector of cutoffs,  $\hat{B} = (\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_T)$ , and let  $Z$  be the space containing vectors  $\hat{B}$ , i.e.,  $\hat{B} \in Z$  if and only if  $\hat{\beta}_t \in [0, \bar{\beta}]$ , for all  $t$ . I begin the proof by constructing a mapping from  $Z$  into  $Z$ . For each  $\hat{B} \in Z$ , there exists a unique sequence of prices  $(v_{N,0}, v_{N,1}, \dots, v_{N,T})$  and  $(v_{S,0}, v_{S,1}, \dots, v_{S,T})$  solving the monopolistic competition equilibrium conditions. With these sequences of prices, one can calculate the discounted costs  $c_{N,t}(\beta)$  and  $c_{S,t}(\beta)$  to a  $\beta$ -type product entering at time  $t$ . If  $c_{N,t}(\bar{\beta}) \geq c_{S,t}(\bar{\beta})$ , let  $\tilde{\beta}_t = \bar{\beta}$ . Otherwise, let  $\tilde{\beta}_t$  be the unique  $\beta \in (0, \bar{\beta})$  where  $c_{N,t}(\beta) = c_{S,t}(\beta)$ . Let  $F(\hat{\beta}_0, \hat{\beta}_1, \dots, \hat{\beta}_T) = (\tilde{\beta}_0, \tilde{\beta}_1, \dots, \tilde{\beta}_T)$  denote the mapping I have constructed. It is straightforward to show that the function  $F(\cdot)$  is increasing; i.e., if for two cutoff vectors  $\hat{B}^\circ$  and  $\hat{B}'$ ,  $\hat{\beta}_t^\circ \leq \hat{\beta}'_t$ , for all  $t$ , then  $\tilde{\beta}_t^\circ \leq \tilde{\beta}'_t$  for all  $t$ , where  $F(\hat{B}^\circ) = \tilde{B}^\circ$  and  $F(\hat{B}') = \tilde{B}'$ . Since this mapping is increasing, continuous, and maps into a bounded set, there exists a fixed point to this mapping which is an equilibrium of the economy. ■

The proof of Proposition 2 uses a lemma. The statement and proof of this lemma require some additional notation. Let  $v_N(\hat{\beta}, a)$  and  $v_S(\hat{\beta}, a)$  denote the prices of the intermediate inputs solving the monopolistic competition equilibrium conditions, given the cutoff of the current entering cohort is  $\hat{\beta}$  and that the average cutoff of the previous entrants that are still alive is  $a$ . It is straightforward to show that  $v_S(\hat{\beta}, a) - v_N(\hat{\beta}, a)$  is strictly decreasing in  $\hat{\beta}$  and  $a$ .

**Lemma 3.** Suppose that for all  $\hat{\beta} \in [0, \bar{\beta}]$ ,

$$(10) \quad \alpha_N - \alpha_S - \beta \left[ v_S(\hat{\beta}, \hat{\beta}) - v_N(\hat{\beta}, \hat{\beta}) \right] > k_0,$$

for some  $k_0 > 0$ . There exists a  $k_1 > 0$ , such that if for any  $\hat{\beta}$

$$(11) \quad \alpha_N - \alpha_S - \hat{\beta} \left[ v_S(\hat{\beta}, \hat{\beta} - x) - v_N(\hat{\beta}, \hat{\beta} - x) \right] \leq 0,$$

then  $x > k_1$ .

*Proof.* Note first that since  $v_S(\hat{\beta}, a) - v_N(\hat{\beta}, a)$  is decreasing in  $a$ , the only way that (10) and (11) can both be true is if  $x \geq 0$ . Suppose that the lemma is not true. Then for each  $n$ , there exists a  $\hat{\beta}^n \in [0, \bar{\beta}]$  and a  $x^n \in [0, \frac{1}{n})$  such that (11) holds. Since the sequence  $\{\hat{\beta}^1, \hat{\beta}^2, \dots\}$  is bounded, there exists a convergent subsequence  $\{\hat{\beta}^{n'}\}$ . Let  $\{x^{n'}\}$  denote the associated subsequence of  $\{x^n\}$ . By

the continuity of  $v_N(\hat{\beta}, a)$  and  $v_S(\hat{\beta}, a)$ , the limit of the left-hand side of (11) as a function of the subsequences  $\{\hat{\beta}^{n'}\}$  and  $\{x^{n'}\}$  is nonpositive. Since the limit of  $\{x^{n'}\}$  is zero, this contradicts (10). ■

I now turn to the proof of Proposition 2. It is convenient to restate the proposition in a slightly more general form by allowing for any arbitrary initial state. This slightly more general result is referred to as Proposition 4.

**Proposition 4.** Assume that  $\alpha_N - \alpha_S > \hat{\Delta}^*$  (Assumption 1). Suppose that either (Case 1)  $h = 2$  and  $\delta \in [0, 1]$  or (Case 2)  $h \geq 2$  and  $\delta = 0$ . Let  $(\hat{\beta}_{-1}^\circ, \hat{\beta}_{-2}^\circ, \dots, \hat{\beta}_{-(h-1)}^\circ)$  be an arbitrary initial state. Let  $\{\hat{\beta}_0^e, \hat{\beta}_1^e, \dots, \hat{\beta}_T^e\}$  be an equilibrium sequence of cutoffs, given this initial state. (1) There exists a  $k > 0$  that is independent of  $T$ , such that in any equilibrium, if  $\hat{\beta}_t^e < \bar{\beta}$ , then  $\hat{\beta}_t^e > a_t + k$ . (2) Let  $T > \frac{\bar{\beta}(h-1)}{k}$ . There exists a  $t' < T$ , such that if  $t \geq t'$ , then  $\hat{\beta}_t^e = \bar{\beta}$ .

*Proof.* The assumption that  $\alpha_N - \alpha_S > \hat{\Delta}^*$  implies that (10) holds. So there must exist the  $k_1 > 0$  so that (11) holds. Set  $k$  equal to this  $k_1$ .

I first show that part (1) of the proposition is true for  $k = k_1$  and for  $T = 0$ , and then by induction, show it is true for all  $T$ .

Suppose  $T = 0$  so there is only a single period in the model. Let  $a_0$  be the average  $\hat{\beta}$  of the  $h - 1$  previous generations as in (9). Let  $c_j(\hat{\beta}_0, a_0)$  be what the equilibrium cost at location  $j$  would be, given that  $\hat{\beta}_0$  is the cutoff in period 0 and given  $a_0$  (analogous to what I did in Section 4.A). Suppose  $\hat{\beta}_0^e < \bar{\beta}$ . A necessary condition for equilibrium is that the difference in cost between the two locations for a type  $\hat{\beta}_0^e$  is zero,

$$0 = \alpha_N - \alpha_S - \hat{\beta}_0^e \left[ v_S(\hat{\beta}_0^e, a_0) - v_N(\hat{\beta}_0^e, a_0) \right].$$

Inequality (11) then implies that  $\hat{\beta}_0^e - a_0 > k_1$ . This completes the proof of part (1) of the proposition for the  $T = 0$  case.

In Case 2 where  $\delta = 0$ , the proof for the case of general  $T$  is the same as the proof for  $T = 0$ . So assume that Case 1 holds where  $h = 2$ .

To prove the result for the case of  $h = 2$  and general  $T$ , assume the result is true for  $T - 1$ . I will show that the result is also true for  $T$ .

Let  $\hat{\beta}_{-1}^\circ$  be the initial state in the  $h = 2$  economy, and let  $\{\hat{\beta}_0^e, \hat{\beta}_1^e, \dots, \hat{\beta}_T^e\}$  be an equilibrium. Consider a new economy in which the initial period is period 1, the final period is period  $T$ , and the initial state is  $\hat{\beta}_0^e$ . This initial state is what the state in period 1 is along the equilibrium path of the original economy. It is immediate that  $\{\hat{\beta}_1^e, \dots, \hat{\beta}_T^e\}$  must be an equilibrium of the new economy. By the induction argument, we know the result must hold for this new economy since the number of

periods in the new economy is one less than the original economy. Hence, for any  $t \geq 1$ , if  $\hat{\beta}_t^e < \bar{\beta}$ , then  $\hat{\beta}_t^e > \hat{\beta}_{t-1}^e + k_1$  (noting that  $a_t^e = \hat{\beta}_{t-1}^e$ , for the case of  $h = 2$ ). It remains to prove that this holds for  $t = 0$ ; i.e., if  $\hat{\beta}_0^e < \bar{\beta}$ , then  $\hat{\beta}_0^e > \hat{\beta}_{-1}^o + k_1$ .

If  $\hat{\beta}_0^e < \bar{\beta}$ , then the discounted difference in cost to type  $\hat{\beta}_0^e$  between the North and the South must be zero; i.e.,

$$(12) \quad 0 = \left\{ \alpha_N - \alpha_S - \hat{\beta}_0^e \left[ v_S(\hat{\beta}_0^e, \hat{\beta}_{-1}^o) - v_N(\hat{\beta}_0^e, \hat{\beta}_{-1}^o) \right] \right\} \\ + \delta \left\{ \alpha_N - \alpha_S - \hat{\beta}_0^e \left[ v_S(\hat{\beta}_1^e, \hat{\beta}_0^e) - v_N(\hat{\beta}_1^e, \hat{\beta}_0^e) \right] \right\}.$$

The first bracketed term is the difference in period-0 cost, the second bracketed term is the difference in period-1 cost. Given the symmetry of  $v_S(\hat{\beta}, a)$  and  $v_N(\hat{\beta}, a)$  for  $h = 2$ , one can rewrite the difference in period-1 cost as

$$\alpha_N - \alpha_S - \hat{\beta}_0^e \left[ v_S(\hat{\beta}_1^e, \hat{\beta}_0^e) - v_N(\hat{\beta}_1^e, \hat{\beta}_0^e) \right] = \alpha_N - \alpha_S - \hat{\beta}_0^e \left[ v_S(\hat{\beta}_0^e, \hat{\beta}_1^e) - v_N(\hat{\beta}_0^e, \hat{\beta}_1^e) \right].$$

Since  $\hat{\beta}_1^e > \hat{\beta}_0^e$ , the lemma implies that the difference in period-1 cost must be strictly positive. Equation (12) then implies that the difference in period-0 cost, the first bracketed term of (12), must be negative. Inequality (11) then implies that  $\hat{\beta}_0^e - a_0 > k_1$ . This completes the proof of part (1) of the proposition.

The proof of part (2) is immediate. ■

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Table 1  
Critical Values for the Normalized Natural Advantage

$\tau$ (Transport Cost)	$\mu$ (Preference for Variety Parameter)					Single-Type Model
	1.10	1.30	1.50	2.00	3.00	
.00	.00	.00	.00	.00	.00	.00
.20	.14	.26	.28	.30	.30	.25
.40	.14	.41	.54	.63	.66	.67
.60	.14	.44	.72	1.01	1.13	1.50
.80	.14	.44	.76	1.46	1.90	4.44
.99	.14	.44	.77	1.97	5.88	99.00
1.00	.14	.44	.77	2.00	$\infty$	$\infty$

Table 2  
Two Equilibrium Transition Paths for the Example

	Period	Demand for Intermediates Fixed in Previous Periods		Cutoff $\hat{\beta}_i$	Number of Specialized-Input Producers		Final-Good Production	
		North	South		North	South	North	South
	-1	.750	.000	.000	1.000	.000	1.000	.000
Slow-Transition Path	0	.750	.000	.102	1.007	.000	.974	.026
	1	.747	.003	.103	1.014	.000	.949	.051
	2	.745	.005	.105	1.021	.000	.923	.077
	3	.742	.008	.122	1.018	.012	.892	.108
	4	.741	.009	.129	1.017	.017	.885	.115
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.
	16	.708	.042	.275	.979	.121	.753	.247
	17	.701	.049	.304	.971	.139	.732	.268
	18	.692	.058	.354	.958	.167	.703	.297
	19	.677	.073	.501	.922	.235	.641	.359
	20	.633	.117	1.000	.737	.497	.460	.540
	21	.406	.344	1.000	.535	.704	.286	.714
	22	.187	.563	1.000	.301	.882	.125	.875
	23	.000	.750	1.000	.000	1.000	.000	1.000
Fast-Transition Path	0	.750	.000	.102	1.007	.000	.974	.026
	1	.747	.003	.104	1.014	.000	.948	.052
	2	.745	.005	.118	1.020	.003	.919	.081
	3	.741	.009	1.000	.825	.384	.669	.331
	4	.494	.256	1.000	.616	.627	.445	.555
	5	.247	.503	1.000	.370	.835	.221	.779
	6	.000	.750	1.000	.000	1.000	.000	1.000

Figure 1

Equilibrium Variety as a Function of the Cutoff

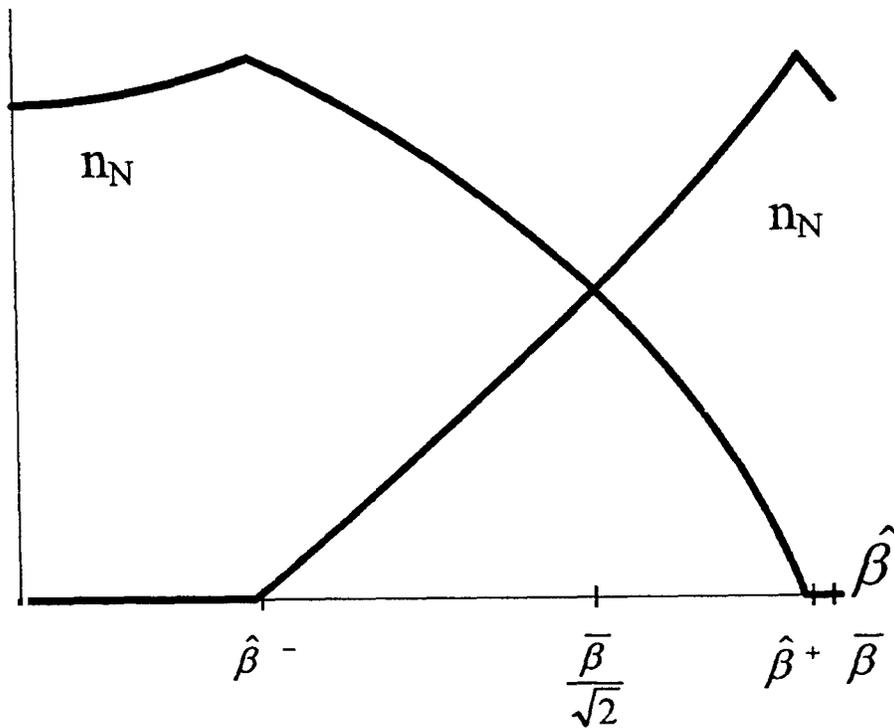


Figure 2  
Graph of the  $\hat{\Delta}$  Function

