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LABOR CONTRACTS IN A MODEL OF IMPERFECT COMPETITION\*

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ABSTRACT

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We propose a definition of involuntary unemployment which differs from that traditionally used in implicit labor contract theory. We say that a worker is involuntarily unemployed if the marginal wage implied by the optimal contract exceeds the marginal rate of substitution between leisure and consumption. We construct a model where risk-neutral firms have monopoly power and show that such monopoly power is necessary for involuntary unemployment to arise in the optimal contract. We numerically compute examples and show that such unemployment occurs for a wide range of parameter values.

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The theory of implicit labor contracts (Martin Baily, 1974; Donald Gordon, 1974; and Costas Azariadis, 1975) was developed to explain the observation that fluctuations in firms' outputs are associated with large employment variability and small wage variability. The theory begins with the observation that diversifying the risk inherent in an uncertain stream of labor income is difficult for workers because of nonconvexities (one can hardly hold several jobs at the same time) and moral hazard problems (one's incentive to work hard is diminished by insurance). The central insight of the theory is that firms have superior access to capital markets and so can insure their workers' incomes. Furthermore, since firms need to monitor work effort anyway, they can also alleviate the moral hazard problems insurance causes. The major finding is that risk-neutral firms offer risk-averse workers contracts which fully insure each worker's consumption while reallocating labor from less favorable to more favorable states of nature. In general, such insurance implies that wage rates are less variable than they would be if no contracts were available. Furthermore, with nonconvexities in labor supply, the contract may involve laying off workers in less favorable states.

Several features of these contracts are troubling. If there are no nonconvexities and leisure is a normal good, then the theory predicts that workers' utility levels are lower in more favorable states of the world. If there are nonconvexities, then for a wide class of utility functions the theory predicts that workers who are temporarily unemployed are better off than those who remain employed. The most questioned aspect of the theory is that it cannot explain involuntary unemployment in the sense that either employed workers are treated better than unemployed workers or workers would like more employment at the going wage rate.

In an attempt to remedy these problems, several authors have explored the idea that adding asymmetric information to the theory might generate involuntary unemployment as part of an optimal contract. (See V. V. Chari, 1983; Jerry Green and Charles Kahn, 1983; Sanford Grossman and Oliver Hart, 1983; and Hart, 1983.) These authors defined unemployment or underemployment to be involuntary if workers and firms get ex post gains from increasing employment which they do not get under the ex ante efficient contract. Unfortunately, these authors found that if firms are risk neutral and workers' utility functions are normal in leisure, then the optimal contract has no involuntary underemployment in this sense. One possibility explored by Russell Cooper (1982) and Grossman and Hart (1983) is to consider risk-averse firms. They show that involuntary underemployment is possible if firms are sufficiently risk averse and leisure is not too normal.

In this paper, we assume that firms are risk neutral, but are monopolists in the product market. We also critically examine the definition of involuntary underemployment and propose a new one. We say that a worker is involuntarily underemployed if the worker would prefer a small increase in employment at the marginal wage implied by the optimal contract. We propose this definition for at least two reasons. One is that, in collecting the U.S. unemployment statistics, the Bureau of Labor Statistics does not ask whether firms would be willing to increase employment at the current wage rate. Another reason follows from an important feature of optimal labor contract theory, that compensation paid to workers is not linear in hours because the contract provides insurance. Given the nonlinearity, it is appropriate to focus on the marginal wage rate, defined as the ratio of the increase in compensation to the increase in employment under

the optimal contract. We show that if the firm does not have monopoly power, then the optimal contract does not display involuntary underemployment in this sense. With monopoly power, however, involuntary underemployment is possible.

We consider particular functional forms for the technology, the demand function facing the firm, and the utility functions of workers. We numerically compute the optimal contract for a wide variety of parameters. We show that the optimal contract has involuntary underemployment (as we define it) for a range of parameter values. In particular, underemployment is more likely as the demand function becomes more inelastic or as the elasticity of substitution between consumption and leisure increases. We also show that, under some circumstances, the optimal contract involves no employment in at least one state and that, at the marginal wage, the worker would strictly prefer to increase employment by a small amount.

Some defensive remarks are appropriate here. First, since we deal with contracts between a single firm and a single worker with no nonconvexities in labor supply, our model is not well suited to addressing involuntary unemployment in the sense of unequal treatment among workers. We don't know yet whether introducing nonconvexities into our model would help explain unemployment of this kind.

Second, if nonconvexities were added, we would still end up with the feature that employed workers are worse off than unemployed workers. To address this problem, we must model the nonconvexity induced by the unemployment insurance system, particularly the requirement that workers be unemployed for two weeks before receiving unemployment compensation, as well as the nature of the optimal contract between firm and worker when moral hazard is present. (See Charles Kahn, 1985, and Dilip Mookherjee, 1986.)

Last, one might ask whether observed unemployment is better explained by other models, such as models of labor market search (Robert Lucas and Edward Prescott, 1974; Dale Mortensen, 1982; and Peter Diamond, 1982 and 1984) or models of coordination failure (Larry Jones and Rodolfo Manuelli, 1987; John Roberts, 1987; and Russell Cooper and Andrew John, 1988). We see these models as complementary rather than conflicting. For purposes such as the analysis of declining and growing industries, part-time labor decisions, and labor market entry decisions, we find search models appealing. However, a sizable fraction of those who lose their jobs in the United States are temporarily laid off. Martin Feldstein (1975, p. 732) reports that 56 percent of all job losses are temporary layoffs rather than permanent separations. This feature of the labor market seems best modeled as part of a contractual arrangement. Precisely because such layoffs are known to be temporary, we do not find the idea that the economy gets stuck in a bad equilibrium an appealing explanation of this phenomenon.

### I. The Model

Here we develop a labor contract model in which the firm is a monopolist in the product market. The model has two agents: a firm and a worker. The worker supplies labor to the firm, which produces an output good. The technology for producing the output is given by the production function  $f(n)$ , where  $n$  denotes the labor services of the worker. The firm faces a downward-sloping demand for its output given by  $D(q, \theta)$ , where  $q$  denotes output and  $\theta$  a demand shock drawn from some distribution function. For simplicity, we assume that the demand shock can take on one of two values, denoted by  $\theta_1$  and  $\theta_2$ . Without loss of generality, we assume that  $\theta_1 < \theta_2$ . The probability that  $\theta = \theta_1$  is denoted by  $p_1$ ; that  $\theta = \theta_2$ , by  $p_2$ .

The preferences of the worker are represented by the utility function  $U(c,n)$ , where  $c$  denotes consumption and  $n$  labor supply. Firms seek to maximize expected profits and the worker to maximize expected utility. We assume that the demand shock is private information to the firm. Before the firm observes the demand shock, the worker and the firm enter into a contract specifying a compensation-employment schedule. Given the private information assumption, it is convenient to represent a contract as four numbers  $(c_1, n_1, c_2, n_2)$  which represent compensation and labor supply in the two states.

The worker has an alternative source of income which yields utility of  $\bar{u}$  if the worker does not agree to the contract. However, once the contract is agreed upon, the worker cannot work for other firms. This assumption is consistent with the evidence that manufacturing firms rehire about 85 percent of the workers they lay off (Feldstein, 1975, p. 726). Unemployment compensation or income from assets can be included in our analysis without changing the results substantially.

An optimal contract solves this problem:

$$(1) \quad \max \sum_{i=1}^2 p_i [q_i D(q_i, \theta_i) - c_i]$$

subject to

$$(2) \quad \sum_{i=1}^2 p_i [U(c_i, n_i)] \geq \bar{u}$$

$$(3) \quad q_i D(q_i, \theta_i) - c_i \geq q_j D(q_j, \theta_i) - c_j, \quad \text{for } i, j = 1, 2$$

$$(4) \quad c_i \geq 0 \text{ and } n_i \geq 0$$

where  $q_i = f(n_i)$ , for  $i = 1, 2$ . The inequalities in (3) are the standard incentive compatibility constraints requiring that the firm not have an incentive to lie about the state of the world.

Note that we do not consider randomized contracts between the firm and the worker, that is, contracts where compensation and employment are determined by the flip of a publicly observed coin. Because the objective function of the firm is not necessarily concave, such randomized contracts can yield ex ante gains if they are feasible. Furthermore, if the firm negotiates a contract with many workers, a publicly observed coin may not be necessary because the desired randomization can be accomplished by employing only a fraction of the workers. We leave this possibility for future research.

At this level of generality, characterizing the optimal contract is difficult. Since our primary interest is in numerically computing the optimal contract, we assume specific functional forms for the demand function, the production function, and the preferences:

$$(5) \quad D(q, \theta) = \alpha + \theta - \gamma q$$

$$(6) \quad U(c, n) = [(c+b)^{1-\beta} + (\bar{n}-n)^{1-\beta}]^{1/(1-\beta)}$$

$$(7) \quad n = a(q-d)^3 + ad^3$$

$$(8) \quad \bar{u} = (b^{1-\beta} + \bar{n}^{1-\beta})^{1/(1-\beta)}.$$

Equation (7) is the inverse production function, or the labor requirement function. Its functional form implies a U-shaped average cost curve for the firm. We have also assumed, in (8), that the worker's reservation utility is given by zero labor supply and zero compensation from the firm. We don't think this assumption affects our results very much.

Before reporting on our results, it is useful to examine the role of monopoly power in the product market as well as the role of private information. Suppose the firm has no market power, that is,  $\gamma = 0$ , and the demand shock,  $\theta$ , is publicly observed. An optimal contract in such an en-

vironment maximizes (1) subject to (2) and (4). It is straightforward to show that an interior contract satisfies

$$(9) \quad U_c(c_1, n_1) = U_c(c_2, n_2)$$

$$(10) \quad - \frac{U_n(c_i, n_i)}{U_c(c_i, n_i)} = (\alpha + \theta_i) f'(n_i), \quad \text{for } i = 1, 2.$$

It can be shown that if the production function and the utility function are strictly concave and leisure is a normal good, then the worker becomes worse off as the demand shock increases and the marginal wage is less than the marginal rate of substitution. That is,

$$(11) \quad \frac{c_2 - c_1}{n_2 - n_1} \leq - \frac{U_n(c_1, n_1)}{U_c(c_1, n_1)}.$$

Let us now add the assumption that the demand shocks are private information to the firm, but retain the assumption that  $\gamma = 0$ . Now the optimal contract maximizes (1) subject to (2)-(4). It follows from Chari's (1983) results that if leisure is a normal good, then the optimal contract has these characteristics:

$$(12) \quad c_2 \geq c_1 \text{ and } n_2 \geq n_1$$

$$(13) \quad (\alpha + \theta_1)q_1 - c_1 = (\alpha + \theta_1)q_2 - c_2$$

$$(14) \quad - \frac{U_n(c_1, n_1)}{U_c(c_1, n_1)} = (\alpha + \theta_1) f'(q_1).$$

Chari (1983) proposed one definition of involuntary underemployment: A worker is involuntarily underemployed if, under the optimal contract, the marginal rate of substitution between labor and consumption is less than the marginal rate of transformation. The idea was that there are unexploited gains to trade. As (14) shows, with normality of leisure, there cannot be involuntary underemployment in this sense in the low state. One can also show that there is no involuntary underemployment in the high state.



We propose a new definition. We say that a worker is involuntarily underemployed if the optimal contract is such that the worker would prefer to increase employment at the marginal wage. One interpretation of the marginal wage is overtime payments. A natural question to ask is, Would a worker be willing to work overtime at, say, time-and-a-half rates? If so, then we say that the worker is involuntarily underemployed.

We can use (13) and (14) to show that the optimal contract with asymmetric information still has the feature that the marginal wage is less than the marginal rate of substitution. Thus, the optimal contract with asymmetric information does not have involuntary underemployment in the marginal wage sense.

## II. Results

Can underemployment in the marginal wage sense be generated when firms have market power? We compute the optimal contract for a range of parameter values. We report here on two simulations.

The parameter values for the first simulation are  $b = 20$ ,  $\beta = 0.8$ ,  $\theta_1 = 0$ ,  $\theta_2 = 10$ ,  $d = 5$ ,  $\bar{n} = 24$ ,  $a = 0.2$ , and  $p_1 = 0.2$ . We vary  $\alpha$  from 10 to 50 in increments of 1 and  $\gamma$  from 0.1 to 3.0 in increments of 0.1. Graphs of the results are displayed nearby. Our measure of unemployment here is the difference between the marginal wage and the marginal rate of substitution. Note that this difference is always positive. Thus, we have underemployment throughout the range of parameter values. Note also that for small values of  $\alpha$ , the labor supply in state 1 is close to zero. However, our measure of unemployment is positive. One interpretation is that the worker is involuntarily unemployed in state 1 in such economies.

The second simulation has these parameters:  $\alpha = 18$ ,  $\theta_1 = 0$ ,  $\theta_2 = 10$ ,  $\gamma = 1$ ,  $d = 5$ ,  $\bar{n} = 24$ ,  $a = 0.2$ , and  $p_1 = 0.2$ . Here the value of  $b$

is varied from 10 to 50 in increments of 1 and the value of  $\beta$  from 0.4 to 1.2 in increments of 0.025. The graphs of the results are also nearby. Note that when  $b$  is large, the labor supply in state 1 is zero, but consumption is clearly still positive. Thus, the contract provides insurance to the worker. Furthermore, our measure of unemployment is clearly positive, so that again involuntary unemployment is possible.

### III. Final Remarks

We have proposed a new definition of involuntary underemployment for implicit contract models. Our definition is consistent in some ways with what the U.S. Bureau of Labor Statistics calls unemployment. The bureau asks whether individuals have looked for jobs in the preceding four weeks. In our model, we ask whether workers would be willing to work more hours at the going wage rate, which we interpret to be the marginal wage in the contract.

One way to use this model to explain unemployment in the sense measured by the bureau is to consider the following scenario. Suppose that the demand shocks are drawn from some distribution, but remain fixed for a period of time, say, a year. Suppose also that efficiency in production requires eight-hour days; that is, there is a nonconvexity in employment. Interpret employment in our model as the number of hours worked in the year. Clearly, then, reductions in employment are reductions in days worked rather than in hours worked per day. During one of the spells of unemployment, let us say we ask a worker whether she or he would prefer to be employed. It would then be appropriate to use the marginal wage as a measure of increased compensation for labor services and to interpret an affirmative response as unemployment.

We should reemphasize that we view our model as appropriate only for certain types of unemployment, such as temporary layoffs. Other models are undoubtedly more useful for understanding, say, the Great Depression or business cycle fluctuations. The assumption of asymmetric information does not seem plausible for analyzing movements in unemployment rates over the business cycle. Furthermore, not all workers who are laid off wait to be recalled; some search for new jobs. An integration of search and contract theory might well lead to better models of unemployment.

We have assumed that firms have market power in the product markets. It would be interesting to embed our model in a general equilibrium setting to ask whether imperfect competition in general equilibrium can magnify the extent of unemployment as well as the response to aggregate shocks. It would also be interesting to examine whether coordination failures can lead to a magnification of the effect on unemployment from asymmetric information.

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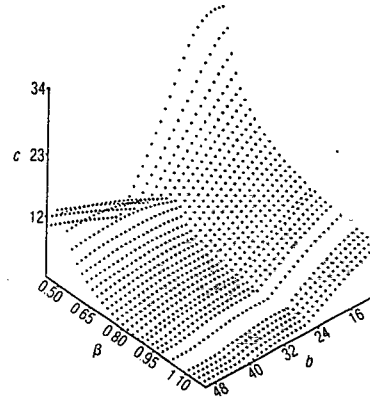
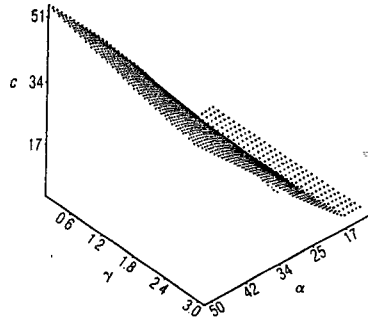
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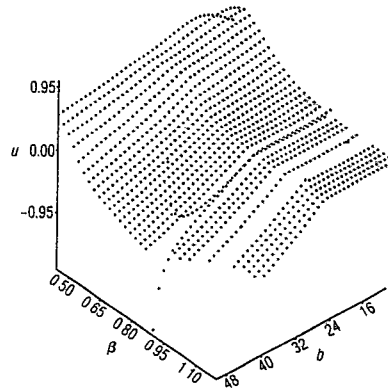
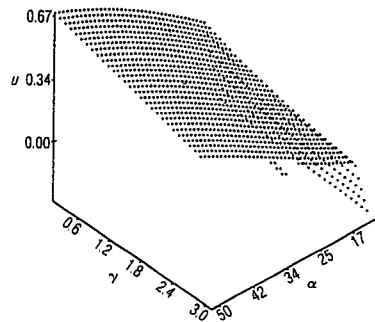
Simulation 1

Simulation 2

Consumption  
in State 1



Measure of  
Unemployment



Labor Supply  
in State 1

