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MONETARY TARGETING IN A DYNAMIC MACRO MODEL

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ABSTRACT

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The consequences of a straightforward monetary targeting scheme are examined for a simple dynamic macro model. The notion of "targeting" used is the strategic one introduced by Rogoff (1985). Numerical calculations are used to demonstrate that for the model under consideration, monetary targeting is likely to lead to a deterioration of policy performance. These examples cast doubt upon the general efficacy of simple targeting schemes in dynamic rational expectations models.

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The views expressed herein are those of the author and not necessarily those of the Federal Reserve Banks of Atlanta or Minneapolis or of the Federal Reserve System.

## 1. Introduction

A topic of considerable policy interest is whether monetary targeting improves upon or detracts from the overall performance of monetary policy. The theoretical debate on this subject has been quite intense, particularly after the Federal Reserve's 1979-82 targeting "experiment." While this debate continues to rage on many fronts, lately it has focused on strategic issues.<sup>1</sup> That is, Can monetary targeting improve the performance of monetary policy by improving the Fed's credibility?

A recent paper by Rogoff (1985) strongly suggests an affirmative answer to this question. In the context of a static macro model, Rogoff demonstrates that various intermediate targeting schemes may be useful in overcoming the credibility problem inherent in a discretionary policy environment.<sup>2</sup>

The present paper is a first attempt to examine the generality of Rogoff's results.<sup>3</sup> In the analysis that follows, the consequences of a simple monetary targeting scheme are traced through for a dynamic macro model. Although analytical characterizations of policy performance are difficult to obtain, numerical results suggest that for the model under consideration, targeting is likely to lead to significant deterioration of policy performance. Nor does targeting lead to improved credibility of monetary policy. Instead, the imposition of targeting often leads to erratic short-term fluctuations in both the money stock and the price level.

The examples below should not be construed as a general condemnation of all monetary targeting schemes. Indeed, recent

work by Whiteman (1985) suggests that policy performance can always be improved by implementing some targeting mechanism. However, the examples considered here suggest that simple, intuitively appealing targeting mechanisms can easily have the opposite of the desired effect.

## 2. The Model

The model considered here can be derived from Cagan's (1956) demand function for real balances:

$$(1) \quad \log (M/P)_t = \alpha \pi_t + \xi y_t + \psi + U_t, \quad \alpha < 0, \xi > 0$$

where  $M$  is the demand for nominal balances,  $P$  is the price level,  $\pi_t$  is the expected inflation rate,  $y_t$  is the log of real income,  $\psi$  is a constant term, and  $U_t$  is a stochastic error term. Following standard practice, the values of all variables are interpreted as deviations about perfectly forecastable trends. In addition, the analysis below abstracts from all real effects by taking  $\xi y_t + \psi$  to be identically zero. The process  $\{U_t\}$  is assumed to follow the stationary first-order autoregressive law

$$(2) \quad U_t = \gamma U_{t-1} - \alpha \varepsilon_t, \quad 0 < \gamma < 1$$

where  $\varepsilon_t$  is Gaussian white noise. Both private agents and policy-makers are assumed to know the values of current and past realizations of  $U_t$ . The money demand shock  $U_t$  is the only source of uncertainty in the model.

By imposing the rational expectations hypothesis and some relatively innocuous side conditions, the money demand equation (1) can be rewritten as<sup>4</sup>

$$(3) \quad p_t = -\rho^{-1} \sum_{j=0}^{\infty} \rho^{-j} E_t(m_{t+j} + u_{t+j})$$

where  $E_t$  denotes the expectation conditional on current and past  $U_t$ 's;  $p_t$  is the log of the price level;  $m_t = \log(M_t)/\alpha$ ;  $u_t = -U_t/\alpha$ ; and  $\rho = (\alpha-1)/\alpha$ . Equation (3) reveals the dependence of the price level on current and all expected future values of nominal money demand. It is this dependence that causes the problem of setting monetary policy to be a dynamic one.

Throughout this paper it is assumed that the Fed can completely control the nominal money stock, which will always equal the nominal money demand in equilibrium. The Fed's objective is taken as to minimize a weighted average of the discounted sum of current and future fluctuations in the logarithms of the money supply and the price level. That is, at time  $t$  the Fed's objective is given by

$$J_t \equiv E_t \sum_{j=0}^{\infty} \beta^j (1/2 p_{t+j}^2 + \lambda/2 m_{t+j}^2), \quad 1 > \beta > 0, \lambda > 0.$$

The objective of the Fed is thus taken as to stabilize the fluctuations of the money supply and price level about their long-term trends, taking equation (3) as given.

The setup of the model described here differs somewhat from most others found in the literature on targeting. A superficial difference is that the price level, rather than some aggregate quantity measure such as GNP, is taken as the target variable. This difference could be resolved by incorporating a real sector for the model, but for reasons of analytical convenience, this is not done. A more important difference is due to the positive weight  $\lambda$  attached to fluctuations in the policy instru-

ment  $m_t$ . Analytically, this term is important in generating a policy tradeoff for the model. If this term were to vanish, the Fed could always attain a value of zero for  $J_t$  by completely offsetting money demand shocks. When  $\lambda > 0$ , the stabilization benefits of such interventions must be weighed against their costs. However, at the level of abstraction assumed by the model, it is difficult to assign an unambiguous economic interpretation to these costs. Still, the inclusion of these costs in the Fed's loss function is justified on an intuitive level, since their exclusion would imply that the Fed is completely indifferent to fluctuations in the policy instrument  $m_t$ .<sup>5</sup>

### 3. Policy Rules Under Precommitment and Discretion<sup>6</sup>

As in virtually all rational expectations models, deriving "optimal" policy rules for the model just described requires that the policy authority's degree of precommitment be specified. To begin, consider the case where the Fed can credibly commit itself to an infinite sequence of policies. In this case, monetary policy is determined once and for all at time  $t = 0$ , conditional on a given sequence of shocks  $\{\varepsilon_t\}$ . This sort of policy environment is sometimes referred to as a precommitment or open loop policy environment.

The optimal precommitment monetary policy for this problem can be found using techniques outlined in Hansen, Epple, and Roberds (1985). Setting the Fed's discount factor  $\beta$  equal to one for convenience,<sup>7</sup> the appropriate Lagrangian for the time  $t = 0$  policy problem is

$$\mathcal{L} \equiv E_0 \sum_{t=0}^{\infty} [1/2(p_t^2 + \lambda m_t^2)] + \theta_t [p_t + \rho^{-1} E_t \sum_{j=0}^{\infty} \rho^{-j} (m_{t+j} + u_{t+j})]$$

where  $\{\theta_t\}$  is a sequence of random Lagrange multipliers. First-order conditions for the precommitment problem are

$$(4) \quad \lambda m_t + \rho^{-1} \sum_{j=0}^t \rho^{-j} \theta_{t-j} = 0$$

$$(5) \quad p_t + \theta_t = 0.$$

Equations (4) and (5) hold for  $t \geq 0$ . Solving for  $m_t$ , we obtain

$$(6) \quad m_0 = (\lambda \rho)^{-1} p_0$$

$$(7) \quad m_t = \rho^{-1} m_{t-1} + (\lambda \rho)^{-1} p_t, \quad t > 0.$$

The time inconsistency of the optimal precommitment policy is manifested in the fact that the representation for  $m_0$  in equation (6) differs from that for  $m_t$  for positive  $t$  in equation (7). Equation (7) requires that  $m_t$  "feed back" on  $m_{t-1}$  after the initial period in which policy is set. If optimal policy were to be reset at some time  $s > 0$ , however, equation (6) would require that  $m_{s-1}$  be ignored in setting  $m_s$ . Thus, without some mechanism to guarantee that the Fed would always stick to its original plan, the precommitment policy is not credible. Nonetheless, it is useful to solve for the precommitment policy as a benchmark against which to compare other policies. Appendix A shows that in equilibrium the sequence of optimal policies follows the law

$$(8) \quad m_t = c_1 m_{t-1} + c_0 u_t, \quad t \geq 0$$

where  $1 > c_1 > 0$  and  $c_0 < 0$ , subject to the initial condition  $m_{-1} = 0$ .

We next consider a policy environment of pure discretion. In a discretionary policy environment, optimal policies are recomputed in every period, so that announcements about time  $t$  policy that are made before time  $t$  are not credible. Given this sort of policy environment, one could think of policy as being set by a sequence of Fed policymakers. The policymaker at time  $t$  has the authority to set time  $t$  policy only. A policymaker may predict what future policymakers will do, but cannot commit them to any predetermined course of action. Accordingly, the appropriate Lagrangian for the time  $t$  policymaker is

$$L_t^* \equiv E_t \sum_{j=0}^{\infty} [1/2(p_{t+j}^2 + \lambda m_{t+j}^2)] + \theta_t [p_t + \rho^{-1} E_t \sum_{j=0}^{\infty} \rho^{-j} (m_{t+j} + u_{t+j})]$$

where again  $\beta = 1$  for convenience. First-order conditions for the time  $t$  policymaker are given by

$$(9) \quad \lambda m_t + \rho^{-1} \theta_t = 0$$

$$(10) \quad p_t + \theta_t = 0.$$

Solving equations (8) and (9) for  $m_t$  in turn yields

$$(11) \quad m_t = (\lambda \rho)^{-1} p_t.$$

The time consistency of monetary policy in this environment is manifested in the fact that the representation for optimal policy given in equation (11) holds for all  $t$ . In Appendix A,

equation (11) is shown to imply the following feedback rule for policy:

$$(12) \quad m_t = f^* u_t, \quad \text{where } f^* = -[1 - \lambda\rho(\gamma - \rho)]^{-1}.$$

Since  $f^*$  lies between 0 and -1, optimal monetary policy in a discretionary environment consists of accommodating some fraction--but not all--of the current (normalized) money demand shock  $u_t$ .

As is true for most policy problems in a rational expectations setting, the performance of the discretionary policy rule given in equation (12) will be dominated by the performance of the precommitment policy rule given in equation (8). That is, the value of the Fed's loss function  $J_t$  will be greater under discretion than under precommitment.<sup>8</sup> However, as Kydland and Prescott (1977) point out, there is no way to recoup this difference given the discretionary policy environment assumed above. To improve the performance of policy, some sort of mechanism must be introduced that will augment the credibility of the policy authority. One candidate for such a mechanism is described in the next section.

#### 4. A Simple Targeting Scheme

There are two major reasons for considering the targeting scheme described below. First, this targeting scheme constitutes a reasonable dynamic generalization of one proposed by Rogoff (1985) in the context of a static model--a generalization made expressly to overcome a "policy credibility" problem similar to the one described in Section 3. Second, the targeting scheme

we consider is designed to mimic, within the confines of the idealized model of this paper, several of the important aspects of monetary targeting as historically practiced under the Humphrey-Hawkins Act.

We should begin by explaining what is meant by targeting in a strategic policy environment. By requiring the Fed to target some aggregate variable, we mean to alter the Fed's objective function  $J_t$  so that the Fed is penalized when that aggregate deviates from its preannounced target value. The idea behind this definition is as follows: altering the Fed's incentives may somehow compel the Fed to take policy actions more closely approximate to the precommitment policies that minimize its true objective  $J_t$ .

The targeting scheme we consider proceeds as follows. For convenience the duration of the time period in the model is taken as a half year. Each year has two months, January and July. In January of each year (i.e., in every even-numbered period), the Fed is required to submit a target value of the nominal money stock for July (i.e., in the subsequent odd-numbered period). In even-numbered periods the Fed is free to set the nominal money stock as it wishes, and its one-period loss function is the same as the one given above:

$$L_e(p_t, m_t) \equiv 1/2(p_t^2 + \lambda m_t^2).$$

During odd periods (i.e., in July of each year), the Fed feels some pressure to meet its preannounced money supply target, so that its one-period loss function becomes

$$L_o(p_t, m_t, m_t^*) \equiv 1/2[p_t^2 + \lambda m_t^2 + \tau(m_t - m_t^*)^2], \quad \tau > 0.$$

where  $m_t^*$  represents the logarithm of the preannounced monetary target, divided by the parameter  $\alpha$ .<sup>9</sup> The Fed's objective(s) is taken as to minimize

$$K_t \equiv E_t \sum_{j=0}^{\infty} \beta^j L_{t+j}$$

where  $L_{t+j} = L_o(\cdot)$  for  $t+j$  odd, and  $L_{t+j} = L_e(\cdot)$  for  $t+j$  even, taking into account the private sector's reaction function given by equation (3). In even periods, the Fed minimizes  $K_t$  by choosing two policy instruments: the current value of the logged money stock  $m_t$  and the monetary target for the next period,  $m_{t+1}^*$ . During odd periods, the Fed can only set one instrument, the current value of the money stock.

Several features of this targeting model deserve discussion. First, it should be emphasized that under this model, the Fed still operates in a policy environment of pure discretion, although its objective function is changed. The monetary targets  $m_t^*$  cannot be interpreted as either binding promises or optimal predictions. Except in the limiting case where  $\tau = \infty$ , there is no constraint that targets must be met exactly. Nor is there an explicit requirement that  $m_{t+1}^*$  must represent an optimal time  $t$  prediction of  $m_{t+1}$ ; i.e., it is not required that  $E_t m_{t+1} = m_{t+1}^*$ . Some pressure to target accurately does exist, however, because the Fed wishes to diminish the penalty term  $\tau/2(m_{t+1} - m_{t+1}^*)^2$  associated with deviations from the targeted money stock. The exact nature of this penalty is left to the reader's imagination, while the parameter  $\tau$  is assumed to be exogenously determined by the institutional setting under which monetary policy is set.

Another important feature of the targeting model is that deviations from target are only subject to penalty at midyear, i.e., in the odd-numbered periods. As will be seen in Section 5, this feature is important methodologically, since it allows the model to be solved using a simple recursive algorithm. More importantly, this feature is meant to reflect the conical shape of the target bands historically announced for monetary aggregates. The notion of conical target bands is captured in an abstract setting by imposing positive costs to such deviations at midyear, while assigning zero costs to deviations at the beginning of each year. In the terminology of Broaddus and Goodfriend (1984), annual "rebasings" of the money stock carries no explicit penalty.

Finally, it should be emphasized that under targeting, the true or "social" objective of the Fed is still  $J_t$ , although its operating objectives are now summarized by the function  $K_t$ . As discussed above, the purpose of using targeting to alter the Fed's operating objective would be to attain lower values of the true loss function  $J_t$ .

## 5. Equilibrium With Targeting

A computationally convenient way of deriving equilibrium policies in a discretionary policy environment is to use the notion of feedback or recursive equilibrium of dominant-player dynamic games, as defined in Kydland (1977). Before using this equilibrium concept to solve for equilibrium policies under targeting, it is perhaps instructive to reconsider the problem of setting discretionary policy without targeting, i.e., the case when  $\tau = 0$ .

We begin by noting that for the model without targeting, there is only one dynamic state variable: the money demand shock  $u_t$ . We wish to consider feedback policies for the Fed that take the form  $m_t = f(u_t)$ . Given the linear-quadratic-Gaussian setup of the model, we can restrict our attention to linear feedback rules of the form  $m_t = fu_t$ . Let  $f_0$  be our initial guess for the value of the optimal feedback parameter  $f^*$ . If, at period  $t$ , private agents believe that policy in all future periods will be set according to the rule  $m_t = f_0 u_t$ , then equation (3) may be evaluated as

$$(13) \quad p_t = [(1 + \rho^{-1}\gamma f_0)/(\gamma - \rho)]u_t - \rho^{-1}m_t.$$

Now define the Fed's value function  $V(u_t)$  as the value of the Fed's objective  $J_t$  when the optimal feedback parameter  $f^*$  is used in the current and all future periods. In equilibrium, the optimal feedback parameter  $f^*$  must satisfy, for any value of  $u_t$ , the requirement that  $\underline{m}_t = f^*u_t$ , where  $\underline{m}_t$  solves

$$(14) \quad \min_{m_t} [1/2(p_t^2 + \lambda m_t^2) + \beta E_t V(u_{t+1})]$$

subject to equations (2) and (13) and where in equilibrium,  $f^* = f_0$ . As shown in Appendix B, solving program (14) and imposing the condition that  $f^* = f_0$  yields a feedback rule  $f^*$  identical to that in equation (12).

For more complex models, it is often difficult to solve for equilibrium feedback rules directly. However, the recursive character of feedback equilibrium suggests a natural algorithm for numerical computation of feedback rules. That is, given an ini-

tial guess  $f_0$  for  $f^*$ , find the feedback rule  $f_1$  that solves program (14), then take  $f_1$  as the next guess for  $f^*$ , and so on. The recursive nature of feedback equilibrium also guarantees that equilibrium policies will be time consistent: in solving the program (14) at time  $t$ , note that the Fed is constrained to take all future policies as given.

We now consider the problem of setting discretionary policy under the targeting scheme described in Section 4. Under targeting, it will be important to distinguish between January (even) and July (odd) periods. In even periods, as in the model without targeting, the Fed's one-period loss function is influenced by only one state variable--the shock  $u_t$ . In odd periods, however, the previously announced logged money stock target  $m_t^*$  must be added to the list of state variables. Two decision variables, the current logged money stock  $m_t$  and the midyear target  $m_{t+1}^*$ , must be set in even periods, while only the current money stock is set at midyear. Consequently we consider policies of the form

$$\left. \begin{aligned} (15) \quad m_t &= f_0 u_t \\ (16) \quad m_{t+1}^* &= f_1 u_t \end{aligned} \right\} \text{ for even } t$$

and

$$(17) \quad m_t = g_0 u_t + g_1 m_t^*, \quad \text{for odd } t.$$

Appendix C shows that when equations (15), (16), and (17) hold, equation (3) may be evaluated as

$$(18) \quad p_t = d_0 u_t + d_1 m_t + d_2 m_{t+1}^*, \quad \text{for } t \text{ even}$$

$$(19) \quad p_t = b_0 u_t + b_1 m_t, \quad \text{for } t \text{ odd}$$

where the b's and d's are complicated functions of  $f_0, f_1, g_0, g_1, \lambda, \rho,$  and  $\gamma$ . Under targeting, equilibrium feedback rules are determined by a 4-tuple  $(f_0^*, f_1^*, g_0^*, g_1^*)$  such that when  $t$  is even,

$$(20) \quad m_t = f_0^* u_t \quad \text{and} \quad m_{t+1}^* = f_1^* u_t$$

and when  $t$  is odd,

$$(21) \quad m_t = g_0^* u_t + g_1^* m_t^*$$

where the  $m_t$ 's in turn solve the program

$$(22) \quad \min_{m_t, m_{t+1}^*} \{L_e(p_t, m_t) + \beta E_t [\min_{m_{t+1}} L_o(p_{t+1}, m_{t+1}, m_{t+1}^*)] + \beta^2 E_t W(u_{t+2})\}, \quad \text{for } t \text{ even}$$

subject to constraints (2), (18), and (19), where  $W(u_t)$  represents the value of  $K_t$  for  $t$  even when optimal policies are in effect and the b's and d's in (18) and (19) are evaluated at  $(f_0^*, f_1^*, g_0^*, g_1^*)$ . The equilibrium feedback parameters can be numerically determined, given values for  $\lambda, \rho,$  and  $\gamma$ , by the iterative procedure outlined in Appendix C. Basically, this procedure takes an initial guess  $(f_0, f_1, g_0, g_1)$  for the feedback parameters, uses these values to obtain equations (18) and (19), and then solves program (22). The feedback rules implied by the solution are in turn used to generate updated versions of equations (18) and (19), and so on, until an approximate fix point is reached.

## 6. Numerical Examples

Because of the somewhat complicated nature of the program (22), analytical characterizations of the targeting equilibrium are difficult to obtain. For this reason, some numerical examples were calculated to obtain an idea of policy performance under targeting. The results of three representative sets of calculations are reported in Table 1.

In each of the numerical examples, arbitrary values were assumed for the parameters  $\lambda$ ,  $\rho$ , and  $\gamma$ . The discount factor  $\beta$  was again taken as equal to one, and the true policy objective  $J_t$  was reinterpreted as an average cost objective. Using formulas derived in Appendix D, the equilibrium value of  $J_t$  was calculated for each set of parameter values under precommitment, discretionary, and various targeting environments. For each policy environment, the performance index  $P \equiv 100(J/J^d)$  was calculated, where  $J^d$  represents the equilibrium value of  $J$  for the same parameter values, given a discretionary policy environment without targeting (i.e., where  $\tau = 0$ ). This index thus gives the performance of policy in a given environment as a percentage of the performance of the best consistent policy without targeting. Note that smaller values of  $P$  are preferred and that a value of  $P$  under 100 indicates improvement due to implementation of targeting.

Before describing the results of these calculations, it may be useful to consider how variations in the parameters  $\lambda$ ,  $\rho$ , and  $\gamma$  affect the potential gains in policy performance due to precommitment. First, recall that setting  $\lambda = 0$  allows the Fed to costlessly offset money demand surprises (if targeting is not in

Table 1  
 Numerical Examples of Policy Performance  
 Under Targeting

Set 1

Parameter Values:  $\lambda = 1.0$ ,  $\rho = 1.5$ ,  $\gamma = 0.95$

| Policy Environment | Performance Index (P) |
|--------------------|-----------------------|
| Precommitment      | 76.57%                |
| Discretionary With |                       |
| $\tau = 0$         | 100.00%               |
| $\tau = 0.1$       | 100.03                |
| $\tau = 1.0$       | 102.88                |
| $\tau = 10.0$      | 114.36                |

Set 2

Parameter Values:  $\lambda = 10.0$ ,  $\rho = 1.1$ ,  $\gamma = 0.95$

| Policy Environment | Performance Index (P) |
|--------------------|-----------------------|
| Precommitment      | 37.87%                |
| Discretionary With |                       |
| $\tau = 0$         | 100.00%               |
| $\tau = 1.0$       | 102.44                |
| $\tau = 10.0$      | 116.12                |

Set 3

Parameter Values:  $\lambda = 0.1$ ,  $\rho = 2.0$ ,  $\gamma = 0.5$

| Policy Environment | Performance Index (P) |
|--------------------|-----------------------|
| Precommitment      | 94.08%                |
| Discretionary With |                       |
| $\tau = 0$         | 100.00%               |
| $\tau = 0.05$      | 99.46                 |
| $\tau = 0.1$       | 102.48                |

effect), so that the global minimum of  $J_t = 0$  can be attained in a discretionary policy environment without targeting. Accordingly, one would expect the gains from precommitment to be small when  $\lambda$  is close to zero. A similar conclusion holds when  $\gamma$  is close to zero. This is because in the limiting case that  $\gamma = 0$ , the dynamic policy game inherent in the model reduces to a sequence of repeated static games, which by definition are immune to dynamic consistency problems.<sup>10</sup> Finally, equation (3) reveals that when  $\rho$  becomes large,  $p_t$  is driven to zero. In the limiting case that  $\rho = \infty$ , the problem of stabilizing  $p_t$  becomes trivial. Hence the benefits of precommitment are likely to be reduced when  $\rho$  is relatively large.

For the first set of numerical examples shown in Table 1, the parameter values  $\lambda = 1.0$ ,  $\rho = 1.5$ , and  $\gamma = 0.95$  were assumed. The performance index  $P$  of about 77 percent for the ideal precommitment environment indicates that the potential gains to precommitment for this example are significant: perfect credibility entails about a 23 percent decrease in the policy loss function. However, attempts to increase policy credibility via targeting were not successful. For the positive values of  $\tau$  that were tried, the implementation of targeting resulted in a deterioration of policy performance (i.e., in values of  $P$  over 100 percent). This deterioration is apparently increasing in the "strictness"  $\tau$  of the targeting mechanism.

In the second set of examples, the parameter values  $\lambda = 10.0$ ,  $\rho = 1.1$ , and  $\gamma = 0.95$  were assumed. As might be inferred from the discussion above, increasing the value of  $\lambda$  and

decreasing the value of  $\rho$ , relative to the first set of parameter values, results in an even greater potential gain in policy performance from precommitment. The value of the performance index  $P$  under precommitment is about 38 percent for this example, implying a 62 percent decrease in the policy loss function under full credibility. Again, attempts to recoup this gain under targeting only resulted in deterioration of policy performance, with the degree of deterioration increasing in  $\tau$ .

For some of the numerical examples considered, implementation of targeting did lead to gains in policy performance. Typical of these examples is the third set shown in Table 1. For this set of examples the parameter values  $\lambda = 0.1$ ,  $\rho = 2.0$ , and  $\gamma = 0.5$  were assumed. As seen from the performance index column of the table, taking  $\tau = 0.05$  in this example results in a decrease of about half a percentage point in the policy objective function. Larger values of  $\tau$  again lead to deterioration of policy performance. However the value of  $P$  for the precommitment case (about 94 percent) reveals that for this example, the magnitude of the dynamic consistency problem is not large. Even in an environment of perfect credibility, only about a 6 percent gain in policy performance can be attained.

In summary, the numerical examples reveal that the effect of targeting on policy performance is somewhat ambiguous. For some parameter values, targeting results in gains in policy performance, while for other values, losses occur. The magnitude of the potential gains (under 2 percent in all the examples tried) tend to be quite small relative to the magnitude of the potential

losses. Moreover, the gains were always present in examples for which the dynamic consistency problem was relatively unimportant (i.e., in examples for which the values of  $\lambda$ ,  $\rho$ , or  $\gamma$  were close to regions where the dynamic consistency problem does not exist). The larger losses were present in examples where potential gains due to increases in credibility were quite large.

Some intuition concerning the failure of the targeting scheme considered in this paper is offered by Figures 1 and 2. These figures depict the responses of  $m_t$  and  $p_t$  to a one-standard-deviation shock to money demand (here corresponding to a minus 0.1 standard deviation realization of  $\varepsilon_t$ ) where the parameter values  $\lambda = 10$ ,  $\rho = 1.1$ , and  $\gamma = 0.95$  are assumed. Responses are plotted for the precommitment case and for the discretionary case where  $\tau = 0$  (no targeting) and  $\tau = 10$  (targeting).

Figure 1 shows the response of  $m_t$  (proportional to the response of the logged money stock to a positive shock to money demand) under the three environments. The optimal precommitment response is seen to require an initial rapid series of increases in  $m_t$ , followed by a series of gradual decreases. The discretionary response without targeting consists of an initial rapid increase, followed by a series of gradual decreases of  $m_t$ . The effect of the targeting scheme considered is to introduce oscillations into the response of  $m_t$ . During the midyear (odd-numbered) periods when targeting is in effect,  $m_t$  is biased towards zero; while during even periods,  $m_t$  is very close to its values under discretion without targeting. Figure 2 shows that under targeting, similar--if somewhat less extreme--oscillations are introduced into the  $p_t$  process.

### Figures 1 and 2

Response of Money Stock and Price Level to a Money Demand Shock  
in Three Policy Environments

(Parameter Values:  $\lambda = 10.0$ ,  $\rho = 1.1$ ,  $\gamma = 0.95$ )

Figure 1: Response of  $m_t$

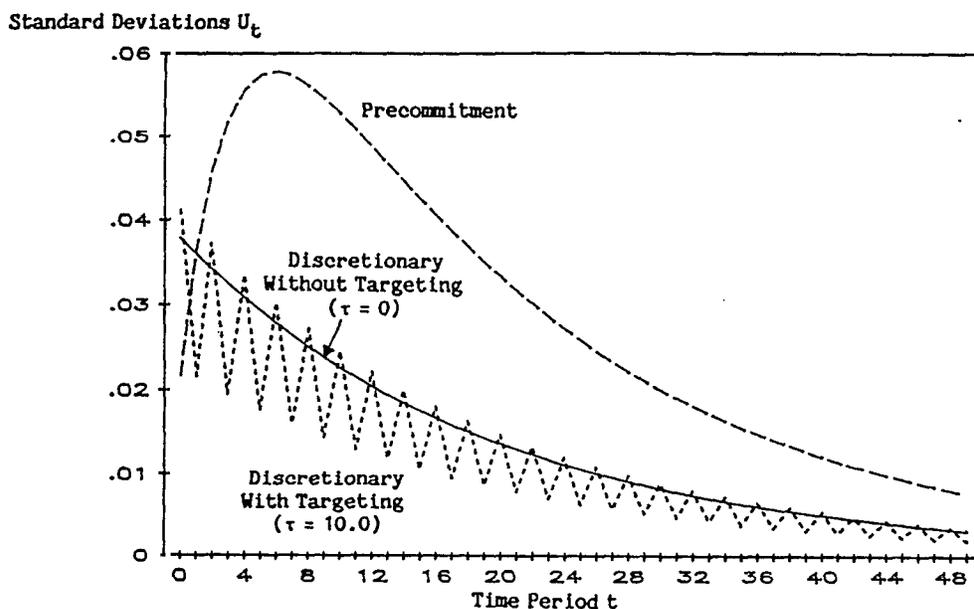
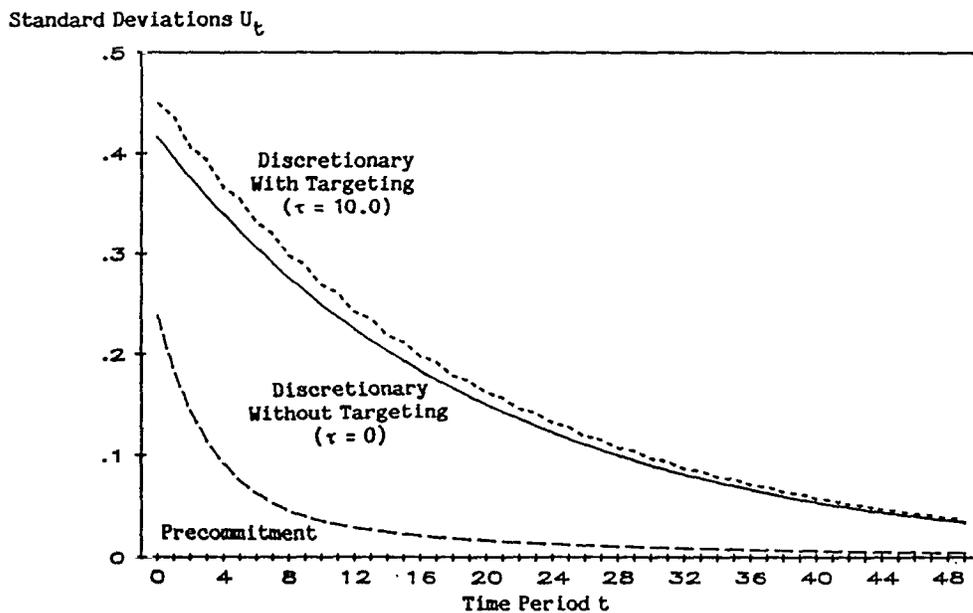


Figure 2: Response of  $p_t$



Some intuition for the example is provided by an earlier result of Whiteman (1986). That paper shows that discretionary policy (without targeting) will dominate a passive policy of always setting  $m_t = 0$ . Hence it is not surprising that targeting, which seems to bias policy responses towards this passive policy, results in a worsening of policy performance. This bias towards zero is a direct result of the targeting mechanism, which assigns positive costs to active policy responses. Of course, these costs are assigned with the idea that they will be more than offset by a resultant increase in credibility. However, the parameter values assumed in this example cause money demand shocks to have very persistent effects, so that the marginal benefit of a one-period-ahead commitment on the part of the Fed is quite small.

## 7. Summary and Conclusion

The consequences of a simple monetary targeting mechanism have been considered for a dynamic macro model under rational expectations. Through the use of numerical examples, the effect of this targeting mechanism on policy performance in this model has been shown to be ambiguous and to be negative for those examples in which policy credibility is an important problem.

These highly stylized examples cannot provide definitive answers concerning the general usefulness of targeting mechanisms in setting government policies. Even in the context of the simple model discussed, it may be possible to design more effective targeting schemes. For example, improvements in policy performance might result from requiring the Fed to target the price level as well as the money supply or from extending the number of

periods over which targeting must occur. Still, the examples considered carry an important message: arbitrarily applied targeting schemes can easily lead to a deterioration of policy performance. These examples also point to the need for more research into the strategic effects of monetary targeting mechanisms.

### Notes

<sup>1</sup>See McCallum (1985) for a survey of the literature on monetary targeting in general and on the strategic aspects of targeting especially.

<sup>2</sup>A similar result is shown in Canzoneri (1985).

<sup>3</sup>The paper can be viewed this way in the sense that both the model and the targeting scheme considered here closely resemble dynamic analogs of those proposed by Rogoff. However, no attempt has been made to nest Rogoff's static setup in the dynamic environments that this paper considers.

<sup>4</sup>See Whiteman (1983) and Watson (1985) for a discussion of the solution of equation (1).

<sup>5</sup>There is also some evidence that such terms are needed to make optimal control models of Fed policy believable. Empirical studies by Litterman (1982, 1986) suggest that assigning costs to fluctuations in the policy instrument (i.e., the interest rate in these studies) is necessary to prevent rapid oscillations in this variable.

<sup>6</sup>The policy problems considered in Section 3 were first proposed and analyzed by Whiteman (1986), using techniques different from the ones employed here. Throughout this paper we will be abstracting from the possibility of reputational effects (see Barro and Gordon 1983) or regime change (see Roberds 1986).

<sup>7</sup>Setting the Fed's discount factor equal to one does not affect the qualitative properties of the models studied below. The government's objective is still well defined if we reinterpret  $J_t$  as an average cost objective, as in Bertsekas (1976). Average

cost objectives are convenient for the numerical simulations reported in Section 6, since they allow estimation of the Fed's objective using sample moments.

<sup>8</sup>For a proof of this statement, see Corollary 3.2 of Whiteman (1986).

<sup>9</sup>This transformation (division of the logged monetary target by  $\alpha$ ) is done purely for notational convenience.

<sup>10</sup>Any potential credibility problems arising in a static context are assumed away in the models of this paper, so as to concentrate on dynamic credibility issues. This assumption seems warranted, given that dynamic credibility issues were the main focus of Kydland and Prescott's (1977) original critique.

Appendix A

Derivation of the Optimal Policy Rule (8)

Substituting the portfolio balance schedule (3) into the Fed's first-order condition (7) yields

$$(A1) \quad -\lambda(L-\rho)m_t = (L^{-1}-\rho)^{-1}(E_t m_t + E_t u_t)$$

where the operator  $L$  is defined as  $L(E_t m_t) \equiv m_{t-1}$ , and  $L^{-1}$  as  $L^{-1}(E_t m_t) \equiv E_t m_{t+1}$ . Equation (A1) can be solved using the method outlined in Sargent (1979). Operating on (A1) with  $(L^{-1}-\rho)$ , we obtain the second-order expectational difference equation

$$(A2) \quad [-\lambda(L^{-1}-\rho)(L-\rho) - 1]E_t m_t = u_t.$$

Applying Sargent's technique then yields the solution for  $m_t$

$$(A3) \quad m_t = c_1 m_{t-1} + [c_3^{-1}/(1-c_2\gamma)]u_t.$$

where  $-\lambda(z^{-1}-\rho)(z-\rho)-1$  can be factored as  $c_2(1-c_1z)(1-c_1z^{-1})$ , when  $c_2 < 0$  and  $c_1 \in (0,1)$ . Equation (8) follows if we substitute  $c_0$  for  $[c_2^{-1}/(1-c_1\gamma)]$  and note that the first-order condition (6) for the initial period may be written as (7), subject to the initial condition  $m_{-1} = 0$ .

Derivation of the Consistent Policy Rule (12)

Using the portfolio balance schedule (3) to eliminate  $p_t$  from the first-order condition (11) yields

$$(A4) \quad [\lambda\rho L^{-1} - (\lambda\rho^2+1)]E_t m_t = u_t.$$

Defining  $d_0 \equiv -(1+\lambda\rho^2)$  and  $d_1 \equiv \lambda\rho/(1+\lambda\rho^2)$ , equation (A4) can be solved using Sargent's (1979) technique to yield

$$(A5) \quad m_t = d_0^{-1} u_t / (1-d_1\gamma).$$

The feedback parameter  $f^*$  may be found by evaluating  $d_0^{-1}/(1-d_1\gamma)$  and simplifying.

Appendix B

Alternative Derivation of  $f^*$  Under Feedback Equilibrium

Begin by writing constraint (13) in abbreviated form as

$$(B1) \quad p_t = a_0 u_t + a_1 m_t.$$

Since the Fed's value function  $V(u_{t+1})$  does not depend on  $m_t$ , solving program (14) is equivalent to solving the simpler program

$$(B2) \quad \min_{m_t} 1/2(\lambda m_t^2 + p_t^2)$$

subject to (B1). The first-order condition for program (B2) is given by

$$(B3) \quad (\lambda + a_1^2)m_t + (a_0 a_1)u_t = 0.$$

Substituting for the  $a$ 's in (B3) and solving for  $m_t$  yields

$$(B4) \quad m_t = [\rho^{-1}(1+\rho^{-1}\gamma f_0)]/[(\lambda+\rho^{-2})(\gamma-\rho)]u_t.$$

Imposing the conditions  $m_t = f^*u_t$ ,  $f^* = f_0$ , and dividing (B4) by  $u_t$  yields

$$(B5) \quad f^* = -[1 - \lambda\rho(\gamma-\rho)]^{-1}.$$

Appendix C

Calculation of Feedback Equilibrium Under Targeting

We begin by evaluating the public's portfolio balance schedule (3) when policies are set using the linear decision rules (15), (16), and (17). Using prediction formulas from Hansen and Sargent (1980), equation (3) may be evaluated for even  $t$  as

$$(C1) \quad p_t = a^*u_t$$

where

$$(C2) \quad a^* = (-\rho^{-1}f_0 - \gamma g_0 - \rho g_1 f_1 + \gamma + \rho)/(\rho^2 - \gamma^2).$$

When  $t$  is odd, equation (3) can be evaluated as

$$(C3) \quad p_t = b_0 u_t + b_1 m_t$$

where  $b_0 = \rho^{-1}(a^*\gamma - 1)$ , and  $b_1 = -\rho^{-1}$ .

Now consider the Fed's optimization problem at some odd time  $t$ , i.e., the inner minimization problem of program (22). Because the next (even) period's value function  $W(u_{t+1})$  does not depend on the choice of  $m_t$ , this minimization problem is equivalent to the simpler program

$$(C4) \quad \min_{m_t} 1/2[p_t^2 + \lambda m_t^2 + \tau(m_t - m_t^*)^2] \quad \text{s.t. (C3), } m_t^* \text{ given.}$$

Solving program (C4) yields the following solution for  $m_t$ :

$$(C5) \quad m_t = (\tau m_t^* - b_1 b_0 u_t)/(\lambda + \tau + b_1^2).$$

Solution (C5) in turn implies the following values for  $g_0$  and  $g_1$ :

$$(C6) \quad g'_0 = -b_0 b_1 / (\lambda + \tau + b_1^2)$$

$$(C7) \quad g'_1 = \tau / (\lambda + \tau + b_1^2).$$

Now consider the Fed's optimization problem at some even time  $t$ , i.e., the outer minimization problem of program (22). Since the public knows that in the next period, policy will be set according to a rule of the form (17), the Fed should take into account the impact of its target announcement on the public's expectation of  $m_{t+1}$ . Substituting (17) into equation (3) and taking expectations then yields

$$(C8) \quad p_t = d_0 u_t + d_1 m_t + d_2 m_{t+1}^*$$

where  $d_0 = -\rho^{-1} \{1 + \rho^{-1} [(1 + g_0 - \gamma a^*) \gamma]\}$ ,  $d_1 = -\rho^{-1}$ , and  $d_2 = -\rho^{-2} g_1$ . Also, the Fed should take into account the impact of its target announcement on its time  $t+1$  loss function via the decision rule (17). Substituting (17) and (C3) into the time  $t+1$  policy loss function yields

$$(C9) \quad L_o(p_{t+1}, m_{t+1}, m_{t+1}^*) = C(m_{t+1}^*, u_{t+1})$$

where

$$C(m_{t+1}^*, u_{t+1}) \equiv 1/2 (c_1 u_{t+1}^2 + c_2 m_{t+1}^{*2} + 2c_3 u_{t+1} m_{t+1}^*)$$

and

$$c_2 = \lambda g_1^2 + (b_1 g_1)^2 + \tau (g_1 - 1)^2$$

$$c_3 = \lambda g_0 g_1 + (b_0 + b_1 g_0)(b_1 g_1) + \tau g_0 (g_1 - 1).$$

Since policy decisions made at time  $t$  (even) do not affect the Fed's value function  $W(u_{t+2})$  at time  $t+2$ , the outer minimization problem of program (22) reduces to the following problem:

$$(C10) \quad \min_{m_t, m_{t+1}^*} \quad 1/2(\lambda m_t^2 + p_t^2) + E_t C(m_{t+1}^*, u_{t+1}) \quad \text{s.t. (C8) and (C9).}$$

Necessary first-order conditions for program (C10) are given by

$$(C11) \quad \begin{bmatrix} (\lambda + d_1^2) & d_1 d_2 \\ d_1 d_2 & (c_2 + d_2^2) \end{bmatrix} \begin{bmatrix} m_t \\ m_{t+1}^* \end{bmatrix} = \begin{bmatrix} -d_1 d_0 \\ -(c_3 \gamma + d_2 d_0) \end{bmatrix} u_t$$

which we abbreviate as  $Dm = du_t$ . Substituting for  $m_t$  and  $m_{t+1}^*$  using equations (15) and (16) and dividing (C11) by  $u_t$  then implies the following values for  $f_0$  and  $f_1$ :

$$(C12) \quad \begin{bmatrix} f_0' \\ f_1' \end{bmatrix} = D^{-1}d.$$

A feedback equilibrium can be calculated by taking some initial guess for the parameters of the equilibrium feedback laws (20) and (21), then iterating on equations (C6), (C7), and (C12) until convergence is reached. In practice, convergence was quite rapid from essentially arbitrary starting values for each of the examples reported. The convergence criterion was that maximal differences between successive approximations be no greater than  $10^{-7}$  in absolute value. For some unreported simulations, convergence was not obtained for large values of  $\tau$ . Similar convergence problems are reported by Kydland and Prescott (1977) for simula-

tions of a policy game in a discretionary environment; this suggests that such problems are not uncommon to this type of model. .

## Appendix D

### Calculation of the Policy Objective

In this section, analytical expressions are derived for the average cost version of the minimized policy objective  $J_t$ , given a targeting policy environment. Analogous expressions for the discretionary and precommitment policy environments can be derived using formulas given in Whiteman (1986); for the sake of brevity, these are omitted here.

We begin by noting that under targeting, the equilibrium vector  $(m_t, p_t)$  process will take on the value of either of two stationary bivariate processes, according to whether  $t$  is odd or even. In even periods,  $m_t$  and  $p_t$  are determined as the processes

$$(D1) \quad m_t^e = f_0 u_t$$

$$(D2) \quad p_t^e = \pi_0 u_t$$

where

$$(D3) \quad \pi_0 = d_0 + d_1 f_0 + d_2 f_2.$$

Similarly, when  $t$  is odd,  $m_t$  and  $p_t$  are determined as

$$(D4) \quad m_t^o = \mu_0 u_t + \mu_1 u_{t-1}$$

$$(D5) \quad p_t^o = \pi_1 u_t + \pi_2 u_{t-1}$$

where

$$(D6) \quad \mu_0 = g_0$$

$$(D7) \quad \mu_1 = f_1 g_1$$

$$(D8) \quad \pi_1 = b_0 + b_1 g_0$$

$$(D9) \quad \pi_2 = b_1 f_1 g_1.$$

It is then straightforward to show that  $J_t$  will take on the constant value  $1/4\{\text{var}(p^e) + \text{var}(p^o) + \lambda[\text{var}(m^e) + \text{var}(m^o)]\}$ . Exploiting stationarity of the  $\{u_t\}$  process,  $J_t$  may then be calculated by evaluating

$$(D10) \quad \text{var}(m^e) = f_0^2 \text{var}(u)$$

$$(D11) \quad \text{var}(p^e) = \pi_0^2 \text{var}(u)$$

$$(D12) \quad \text{var}(m^o) = (\mu_0^2 + \mu_1^2 + 2\gamma\mu_0\mu_1) \text{var}(u)$$

$$(D13) \quad \text{var}(p^o) = (\pi_1^2 + \pi_2^2 + 2\gamma\pi_1\pi_2) \text{var}(u).$$

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