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A Defense of *AK* Growth Models

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Over the past 200 years, many countries have experienced sustained growth in gross domestic product (GDP) per capita. Accounting for this sustained growth has been a central goal of modern economic growth theory. Early models simply assumed some positive rate of technological progress which translated into positive GDP growth. Now models have been developed that generate growth endogenously. One class of such models, commonly called *AK* models,[1] relies on the assumption that returns to capital do not diminish as the capital stock increases. Without diminishing returns, a country with a high stock of capital is not deterred from continued investment and, therefore, continued growth.

The *AK* class of models has been heavily criticized. Most critics have attacked the main assumption, the absence of diminishing returns, as having little empirical support.[2] However, such criticisms are themselves difficult to support if capital is viewed broadly to include human capital and intangible capital, both of which are difficult to measure. More serious critiques analyze the testable predictions of *AK* models. Jones (1995), for example, argues that a key prediction of *AK* models is inconsistent with the data. Unlike the earlier exogenous growth models, *AK* models predict that permanent changes in government policies affecting investment rates should lead to permanent changes in a country's GDP growth. Jones tests this prediction by comparing investment as a share of GDP and the growth rate of GDP for 15 countries that belong to the Organisation for Economic Co-operation and Development (OECD). Using data for the post–World War II period, Jones (1995) argues that *AK* models are inconsistent with the time series evidence because during the postwar period, rates of investment, especially for equipment, have increased significantly, while GDP growth rates have not.

Here I defend *AK* growth models against that critique: I demonstrate that the key prediction of *AK* theory is consistent with the data. Using historical data going back to the 19th century, I show that the patterns Jones points to—episodes in which investment rates rose while growth rates remained constant or fell—were short-lived. Yet the simple model Jones tests predicts not short-run patterns, but long-run trends. The longer time series show that periods of high investment rates roughly coincide with periods of high growth rates, just as *AK* models predict. This is true for OECD countries and for three Asian non-OECD countries for which historical data are available. A positive relationship is also clear in the data for a larger number of countries than Jones examines. Cross-sectional data for a range of countries at different stages of development reveal a strong positive relationship between average investment

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[1]The name *AK* comes from the simplest form of the models' production function in the simplest case, namely, $Y = AK$. Here A is a positive constant representing the economy's level of technology and K is the economy's stock of capital.

[2]For reviews, see Barro and Sala-i-Martin 1995 and Aghion and Howitt 1998.

rates and average growth rates, again, just as AK models predict.

To account for the short-run deviations that Jones finds in investment and growth trends, I consider a version of an AK-style model that is slightly more general than the one he tests. The version Jones tests assumes that government policies affecting investment and growth do not affect key factors like capital/output ratios or labor/leisure decisions. Since those factors are not changing, the model predicts a stark relationship between the rate of physical investment and growth: they should move in lockstep. If, instead, the model assumes that these factors are affected by changes in government policies, then the model does not necessarily predict that growth rates will change one-for-one with investment rates. I construct simple examples in which changes in policy variables directly affect capital/output ratios and the labor/leisure tradeoff. These AK-style models can predict deviations in trends of investment rates and growth rates consistent with the patterns in postwar data.

Basic Theory

To start, let's look at several simple AK growth models to highlight the link between investment rates and growth rates that this class of models predicts. As we shall see, the simplest versions of AK models imply a tight positive relationship between investment as a share of output and the growth rate of output.

Consider a simple AK model of growth. The model has a representative household that chooses per capita consumption c and per capita investment x in each period to maximize lifetime utility U; that is,

$$(1) \qquad \max_{\{c_t, x_t\}} \sum_{t=0} \beta^t U(c_t)$$

for $0 < \beta < 1$, where t is an index for time. The optimization problem (1) is subject to a resource constraint, a capital accumulation constraint, and inequality constraints:

$$(2) \qquad c_t + x_t = Ak_t$$

$$(3) \qquad k_{t+1} = (1-\delta)k_t + x_t$$

$$(4) \qquad c_t \geq 0 \text{ and } x_t \geq 0$$

given k_0, where k_t is the stock of capital at time t, A is the level of technology, and δ is the rate of depreciation of the capital stock. Per capita output in this model is simply

$$(5) \qquad y_t = Ak_t.$$

The production technology in equation (2) has constant returns to scale; clearly, doubling the stock of capital doubles output. Without diminishing returns to scale, a country with a high stock of capital will continue to invest and continue to grow. To justify the constant returns assumption, we typically interpret the capital stock as a broad measure that includes not only physical capital, but also human capital and intangible capital.

If the level of technology does not change over time, then in this simple version of the model, the growth rate of output equals the growth rate of the capital stock. If we divide both sides of equation (3) by the current capital stock k_t, then we have

$$(6) \qquad \gamma_t = 1 - \delta + x_t/k_t$$

$$(7) \qquad \gamma_t = 1 - \delta + Ax_t/y_t$$

where γ_t is the growth rate of capital and of output at time t. Equation (7) illustrates the tight link predicted between the investment rate and output growth. This theory predicts that sustained increases in the investment/output ratio should be accompanied by sustained increases in the growth rate of output.

Now let's extend the model slightly. As noted above, the capital stock in the AK model is usually interpreted broadly. If we include the components separately in the model, but still retain the linear structure, we end up with the same implications for investment and growth.

To see this, consider an extension of the model above which includes, explicitly, both physical and human capital. Let k denote the stock of physical capital and h denote the stock of human capital, with x_k and x_h denoting the investments in the two stocks. Now the problem is to maximize the utility in (1) subject to

$$(8) \qquad c_t + x_{kt} + x_{ht} = Ak_t^{\alpha}h_t^{1-\alpha}$$

$$(9) \qquad k_{t+1} = (1-\delta)k_t + x_{kt}$$

$$(10) \qquad h_{t+1} = (1-\delta)h_t + x_{ht}$$

$$(11) \qquad c_t \geq 0,\ x_{kt} \geq 0, \text{ and } x_{ht} \geq 0$$

where α is the share of physical capital in production. For simplicity, assume that both types of capital depreciate at the same rate δ. In this example, output is given by

$$(12) \qquad y = Ak^{\alpha}h^{1-\alpha}$$

where the exponents on the two accumulable factors sum to 1. Here, as before, doubling the capital stocks doubles output.

In this model, households choose investments so as to achieve a constant ratio of human to physical capital. This is the ratio of the components' relative shares: $(1-\alpha)/\alpha$. Thus, total output can be written as a linear function of k, or as

$$(13) \quad Ak^{\alpha}h^{1-\alpha} = A[(1-\alpha)/\alpha]^{1-\alpha}k$$

and the growth rate of output still equals the growth rate of physical capital. From (9), we can derive the growth rate of physical capital by dividing both sides of the equation by k_t. If h/k does not start at $(1-\alpha)/\alpha$, it rapidly adjusts to this ratio if the inequalities in (11) are not binding. After the adjustment, the variables c, x_k, x_h, k, and h all grow at a constant rate γ. This rate is given by

$$(14) \quad \gamma = 1 - \delta + x_k/k = 1 - \delta + A[(1-\alpha)/\alpha]^{1-\alpha}x_k/y.$$

Here again we find a tight link between the rate of physical investment and growth.

The two models we have considered are special cases of the broad class of *AK* models that allow for sustained growth in consumption, capital, and output. In these models, the production technology either was linear ($y = Ak$) or had constant returns in accumulable factors ($y = Ak^{\alpha}h^{1-\alpha}$). In such cases, the link between investment and growth can be made very stark. However, a strong link remains in *AK* models even with more general production technologies, $y = f(k)$, that have the property that $\lim_{k\to\infty} f'(k) = A$. If $A > \delta$, then the model generates sustained growth. (For more details on the mathematics, see Jones and Manuelli 1990.) This specification of the production function still implies that returns to capital are bounded below. Thus, higher capital stocks do not deter a country from further investment, and higher investment implies higher growth rates.

Finally, we could extend the model a bit more by allowing for a more general industrial specification. Typical in the growth literature is a model in which different types of capital are produced in different sectors of the economy. A standard assumption is that production of human capital requires a different type of technology than production of consumption or physical investment goods. For example, the main input to production of investment of human capital might be assumed to be human capital (teachers) rather than physical capital (buildings). Assuming different technologies allows for more flexibility in the model, but it does not change the model's main implication: Investment is the engine of growth. If investment rates are high, growth rates should be too.[3]

A Case Against *AK* Theory

Jones (1995) argues that this main implication of *AK* models is not supported by the data. In particular, he points out that while investment/output ratios have risen in many countries over the postwar period, output growth rates have stayed roughly constant or have fallen.

The evidence Jones (1995) uses to make a case against *AK* theory is summarized in Tables 1–3. In Table 1 are five-year growth rates of GDP per worker for eight OECD countries.[4] The growth rates have been annualized and are reported for the period 1950–89. These data show that in these countries, over these 40 years, growth rates have fallen somewhat or have remained roughly constant. Japan, for example, had high growth rates in the 1950s and 1960s; but more recently, its growth rates have fallen. Although France's and Germany's growth rates have not been as high, the patterns in these countries have been similar to that in Japan. Countries like the United States, however, have experienced quite steady growth. Still, Table 1 clearly shows that none of these countries has had a significantly positive growth trend over the postwar period.

The investment data appear to tell a different story. In Table 2 are Jones' (1995) data on average investment/output ratios for producers' durable equipment.[5] For most countries, this ratio has increased significantly over the postwar period. For example, in Canada, France, Germany, the United Kingdom, and the United States, the producers' durables investment rate nearly doubled over the 40-year period. In Japan, meanwhile, the rate nearly tripled. Jones interprets these increases as evidence against *AK*-style models since the investment rate increases do not

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[3]For more details about two-sector endogenous growth models, see Lucas 1988, 1990; King and Rebelo 1990; Rebelo 1991; Kim 1992; Jones, Manuelli, and Rossi 1993; Barro and Sala-i-Martin 1995; Stokey and Rebelo 1995; Jones and Manuelli 1997; and McGrattan and Schmitz 1998.

[4]Jones (1995) also includes Austria, Belgium, Denmark, Finland, Italy, Norway, and Sweden, all of which have similar investment and growth experiences as the countries reported in Tables 1–3. Here, I report statistics for the smaller set of countries that have more historical data available than Jones examines.

[5]Jones focuses on producers' durable equipment because this component has been found to be strongly correlated with growth in cross-country regressions. See De Long and Summers 1991. The data in Table 2 were constructed by Robert Summers. See the appendix in Jones 1995.

Tables 1–3 The Evidence Jones Uses Against *AK* Models

5-Year Annualized Growth Rates of Gross Domestic Product per Worker (%)
and Average Investment Shares of Gross Domestic Product (%), 1950–89

		Values for Countries							
Variables	Years	Australia	Canada	France	Germany	Japan	Netherlands	United Kingdom	United States
Table 1	1950–54	1.83	2.42	4.02	8.32	6.74	4.31	2.82	2.64
Growth Rates	1955–59	1.82	1.42	4.76	4.93	6.81	3.96	1.94	.92
per Worker	1960–64	1.98	2.69	4.79	4.42	7.97	3.80	2.44	2.80
	1965–69	3.49	2.29	4.87	4.22	9.47	4.26	2.20	1.67
	1970–74	.60	1.89	1.98	1.89	3.03	1.53	1.35	–.22
	1975–79	.97	.99	2.39	3.16	4.01	1.29	1.38	1.02
	1980–84	1.21	1.63	.19	–.02	2.94	–.46	1.58	1.28
	1985–89	.92	1.99	2.32	1.60	3.75	1.81	3.09	1.71
Table 2	1950–54	8.14	3.06	4.34	4.81	3.37	6.34	4.79	4.43
Producers'	1955–59	7.86	2.88	5.14	5.51	3.82	8.22	5.47	4.26
Durables	1960–64	9.24	2.56	6.27	6.84	5.57	8.89	6.04	4.23
Investment	1965–69	10.02	3.15	6.88	6.85	6.03	9.17	6.55	5.23
Shares	1970–74	8.91	3.39	8.09	7.75	7.42	9.37	6.91	5.38
	1975–79	8.34	3.84	7.97	7.32	6.44	7.34	6.86	5.87
	1980–84	9.33	5.03	7.89	7.57	7.47	6.65	6.63	6.15
	1985–89	9.51	5.69	8.05	8.13	9.81	8.65	7.49	7.21
Table 3	1950–54	26.5	24.0	20.1	27.5	18.2	22.8	13.2	24.0
Total Physical	1955–59	27.1	26.0	22.8	31.0	21.2	25.2	15.5	23.7
Investment	1960–64	28.3	22.4	25.3	30.8	28.1	26.5	17.3	22.5
Shares	1965–69	28.9	23.1	26.8	28.8	30.6	27.0	18.6	23.0
	1970–74	28.4	22.7	29.6	28.9	36.9	27.3	19.5	22.9
	1975–79	27.1	23.9	27.1	25.8	34.2	23.6	18.5	22.9
	1980–84	27.2	24.0	25.0	24.7	32.6	20.4	15.8	22.5
	1985–89	27.0	26.5	25.1	23.4	33.7	22.0	18.7	23.0

Sources: Tables 1 and 3, Summers and Heston 1991 and Penn World Table, Mark 5.6 (http://www.nber.org/pwt56.html); Table 2, Robert Summers (Jones 1995, p. 506)

coincide with increases in GDP growth rates. In fact, for some countries, the investment rate increases coincide with decreases in GDP growth rates.

Table 3 shows the postwar average investment rates for total physical investment (producers' durable equipment plus structures). For two countries—Australia and the United States—this investment rate is roughly constant over the period. But rates for the other countries display trends. For some countries, like Germany and the Netherlands, significant increases in investment occurred in the 1950s and 1960s, followed by significant decreases. For other countries, trends are more persistent. In the United Kingdom, for example, total investment rose from about 13 percent of GDP in the early 1950s to about 19 percent in the late 1980s. For Japan, the increase is even greater. Japan had an investment rate of about 18 percent in the

Charts 1–4 **Another Look at Jones' Evidence**

Growth Rate of Gross Domestic Product per Worker
and Investment as a Percentage of Gross Domestic Product, 1950–92

Variable Plotted	Country: United States	Country: Japan
Growth Rate	Chart 1	Chart 3

Chart 1

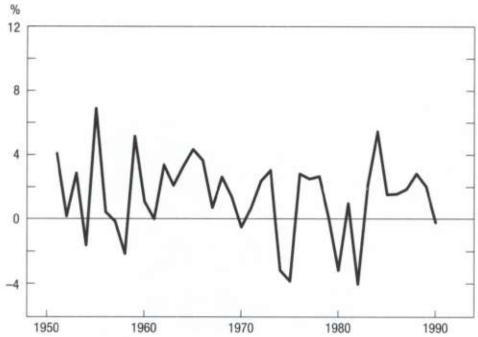


Chart 3

Variable Plotted	Country: United States	Country: Japan
Investment Share	Chart 2	Chart 4

Chart 2

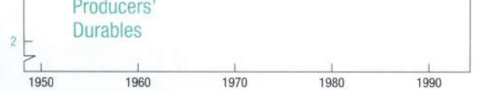
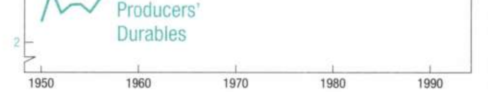


Chart 4



Sources: Summers and Heston 1991, Penn World Table, Mark 5.6 (http://www.nber.org/pwt56.html); Robert Summers (Jones 1995, p. 506)

early 1950s which doubled by the early 1970s. Investment subsequently fell, but not much; it was still relatively high in the 1980s.

To see these patterns more clearly, consider the data for the United States and Japan plotted in Charts 1–4. Chart 1 shows annual growth rates for GDP per worker for the United States; Chart 2, the producers' durables investment share of GDP and the total investment share of GDP for that country. Notice the relative movements in these data. For the United States, the growth rate and the total investment rate display no obvious trends, while producers' durables investment is clearly trending upward.

Japan's data plots are even more striking. Chart 3 shows Japan's annual growth rates for GDP per worker.

Between 1950 and 1973, growth rates averaged around 7 percent per year. Between 1973 and 1988, the average fell to around 3 percent per year. Chart 4 shows Japan's investment shares. Unlike the United States, Japan had dramatic increases in both producers' durables investment and total investment.

Jones (1995) runs a battery of time series tests on the data for 15 OECD countries to look statistically for trends in investment rates and growth rates.[6] He finds empirical support for positive trends in investment rates—especially producers' durable equipment rates—but not in growth rates. (The United States is the only country for which Jones finds no trend in the total investment/output ratio.) Because Jones generally finds positive trends in investment/output ratios and no trends in growth rates, he concludes that the main prediction of *AK*-style models is not consistent with the data.

Reevaluating *AK* Theory

Now I reevaluate *AK* theory from a different empirical and theoretical standpoint. Empirically, I consider the investment and growth evidence over longer time periods and more countries than Jones does. Theoretically, unlike Jones, I allow government policy changes to affect selected key factors in the model. These differences with Jones' analysis lead me to a different conclusion.

Another Look at the Data . . .

To evaluate the main prediction of *AK* theory, we need to look in the data for trends in investment rates and growth rates. Using only postwar data for countries at similar stages of development is likely to emphasize temporary movements in the data and so hide trends, not reveal them. We can expand our view: Longer time series are available for many of the countries that Jones studies, and data are available for countries at different stages of development. This broader view reveals the long-run trends that *AK* theory predicts.

□ *Historical Data*

One obvious way to capture trends is to examine data over long time horizons. Here, I extend the sample back more than a century to see if it contains any relationship between trends in investment/output ratios and growth rates. Using data from Maddison (1992, 1995)[7] for 1870–1989, I find that Jones' deviations from investment and growth trends are relatively short-lived, and periods of high investment rates roughly do coincide with periods of high growth.

In Charts 5–15, I plot 120 years of investment and growth rates for 11 countries.[8] For the investment/output ratio, I use gross fixed domestic investment as a percentage of GDP valued in current prices. (See Maddison 1992.) For the growth rates, I construct nine-year moving averages of per capita GDP growth using equal weights for the current year, four lags, and four leads. (See Maddison 1995, Table D.) This averaging is meant to smooth out some of the large swings that occurred during the world wars.

The charts show similar patterns across the 11 countries plotted. During the prewar period (1870–1914), both investment and growth rates fluctuate considerably, but for most countries, they exhibit no persistent deviations from trends. (One exception is seen in Chart 10; Canada's domestic investment ratio rose dramatically at the turn of the century while its growth rate did not. However, Canada's foreign investment fell as domestic investment rose, so total investment in the country does move in parallel with growth.)[9] The charts also show that for most countries, the war period (1915–49) was a time of major economic disruption: the charts show huge swings in growth during that period despite the smoothing of rates. Finally, the charts show that most of the increases in investment and growth occurred during the postwar period (1950–89). This is most evident for the Asian non-OECD countries (Charts 13–15).

For the OECD countries (Charts 5–12), the same patterns emerge from these data of Maddison as from the data of Summers and Heston (1991) that Jones analyzes and that we have seen in Table 1. (One exception is in Chart 12; Maddison's estimates for Japan show a more moderate increase in the share of investment than do Summers and Heston's.) As Jones points out, across these countries, sometimes investment rates are rising while growth rates are not. However, as the charts reveal, the deviations from

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[6]In particular, Jones (1995) tests for unit roots in the time series data. A process z_t is called a *unit root* if its first difference $z_t - z_{t-1}$ is stationary. A common test for unit roots is that proposed by Dickey and Fuller (1979), who estimate the regression equation $z_t = \mu + \rho z_{t-1} + B(L)\Delta z_{t-1} + \varepsilon_t$ and test the hypothesis that $\rho = 1$, where μ is a constant, $\Delta z_t = z_t - z_{t-1}$, $B(L) = B_1 + B_2 L + \ldots + B_q L^{q-1}$, L is a lag operator (that is, $Lz_t = z_{t-1}$), and ε_t is a stochastic process that is uncorrelated over time and has a mean of zero.

[7]The statistical appendix for Maddison's 1992 paper is Maddison 1991.

[8]Labor force data are not available before 1950, so I report per capita rather than per worker growth.

[9]See Maddison 1992, which also reports gross national saving as a percentage of GDP. For the other countries, there are no noticeable differences in the trend patterns of gross domestic investment and gross national saving.

Charts 5–15 **A Longer Look Back at Investment and Growth**

Gross Fixed Domestic Investment as a Percentage of Gross Domestic Product
and Growth Rate of Per Capita Gross Domestic Product (9-Year Moving Average)
During 1870–1989 in 11 Countries

Charts 5–8 **In Western European OECD Countries . . .**

Chart 5 **France**

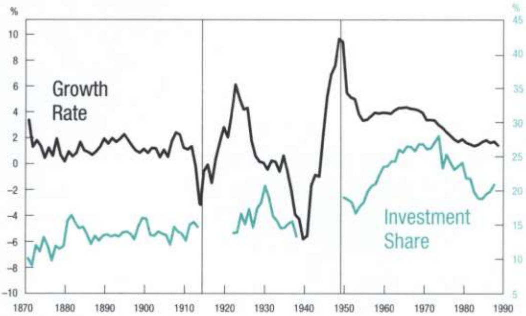


Chart 6 **Germany**

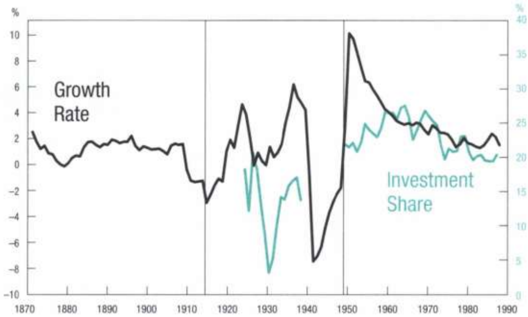


Chart 7 **Netherlands**

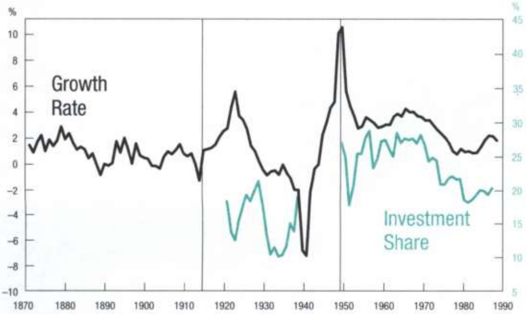


Chart 8 **United Kingdom**

Sources: Maddison 1992, 1995

trends are small relative to year-by-year or even decade-by-decade movements, and the deviations are not persistent.

Overall, the charts reveal a general upward movement in both investment rates and growth rates during and after the world wars. To show that more directly, I display in Charts 16–18 the averages of the time series plotted in Charts 5–15 for the three subperiods—before, during, and after the wars.

Chart 16 shows data for the Western European OECD countries. Notice that the average growth rates for all of these countries are two or three times higher in the postwar period than in the prewar period. Similarly, investment rates are highest in the postwar period. For France and the United Kingdom, the investment rates are close to twice as high after the wars as before them. And these rates likely underestimate the increases in investment since the data do not include human capital investment. By most measures, human capital investment has increased during the 20th century. (See Mitchell 1981, 1995 and Becker 1993.)

Charts 9–12 . . . In Non-European OECD Countries . . .

Chart 9 **Australia**

Chart 10 **Canada**

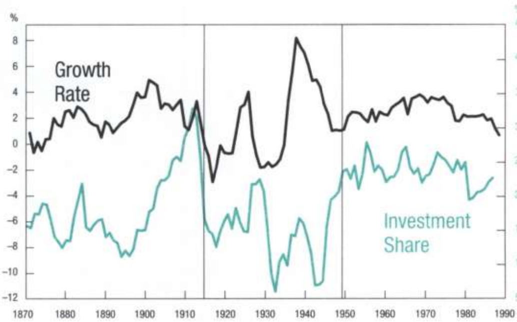


Chart 11 **United States**

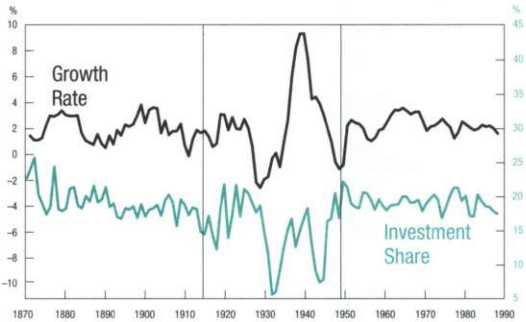


Chart 12 **Japan**

Chart 17 shows averages of investment and growth rates in the non-European OECD countries that Jones studies. Here we see the same basic patterns as those for the Europeans. One exception is the United States. The average U.S. growth rate is roughly the same in all three periods between 1870 and 1989. Furthermore, the average U.S. investment rate is about the same in both the prewar and postwar periods.

Still, across the OECD countries, the general trends are clear: higher investment rates correspond to higher growth rates. During the prewar period, average investment/output ratios for the OECD countries range from about 10 percent to about 20 percent. During the postwar period, most are higher than 20 percent. Average growth rates, meanwhile, mostly move from about 1 percent to about 2 percent. A striking example of the upward shift in growth rates is Japan. In the postwar period, Japan's average annual growth rate is 6 percent, whereas in the prewar period, it is only 2 percent.

Finally, Chart 18 displays data for three Asian non-OECD countries that Jones does not study. The data for these countries show the same familiar pattern. Korea and Taiwan, like Japan, had phenomenal growth experiences after World War II, both averaging about 6 percent per year. Compared to rates in OECD countries, investment rates in these Asian countries were very low in the prewar period, but they have increased significantly since. India's average growth rate dramatically increased from near-zero levels to nearly 2 percent per year. At the same time, the investment rate in India nearly tripled.

In summary, Charts 5–18 show that Jones' negative conclusion is not supported by the longer time series.

Charts 13–15 **. . . And in Asian Non-OECD Countries**

Chart 13 **India**

Chart 14 **Korea**

Chart 15 **Taiwan**

Rather, in 11 countries over the last century, the *AK* models' prediction of simultaneous long-run movements in investment and growth is confirmed.

□ *Cross-Country Data*

Another way to capture trends is to extend the data to many more countries, to a wider range of development experiences than that in the relatively advanced OECD countries. Cross-country averages of such data also reveal a positive correlation between investment rates and growth rates, just as *AK* models predict.

The data I analyze are from Summers and Heston 1991. I include all countries with available data for the share of investment in GDP and for GDP per worker. To avoid eliminating many poor countries, I restrict the sample to the time period for which most countries have data: 1960–85.

Summers and Heston (1991) have data for 125 countries during this 26-year period. I sort these countries by their annualized 25-year growth rates and construct an average for the five slowest-growing countries, one for the five next-to-slowest–growing countries, and so on. For each group of countries, then, I construct average investment/output ratios by first constructing an average rate over the 26-year period for each country and then averaging over the five countries in the group. This procedure is meant to illustrate more clearly the pattern between investment rates and growth rates.

Chart 19 shows the result: a definite positive correlation between investment rates and growth rates. The slowest-growing countries have an average investment rate around 7 percent. The fastest-growing countries have an average rate almost four times higher, close to 25 percent.

Charts 16–18 **The Long-Run Trends**

Gross Fixed Domestic Investment's Average Annual Percentage of Gross Domestic Product
and Average Annual Growth Rate of Per Capita Gross Domestic Product
During Three Periods of 1870–1989 in 11 Countries

■ 1870–1914 ■ 1915–49 ■ 1950–89

Chart 16 **In Western European OECD Countries . . .**

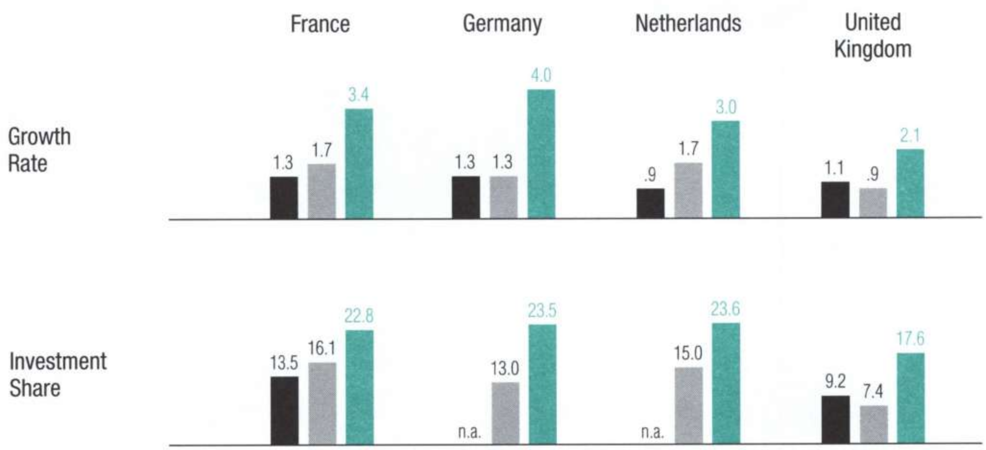
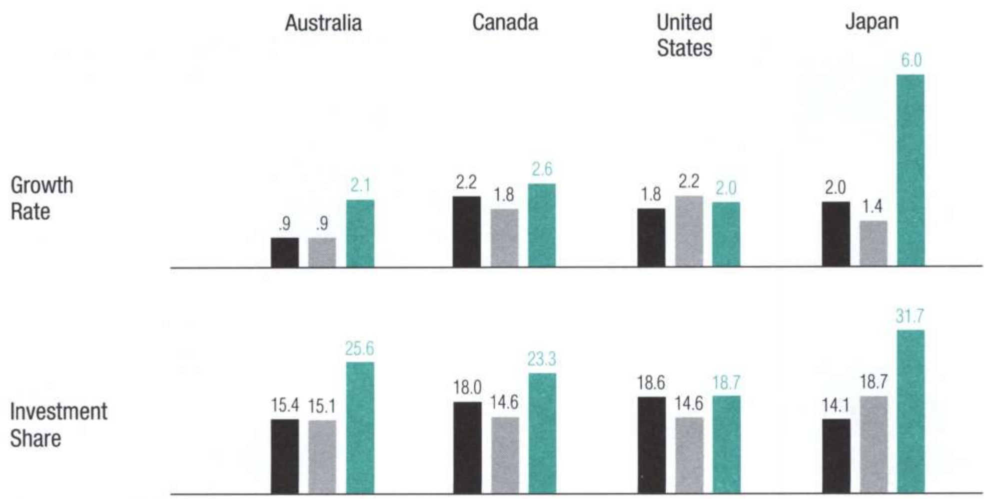


Chart 17 **. . . In Non-European OECD Countries . . .**



n.a. = not available
Sources: Maddison 1992, 1995

Chart 18 ... And in Asian Non-OECD Countries

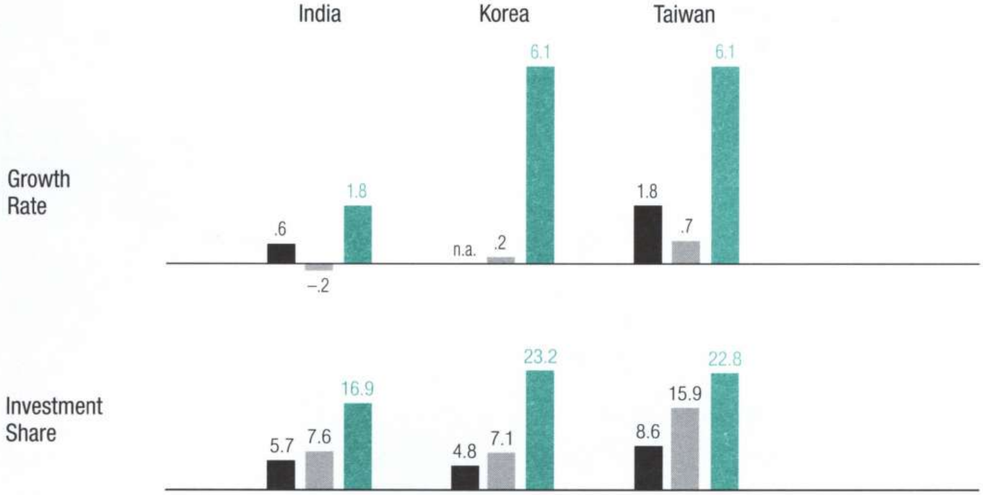
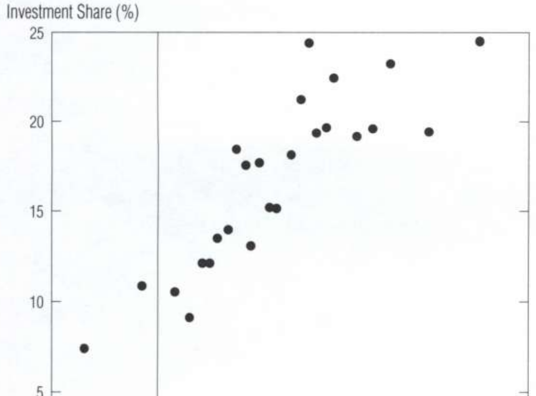


Chart 19 The Cross-Country Relationship Between Investment and Growth

Growth Rate of Gross Domestic Product per Worker
and Investment Share of Gross Domestic Product for 125 Countries,
Ranked by Annualized 25-Year Growth Rates,
Then Averaged in Groups of Five, 1960–85



Source: Summers and Heston 1991

The correlation among all the average rates is 0.87.

As with the historical time series, these cross-country data confirm the main prediction of *AK*-style growth models. Higher investment rates coincide with higher growth rates, both across time and across countries.

. . . And the Theory

So far, we have focused on *AK* models' predictions of long-run trends. Now let's see if this type of model can account for the short-run deviations in the investment and growth trends that Jones isolates. To investigate that, we need to extend the basic theory in such a way as to break the tight connection between investment and growth derived in equation (14). The connection can be broken simply by assuming that government policies affect two key factors: the capital/output ratio and the labor/leisure choice. The resulting models do predict short-run deviations from trends consistent with the postwar data.

☐ *Policies Affecting the Capital/Output Ratio*

First, I consider a version of an *AK*-style model with different tax rates on structures and on producers' durable equipment to show that this type of model can predict a pattern like that Jones finds: producers' durables investment rising, but output growth rates roughly constant.

Consider again the problem of a representative household choosing consumption and investment to maximize utility (1). Suppose that now the household earns income by renting out its capital to firms. The household has two types of capital: structures k_s and equipment k_e. Denote the investment in structures and equipment as x_s and x_e, respectively. Suppose also that the income a household receives is taxed. The budget constraint for the households in period t is then given by

$$(15) \quad c_t + x_{st} + x_{et} = (1-\tau_{st})r_{st}k_{st} + (1-\tau_{et})r_{et}k_{et} + T_t$$

where r_s and r_e are rental rates on structures and equipment, respectively; τ_s and τ_e are tax rates on structures and equipment, respectively; and T is transfer payments to households from the government.[10] The optimization problem here is to maximize (1) subject to laws of motion for capital accumulation and the budget constraint in (15). Assume that the processes for r_s, r_e, and T are given.

Now the growth rate can be written in terms of the equipment investment/output ratio x_e/y. If output is given by $y = Ak_e^{\alpha}k_s^{1-\alpha}$, then its growth rate is given by

$$(16) \quad \gamma = 1 - \delta + x_e/k_e$$
$$= 1 - \delta + A\{(1-\alpha)(1-\tau_s)/[\alpha(1-\tau_e)]\}^{1-\alpha}x_e/y$$

where the ratio of tax rates now enters because the capital/output ratio depends on the tax rates. Notice that changes in tax rates affect growth indirectly through their effects on the investment/output ratio and directly through the term $[(1-\tau_s)/(1-\tau_e)]^{1-\alpha}$. This simple example shows that the relationship Jones tests in the simple *AK* model [equation (14)] is not a relationship common to all *AK* models. In the extended *AK* model, if tax rates change differentially, then the investment rates for components of investment do not move in lockstep with the growth rates.[11]

What about Jones' (1995) prediction that policy changes having a positive effect on investment/output ratios should have a positive effect on long-run output growth? If effective tax rates on equipment were to fall while effective tax rates on structures rose, this model would predict an increase in the producers' durable equipment investment rate x_e/y and a decrease in the ratio $(1-\tau_s)/(1-\tau_e)$. These effects might be roughly offsetting, which would imply that the growth rate would change little. Furthermore, since the structures investment rate would fall, the total investment rate would change little.

To determine the exact effect on the growth rate in this model, we must express the growth rate in terms of inputs to the model. Suppose that the utility function is given by

$$(17) \quad U(c) = c^{1-\sigma}/(1-\sigma)$$

where σ is a measure of risk aversion. If we write the growth rate entirely in terms of fixed parameters and policy parameters (τ_s and τ_e), then we have

$$(18) \quad \gamma = (\beta\{1 - \delta + A[\alpha(1-\tau_e)]^{\alpha}[(1 - \alpha)(1 - \tau_s)]^{1-\alpha}\})^{1/\sigma}.$$

This expression depends only on exogenous factors, inputs chosen by the modeler. Policy changes that imply that the growth rate remains constant are those with the term $(1-\tau_e)^{\alpha}(1-\tau_s)^{1-\alpha}$ constant. This occurs when one tax rate falls and the other rises in such a way as to leave this term fixed. If one rate falls and the other rises, one investment rate falls while the other rises. The key, however, is that total investment does not change much.

This example uses a shift in tax rates favoring equipment investment to produce an increase in the producers' durable equipment investment rate. This is not merely a hypothetical example. The United States experienced such a shift in tax rates with the introduction of the investment tax credit in 1962, the year that the U.S. producers' durables investment rate started to drift upward. (See Chart 2.) This policy change gave firms a tax credit that was proportional to their purchases of equipment but that could not be applied to structures. The subsidy changed frequently, ranging from 0 to 10 percent, and was in effect until 1986. According to Cummins, Hassett, and Hubbard (1995), the major U.S. tax reforms enacted in 1962, 1971, 1981, and 1986 had a significant positive effect on firms' equipment investment.

Greenwood, Hercowitz, and Krusell (1997) argue that more important than tax changes is investment-specific technological change favoring equipment. These researchers assume that the accumulation equation for equipment is given by

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[10] The results are the same if we assume that investment in structures or equipment is taxed.

[11] Kocherlakota and Yi (1997) test whether the data are better described by an endogenous or an exogenous growth model. Unlike Jones (1995), they explicitly incorporate fiscal variables in their time series regressions.

$$(19)\quad k_{et+1} = (1-\delta)k_{et} + x_{et}q_t$$

where q_t represents the current state of technology for producing equipment. In effect, $1/q$ is the cost in terms of final output of producing a new unit of equipment. A shift in q here has the same effect as a shift in $1-\tau_e$ in our model above. Therefore, taking account of this sort of technological change does not change the basic analysis.[12] All we need to do is substitute q for $1-\tau_e$.

We would also predict deviations in equipment investment rates and output growth rates if the simple one-sector *AK* model were extended to allow for two sectors, one for consumption goods and structures and one for equipment. If the equipment-producing sector is more capital-intensive than the consumption goods sector, then the equipment/output ratio will rise over time. (For a description of this version of the two-sector model, see Rebelo 1991.)

What all of these examples show is that the relationship in (14), which forms the basis of Jones' (1995) time series tests, does not generally hold for the *AK* model.

□ *Policies Affecting the Labor/Leisure Decision*

So, in an *AK* model, growth rates can be constant while some components of investment are rising. But can the theory account for countries in which growth rates are constant or fall while total investment is rising? Yes, the theory can, in fact, account for different trends in growth and total investment. The key to the result has to do with how labor is supplied. Earlier we assumed it was supplied inelastically. Now we allow households to choose how much time to devote to work or leisure. With this assumption, some policies turn out to have a negative effect on labor supply, and hence growth, but a positive effect on investment rates.

Assume that households choose consumption c, investment in physical capital x_k, investment in human capital x_h, and hours of work l to maximize lifetime utility given by

$$(20)\quad \max_{\{c_t, x_{kt}, x_{ht}, l_t\}} \sum_{t=0} \beta^t U(c_t, l_t)$$

where l is the fraction of time at work and $\partial U(c,l)/\partial l < 0$. Also assume that consumption and income can be taxed. The budget constraint now is given by

$$(21)\quad (1+\tau_{ct})c_t + x_{kt} + x_{ht} = (1-\tau_{kt})r_t k_t + (1-\tau_{ht})w_t h_t l_t + T_t$$

where r is the rental rate for capital and w is the wage rate. The proceeds of the taxes on these incomes are used either to finance government purchases of goods or for transfer payments to the households. For the calculations below, I assume that government purchases are equal to a share s_g of total output.

For this example, total output is given by

$$(22)\quad y = Ak^{\alpha}(hl)^{1-\alpha}$$

and the equilibrium growth rate on a constant growth path satisfies

$$(23)\quad \gamma = 1-\delta + x_k/k$$

$$(24)\quad \gamma = 1-\delta + [Ak^{\alpha}(hl)^{1-\alpha}/k]x_k/y$$

$$(25)\quad \gamma = 1-\delta + A\{(1-\alpha)(1-\tau_h)/[\alpha(1-\tau_k)]\}^{1-\alpha} l^{1-\alpha} x_k/y.$$

Although the growth rate γ in equation (25) does not depend directly on the consumption tax rate τ_c or on the government share s_g, it does depend indirectly on these policy variables through their effects on the labor input and the investment rate.

To derive a more reduced-form relationship between investment rates and growth rates, we must specify a functional form for preferences. Assume that

$$(26)\quad U(c,l) = [c(1-l)^{\psi}]^{1-\sigma}/(1-\sigma)$$

which is the same function used earlier if $\psi = 0$. From the first-order conditions of the household's maximization problem, we can show that on the constant growth path, the labor supply is related to the growth rate as follows:

$$(27)\quad l = [\gamma^{\sigma} - \beta(1-\delta)]^{1/(1-\alpha)} \times \{[\alpha(1-\tau_k)]^{-\alpha/(1-\alpha)}/[(\beta A)^{1/(1-\alpha)}(1-\alpha)(1-\tau_h)]\}.$$

Holding τ_k and τ_h fixed, we can see that policies which have a positive effect on the growth rate must also have a positive effect on the labor supply since $\partial l/\partial\gamma > 0$. If we substitute (27) into (25) and the analog of the human capital accumulation equation, then we can also derive relationships between the investment rates and the growth rate as follows:

[12] Greenwood, Hercowitz, and Krusell (1997) generate growth by exogenous changes in technology. The example here generates long-run growth by capital accumulation. However, temporary changes in either tax rates or technology will imply temporary changes in growth rates from their long-run trend, as is true in the exogenous growth model.

$$(28)\quad x_k/y = \{(\gamma - 1 + \delta)/[\gamma^\sigma - \beta(1-\delta)]\}\beta\alpha(1-\tau_k)$$

$$(29)\quad x_h/y = \{(\gamma - 1 + \delta)/[\gamma^\sigma - \beta(1-\delta)]\}\beta(1-\alpha)(1-\tau_h).$$

Taking the derivatives of (28) and (29) with respect to the growth rate γ (with τ_k and τ_h held fixed) gives $\partial(x_k/y)/\partial\gamma < 0$ and $\partial(x_h/y)/\partial\gamma < 0$, if $\sigma > 1$ (that is, if households are sufficiently averse to risk). In other words, these relationships imply that policy changes having a negative impact on growth (with tax rates τ_k and τ_h held constant) have a negative impact on the labor input and a positive impact on both investment rates. We can, therefore, construct examples in which the investment rates rise, the labor input falls, and the growth rate falls.

For example, suppose the consumption tax rate τ_c increases. Such a policy change causes households to shift their purchases from consumption to investment, which is why the investment rates rise. The tax also has a negative impact on employment and, thus, on growth.

For another example, suppose, instead, that the ratio of government consumption to output s_g falls. The fall in spending acts like a positive wealth effect that increases consumption and leisure. Thus, households work less, and the growth rate falls. Purchases of investment fall, but output falls more. Therefore, as equations (28) and (29) show, the investment rates rise as the growth rate falls. If factor tax rates are also changing, then the changes in investment rates and growth rates could potentially be larger since they are affected indirectly by changes in the capital/output ratio.

The consumption tax rate and the government share of total spending are two examples of policy variables that have an indirect effect on growth rates through their effect on the labor supply decision. Clearly, these examples show that ignoring changes in labor supply may lead to the wrong inferences. Countries such as France, the Netherlands, and the United Kingdom have all experienced significant decreases in their labor inputs over the postwar period.[13] These countries have also had increases in their investment rates during the 1950s and 1960s with no comparable increase in growth rates.

Cooley and Ohanian (1997) estimate the effects of alternative government policies for the United Kingdom over the postwar period. Their benchmark model is a two-sector version of the extended model we have considered. They show that their model fits the data on investment and growth remarkably well. But unlike Jones, they do not ignore the effects that policies have on capital/output ratios and labor inputs.

The examples of this section demonstrate that the relationship between investment and growth that Jones (1995) tests [equation (14)] is not a relationship generic to *AK* models. In his simple *AK* model, Jones ignores the fact that some policy changes affect the capital/output ratio as well as the investment/output ratio. Jones also ignores the effects of many policy changes because he assumes that labor is inelastically supplied. By not ignoring these effects, I have shown, at least over short horizons, that more than one possible pattern in growth and investment is consistent with an *AK*-style endogenous growth model—including the patterns observed for the OECD countries in the post–World War II period.

Of course, these results should not be interpreted to mean that anything is possible. As we have seen, over long horizons, *AK*-style models do predict that countries following policies promoting investment should have high growth rates. In the historical and cross-country data, this is exactly what we see.

Conclusion

My work here is in large part a reaction to critiques of *AK* theory that are based on fragile predictions of the models and movements in the investment/output ratio and output growth rates over short samples. I have presented data on the investment share and GDP growth and argued that the key prediction of *AK* theory is consistent with the data when versions of the model and the data are compared appropriately.

But I have taken only one necessary step in defending *AK* theory. Showing that the theory does not appear to be inconsistent with the available data falls short of showing that the theory's quantitative implications are in line with observations. Further work is needed to definitively establish that *AK* theory is a good theory of growth—or to definitively dismiss it.

[13]Maddison (1995) reports population, total employment, and annual hours of work per employed persons for various dates between 1870 and 1992. These series can be used to estimate the changes in the labor input.

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