Discussion Paper 94

Institute for Empirical Macroeconomics
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October 1994

Fiscal Spending Shocks, Endogenous Government Spending, and Real Business Cycles

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ABSTRACT

We analyze a real business cycle model in which the government optimally chooses public investment and nonmilitary current expenditures, to maximize the welfare of the representative private agent. We characterize the optimal response of endogenous spending to shocks to technology and to military expenditures. Comovements between the components of government spending and other macroeconomic aggregates predicted by the model are compared with the corresponding comovements in the U.S. data. The model captures the qualitative features of the relative volatilities of the components of government spending quite well, but predicts too high correlations between the components of government spending and output.

*Ambler acknowledges financial support from the SSHRC, and both authors thank the FCAR Fund of the Québec government for financial assistance. We thank Anton Braun, Louis Phaneuf, Sergio Rebelo, Richard Rogerson, Randall Wright, and two anonymous referees for comments on previous versions of this paper. The usual caveat applies. Any opinions, findings, conclusions,or recommendations expressed herein are those of the authors and not necessarily those of the National Science Foundation, the University of Minnesota, the Federal Reserve Bank of Minneapolis, or the Federal Reserve System.

1. Introduction

The real business cycle (RBC) approach to macroeconomic fluctuations seeks to explain the main stylized facts of the business cycle by building artificial economy models in which economic equilibria are the outcomes of the interaction of rational agents who solve explicit intertemporal maximization problems. Recent contributions to the RBC literature have analyzed the role of the government sector.[1] Aiyagari, Christiano and Eichenbaum (1992) and Baxter and King (1993) analyze the multiplier effects of shocks to different types of government expenditure in a general equilibrium setting. Lucas and Stokey (1983); Paquet (1989) and Chari, Christiano and Kehoe (1991) consider the optimal financing of a given pattern of government expenditure when only distortionary taxation and borrowing are available. Braun (1994) and McGrattan (1994) model shocks to both expenditures and to distortionary tax rates on labor and capital income and analyze the relative contributions of the different types of shocks to explaining the fluctuations of aggregate output. Cho and Phaneuf (1993) model fiscal spending, monetary and technological shocks in a model with nominal contracts. Christiano and Eichenbaum (1992) consider the effects of exogenous spending shocks on the predictions of equilibrium business cycle models for the correlation between total hours worked and labor productivity.

In all of these contributions, the pattern of government expenditures is taken as given. In this paper, we analyze the effects of endogenizing the government's expenditure decisions, so that its actions are also those of an agent that solves an explicit intertemporal optimization problem. We analyze a model in which government spending contains two endogenous components in addition to an exogenous component which is beyond its immediate control. For the purposes of calibrating the model, we take the latter component to be equal to military expenditures, and assume that exogenous expenditures do not directly affect private agents' marginal utility. The first endogenous component affects directly private agents' marginal utilities. The second component consists of public investment expenditures and affects the public capital stock, which enters into the aggregate production function. The government finances its expenditures through a combination of a proportional tax on total income and a lump-sum tax, and its feedback rules for the endogenous components of its spending are chosen

[1] See Paquet (1994) for an extensive literature survey.

to maximize the utility of the representative private agent. Hence, government spending responds optimally to technology shocks and to shocks to military expenditures. Since spending is financed by distortionary taxation, government intervention can be used to attain a second-best optimum for the economy. Using this setup, we characterize optimal fiscal policy. We also analyze whether the comovements between the different components of government spending and other macroeconomic aggregates resemble those observed in the U.S. data. Under the maintained hypothesis that the model is an adequate representation of the economy, this allows us to evaluate to what extent U.S. government expenditures respond optimally to economic shocks. On the other hand, under the maintained hypothesis that the government is an optimizing agent, the model's ability to reproduce the observed comovements of the components of government expenditures can be interpreted as a test of the adequacy of the model's specification.

The rest of the paper is structured as follows. The model is described in the second section, along with a detailed discussion of the optimization problems of the representative private agent and of the government, of the conditions for a competitive equilibrium, and of the model's calibration and solution. In the third section, we discuss the stochastic properties of the model and compare them with the data. We also analyze the impulse response functions of the government's endogenous policy variables and of other macroeconomic aggregates to technology shocks and to shocks to the exogenous component of government spending. Conclusions are drawn in the fourth section.

2. *The Model*

2.1 *Preferences and Technology*

The model is of an economy with a representative private agent whose preferences can be represented by a time-separable utility function over an infinite horizon. The discounted utility at time t for the representative individual is given by

$$U(t) = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \ln(c_{t+i}) - \frac{\gamma_1}{1+\psi_1} h_{t+i}^{1+\psi_1} - \frac{\gamma_2}{1+\psi_2} e_{t+i}^{1+\psi_2} + \phi \ln(G_{1\,t+i}) \right\}. \quad (1)$$

E_t is the mathematical expectations operator conditional on information at time t, β is a subjective discount factor, c_{t+i} is the flow of consumption at time $t+i$, h_{t+i} is the average number of hours worked, e_{t+i} denotes the employment rate for quarter $t+i$, and G_{1t+i} denotes the *per capita* value of the endogenous component of government consumption expenditures. Lower case variables are used for quantities that are associated with individual agents, and upper case variables denote economy-wide *per capita* quantities. As specified in equation (1), following Cho and Cooley's (1994) motivation, private agents make decisions about labor supply along both the intensive (hours) and the extensive (employment) margins. The U.S. data suggest that most of the variation in total hours of employment comes from movements in and out of the labor force rather than changes in average hours of work. This specification allows us to parameterize the model to replicate the observed relative volatilities of employment and average hours rather than reporting results based on the extreme cases of divisible labor (with movements along the intensive margin only) or indivisible labor (with movements along the extensive margin). The effective number of hours worked in a quarter is given by

$$n_t = h_t \cdot e_t. \tag{2}$$

We consider two types of current government expenditures in this economy. Type-1 spending, G_{1t}, may substitute for private consumption spending, so that[2]

$$c_t = c_t^p + \alpha G_{1t} \ , \tag{3}$$

where c_t^p is the amount of private consumption. Since we will assume that type-1 fiscal spending is endogenously determined, the extra term $\phi \ln(G_{1t})$ in the utility function is needed. For $\alpha<1$, the optimal level of fiscal spending of this type would be zero without it. Type-2 government spending, G_{2t}, takes up resources without affecting directly the representative agent's marginal utility. Furthermore, it is exogenous.[3] In the data, we equate type-2 spending with military expenditures.

[2] Note that G_1 does not necessarily represent a public good, which is characterized by nonexclusion and nonrival consumption. The former means that it is prohibitively costly to limit the consumption of the good to a selected number of people, while the latter means that consumption of the good by one person need not decrease the quantity consumed by others. In a representative agent model, nonexclusion is trivially satisfied, but non rival consumption must be reflected by joint benefits would correspond to $\alpha > 1$, as an increase in G_1 by one unit would replace more than one unit of private spending at the margin.

[3] With only type-2 government spending, our model becomes similar to that of Christiano and Eichenbaum (1992) with their α parameter set equal to zero. With only type-2 government spending and public investment

The representative agent faces the following flow budget constraint:

$$c_t^p + i_t \leq (1-\tau)(w_t n_t + q_t k_t) - T_t \ . \tag{4}$$

This states that the agent's expenditures on private consumption goods, c^p_t , and private investment, i_t, are bounded by after-tax total income. The agent's gross income is equal to the sum of labor income and capital income, where w_t and q_t are respectively the wage and the rental rates. Agents face a proportional tax rate τ on total income that we assume to be time invariant, as well as a lump-sum tax T_t.[4] Individual agents own the capital stock, and the individual's holdings of capital evolve according to

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{5}$$

where δ is the constant rate of depreciation of capital.

Competitive firms in this economy use capital and labor services they purchase from households to produce goods according to a production function that exhibits constant returns to scale in privately-owned inputs. The aggregate production function is given by

$$Y_t = (z_t N_t)^{(1-\theta)} K_t^{\theta} K_{gt}^{\theta_g} \ , \tag{6}$$

where z_t is the state of technology at time t and K_{gt} is the stock of public capital. Technology evolves according to

$$z_{t+1} = z_t \ \exp(\lambda_{t+1}), \tag{7}$$

where λ_{t+1} is white noise with mean λ. The log of technology is thus a random walk with drift. From the firm's profit maximization problem, factors are paid their real marginal products, so that

$$w_t = (1-\theta)\, z_t^{(1-\theta)} \left(\frac{K_t}{N_t}\right)^{\theta} K_{gt}^{\theta_g}, \tag{8}$$

spending, our model is similar to that of Baxter and King (1993).

[4] A further extension in the modeling of taxation would allow for a component subject to stochastic shocks as in Braun (1994) and McGrattan (1994), and a component that could vary optimally in response to exogenous shocks as in Chari, Christiano and Kehoe (1991). This goes beyond the scope of the present paper. Nonetheless, the introduction of a distorting tax on total income, though constant, is not innocuous in mimicking some key properties of the data. See footnote 12.

and

$$q_t = \theta\, z_t^{(1-\theta)} \left(\frac{K_t}{N_t}\right)^{(\theta-1)} K_{gt}^{\theta_g}. \tag{9}$$

The aggregate resource constraint is given by

$$C_t^p + I_t + I_{gt} + G_{1t} + G_{2t} \leq Y_t, \tag{10}$$

where I_{gt} is public investment. When this relation holds with strict equality, we have the goods market clearing condition. The aggregate private and public capital stocks evolve according to

$$K_{t+1} = (1-\delta)K_t + I_t\ , \tag{11}$$

and

$$K_{gt+1} = (1-\delta)K_{gt} + I_{gt}\ , \tag{12}$$

where it is assumed that the depreciation rate is the same for both private and public capital stocks.

2.2 *The private agent's problem*

The representative private agent's problem involves choosing time paths $\{c_{t+i}, h_{t+i}, e_{t+i}, i_{t+i}\}_{i=0}^{\infty}$ to maximize the utility function (1) subject to the budget constraint (4), and the initial stock of capital. When solving this optimization problem, the private agent takes as given the wage rate, the gross marginal return on capital, the tax rate on total income, the amount of lump-sum tax (or transfer), and the laws of motion of the different components of government spending. Since technology follows a random walk with drift, the private agent's optimization problem is not stationary. However, it can easily be converted into a stationary problem in the following manner. For any variable X_t, define the following normalized variable:

$$\bar{X}_t \equiv \frac{X_t}{(z_t)^{(1-\theta)/(1-\theta-\theta_g)}}. \tag{13}$$

The aggregate production function can be rewritten as

$$\bar{Y}_t = N_t^{\,(1-\theta)}\, \bar{K}_t^{\theta} K_{gt}^{\theta_g}\, \exp\left(\frac{-\lambda_t(\theta+\theta_g)(1-\theta)}{1-\theta-\theta_g}\right). \tag{6'}$$

The agent's optimization problem can in turn be written as the following dynamic programming problem with stationary state variables:

$$v^p\left(\zeta_t^p, S_t^p, s_t^p\right) = \max_{d_t} \left\{r^p(\zeta_t^p, S_t^p, s_t^p, D_t, d_t) + \beta E_t\left[v^p(\zeta_{t+1}^p, S_{t+1}^p, s_{t+1}^p)\right]\right\} . \tag{14}$$

The value function depends on a vector of state variables which are exogenous from the point of view of the representative private agent,

$$\zeta_t^p \equiv \left[\lambda_t, \bar{G}_{1t}, \bar{G}_{2t}, \bar{I}_{gt}, \bar{K}_{gt}\right]',$$

a single state variable which is under the control of the representative agent,

$$s_t^p \equiv \bar{k}_t,$$

and its aggregate *per capita* equivalent

$$S_t^p \equiv \bar{K}_t.$$

The one-period return function $r^p(\cdot)$ depends also on a vector of the private agent's control variables,

$$d_t \equiv \left[h_t, e_t, \bar{i}_t\right]',$$

and their aggregate *per capita* equivalents

$$D_t \equiv \left[H_t, E_t, \bar{I}_t\right]'.$$

The one-period return function can be written as follows:

$$r^p(\zeta_t^p, S_t^p, s_t^p, D_t, d_t) = \ln(\bar{c}_t) - \frac{\gamma_1}{1+\psi_1} h_t^{1+\psi_1} - \frac{\gamma_2}{1+\psi_2} e_t^{1+\psi_2} + \phi \ln(\bar{G}_{1t}), \tag{15}$$

with

$$\bar{c}_t = (1-\tau)\left(\bar{w}_t h_t e_t + q_t \bar{k}_t\right) - \bar{i}_t - \bar{T}_t + \alpha \bar{G}_{1t} ,$$

where

$$\bar{w}_t = (1-\theta)\left(\frac{\bar{K}_t}{N_t}\right)^{\theta} \bar{K}_{gt}^{\theta_g} \exp\left(\frac{-\lambda_t(\theta+\theta_g)(1-\theta)}{1-\theta-\theta_g}\right).$$

and where lump sum taxes satisfy the government's flow budget constraint given in (18) below. The maximization is subject to the following constraints:

$$\bar{k}_{t+1} = (1-\delta)\bar{k}_t \exp\left(\frac{-\lambda_t(1-\theta)}{1-\theta-\theta_g}\right) + \bar{i}_t, \tag{16a}$$

$$\bar{K}_{t+1} = (1-\delta)\bar{K}_t \exp\left(\frac{-\lambda_t(1-\theta)}{1-\theta-\theta_g}\right) + \bar{I}_t, \tag{16b}$$

$$\bar{K}_{gt+1} = (1-\delta)\bar{K}_{gt} \exp\left(\frac{-\lambda_t(1-\theta)}{1-\theta-\theta_g}\right) + \bar{I}_{gt}, \tag{16c}$$

$$\bar{I}_{gt} = \bar{I}_{gt}\left(\lambda_t, \bar{G}_{2t}, \bar{K}_t, \bar{K}_{gt}\right), \tag{16d}$$

$$\bar{G}_{1t} = \bar{G}_{1t}\left(\lambda_t, \bar{G}_{2t}, \bar{K}_t, \bar{K}_{gt}\right), \tag{16e}$$

and the laws of motion for λ_t and $\bar{G}_{2t}$. We also require the aggregate consistency condition, which states that in equilibrium the representative agent's holdings of capital equal the aggregate *per capita* capital stock. Also, in equilibrium, the laws of motion for the government's control variables $\bar{I}_{gt}$ and $\bar{G}_{1t}$ must be compatible with the solution to the government's optimization problem. We now turn to the specification of that problem.

2.3 *The government's problem*

The exogenous component of government expenditures is governed by the following law of motion:

$$\ln\left(\bar{G}_{2t}\right) = (1-\rho)\ln\left(\bar{G}_2\right) + \rho\ln\left(\bar{G}_{2t-1}\right) + \mu_t. \tag{17}$$

Here, $\bar{G}_2$ is the steady-state level of $\bar{G}_{2t}$, and μ_t is a zero mean white noise shock. Detrended type-2 spending thus follows a first-order autoregressive process about its steady-state level.

The government is required to maintain a balanced budget every period while it finances its total spending through a combination of a lump-sum tax and a fixed proportional tax rate on total income. The government flow budget constraint is given by

$$\bar{G}_{1t} + \bar{G}_{2t} + \bar{I}_{gt} = \tau\bar{Y}_t + \bar{T}_t \tag{18}$$

This setup is consistent with a Ricardian world as a first-order approximation, with deviations from pure Ricardian equivalence arising because of distortionary taxation on income.

The government chooses type-1 current spending and investment spending taking into account the effects of its choices on the decisions of the private agent. Because of the proportional income tax, the economy's competitive equilibrium is not Pareto optimal. The interaction between the government and the private sector is not a "team game" problem such as described by Chari, Kehoe and Prescott (1989) and Ambler and Desruelle (1991), and the competitive equilibrium is not equivalent to an appropriately defined social planning problem.[5]

The government maximizes the welfare of the representative private agent. We formulate its maximization problem as a dynamic programming problem. As long as the private agent expects the government to use dynamic programming in its optimization, which we assume, this guaranties that the government's optimal policy is time consistent, which is appropriate in an environment where precommitting to an announced time path for its policy variables is not possible.[6] The value function for the government's problem is of the form

$$v^g\left(\zeta_t^g, S_t^g\right) = \max_{g_t} \left\{r^g(\zeta_t^g, S_t^g, D_t, g_t) + \beta E_t\left[v^g(\zeta_{t+1}^g, S_{t+1}^g)\right]\right\}, \tag{19}$$

where the vector of state variables which are exogenous from the point of view of the government is given by

$$\zeta_t^g \equiv \left[\lambda_t, \bar{G}_{2t}\right]',$$

the state variables which are under its control are given by

$$S_t^p \equiv \left[\bar{K}_t, \bar{K}_{gt}\right]',$$

and its control variables are

$$g_t \equiv \left[\bar{I}_{gt}, \bar{G}_{1t}\right]'.$$

The one-period return function can be written as follows:

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[5] As Stokey and Lucas (1989, p.542) note, "there is no single method of analysis that succeeds for all distorted economies."

[6] For a discussion, see Blanchard and Fischer (1989), pp.594-595.

$$r^g(\zeta_t^g, S_t^g D_t, g_t) = \ln\left(\bar{C}_t\right) - \frac{\gamma_1}{1+\psi_1} H_t^{1+\psi_1} - \frac{\gamma_2}{1+\psi_2} E_t^{1+\psi_2} + \phi \ln(\bar{G}_{1t}), \tag{20}$$

with

$$\bar{C}_t = \bar{Y}_t - \bar{I}_t - \bar{I}_{gt} - \bar{G}_{2t} - (1-\alpha)\bar{G}_{1t}.$$

The maximization is subject to the constraints given in (16b) and (16c), as well as the private-sector feedback rules which follow from the solution to the representative agent's maximization problem and the aggregate consistency condition:

$$H_t = H\left(\zeta_t^p, S_t^p\right), \tag{21a}$$

$$E_t = E\left(\zeta_t^p, S_t^p\right), \tag{21b}$$

$$I_t = I\left(\zeta_t^p, S_t^p\right). \tag{21c}$$

2.4 Solution Method and Calibration

The dynamic programming problems of the private agent and government have no exact analytical solutions. We follow the methodology described in Hansen and Prescott (1994) to simulate the model numerically, suitably extended to take into account the dynamic Stackelberg game between the government and the representative private agent (see Ambler and Paquet, 1994b, for further details). The methodology involves eliminating all nonlinear constraints from the model by substituting them into the return function and then using a quadratic approximation to the return function around the steady state levels of the detrended variables. The steady-state values for the variables of the model are found by solving the first-order conditions with respect to the decision variables of the dynamic programming problems set up above, using the envelope conditions with respect to the state variables, and then imposing the aggregate consistency conditions and the steady state.

The parameter values used to simulate the model are shown in Table 1. These values are similar to those used elsewhere in the literature and their choice is motivated as follows. The subjective discount parameter β was chosen as to give a three per cent annual rate of time preference, while the depreciation rate was set at an 8.7 per cent annual rate. The value of α was chosen to equal 0.3, which is in the middle of the range attributed by Barro (1989). The value of θ is compatible with the average share of profits in total income in the U.S. data. The total time endowment per quarter was normalized to equal one. The values of γ_1, γ_2, ψ_1 and ψ_2 were chosen as follows. With ψ_1 normalized to equal one, the other three parameters were chosen to replicate three facts of the U.S. economy; the average values of e_t and h_t and the relative volatilities of employment and hours per person (i.e. $\sigma_e/\sigma_h = 1.87$). The tax rate τ was set so that the government's budget is balanced in the steady state. Setting its value to replicate the weighted average of the effective marginal tax rates on labor and capital income in the data would have led to a slightly higher value, without substantially changing the results of the numerical simulations.[7]

The main parameters that affect the government's choice of its policy instruments are θ_g, the weight of public capital in the production function, and ϕ, the weight on endogenous government current expenditures as a separable argument in the utility function. The values of these parameters were chosen so that the steady-state ratios of government capital expenditures to total output and of type-1 current government expenditures to total output match the corresponding average values in our data set.

The remaining free parameters are those of the exogenous stochastic processes governing technology and exogenous government spending. The standard deviation of technology shocks σ_λ was set to 0.0114, which is the value inferred from our data set for the 1959:1-1992:3 sample.[8] The data were compatible with our specification of a random walk with drift: the log of the generated series was highly persistent, while its first difference was close to white noise. The drift parameter λ was chosen so that the steady-state rate of growth of the artificial economy

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[7] See McGrattan (1994) for data on effective marginal tax rates on labor and capital income.

[8] From the Citibase databank, we defined C_t as the sum of real expenditures on nondurable goods plus services, I_t as the sum of private fixed investment plus real expenditures on durable goods, G_t as real government purchases of goods and services (federal, state and local). Total income Y_t is the sum of C_t, I_t, and G_t. Hours worked, N_t, were taken from the Household Survey, and the employment rate was computed as the share of the employed civilian labour force in the population over 16 years old. To construct a series for the private and public capital stocks compatible with our framework, we used equations (11) and (12) along with the data on private and public investment and the value of the depreciation rate. The initial values of private and public capital stocks were chosen to yield capital to output ratios that were on average equal to the ratios in the nonstochastic steady state of the model. Given the capital stock series, Solow residuals were generated by imposing our adopted specification of the production function on the data. The standard deviation of the first-differenced logarithm of z_t could thus be inferred. A similar method is used to construct capital stock series in Ambler and Paquet (1994a).

matched the average rate of growth of per capita output in our data set. We took the exogenous component to be equal to military spending, which averaged 34.5% of total spending over our sample period. Public investment spending was taken to be equal to federal, state and local government non-defense expenditures on durable goods, equipment and structures.[9] The data were compatible with an AR(1) process for normalized exogenous spending, with an autoregressive parameter ρ of 0.98 and a standard deviation σ_μ of 0.0237. The calculated correlation $corr(\lambda,\mu)$ between the innovations in the technology process and exogenous government spending was imposed when generating the simulation results reported below.

3. *Simulation Results*

Given the above specification and calibration, we subjected the model to a series of stochastic simulations and computed a set of comovement statistics. The results are reported in Table 2. We focused on the model's predictions concerning the volatility of output, the relative volatilities of various macroeconomic aggregates (including the components of government spending), and the contemporaneous correlations of various aggregates. In each case, the results are based on 500 replications of 200 identically and independently distributed values for the exogenous shocks, truncated to a sample of 135 observations so that the results are not affected by initial conditions. Each comovement statistic is the average across the 500 replications of the simulated series, and the reported standard deviation gives a rough idea of the statistical significance of the results in a sample of the same length as the data. Comovements observed in the data are reported in the second column of the table.[10] The third column gives the predictions of the model given the estimated properties of the two stochastic shocks. The fourth column illustrates the stochastic properties of the model with the shocks to the exogenous component of government spending shut down, and the last column illustrates the model's properties when the technology shocks are shut down.

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[9] Data on federal non-defense spending on durable goods is not available before 1972:1. We assume that the share of federal non-defense spending in total public expenditure was equal to 2.06% for the earlier subperiod, the same as its share from 1972:1 to 1974:1. The share of state and local durable expenditures in total public investment was close to constant over the earlier subperiod. Our measure gives an average share of public investment in output very close to that used by Easterly and Rebelo (1993) for the United States. Their measure comes from annual data compiled by the World Bank.

[10] The data sources are described in footnote 8 above.

The sample period runs from 1959:1 to 1992:3. The U.S. time series are measured in logs and passed through the Hodrick-Prescott filter.[11] Those simulated series which were normalized by dividing by the level of aggregate technology z_t were first denormalized by multiplying by z_t. Then, all of the simulated series were converted to logs and passed through the Hodrick-Prescott filter.

The model's predictions concerning comovements other than with the components of government spending are close to those of standard RBC models without a government sector. The volatilities of investment, consumption, and employment relative to output volatility are quite close to those in the data. The predicted volatility of output is slightly too high, but is within two standard deviations of its value in the data.[12] The predicted volatility of average labor productivity is too low relative to both output volatility and the volatility of total hours. This reflects our parameterization of the individual's choice of hours and employment. Our specification allows both the divisible-labor model (with all movements in labor supply along the intensive margin) and the indivisible-labor model of Rogerson (1988) and Hansen (1985) (with all movements in labor supply along the extensive margin) as special cases. The model is parameterized to reproduce the relative volatility of employment and hours, and this yields predictions for productivity fluctuations that are closer to the indivisible labor model than the divisible labor model (see Hansen and Wright, 1992, for a comparison of the predictions of the two models). The predicted correlations of consumption, investment, and total hours with output match the data quite well, with the consumption-output correlation slightly too low and the total hours-output correlation slightly too high. The model predicts correlations between hours and output and between employment and output that are considerably higher than their observed counterparts. The predicted correlation between average labor productivity and output

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11 See Prescott (1986) for a description. The Hodrick-Prescott filter removes low-frequency components of the data. Filtering the data in this way is now standard practice, and we adopt it here to facilitate comparison with other, similar studies.

12 We have also simulated the model with the distorting tax rate being set to zero. This deteriorated the model's predictions with respect to some of its stochastic properties. In this case, compared to the data, the predicted volatility of output was even higher, total government spending was not enough volatile relative to output volatility, and labor productivity was not contemporaneously correlated with output. Hence, this emphasizes the distinct role of distorting taxation in the business cycle, which changes the private agents' responses to exogenous shocks.

matches the data very well, and the predicted correlation between total hours and average labor productivity is statistically insignificant, which agrees with the consensus view of the data.[13]

Turning to the comovements between the components of government spending and other aggregates, the model captures the qualitative features of the relative volatilities observed in the data quite well. The data tell us that total government spending, *ag*, is about as volatile as aggregate output, which is close to what the model predicts. Quantitatively, the predicted volatility of total government spending relative to output volatility is about three standard deviations less than the observed value. The data tell us that both military expenditures and public investment are significantly more volatile than aggregate output, while nonmilitary current expenditures are less volatile than output. Once again, this is what the model predicts. Quantitatively, the model predicts a volatility of public investment relative to aggregate output that is quite close to that of private investment. The figure in the data is somewhat lower. The model also underpredicts the relative volatility of the exogenous component of spending: in the data military expenditures are actually more volatile than public investment spending, while in the model type-2 current expenditures are less volatile.

The model predicts that public investment should be more correlated with output than total government spending, and that exogenous spending should be significantly less correlated with output than total government spending. This is what we see in the data. Unfortunately, the model significantly overpredicts the correlation of total government spending and of each of its components with output.

Comparing the third and fourth columns of Table 2 with the second column tells us that the stochastic properties of the model are not greatly affected by the addition of shocks to the exogenous component of government spending. This explains the low predicted correlation between type-2 expenditures and output: spending shocks have little impact on output, and have an effect on the correlation between the two series which is similar to adding noise to the spending series while leaving the output series unchanged. The presence of exogenous spending

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[13] The calculated value in the data is sensitive to the sample period and to how total hours are measured. Hansen and Wright (1992) report values which range from -0.35 to 0.10. The statistically insignificant correlation between productivity and total hours results from nonlinearities in the impulse response functions of employment and average hours to technology shocks.

shocks also dampens the correlation between total government spending and output as well as the correlation between average labor productivity and total hours.

Table 3 gives the correlation between output on the one hand and either total government spending, or its various components on the other hand. The model predicts noncontemporaneous correlations that are significantly less than the contemporaneous correlations. Moreover, the correlations at corresponding leads and lags are of similar value, so that the distribution around the contemporaneous correlation is approximately symmetric for all of the components of spending. In contrast, the data display a highly asymmetric distribution, with output leading total spending and its components. In each case, the strongest correlation occurs with a five-period lead. This is true even for military spending, even though the correlation remains relatively low at all leads and lags. This suggests that there may be implementation lags for the endogenous components of spending which are not captured by the model; our specification allows the government to react to current shocks.

Figures 1 and 2 give the impulse responses of various endogenous variables to, respectively, a one-standard-deviation shock to technology and a one-standard-deviation shock to the exogenous component of fiscal spending. All variables are measured in logs and are the deviations of the variables from their reference steady-state growth paths in the absence of any shocks. All of the illustrated series contain a common trend. A technology shock causes a permanent deviation of all of the series from their reference growth paths, so the series in Figure 1 converge towards 0.0114, the calibrated standard deviation of the technology shock. The effect of a spending shock on all of the series is temporary, so that all of the series in Figure 2 converge towards zero.

The impulse response functions give a clear idea of what is driving optimal government policy in the model. In response to both types of shocks, the response functions of public and private investment are very similar, and the response functions of endogenous government current expenditures and private consumption are practically indistinguishable from one another. This tells us that, despite the presence of the separate term in G_1 in the utility function, private consumption and type-1 government expenditures affect utility in a similar manner. Also, despite different weights in the production function, the effects of private and public capital on output are qualitatively very similar. Both public and private consumption are smoothed over time, and public and private investment are used to achieve the goal of consumption smoothing via capital accumulation.

Given these impulse response functions, the model's predictions concerning the relative volatilities of the components of government spending are not surprising. Also, the model's predictions concerning the correlations of private consumption and current endogenous spending with output are quite similar, as are its predictions concerning the correlations of private and public investment spending with output. For example, in response to military spending shocks, the model predicts that both private consumption and endogenous current government expenditures should be negatively correlated with output.

4. Conclusions

We have analyzed the properties of a neoclassical growth model with a government sector in which the government chooses its public investment expenditures and its nonmilitary current expenditures to maximize the welfare of a representative private agent. We show that optimal policy leads to the prediction that current expenditures should behave similarly to private consumption and that public investment should behave very much like private investment spending. The relative volatilities of macroeconomic aggregates in the data are broadly compatible with this prediction. Government nonmilitary current expenditures, like private consumption expenditures, are less volatile than output in the data. Public investment spending is substantially more volatile than output in the data, as is private investment spending. Our model predicts that total government spending should have about the same volatility as total output, which is compatible with the data.

The model's main shortcomings are its predictions of high correlations between the different components of spending and aggregate output. The observed correlations are all quite low. One possible explanation of this result is measurement error: government nonmilitary current expenditures no doubt contain an (unobservable) component that is beyond its immediate control and which therefore behaves more like military expenditures. Also, it is reasonable to suppose that the decision-making process is much more time-consuming for the government than for private agents. This would introduce a delay between the occurrence of a shock and the endogenous reaction of the government, which could reduce the contemporaneous correlation between output and the endogenous components of spending. This hypothesis is supported by the asymmetric distribution of the noncontemporaneous correlations between output and spending in the data. Finally, governments may well have arguments in their utility functions other than the welfare of their private citizens. Introducing political economy considerations into the analysis would almost certainly break the tight link between the shocks that generate fluctuations in output and the endogenous fluctuations of the components of government spending.

Our introduction of endogenous government spending is therefore far from the final word on the determination and the role of endogenous fiscal policy. Further research will also need to consider the interaction of the endogenous determination of current fiscal spending and public investment with the optimal determination of marginal tax rates. Nevertheless, we think this paper constitutes a useful step in modeling the dynamic interaction between the private sector and the government.

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Table 1
Parameter values.[a]

β	α	δ	θ	τ	ψ_1	ψ_2	γ_1
0.9926	0.3	0.021	0.347	0.2094	1	0.1	6.7848

γ_2	θ_g	ϕ	ρ	λ	σ_λ	σ_μ	$corr(\lambda,\mu)$
1.5425	0.0467	0.107	0.98	0.0031	0.0114	0.0237	-0.25

[a]All parameters are defined in the text.

Table 2
Stochastic properties of the model.[a]

Statistic	U.S. data	Both shocks	Technology shocks only	Spending shocks only
σ_y	0.0159	0.0193 (.002)	0.0198 (.002)	0.0043 (.0005)
σ_i/σ_y	3.1826	3.1838 (.278)	3.1572 (.231)	2.3788 (.023)
σ_c/σ_y	0.5189	0.5067 (.058)	0.4731 (.042)	0.1433 (.041)
σ_{ag}/σ_y	1.1189	0.9088 (.065)	0.8120 (.014)	2.6603 (.130)
σ_{g1}/σ_y	0.7228	0.4080 (.021)	0.3925 (.014)	0.1092 (.005)
σ_{g2}/σ_y	2.6093	1.5630 (.219)	0.7180 (.006)	6.9013 (.050)
σ_{ig}/σ_y	1.9826	2.8488 (.156)	2.8133 (.138)	2.2509 (.030)
σ_n/σ_y	0.8776	0.9212 (.024)	0.9130 (.009)	1.5257 (.006)
σ_e/σ_h	1.8705	1.8194 (.008)	1.8190 (.007)	1.8187 (.005)
$\sigma_{y/n}/\sigma_y$	0.5934	0.2746 (.033)	0.2297 (.025)	0.5336 (.005)
$\sigma_n/\sigma_{y/n}$	1.4789	3.4046 (.439)	4.0274 (.480)	2.8593 (.016)
$corr(c,y)$	0.8979	0.7373 (.090)	0.7830 (.070)	-0.5257 (.329)
$corr(i,y)$	0.9266	0.9504 (.017)	0.9569 (.014)	0.9849 (.003)
$corr(ag,y)$	0.2317	0.8537 (.040)	0.9707 (.011)	0.9962 (.001)
$corr(g_1,y)$	0.1737	0.9036 (.019)	0.9301 (.011)	-0.6697 (.064)
$corr(g_2,y)$	0.0951	0.4334 (.127)	0.9958 (.001)	0.9975 (.001)

Table 2 (Continued)
Stochastic properties of the model.[a]

Statistic	U.S. data	Both shocks	Technology shocks only	Spending shocks only
$corr(i_g,y)$	0.2733	0.8999 (.046)	0.9056 (.036)	0.9787 (.004)
$corr(n,y)$	0.8079	0.9622 (.010)	0.9749 (.006)	0.9973 (.001)
$corr(h,y)$	0.6401	0.9621 (.010)	0.9749 (.006)	0.9973 (.001)
$corr(e,y)$	0.7777	0.9621 (.010)	0.9749 (.006)	0.9973 (.001)
$corr(y/n,y)$	0.4903	0.4138 (.081)	0.4785 (.026)	-0.9774 (.005)
$corr(y/n,n)$	-0.1174	0.1532 (.098)	0.2725 (.042)	-0.9903 (.002)

[a]The notation σ_x refers ot the standard deviation of a variable x, and $corr(x,y)$ to the contemporaneous correlation between variables x and y. For each predicted comovement statistic, we report its mean across 500 replications with the associated standard error in parentheses beneath. The variable ag is total government spending, and other definitions of variables are given in the text. Data sources are described in footnote 8 of the text.

Table 3

Cyclical Behaviour of Real Output and Government Spending Variables.[a]

Variable x		Cross Correlation of Real Ouput With										
		x(t-5)	*x(t-4)*	*x(t-3)*	*x(t-2)*	*x(t-1)*	*x(t)*	*x(t+1)*	*x(t+2)*	*x(t+3)*	*x(t+4)*	*x(t+5)*
ag	US data	-0.0437	-0.0220	0.0081	0.0737	0.1538	0.2317	0.2440	0.2446	0.2610	0.3040	0.3522
	Both shocks	-0.0731	0.0253	0.1610	0.3404	0.5707	0.8537	0.7230	0.4816	0.2872	0.1393	0.0270
		(0.119)	(0.122)	(0.119)	(0.107)	(0.081)	(0.040)	(0.067)	(0.094)	(0.114)	(0.124)	(0.129)
g_1	US data	-0.1098	-0.0696	-0.0336	0.0280	0.0962	0.1737	0.2501	0.2753	0.2784	0.2960	0.3435
	Both shocks	-0.1959	-0.0881	0.0656	0.2774	0.5535	0.9036	0.7331	0.5735	0.4281	0.3017	0.1923
		(0.101)	(0.100)	(0.094)	(0.082)	(0.057)	(0.019)	(0.056)	(0.091)	(0.114)	(0.131)	(0.138)
g_2	US data	0.0179	0.0127	0.0091	0.0239	0.0554	0.0951	0.0938	0.0972	0.1134	0.1355	0.1574
	Both shocks	-0.0410	0.0090	0.0784	0.1686	0.2871	0.4334	0.3153	0.2157	0.1342	0.0751	0.0292
		(0.149)	(0.151)	(0.148)	(0.143)	(0.136)	(0.127)	(0.131)	(0.135)	(0.141)	(0.144)	(0.149)
i_g	US data	-0.0697	-0.0577	-0.0241	0.0808	0.1861	0.2733	0.2364	0.1919	0.1952	0.2847	0.3634
	Both shocks	-0.0073	0.0947	0.2313	0.4098	0.6331	0.8999	0.8353	0.5020	0.2445	0.0529	-0.0839
		(0.126)	(0.125)	(0.119)	(0.103)	(0.078)	(0.046)	(0.062)	(0.083)	(0.099)	(0.105)	(0.107)

[a] We report the mean of each cross-correlation across 500 replications with its associated standard error in parentheses beneath. The variable *ag* is total government spending, and other definitions of variables are given in the text. Data sources are described in footnote 8 of the text.

Figure 1
Impulse response functions of selected variables
to a one standard deviation technology shock

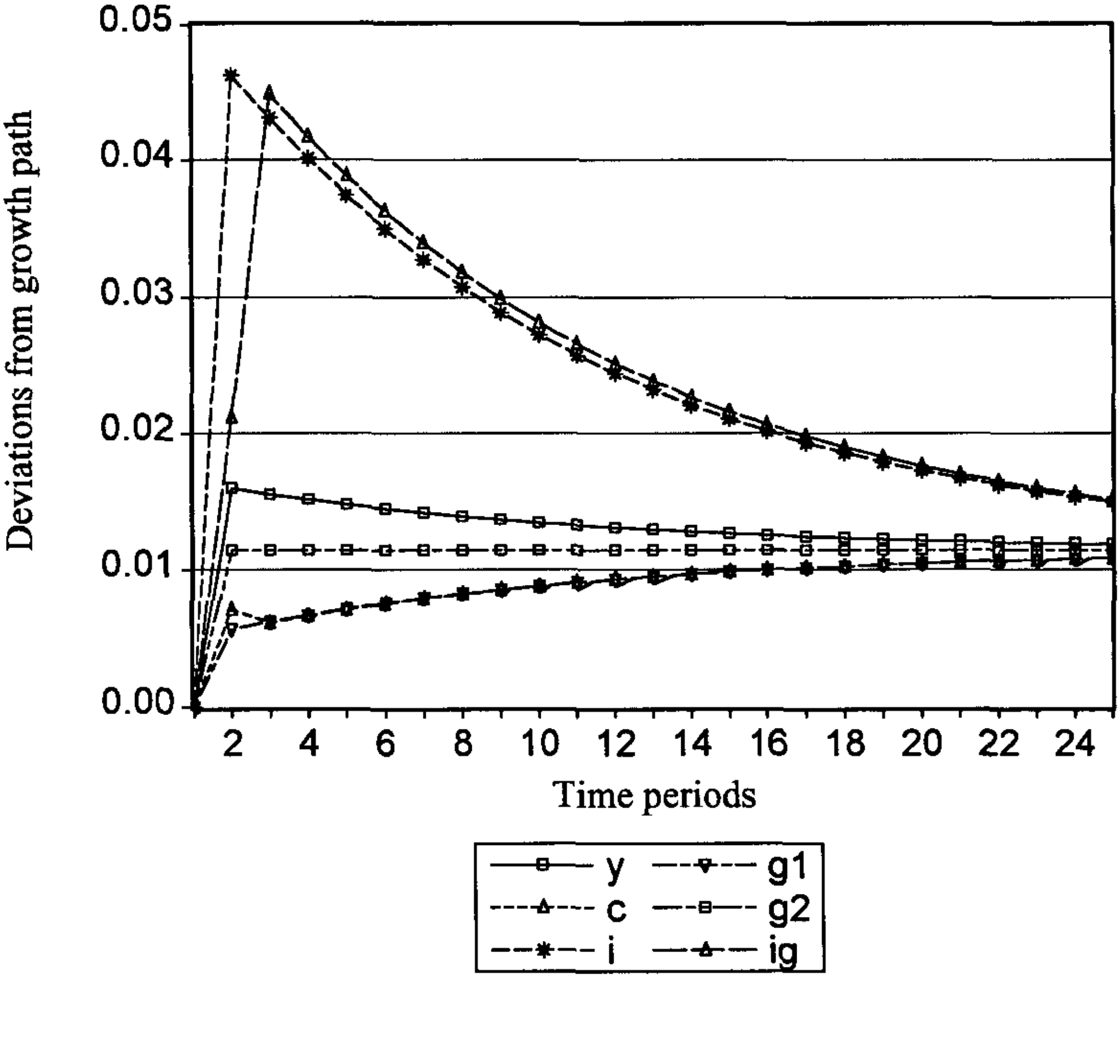

Figure 2
Impulse response functions of selected variables
to a one standard deviation spending shock