Information, Misallocation and Aggregate Productivity*

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Abstract

We propose a theory linking imperfect information to resource misallocation and hence to aggregate productivity and output. In our setup, firms learn from both private sources and imperfectly informative stock market prices. We devise a novel empirical strategy that uses a combination of firm-level production and stock market data to pin down the information structure in the economy. Applying this methodology to data from the US, China, and India reveals substantial losses in productivity and output due to informational frictions - even when only one factor, namely capital, is subject to the friction. Our estimates for these losses range from 7-10% for productivity and 10-14% for output in China and India, and are smaller, though still significant, in the US. Losses are substantially higher when labor decisions are also made under imperfect information. Private learning plays a significant role in mitigating uncertainty and improving aggregate outcomes; learning from financial markets contributes little, even in the US.

1 Introduction

In a frictionless environment, the optimal allocation of factor inputs across productive units requires the equalization of marginal products. Deviations from this outcome represent a misallocation of resources and translate into sub-optimal aggregate outcomes, specifically, depressed levels of productivity and output. A recent literature empirically documents the presence of substantial misallocation and points out its potentially important role in accounting for large observed cross-country differences in productivity and income per-capita. With some notable exceptions, however, the literature has remained largely silent about the underlying factors driving this misallocation.

In this paper, we propose just such a theory, linking imperfect information to resource misallocation and hence to aggregate productivity and output. Our point of departure is a standard general equilibrium model of firm dynamics along the lines of Hopenhayn (1992). The key modification here is that firms choose inputs under limited information about their idiosyncratic fundamentals, i.e., either productive efficiency or demand conditions. This informational friction leads to a misallocation of factors across in an ex-post sense. The extent of this misallocation depends on the residual uncertainty at the time of the input choice, which in turn, is a function of the volatility of the fundamental shocks and the quality of the information at the firm level. Through this channel, uncertainty reduces aggregate productivity and output. The parsimonious nature of our analytical framework enables a sharp characterization of these relationships and yields simple closed-form expressions linking informational frictions at the micro-level to aggregate outcomes.

The second piece of our theoretical framework focuses on the firm's learning problem. Here, we develop a flexible information structure in which firms learn not only from their own private sources of information, but also from their own stock prices. This captures the idea that financial markets aggregate dispersed information across investors to generate informative prices, which in turn guides firm decisions.[1] This informational role of financial markets dates back at least to Tobin (1982) and continues to be the subject of much study - a recent body of work documents a "feedback effect" from stock prices to real activity.[2] In our framework, this is modeled with an explicit description of financial market trading in the noisy rational expectations paradigm of Grossman and Stiglitz (1980).[3] Imperfectly informed investors and noise traders buy and sell shares of the firm's stock, generating equilibrium prices which aggregate informa-

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[1]Note that this does not require investors to have better information than firms; only that they are privy to different information that may also be relevant for firm decisions.

[2]We discuss a few particularly relevant examples of such work below and refer the reader to Bond et al. (2012) for an excellent survey.

[3]We rely particularly on recent work by Albagli et al. (2011b) for our specific modeling structure.

tion imperfectly. This provides firms with an additional, albeit noisy, signal of fundamentals, which is combined with their own private information to guide input decisions.

The presence of learning from financial markets serves two purposes in our analysis: first, we are able to quantitatively evaluate the contribution of financial markets to allocative efficiency through an informational channel, i.e., by providing higher quality information to decision-makers within firms. Our analysis is, to the best of our knowledge, the first to measure and shed light on the aggregate consequences of this channel in a standard macroeconomic framework. Second, as we describe next, the informational content of observed market prices is at the core of our empirical approach and allows us to identify the severity of otherwise unobservable informational frictions in the economy.

Quantifying uncertainty is challenging because we do not observe the entire information set of firms. We develop a novel empirical strategy that combines firm-level production and financial market data to infer the extent of uncertainty at the firm-level. Our key insight is that asset prices allow us to observe a subset of firm information. Combining this with firm-level production data (specifically, investment decisions and measured fundamentals), we are able to gauge both the noisiness of this signal (by measuring the correlation of returns with future fundamentals) and the responsiveness of firm decisions to it (by measuring the correlation of returns with investment). Intuitively, the correlation of stock returns with fundamentals provides information on the magnitude of the noise in prices. Given the noise, the extent to which firms adapt their decisions to the signal then allows us to pin down the overall quality of their information (from all sources, including those we do not observe). The lower this quality, the greater the reliance on financial markets for information and therefore, the higher will be the correlation between investment decisions and stock market returns. It is worth emphasizing the need to analyze these moments together - the correlation of returns with investment alone does not tell us much about the extent of uncertainty.[4]

We prove that our moments identify the estimated parameters for two polar cases: when firm level shocks are iid and when they follow a random walk. We make additional use of these analytically tractable cases to demonstrate the validity of our approach under several variants to our baseline framework, in particular, with additional "distortions" that contribute to observed misallocation, as well as with a more complex correlation structure between market and firm information. In our quantitative work, we show that the relationships between moments and parameters go through almost exactly.

We apply our empirical methodology to data from 3 countries - the US, China and India.

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[4]To give a simple example, the correlation between returns and investment can be high either because firms and investors are both perfectly informed, in which case all firm-level variables are functions of a single fundamental shock, or alternatively, because firms are poorly informed and therefore learn much from market prices.

Our results point to substantial uncertainty at the micro level, particularly in China and India. Even in the US, which has the highest degree of learning, our most conservative estimate for the posterior variance of the firm is about 40% of the ex-ante, or prior, uncertainty.[5] The corresponding estimates for the other two countries ranges from about 60-90%. The associated implications for aggregate productivity and output are then quite significant. In China and India, TFP losses (in log points, relative to the first best) are in the range of 7-10%, while losses in steady state output (again, in log points relative to the first best) range from 10% to almost 15%. The corresponding values in the US are noticeably smaller but still significant - 4% for productivity and 5% for output. Importantly, these baseline calculations assume that only investment decisions are made under imperfect information, while labor is assumed to adjust perfectly to contemporaneous conditions. In this sense, they are conservative estimates of the total impact of informational frictions. Assuming that the friction affects labor inputs to the same degree as capital leads to losses that are substantially higher. For example, in this case, the gap between status quo and first best increases to 55-80% in TFP for India and China.[6] We interpret this as an upper bound on the total effect of the friction, with reality likely falling somewhere in between this and the baseline version.[7] To put these numbers in context, we compare them to direct measures of misallocation in our sample and find that informational frictions account for anywhere from 20-60% of observed dispersion in the marginal product of capital. This fraction goes up once we control for firm-fixed effects.

Our framework also enables us to quantify the extent of total learning, particularly the contribution of financial markets. Here, we arrive at a striking conclusion - learning from stock prices is only a small part of total learning at the firm level, even in a relatively well-functioning financial market like the US. Thus, the impact of this channel on overall allocative efficiency and hence aggregate performance of the economy is actually quite limited.[8] We show that this is primarily due to the high levels of noise in market prices, making them relatively poor signals of fundamentals, even without taking into account the relatively better quality of firm-level private information.[9] A counterfactual experiment delivering access to US-quality financial markets (in a purely informational sense) to firms in China and India generates only small improvements in allocative efficiency. In contrast, a significant amount of learning occurs from

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[5]In our autoregressive structure, this prior uncertainty is the variance of contemporaneous innovations. Firms are able to infer the persistent component in their fundamental perfectly from their history.

[6]The corresponding numbers for steady state output are 80-100%.

[7]We provide suggestive evidence that this is the case using dispersion in the marginal product of labor among US firms.

[8]Of course, we abstract from other channels through which informative prices, and more generally, well-functioning stock markets improve efficiency. See also the discussion in Section 4.4.

[9]This distinction is related to the concepts of forecasting price efficiency (to what extent do prices reflect and predict fundamentals) versus revelatory price efficiency (to what extent do prices promote real efficiency by revealing new information to the firm) as put forth in Bond et al. (2012).

private, or internal, sources within the firm. Moreover, disparities along this dimension (i.e. in the quality of such information) are the primary drivers of cross country differences, not access to well-functioning financial markets. This is reminiscent of Bloom et al. (2013), who highlight the role of better management practices and/or manager skill in explaining cross-country differences in performance. Finally, we show that differences in the volatility of the shocks to firm-level fundamentals also play a role in generating cross-country differences in the severity of informational frictions. Specifically, firms in China and India are subject to larger shocks to fundamentals than firms in the US, making the inference problem more difficult in those countries even without the effect of differences in signal qualities.

Our paper relates to several existing branches of literature. We bear a direct connection to recent studies on the aggregate implications of misallocated resources, for example, Hsieh and Klenow (2009), Restuccia and Rogerson (2008), Guner et al. (2008), and Bartelsman et al. (2013). Indeed, we can map our measure of informational frictions directly into the measures of misallocation studied in these papers, i.e., into the dispersion in marginal products and the covariance between firm-level fundamentals (productivity, for example) and activity (i.e, factor use or output). We differ from these papers in our explicit modeling of a specific friction as the source of misallocation, a feature we share with Midrigan and Xu (2013), Moll (2014), Buera et al. (2011), and Asker et al. (2012), who study the role of financial frictions and capital adjustment costs, respectively. Our focus on the role of imperfect information is related to that of Jovanovic (2013), who studies an overlapping generations model where informational frictions impede the efficient matching of entrepreneurs and workers.

Our structure of firm learning holds some similarity with Jovanovic (1982) and our linking of financial markets, information transmission, and real outcomes is reminiscent of Greenwood and Jovanovic (1990). As mentioned earlier, the informational role of stock markets is the subject of a large body of work in empirical finance. One strand of this literature focuses on measuring the information content of stock prices. Durnev et al. (2003) show that firm-specific variation in stock returns, i.e., "price-nonsynchronicity," is useful in forecasting future earnings and Morck et al. (2000) find that this measure of price informativeness is higher in richer countries. A related body of work closer to our own and recently surveyed by Bond et al. (2012) looks directly at the feedback from stock prices to investment and other decisions. Chen et al. (2007), Luo (2005), and Bakke and Whited (2010) are examples of studies that find evidence of managers learning from markets while making investment decisions. Bai et al. (2013) combines a simple investment model with a noisy rational expectations framework to assess whether US stock markets have become more informative over time. Our analysis complements these papers by placing information aggregation through financial markets into a standard macroeconomic setting, which allows us to make precise statements about the quantitative

importance of this channel for information transmission, real activity and aggregate outcomes. Our results on the limited role for stock market information bear some resemblance to the conclusions reached by Morck et al. (1990), who find a limited incremental role for stock prices in predicting investment, once fundamentals are controlled for.[10] Our focus here is different - we are interested in measuring the contribution of stock market information to aggregate allocative efficiency and our use of a structural model enables us to interpret these types of reduced-form results in terms of their implications for firm and market uncertainty and to quantify the aggregate consequences.

The remainder of the paper is organized as follows. Section 2 describes our model of production and financial market activity under imperfect information. Section 3 spells out our approach to identifying informational frictions in two analytically tractable cases of our model, while Section 4 details our numerical analysis and presents our quantitative results. We summarize our findings and discuss directions for future research in Section 5. Details of derivations and data work are provided in the Appendix.

2   The Model

In this section, we develop our model of production and financial market activity under imperfect information. We turn first to the production side of the economy, where we derive sharp relationships linking the extent of micro level uncertainty to aggregate outcomes. Next, we flesh out the information environment, and in particular, lay out a fully specified financial market in which dispersed private information of investors and noise trading interact to generate imperfectly informative price signals.

2.1   Production

We consider an infinite-horizon economy set in discrete time. The economy is populated by a representative large family endowed with a fixed quantity of labor that is supplied inelastically to firms. The aggregate labor endowment is denoted by N. The household has preferences over consumption of a final good and accumulates capital that is then rented to firms. We purposely keep households simple as they play a limited role in our analysis.

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[10]More precisely, they find very small improvements in R^2 when adding stock returns to an investment regression that already includes fundamentals.

Technology. A continuum of firms of fixed measure one, indexed by i, produce intermediate goods using capital and labor according to

$$Y_{it} = K_{it}^{\widehat{\alpha}_1} N_{it}^{\widehat{\alpha}_2}, \qquad \widehat{\alpha}_1 + \widehat{\alpha}_2 \leq 1$$

Intermediate goods are bundled to produce the single final good using a standard CES aggregator

$$Y_t = \left(\int A_{it} Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

where A_{it} is the idiosyncratic quality or productivity component of good i and represents the only source of uncertainty in the economy (i.e., we abstract from aggregate risk). We assume that A_{it} follows an AR(1) process in logs:

$$a_{it} = (1-\rho)\,\overline{a} + \rho a_{it-1} + \mu_{it}, \qquad \mu_{it} \sim \mathcal{N}\left(0, \sigma_\mu^2\right) \tag{1}$$

where we use lower-case to denote natural logs, a convention we follow throughout, so that, e.g., $a_{it} = \log A_{it}$. In this specification, $\overline{a}$ represents the unconditional mean of a_{it}, ρ the persistence, and μ_{it} an i.i.d. innovation with variance σ_μ^2.

Market structure and revenue. The final good is produced by a competitive firm under perfect information. This yields a standard demand function for intermediate good i

$$Y_{it} = P_{it}^{-\theta} A_{it}^{\theta} Y_t \quad \Rightarrow \quad P_{it} = \left(\frac{Y_{it}}{Y_t}\right)^{-\frac{1}{\theta}} A_{it}$$

where P_{it} denotes the relative price of good i in terms of the final good, which serves as numeraire.

The elasticity of substitution θ indexes the market power of intermediate good producers. Our specification nests various market structures. In the limiting case of $\theta = \infty$, we have perfect competition, i.e., all firms produce a homogeneous intermediate good. In this case, the survival of heterogenous firms requires decreasing returns to scale in production to limit firm size, that is, $\widehat{\alpha}_1 + \widehat{\alpha}_2 < 1$. When $\theta < \infty$, we have monopolistic competition, with constant or decreasing returns to scale. No matter the assumption here, however, firm revenue can be expressed as

$$P_{it} Y_{it} = Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} \tag{2}$$

where

$$\alpha_j = \left(1 - \frac{1}{\theta}\right) \widehat{\alpha}_j$$

Our framework accommodates two alternative interpretations of the idiosyncratic component A_{it}, either as a firm-specific level of demand or productive efficiency. The analysis is identical under both interpretations, though one could argue that learning from markets may be more plausible for demand-side factors. Neither the theory nor our empirical strategy requires us to differentiate between the two, so we will simply refer to A_{it} as a firm-specific fundamental.

Input choices under imperfect information. The key element of our theory is the effect of imperfect information on the firm's choice of factor inputs, that is, capital and labor. These are modeled as static and otherwise frictionless decisions, i.e., firms rent capital and/or hire labor period-by-period, but with potentially imperfect knowledge of their fundamentals A_{it}. Clearly, the impact of the information friction will depend on whether it affects both inputs or just one. Rather than take a particular stand on this important issue regarding the fundamental nature of the production process, we present results for two cases: in case 1, both factors of production are chosen simultaneously under the same (imperfect) information set; in case 2, only capital is chosen under imperfect information whereas labor is freely adjusted after the firm perfectly learns the current state.

Case 1: Both factors chosen under imperfect information. In this case, the firm's profit-maximization problem is given by

$$\max_{K_{it}, N_{it}} \quad Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it} - R_t K_{it} \tag{3}$$

where $\mathbb{E}_{it} [A_{it}]$ denotes the firm's expectation of fundamentals conditional on its information set $\mathcal{I}_{it}$, which we make explicit below. Standard optimality and market clearing conditions imply

$$\frac{N_{it}}{K_{it}} = \frac{\alpha_2 R}{\alpha_1 W} = \frac{N}{K_t} \tag{4}$$

i.e., the capital-labor ratio is constant across firms.

Our empirical analysis relies on moments of firm-level investment data and with this in mind, we use the optimality conditions characterized in (4) to rewrite (3) simply as a capital input choice problem:

$$\max_{K_{it}} \quad \left(\frac{N}{K_t} \right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it} [A_{it}] K_{it}^{\alpha} - \left(1 + \frac{\alpha_2}{\alpha_1} \right) R K_{it} \tag{5}$$

where

$$\alpha = \alpha_1 + \alpha_2 = (\widehat{\alpha}_1 + \widehat{\alpha}_2)\left(\frac{\theta - 1}{\theta}\right)$$

Notice that the firm's expected revenues depend only on the aggregate capital-labor ratio, its conditional expectation of A_{it}, and the chosen level of its capital input. The curvature parameter α depends both on the returns to scale in production as well as on the elasticity of demand, and will play an important role in our quantitative analysis below. Solving this problem and imposing capital market clearing gives the following expressions for the firm's capital choice (the labor choice exactly parallels that of capital):

$$K_{it} = \frac{(\mathbb{E}_{it}\left[A_{it}\right])^{\frac{1}{1-\alpha}}}{\int (\mathbb{E}_{it}\left[A_{it}\right])^{\frac{1}{1-\alpha}} di} K_t \tag{6}$$

Case 2: Only capital chosen under imperfect information. The firm's problem now is

$$\max_{K_{it}} \quad \mathbb{E}_{it}\left[\max_{N_{it}} \quad Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it}\right] - R_t K_{it}$$

and optimizing over N_{it} gives

$$N_{it} = \left(\frac{\alpha_2}{W} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}} \tag{7}$$

Note that in contrast to (4), capital-labor ratios are now functions of the firm's fundamental A_{it} and chosen level of capital K_{it}, the former fully observed when making the labor choice and the latter fixed. Imposing labor market clearing and substituting, we can write the firm's capital choice problem as:

$$\max_{K_{it}} \quad (1 - \alpha_2)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \mathbb{E}_{it}\left[\tilde{A}_{it}\right] K_{it}^{\tilde{\alpha}} - R_t K_{it} \tag{8}$$

where

$$\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}}, \qquad \tilde{\alpha} = \frac{\alpha_1}{1 - \alpha_2}$$

Thus, the firm's capital choice problem here has the same structure as in case 1 (compare equations (5) and (8)), but with a slightly modified fundamental and overall curvature. This will make the two cases qualitatively very similar, though, as we will see, the quantitative implications will be quite different. We mark with a $\sim$ the transformed objects that are relevant in case 2, a convention we will carry thoughout this section. The firm's input choices can be

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shown to satisfy

$$K_{it} = \frac{\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}}{\int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di} K_t, \qquad N_{it} = \frac{\tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \tilde{A}_{it}\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di} N \qquad (9)$$

While the capital choice looks similar to case 1, the labor choice now depends on the joint distribution of $\tilde{A}_{it}$ and $\mathbb{E}_{it}\left[\tilde{A}_{it}\right]$. Despite this, the analysis remains quite tractable and we will derive simple expressions for the economic aggregates as functions of the underlying uncertainty.

To complete our characterization of the firm's problem and therefore of the production-side equilibrium in the economy, we must explicitly spell out the information set $\mathcal{I}_{it}$ on which the firm relies to form its expectations. We defer this discussion to the following subsection and for now directly make conjectures about firm beliefs, which we will later verify under our information structure. Specifically, we assume the conditional distribution of the fundamental to be log-normal in both case 1 and 2, i.e.,

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\begin{aligned}
a_{it}|\mathcal{I}_{it} &\sim \mathcal{N}\left(\mathbb{E}_{it}\left[a_{it}\right], \mathbb{V}\right) \\
\tilde{a}_{it}|\mathcal{I}_{it} &\sim \mathcal{N}\left(\mathbb{E}_{it}\left[\tilde{a}_{it}\right], \tilde{\mathbb{V}}\right)
\end{aligned}
$$

where $\mathbb{E}_{it}\left[a_{it}\right]$ and $\mathbb{V}$ denote the posterior mean and variance of a_{it} in case 1, respectively, and similarly $\mathbb{E}_{it}\left[\tilde{a}_{it}\right]$ and $\tilde{\mathbb{V}}$ in case 2. Further, as we will show, the cross-sectional distribution of the posterior mean $\mathbb{E}_{it}\left[a_{it}\right]$ is also normal, centered around the true mean $\bar{a}$ with associated variance $\sigma_a^2 - \mathbb{V}$. Focusing on case 1 for a moment, the variance $\mathbb{V}$ indexes the severity of informational frictions in the economy and will turn out to be a sufficient statistic for misallocation and the associated productivity/output losses. It is straightforward to show that $\mathbb{V}$ is closely related to commonly used measures of allocative efficiency. For example, it maps exactly into the dispersion of the marginal revenue product of capital (in logs), measured along the lines of Hsieh and Klenow (2009), i.e., $\sigma_{mrpk}^2 = \mathbb{V}$. Similarly, it has a negative effect on the covariance between fundamentals and firm activity as examined, for example, in Bartelsman et al. (2013) and Olley and Pakes (1996), i.e., the covariance between a_{it} and k_{it} satisfies $\sigma_{ak} = \frac{\sigma_a^2 - \mathbb{V}}{1-\alpha}$. Thus, our measure of informational frictions is easily related to measures of misallocation studied in the literature. An analogous correspondence holds for case 2.

Aggregation. We now turn to the aggregate economy, and in particular, measures of total factor productivity (TFP) and output. Given our focus on misallocation, we abstract from aggregate risk and restrict our attention to the economy's stationary equilibrium, in which all aggregate variables remain constant through time. Relegating the rather lengthy details to the

Appendix, we use (6) and (9) along with the fact that $Y = \int P_{it} Y_{it} di$ as well as standard properties of the log-normal distribution to derive the following simple representation for aggregate output, where the reader should recall that lower-case denotes natural logs:

$$y = a + \widehat{\alpha}_1 k + \widehat{\alpha}_2 n \tag{10}$$

Aggregate TFP, denoted a, is endogenous and is given by

$$\text{Case 1:} \qquad a = a^* - \frac{1}{2} \left(\frac{\theta}{\theta - 1} \right) \frac{\alpha}{1 - \alpha} \mathbb{V} \tag{11}$$

$$\text{Case 2:} \qquad a = a^* - \frac{1}{2} \left(\frac{\theta}{\theta - 1} \right) \frac{(1 - \alpha_2) \alpha_1}{1 - \alpha} \widetilde{\mathbb{V}} \tag{12}$$

where

$$a^* = \frac{\theta}{\theta - 1} \overline{a} + \frac{1}{2} \left(\frac{\theta}{\theta - 1} \right) \frac{\sigma_a^2}{1 - \alpha}$$

is aggregate TFP under full information, which is identical in the two cases. These expressions are at the heart of our mechanism and reveal a sharp connection between the micro-level uncertainty summarized by $\mathbb{V}$ (or $\widetilde{\mathbb{V}}$) and aggregate productivity: in both cases, aggregate productivity monotonically decreases in uncertainty, with the magnitude of the effect depending in a similar fashion on the curvature parameters α and θ (the common component across the cases is $\frac{1}{2} \left(\frac{\theta}{\theta - 1} \right) \frac{1}{1 - \alpha}$). The higher is α or θ, that is, the closer to constant returns/perfect competition, the more severe are the losses from misallocated resources. In case 1, it is straightforward to see that only these aggregate curvature parameters play a role. In contrast, in case 2, the relative shares of capital and labor matter, captured by the term $(1 - \alpha_2) \alpha_1$. The greater is labor's share α_2, the greater the ability of firms to mitigate the effects of capital misallocation due to the informational friction by reallocating labor in a compensating fashion. Any such flexibility is absent in case 1, in which both inputs are subject to the same friction. Thus, for a given level of uncertainty, case 1 will lead to more severe losses in aggregate TFP from misallocated resources than will case 2.

Holding the aggregate factor stocks fixed, the effect of informational frictions on aggregate productivity a is also the effect on aggregate output y. However, the aggregate capital stock in the steady state is not invariant to the severity of the friction: informational frictions and the resulting misallocation reduce incentives for capital accumulation and so the steady state stock of capital. This is a well-known effect in this class of models (i.e., consider the effect of a change in TFP on steady state output in a standard neoclassical model with fixed labor supply). Incorporating this additional effect of uncertainty amplifies the impact of allocative

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inefficiencies on output:

$$\frac{dy}{d\mathbb{V}} = \frac{da}{d\mathbb{V}}\left(\frac{1}{1-\widehat{\alpha}_1}\right) \tag{13}$$

$$\frac{dy}{d\widetilde{\mathbb{V}}} = \frac{da}{d\widetilde{\mathbb{V}}}\left(\frac{1}{1-\widehat{\alpha}_1}\right) \tag{14}$$

2.2 Information

We have shown that $\mathbb{V}$ (or in case 2, by $\widetilde{\mathbb{V}}$), i.e., the variance of the firm's posterior beliefs, is a sufficient statistic for the impact of informational frictions on resource misallocation and the resulting consequences for aggregate outcomes. We now make explicit the information structure in the economy, that is, the elements of the information set $\mathcal{I}_{it}$, which in turn will allow us to characterize $\mathbb{V}$ in terms of the primitives of the economy - specifically, the variances of fundamentals and signal errors.

The firm's information set $\mathcal{I}_{it}$ is composed of three elements. The first is the entire history of its fundamental shock realizations and stock prices, i.e., $\{a_{it-s}\}_{s=1}^{\infty}$. Second, the firm also observes a noisy private signal of its contemporaneous fundamental

$$s_{it} = a_{it} + e_{it}, \qquad e_{it} \sim \mathcal{N}\left(0, \sigma_e^2\right)$$

where e_{it} is an i.i.d. mean-zero and normally distributed noise term. The third and last element of the information available to the firm is the price of its own stock, P_{it}. The final piece of our theory then is to outline how the stock price is determined and to characterize its informational content.

The stock market. Our formulation of the stock market and its informational properties follows the noisy rational expectations paradigm in the spirit of Grossman and Stiglitz (1980). For our specific model structure, we draw heavily from recent work by Albagli et al. (2011a) and Albagli et al. (2011b). For each firm i, there is a unit measure of outstanding stock or equity, representing a claim on the firm's profits. These claims are traded by two groups of agents - imperfectly informed investors and noise traders. [11]

There is a unit measure of risk-neutral investors for each stock. Every period, each investor decides whether or not to purchase up to a single unit of equity in firm i at the current market

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[11]One interpretation is that these are intermediaries investing on behalf of a representative household. Some of them are rational, optimizing investors while others trade in a random fashion. The household cannot distinguish between these two types and hence, they co-exist. The exact rationale for the presence of noise traders is not crucial for our purposes - their role in our setup is solely to prevent perfectly informative prices.

price P_{it}. The market is also populated by noise traders who purchase a random quantity $\Phi\left(z_{it}\right)$ of stock i each period, where $z_{it} \sim \mathcal{N}\left(0, \sigma_z^2\right)$ is iid and Φ denotes the standard normal CDF.

Like firms, investors also observe the entire history of fundamental realizations, and in particular, know a_{it-1} at time t. They also see the current stock price P_{it}. Finally, each investor j is endowed with a noisy private signal about the firm's contemporaneous fundamental a_{it}:

$$s_{ijt} = a_{it} + v_{ijt}, \qquad v_{ijt} \sim \mathcal{N}\left(0, \sigma_v^2\right)$$

The total demand of investors for stock i is then given by

$$D\left(a_{it-1}, a_{it}, P_{it}\right) = \int d\left(a_{it-1}, s_{ijt}, P_{it}\right) dF\left(s_{ijt}|a_{it}\right)$$

where $d\left(a_{it-1}, s_{ijt}, P_{it}\right) \in [0,1]$ is the demand of investor j and F is the conditional distribution of investors' private signals. The expected payoff to investor j from purchasing the stock is given by

$$\mathbb{E}_{ijt}\left[\Pi_{it}\right] = \int \left[\pi\left(a_{it-1}, a_{it}, P_{it}\right) + \beta\bar{P}\left(a_{it}\right)\right] dH\left(a_{it}|a_{it-1}, s_{ijt}, P_{it}\right)$$

The term $\pi\left(a_{it-1}, a_{it}, P_{it}\right)$ denotes the expected current profit of the firm as a function of history[12], the current realization of the fundamental a_{it} and the current stock price P_{it}. This is a function of the current price P_{it} because it enters the firm's information set and through that, influences firm decisions[13]. The distribution $H\left(a_{it}|a_{it-1}, s_{ijt}, P_{it}\right)$ is the investor j's posterior over a_{it} and $\bar{P}\left(a_{it}\right)$ is the expected price in period $t+1$, conditional on the current fundamental a_{it}. Formally,

$$\bar{P}\left(a_{it}\right) = \int \mathcal{P}\left(a_{it}, a_{it+1}, z_{it+1}\right) dG\left(a_{it+1}, z_{it+1}|a_{it}\right)$$

is obtained by integrating the price function $\mathcal{P}\left(\cdot\right)$ over $\left(a_{it+1}, z_{it+1}\right)$, conditional on a_{it} (the distribution is denoted G). Clearly, optimality implies:

$$d\left(a_{it-1}, s_{ijt}, P_{it}\right) = \begin{cases} 1 & \text{if } \mathbb{E}_{ijt}\left[\Pi_{it}\right] > P_{it} \\ \in [0,1] & \text{if } \mathbb{E}_{ijt}\left[\Pi_{it}\right] = P_{it} \\ 0 & \text{if } \mathbb{E}_{ijt}\left[\Pi_{it}\right] < P_{it} \end{cases}$$

that is, an investor purchases the maximum quantity allowed (1 share) when the expected payoff (conditional on her information) strictly exceeds the price, does not purchase any shares when the expected payoff is (strictly) less than the price, and is indifferent when the two are equal.

---

[12]Given the assumption of an AR(1) structure for fundamentals and an iid process for z_{it}, the most recent realization, a_{it-1}, is a sufficient statistic for historical information.

[13]The Appendix explicitly characterizes $\pi\left(\cdot\right)$ in terms of the firm's problem studied in the previous subsection.

We focus on threshold equilibria, where (a) trading decisions of investors are characterized by a threshold rule: there is a signal $\widehat{s}_{it}$ such that only investors observing signals higher than $\widehat{s}_{it}$ choose to buy a share and (b) the market price P_{it} is an invertible function of $\widehat{s}_{it}$.[14]

Aggregating the demand decisions of all investors, market clearing implies

$$1 - \Phi\left(\frac{\widehat{s}_{it} - a_{it}}{\sigma_v}\right) + \Phi\left(z_{it}\right) = 1$$

which leads to a simple characterization of the threshold signal

$$\widehat{s}_{it} = a_{it} + \sigma_v z_{it} \tag{15}$$

In a threshold equilibrium, this defines a monotonic relationship between P_{it} and $\widehat{s}_{it}$, which implies that observing the stock price is informationally equivalent to observing $\widehat{s}_{it}$: in other words, the stock price serves as an additional noisy signal of firm fundamentals as defined in (15). The precision of this signal is $\frac{1}{\sigma_v^2 \sigma_z^2}$, i.e., it is decreasing in both the variance of the noise in investors' private signals and the size of the noise trader shock.

This simple expression for price informativeness in (15) is the key payoff of the structure we have imposed on stock market trading: we now have a complete characterization of the firm's information set and hence the posterior variance $\mathbb{V}$, even without an explicit solution for the price function.[15] Formally, the firm's information set is then defined by $\mathcal{I}_{it} = (a_{it-1}, s_{it}, \widehat{s}_{it})$, where a_{it-1} is the relevant history, s_{it} the firm's private signal, and $\widehat{s}_{it}$ the market signal defined in (15), which in turn yields a simple expression for $\mathbb{V}$:

$$\mathbb{V} = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2 \sigma_z^2}\right)^{-1}$$

The firm's posterior variance is thus increasing in the noisiness of the firm's private signal, σ_e^2, and of stock market prices, $\sigma_v^2 \sigma_z^2$. In the absence of any learning, $\mathbb{V} = \sigma_\mu^2$, that is, all fundamental uncertainty remains unresolved at the time of the firm's input choice. At the other extreme, under perfect information, $\mathbb{V} = 0$.

Finally, note that the marginal investor, i.e., the investor satisfying $s_{ijt} = \widehat{s}_{it}$, must be exactly indifferent between buying and not buying. It follows then that the price P_{it} must

---

[14]Albagli et al. (2011a) provide sufficient conditions for existence and uniqueness in a similar environment, but their conditions cannot be directly applied to our setup. Therefore, we have to resort to numerically verifying the optimality of the threshold rule.

[15]It is straightforward to show that the conditional and cross-sectional distributions are log-normal under this information set, exactly as conjectured.

equal her expected payoff from holding the stock:

$$
\begin{aligned}
P_{it} &= \int \left[\pi \left(a_{it-1}, a_{it}, P_{it} \right) + \beta \bar{P} \left(a_{it} \right) \right] dH \left(a_{it} | a_{it-1}, \widehat{s}_{it}, P_{it} \right) \\
&= \int \left[\pi \left(a_{it-1}, a_{it}, \widehat{s}_{it} \right) + \beta \bar{P} \left(a_{it} \right) \right] dH \left(a_{it} | a_{it-1}, \widehat{s}_{it}, \widehat{s}_{it} \right)
\end{aligned}
$$

where, with a slight abuse of notation, we replace P_{it} with its informational equivalent $\widehat{s}_{it}$ in the arguments of $H(\cdot)$. Rewriting this equation in recursive form yields a fixed-point characterization of the price function:

$$
\begin{aligned}
\mathcal{P} \left(a_{-1}, a, z \right) &= \int \pi \left(a_{-1}, a, a + \sigma_v z \right) dH \left(a | a_{-1}, a + \sigma_v z, a + \sigma_v z \right) \\
&\quad + \beta \int \left[\int \mathcal{P} \left(a, a', z' \right) dG \left(a', z' | a \right) \right] dH \left(a | a_{-1}, a + \sigma_v z, a + \sigma_v z \right) \quad (16)
\end{aligned}
$$

In our numerical analysis, we solve this functional equation numerically using a standard iterative procedure and then verify the invertibility and threshold properties of the equilibrium.

3 Identifying Informational Frictions

The main hurdle in quantifying the effects of imperfect information is imposing discipline on the information structure in the economy, given that we do not directly observe signals at the firm level. We overcome this difficulty with a novel empirical strategy that combines moments from firm-level production and stock market data to pin down the informational parameters of our model. In this section, we develop the intuition for that strategy by analyzing two special cases of our model - when firm level shocks are iid and when they follow a random walk. When we turn to our quantitative model below, we will verify numerically that the insights highlighted in this section extend to the general model used there.

We perform two sets of exercises: first, we prove that the informational parameters of our model are identified by three key moments of the data - the correlations of stock returns with both fundamentals and investment, and the volatility of returns. In both cases, there is a simple mapping between these moments and the informational parameters, yielding a good deal of intuition as to why these particular moments are informative about the information structure. Additionally, we highlight a tight connection between our identification approach and the reduced-form regression-based strategies that have been used extensively in previous studies on firm learning from prices. Our structural model then allows enables to use these relationships to quantify the extent of firm and market uncertainty. In a second set of exercises, we further exploit the tractability of these settings to demonstrate the validity of our approach under two

variants of the baseline framework: first, we introduce distortions other than informational frictions - both correlated and uncorrelated with firm fundamentals; second, we allow for a more general correlation structure between firm and market information.

3.1 Identification

Transitory shocks. First, we consider the case where shocks to fundamentals are i.i.d., i.e., $\rho = 0$ in equation (1). In this case, a log-linear approximation of the price (around the deterministic case) leads to[16]

$$p_{it} \equiv \log P_{it} \approx \xi \tilde{\mathbb{E}} \left[a_{it} \right] + \text{Constant}$$

where $\xi = \frac{1-\beta}{1-\alpha}$ and $\tilde{\mathbb{E}} \left[a_{it} \right]$ is the expectation of a_{it}, conditional on the marginal investor's information set. It is then straightforward to derive[17]

$$\tilde{\mathbb{E}} \left[a_{it} \right] = \tilde{\mathbb{E}} \left[u_{it} \right] = \psi \left(u_{it} + \sigma_v z_{it} \right)$$

where

$$\psi = \frac{\frac{1}{\sigma_v^2} + \frac{1}{\sigma_v^2 \sigma_z^2}}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_v^2} + \frac{1}{\sigma_v^2 \sigma_z^2}}$$

Similarly, capital is a log-linear function of the firm's expectation of the current innovation

$$k_{it} = \frac{\mathbb{E} \left[u_{it} \right]}{1 - \alpha} + \text{Constant} \tag{17}$$

which is a precision-weighted average of its private signal and the information in prices:

$$\mathbb{E} \left[u_{it} \right] = \phi_1 \left(u_{it} + e_{it} \right) + \phi_2 \left(u_{it} + \sigma_v z_{it} \right)$$

where

$$\phi_1 = \frac{\mathbb{V}}{\sigma_e^2}, \qquad \phi_2 = \frac{\mathbb{V}}{\sigma_v^2 \sigma_z^2}$$

In the Appendix, we derive the following expressions for the two correlations of interest, that between returns and changes in fundamentals, denoted ρ_{pa}, and between returns and

---

[16]See the Appendix for derivations. From here on, in a slight abuse of notation, we use $\mathbb{V}$ to denote the uncertainty in both case 1 and case 2, where it should be understood that $\mathbb{V}$ in case 2 corresponds to $\tilde{\mathbb{V}}$ in the theory. We similarly use a_{it} to denote the fundamental in both cases and α the relevant curvature parameter.

[17]Note that both of the signals in the marginal investor's information set are equal to $u_{it} + \sigma_v z_{it}$.

investment, denoted ρ_{pk}:

$$\rho_{pa} \equiv \text{Corr}\left(p_{it} - p_{it-1}, a_{it} - a_{it-1}\right) = \frac{1}{\sqrt{1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}}} \tag{18}$$

$$\rho_{pk} \equiv \text{Corr}\left(p_{it} - p_{it-1}, k_{it} - k_{it-1}\right) = \frac{1}{\sqrt{\left(1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}\right)\left(1 - \frac{\mathbb{V}}{\sigma_\mu^2}\right)}} \tag{19}$$

Equation (18) shows that the higher is $\frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}$, the noise-to-signal ratio in prices, the lower is the correlation of returns with fundamentals. Equation (19) then implies that for a given level of noise in prices, ρ_{pk} is increasing in the firm's posterior variance $\frac{\mathbb{V}}{\sigma_u^2}$ - investment choices covary more strongly with the signal when firms are more uncertain. Note that we work with $\frac{\mathbb{V}}{\sigma_u^2}$ for convenience - combined with σ_u^2 (which can be directly obtained from firm level revenues and capital) and $\sigma_v^2 \sigma_z^2$ (from the expression for ρ_{pa}), this bears a one-to-one relationship with σ_e^2, the noise in the firm's private signal.

Notice that a high ρ_{pk} does not by itself indicate a high level of uncertainty. Firm choices can be highly correlated with returns either because they both track fundamentals very closely or because firms are uncertain.[18] Observing ρ_{pa} allows us to isolate the effect of the latter. To see this more clearly, substitute for ρ_{pa} in (19) to derive,

$$\frac{\rho_{pa}}{\rho_{pk}} = \sqrt{1 - \frac{\mathbb{V}}{\sigma_\mu^2}} \qquad \Rightarrow \qquad \frac{\mathbb{V}}{\sigma_\mu^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2 \tag{20}$$

Thus, the severity of informational frictions is pinned down directly by the relative correlation $\frac{\rho_{pa}}{\rho_{pk}}$. Full information implies a value of 1 for this ratio. This sharp link between the relationship between the correlations and the severity of informational frictions guides our empirical approach in our quantitative analysis below. The more general version of the model there will preclude an analytical mapping between these correlations and $\mathbb{V}$, but numerical simulations reveal a very similar positive relationship. Given our assumptions on production and demand, we can back out the fundamental a_{it} from data on revenues and capital, enabling us to measure the two correlations in the data. Combining them yields $\mathbb{V}$, the sufficient statistic for firm-level uncertainty.[19]

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[18]For example, suppose we make prices more informative, i.e., decrease $\sigma_v^2 \sigma_z^2$. Then ρ_{pk} rises even though uncertainty decreases.

[19]For completeness, we show in the Appendix that the volatility of returns and their correlation with fundamentals can be used in a final step to separately identify σ_v^2 and σ_z^2.

Permanent shocks. Adding persistence in fundamental shocks to this version of the model is straightforward. As an additional special case, we focus on that of permanent shocks, i.e., $\rho = 1$, and show that the main insights from the i.i.d. case extend to this scenario as well.[20] We first derive the expression for the ex-dividend price of the stock, which takes a similar form to the i.i.d. case:[21]

$$p_{it} = \frac{1}{1-\alpha}\tilde{\mathbb{E}}\left[a_{it}\right] + \text{Constant}$$

We can then derive the following expressions, which again demonstrate a sharp mapping between the three moments - ρ_{pk}, ρ_{pa} and σ_p^2 - and the informational parameters:

$$
\begin{aligned}
\frac{\mathbb{V}}{\sigma_\mu^2} &= \frac{\rho_{pk} - \rho_{pa}}{\eta} \\
\frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2} &= \frac{(1-\eta^2)}{2\rho_{pa}^2} + \frac{\eta}{\rho_{pa}} - 1 \\
\frac{\sigma_z^2 + 1}{\sigma_z^2 + 1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}} &= \frac{1}{\eta}
\end{aligned}
\tag{21}
$$

where $\eta = \frac{1}{1-\alpha}\frac{\sigma_\mu}{\sigma_p}$. In contrast to the i.i.d. case, all three moments are now necessary to infer the extent of uncertainty, yet the intuition for identification is quite similar: as before, all else equal, a higher relative correlation (ρ_{pk} vs ρ_{pa}) implies greater uncertainty, a lower ρ_{pa} higher levels of noise in prices, and higher return volatility larger noise trader shocks.

Relationship to investment-Q regressions. In the cases of our model analyzed here, we can write a reduced-form representation of investment as a log-linear function function of fundamentals, signal errors and prices:

$$\Delta k_{it} = \lambda_1 \left(\Delta\mu_{it} + \Delta e_{it}\right) + \lambda_2 \Delta p_{it} \tag{22}$$

where Δ's denote changes. This is in the spirit of specifications widely used in the empirical corporate finance literature to examine the role of learning from stock markets. For example, Morck et al. (1990) regress investment growth on stock returns, sales growth and other controls.[22] Our key departure from these studies is not primarily in the intuition underlying our approach, but rather in our use of a structural model, which enables us to interpret the coefficients from these reduced-form regressions in terms of the informational primitives of the economy. For example, consider the coefficient on stock returns, λ_2: in the case of iid shocks,

---

[20]With permanent shocks and no exit, there is no stationary distribution. Since our goal here is primarily to provide intuition for our empirical strategy, we ignore this complication and interpret this as a limiting case.

[21]All derivations are in the Appendix.

[22]Chen et al. (2007) use Q and cash flow growth as their independent variables.

this is given by

$$\lambda_2 = \frac{1}{(1-\beta)\,\psi} \left(\frac{\mathbb{V}}{\sigma_v^2 \sigma_z^2} \right) \qquad (23)$$

Equation (23) reveals the same intuition as in (19): λ_2 can be high either because firms are subject to a good deal of uncertainty, i.e., $\mathbb{V}$ is large, and so rely heavily on information from markets, or because markets are highly informative, i.e., $\sigma_v^2 \sigma_z^2$ is low.

Of course, the regression implied by (22) consistently identifies the coefficients only in the case of orthogonality between the error e_{it} and the regressors a_{it} and p_{it}. If the noise terms in the signals of firms and investors are correlated, this orthogonality assumption is violated, leading to endogeneity biases in the regression estimates. A similar issue arises if the choice of capital is affected by additional factors that are correlated with fundamentals.[23] As we demonstrate next, our approach is robust to these concerns.

3.2   Robustness

Thus far, we have assumed that informational frictions are the only impediment to marginal product equalization. This is clearly a rather extreme assumption - in reality, investment decisions are likely affected, or "distorted," by a number of other factors. These may originate, for example, from technological limitations (e.g., adjustment costs), contracting frictions (e.g., financial constraints), or distortionary government policies. All of these can lead to dispersion in marginal products. Quantifying their contribution - whether individually or jointly - to observed cross-country differences is certainly the overall objective of the growing literature on misallocation, but one that is well beyond the scope of this paper. Our more limited goal here is to isolate and quantify the degree of firm-level uncertainty and its particular role in generating misallocation. From this perspective, our interest in these alternative sources of misallocation is to assess the extent to which our measurement strategy remains valid in their presence.

To this end, we introduce other distortions drawn from a flexible, albeit stylized, class, which allows us to incorporate some essential features of the factors listed above without sacrificing analytical tractability. Specifically, we directly modify the firm's optimality condition (17):

$$k_{it} = \frac{(1+\gamma)\,\mathbb{E}\,[u_{it}] + \varepsilon_{it}}{1-\alpha} + \text{Constant}$$

---

[23]Consider, for example, the effects of introducing a correlated distortion $\tau_{it} = \gamma u_{it}$ into the firm's capital choice in (17) so that $k_{it} = \frac{(1+\gamma)\mathbb{E}[\mu_{it}]}{1-\alpha} + \text{Constant}$. The coefficient λ_2 is given by $\frac{1+\gamma}{(1-\beta)\psi} \frac{\mathbb{V}}{\sigma_v^2 \sigma_z^2}$. In other words, inferring $\mathbb{V}$ from λ_2 requires knowledge of (or at the very least, an adjustment for) γ. Note that uncorrelated distortions will not have any effect on λ_2.

This is equivalent to introducing a 'distortion' τ_{it}, comprising two terms:

$$\tau_{it} = \gamma u_{it} + \varepsilon_{it} \qquad \varepsilon_{it} \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$$

The parameter γ indexes the extent to which τ_{it} covaries with the fundamental u_{it}. For example, if $\gamma < 0$, τ is to be interpreted as a factor that discourages (encourages) investment by firms with stronger (weaker) fundamentals. The second term, ε_{it}, represents factors that are uncorrelated with firm-specific fundamentals. The specification above assumes that they are idiosyncratic (though it is straightforward to add a common component) and perfectly observed by the firm (though not by the econometrician). The two parameters γ and σ_ε^2 together pin down the amplitude (measured by the standard deviation) of this distortion and its correlation with firm-level fundamentals. To understand how the presence of τ_{it} affect our inference strategy, we use equation (20) to estimate $\mathbb{V}$ and compare it to the true measure of uncertainty.

Correlated distortions. Consider first the case with only correlated distortions, i.e., $\sigma_\varepsilon^2 = 0, \gamma \neq 0$. In this case, we can show that equation (20) is unaffected, that is, the ratio of the two correlations still uncovers the true $\mathbb{V}$. This is despite the fact that τ_{it} does have an effect on more direct measures of misallocation. For example, the dispersion in the marginal revenue product of capital (MRPK) is given by

$$\sigma_{mrpk}^2 = \gamma^2 \left(\sigma_\mu^2 - \mathbb{V}\right) + \mathbb{V}$$

which is increasing in the absolute value of γ. Similarly, the covariance between fundamentals and investment is also affected by γ:

$$\sigma_{ak} = \left(\frac{1+\gamma}{1-\alpha}\right)\left(\sigma_\mu^2 - \mathbb{V}\right)$$

Thus, a strategy which targets these measures directly (i.e., chooses $\mathbb{V}$ to match these moments) would lead to a biased estimate of the severity of informational frictions. In particular, inferring $\mathbb{V}$ from σ_{mrpk}^2 without adjusting for γ overstates the extent of uncertainty, while using σ_{ak} can lead to an upward or downward bias, depending on the sign of γ. Similar concerns also apply to inferences made directly from the cross-sectional variance of investment or revenue - these moments also confound the effects of uncertainty with other factors, and so using them without taking a stand on the nature of these other distortions could be problematic. Using the relative correlation as outlined above, on the other hand, continues to identify the true level of uncertainty. In this sense, our identification strategy is robust to the presence of such

correlated distortions.[24]

Uncorrelated distortions. Next, we turn to the case where distortions are uncorrelated with fundamentals, i.e., $\sigma_\varepsilon^2 \neq 0, \gamma = 0$. In this case, we can show

$$\frac{\mathbb{V}}{\sigma_\mu^2} = 1 - \left(\frac{\rho_{pa}}{\rho_{pk}}\right)^2 + \frac{\sigma_\varepsilon^2}{\sigma_\mu^2}$$

The first two terms on the right hand side correspond to our measure of $\mathbb{V}$ using (20) and the last additional term is strictly positive. Thus, in this case, we underestimate the true extent of uncertainty. In other words, the presence of factors uncorrelated with fundamentals would tend to make our estimate of $\mathbb{V}$ more conservative, in that we would infer less uncertainty than is truly the case. This again is despite the fact that these uncorrelated distortions do indeed exacerbate misallocation, as the following expression shows

$$\sigma_{mrpk}^2 = \mathbb{V} + \sigma_\varepsilon^2$$

and so, again, a strategy of choosing $\mathbb{V}$ to directly match this moment would lead to an overstatement of the extent of informational frictions.

Note that in both instances (correlated and uncorrelated), if the true $\mathbb{V}$ were 0 (i.e., firms were perfectly informed), using (20) would not lead us to a positive estimate. In other words, we will not find evidence of informational frictions in a full information economy, even one with marginal product dispersion.

Correlated signals. Another rather stark assumption in our baseline model is independence of the various noise terms in the signals. Correlation in these errors (specifically, e_{it}, v_{ijt}, z_{it}), that is, some commonality in signals even conditional on a_{it}, could be another source of co-movement between firm choices and prices and so may affect the moments used above (most directly ρ_{pk}), leading to incorrect conclusions about uncertainty. However, this turns out not to be the case - even with correlated errors, equation (20) leads us to the true $\mathbb{V}$. Indeed, it can be shown that (20) holds for an arbitrary correlation structure between stock market and firm information.[25] Note that this only applies to our estimate of $\mathbb{V}$ - our conclusions about the individual variances (σ_e^2, σ_v^2 and σ_z^2) are sensitive to assumptions about the correlation structure.[26]

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[24]Note, however, that the effect of uncertainty on aggregate outcomes does depend on γ.

[25]See the Appendix for the derivation.

[26]Numerical simulations suggest a similar robustness in the general model, i.e., our conclusions about the extent of uncertainty do not change significantly for alternative correlation assumptions. Specifically, we solved a version of our model in which each investor sees a noisy signal of the firm's private signal. Note that this rules

4 Quantitative Analysis

Our analytic results in the previous section demonstrated a tight relationship between the three moments $\left(\rho_{pk}, \rho_{pa}, \sigma_p^2\right)$ on which we focus and the three informational parameters in our economy $(\sigma_e^2, \sigma_v^2, \sigma_z^2)$, and through them, the degree of micro-level uncertainty. With this intuition in hand, we now return to our general model and, following an identification approach quite similar in spirit, proceed to quantitatively to infer these parameters using data from 3 countries - the US, China, and India. We use the results to quantify the extent of informational frictions, the degree of resulting misallocation, and the impact on aggregate outcomes across these countries. Our analysis also sheds light on the role of learning from various channels, and perhaps of particular interest, on the importance of financial markets in improving allocative efficiency by delivering useful information to firms. We make use of the multi-country aspect of our analysis to perform a number of counterfactual experiments assessing the sources of cross-country variation in uncertainty and their respective effects on allocative efficiency.

4.1 Parameterization

We begin by assigning values to the more standard parameters in our model - specifically, those governing the preference and production structure of the economy. Throughout our analysis, we will hold these constant across countries; cross-country differences will come only from the parameters governing the stochastic processes on firm fundamentals and learning. Although a simplification, we feel that this is a natural starting point and allows us to focus on the role of imperfect information in leading to differences in aggregate outcomes across countries.

An important issue here is the choice of period length in the model. The focus of this paper - investment decisions - and our modeling choices push us towards larger period lengths. There is significant empirical evidence of long lags in planning and implementing investment projects, with estimates of the mean duration time between the planning stage and project completion of between 2 and 3 years.[27] It seems reasonable then to assume that firms are required to forecast fundamentals over a relatively long horizon (2-3 years) when making large investment decisions and have only limited flexibility to adjust capital choices ex-post in response to additional information. One approach would be to explicitly model these lags as well as other features (such as adjustment costs/irreversibilities) which are likely relevant for investment decisions

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out learning from markets by construction. This had only a small impact on $\mathbb{V}$.

[27]The classic reference here is Mayer (1960), who uses survey data on new industrial plants and additions to existing plants to find a mean gestation lag between the drawing of plans and the completion of construction of about 22 months. More recently, Koeva (2000) finds the average length of time-to-build lags to be about 2 years in most industries, defined as the period between the announcement of new construction and the ensuing date of completion. Given that this excludes the planning period prior to the announcement date, the total gestation lag is likely somewhat longer.

over short horizons. Largely for tractability, we take a different route and model the capital choice as a static one, but interpret a period in the model as spanning a relatively long horizon, specifically 3 years. This makes the omission of explicit lags/irreversibilities, etc. somewhat less of a concern.[28] More importantly, this approach allows us to preserve the analytical tractability on the production side, particularly the simple expressions linking uncertainty to aggregate outcomes.

In line with our choice of period length, we set the discount rate β equal to 0.9. We assume constant returns to scale in production and set the production parameters $\widehat{\alpha}_1$ and $\widehat{\alpha}_2$ to standard values of 0.33 and 0.67, respectively. Given our CRS assumption, firm scale is then limited only by the curvature in demand, captured by the elasticity of substitution θ. This will be a key parameter in influencing the quantitative impact of informational frictions. The literature contains a wide range of estimates for this parameter. We set our baseline estimate at 6, roughly in the middle of the commonly used range.[29] Our choice of θ translates into an $\tilde{\alpha} = 0.62$ in case 2 and $\alpha = 0.83$ in case 1.

Next, we turn to the country-specific parameters. We begin with those governing the process on firm fundamentals a_{it}: the persistence ρ and variance of the innovations σ_μ^2. In both of our cases, we can directly construct the fundamental for each firm (up to an additive constant) as $rev_{it} - \alpha k_{it}$, where rev_{it} denotes the log of revenues, and α depends on whether we are in case 1 or case 2.[30] We then estimate the parameters of the fundamental process by performing the autoregression implied by (1), additionally including a time fixed-effect in order to isolate the idiosyncratic component of the innovations. The resulting coefficient delivers an estimate for ρ and the variance of the residuals for σ_μ^2.[31]

Finally, it remains to pin down the three informational parameters of our model, i.e., the variances of the error terms in firm and investor signals σ_e^2 and σ_v^2, and the variance of the noise trader shock σ_z^2. We follow almost directly the strategy outlined above, i.e., target second moments in the growth rates of firm-level investment, fundamentals, and prices.[32] The precise moments we use are the cross-sectional correlations of stock returns with investment growth (denoted ρ_{pi}) and changes in fundamentals, as well the variance of returns (as above, denoted ρ_{pa} and σ_p^2, respectively). Importantly, stock returns are lagged by one period, so that the

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[28]Morck et al. (1990) make a similar argument and perform their baseline empirical analysis using 3-year spans. They also point out that the explanatory power of investment growth regressions at shorter horizons (e.g., 1 year) are quite low.

[29]Additionally, we report our results, at least in case 2, for two other values of θ, namely 4 and 10.

[30]Substitute either (4) or (7) into (2).

[31]See the Appendix for details.

[32]We follow the literature examining the feedback effects of stock prices and use growth rates of investment (in the analytical cases studied earlier, we used investment, i.e. the growth rate of capital). By working with growth rates, we partly cleanse the data of firm fixed-effects, which are a significant component in cross-sectional differences in these variables. See Morck et al. (1990) for a more detailed discussion of these issues.

correlations reflect the comovement of investment and fundamentals with stock returns over the preceding period. This avoids feedback from investment and fundamentals to returns, the reverse of the relationship in which we are interested. Table 1 summarizes our empirical approach.

Table 1: Parameterization - Summary

Parameter	Description	Target/Value
Common		
	Time period	3 years
β	Discount rate	0.90
$\widehat{\alpha}_1$	Capital share	0.33
$\widehat{\alpha}_2$	Labor share	0.67
θ	Elasticity of substitution	6
Country-specific		
ρ	Persistence of fundamentals	} Estimates of (1):
σ_μ	Shocks to fundamentals	$a_{it} = (1 - \rho)\,\overline{a} + \rho a_{it-1} + \mu_{it}$
σ_e^2	Firm private info	} ρ_{pi}
σ_v^2	Investor private info	ρ_{pa}
σ_z^2	Noise trading	σ_p^2

We use a simulated method of moments (SMM) approach to assign values to the informational parameters. Formally, we search over the parameter vector $(\sigma_e^2, \sigma_v^2, \sigma_z^2)$ to find the combination that minimizes the (unweighted) sum of squared deviations of the model-implied moments from the target moments. Before reporting the parameter estimates, we provide a numerical analogue of the identification argument in the previous section. This more general version of the model does not yield analytical expressions for the moments of interest, but we show in Figure 1 that the relationships between moments and parameters highlighted above go through almost exactly. The first panel shows a positive relationship between $\mathbb{V}$ and the relative correlation, as suggested by equations (20) and (21).[33] Similarly, in the second panel, we see that higher levels of noise in prices are associated with lower ρ_{pa}, exactly as we saw in the analytical cases.[34] Finally, the bottom panel shows that, holding fixed the total noise $(\sigma_v^2 \sigma_z^2)$, increasing the size of the noise trader shock makes returns more volatile.

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[33]Here, we plot the difference in correlations, i.e. $\rho_{pi} - \rho_{pa}$, but using the ratio yields a very similar picture.

[34]Here, we hold V and the ratio $\frac{\sigma_v}{\sigma_z}$ fixed, so as to focus on the information content of prices.
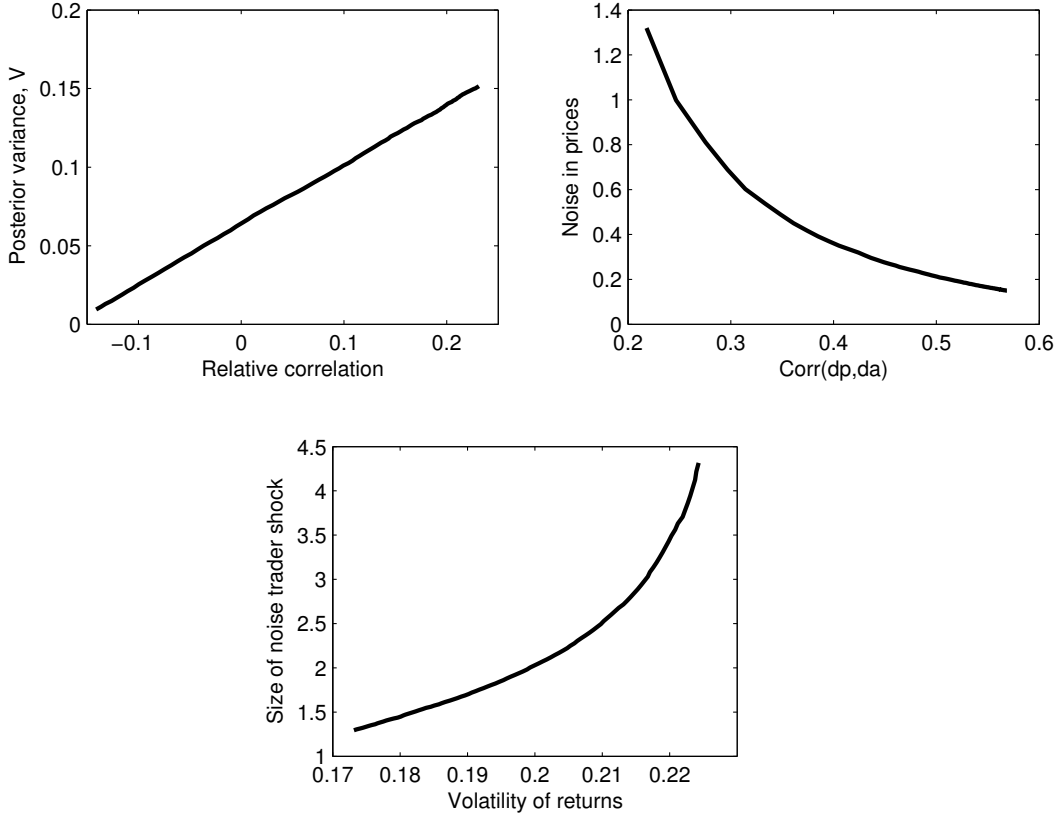
Figure 1: Identification - Quantitative Model

4.2 Data and Parameter Values

We construct the target moments using data on firm-level production variables and stock returns from Compustat North America for the US and Compustat Global for China and India. We focus on a single cross-section of firms in each country for the year 2012. This is the period with the largest number of observations, particularly so for China and India.[35] Note that our data requirements are quite stringent: due to our focus on 3 year horizons and our use of growth rates, we require at least 9 consecutive years of data for a firm in order to construct two 3 year periods and include it in our sample, with 2012 representing the final year of the second period.

We compute the firm's capital stock k_{it} in each period as the average of its property, plant and equipment (PP&E) over the relevant 3 years, and investment as the change in the capital stock relative to the preceding 3 year period. To construct our measure of the fundamental a_{it}, we compute sales/revenue analogously as the average over the 3 year period and calculate $a_{it} = rev_{it} - \alpha k_{it}$. First-differencing gives investment and changes in fundamentals between the

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[35]This is less of an issue for the US, and so as a robustness exercise, we recomputed the moments using a larger sample with more years. While there is some time variation, the results are quite similar to those from the single cross-section.

two periods. Stock returns are constructed as the change in the firm's stock price over each 3 year period, adjusted for splits and dividend distributions. In order to be comparable to the unlevered returns in our model, stock returns in the data need to be adjusted for financial leverage. To do so, we assume that claims to firm profits are sold to investors in the form of debt and equity in a constant proportion (within each country). Observed return variances must then be multiplied by a constant factor in order to make them comparable to returns in the model, where the factor depends on the ratio of debt to total assets (or alternatively, debt to equity). Computing the debt-asset ratio from our sample gives average values of about 0.30 in both the US and India, and about 0.16 in China, with corresponding adjustment factors of about 0.5 and 0.7, respectively, and all values reported below reflect this adjustment.[36] To isolate the firm-specific variation in our data series, we extract a time fixed-effect from each and utilize the residual as the component that is idiosyncratic to the firm. This is equivalent to demeaning each series from the unweighted average in each time period.[37] As mentioned above, the target moments are computed using returns lagged by one period.[38] This avoids the simultaneity problems associated with comparing price movements to contemporaneous investment/revenue numbers. We trim the 2% tails of each series in order to mitigate measurement error. The Appendix provides further details on how we build our sample and construct the variables.

We report the three target moments in the left hand panel of Table 2 both for case 2 in the upper panel and case 1 in the lower.[39] In both cases, the moments exhibit significant cross-country variation. The US and India show similar levels of return volatility and a similar relation between returns and investment growth, but quite different comovements with fundamentals - returns in the US are more highly correlated with future changes in fundamentals. China has a return variance that is almost half that of the other two countries, along with the lowest correlations of returns with investment and fundamentals. As made clear by the analytic results in Section 3, none of these moments is a sufficient statistic to identify the informational role of markets or infer the extent of micro-level uncertainty; rather, it is the joint pattern of the three moments that matters and our explicit modeling of both production and financial market activity is precisely what allows us to tease this out.

In the right hand panel of Table 2, we report the resulting parameter values. The parameters governing the process on fundamentals ρ and σ_μ are inferred from the regression implied by (1) as detailed above and the informational parameters from the target moments and SMM procedure

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[36]In brief, letting d denote the debt-asset ratio, the observed return variances must be multiplied by $(1 - d)^2$ to obtain the variance of unlevered returns. We describe the details of the calculations in the Appendix.

[37]An alternative is to use CAPM $\beta's$ to remove the aggregate component from individual stock returns. This approach yields very similar results.

[38]For example, ρ_{pi} is the correlation between 2006-09 returns and investment growth during 2009-12.

[39]ρ_{pa} changes across cases since α affects our estimates of a. The remaining two moments change almost imperceptibly due to trimming.

just described. As we would expect from the cross-country variation in the target moments, the parameter estimates also vary markedly across countries. The US has less volatile fundamental shocks, better private information at the firm level (lower σ_e), and generally lower levels of noise in the stock market (lower σ_v and σ_z). In the next section, we gauge the detrimental impact of the estimated frictions on productivity and output and in leading to differences in these aggregates across countries.

Table 2: Target Moments and Parameter Estimates

	Target moments			Parameters				
	ρ_{pi}	ρ_{pa}	σ_p^2	ρ	σ_μ	σ_e	σ_v	σ_z
Case 2								
US	0.23	0.18	0.23	0.92	0.45	0.39	0.37	3.50
China	0.16	0.06	0.14	0.78	0.51	0.67	0.74	4.24
India	0.25	0.08	0.23	0.93	0.53	1.04	0.69	4.36
Case 1								
US	0.24	0.10	0.23	0.88	0.46	0.63	0.65	3.16
China	0.15	0.02	0.14	0.75	0.53	0.74	1.18	3.14
India	0.26	0.00	0.22	0.88	0.55	1.39	1.69	4.14

4.3 Results

We report our baseline results in Table 3. The first three columns present the implied value for $\mathbb{V}$ based on the parameter estimates in Table 2, both in absolute terms and as a percentage of the underlying fundamental uncertainty, σ_μ^2, and of the total dispersion in the MRPK in our sample, σ_{mrpk}^2.[40] In the last two columns, we compute the implied losses in aggregate TFP and output relative to a full information benchmark. The top panel contains results for case 2, in which only capital is chosen under imperfect information, and the bottom panel the analogous results for case 1, in which both capital and labor are. Case 2 is the more conservative scenario (in the sense that it leads to lower TFP/output losses). We return to this issue in our discussion below and provide some suggestive evidence that reality likely falls in the middle.

Turning to the first two columns, there is substantial uncertainty in all cases: as a percent of the fundamental uncertainty σ_μ^2, the residual uncertainty ranges from a low of 41% in the US to a high of 77% in India in case 2, and similarly in case 1, although China and the US move much closer together on this score. In other words, firm learning eliminates from a high of about 60% of total uncertainty in the US to a low of about 20% in India in case 2, with a

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[40]σ_{mrpk}^2 is computed as the average of within-industry dispersion in each country.

Table 3: The Impact of Informational Frictions

	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_\mu^2}$	$\frac{\mathbb{V}}{\sigma_{mrpk}^2}$	$a^* - a$	$y^* - y$
Case 2 ($\alpha = 0.62$)					
US	0.08	0.41	0.22	0.04	0.05
China	0.16	0.63	0.34	0.07	0.10
India	0.22	0.77	0.48	0.10	0.14
Case 1 ($\alpha = 0.83$)					
US	0.13	0.63	0.35	0.40	0.60
China	0.18	0.65	0.39	0.55	0.83
India	0.26	0.86	0.56	0.77	1.15

generally similar pattern in case 1, although the estimated degree of learning falls in the US and India. As we would expect from the cross-country variation in the parameter estimates in Table 2, the level of uncertainty varies systematically across countries: US firms are the most informed and Indian firms the least, with Chinese firms falling in the middle.

Next, we ask, how much of total MRPK dispersion do informational frictions account for? The answer is a significant portion: as a percentage of the total σ_{mrpk}^2, $\mathbb{V}$ represents between about 20% and almost 60%, with the share generally lower in the US than in the two emerging markets. Later, we will decompose observed MRPK into a firm fixed effect and a transitory component and assess the role of informational frictions in explaining the latter. In a sense, this is a more meaningful comparison because informational frictions really cannot speak to fixed effects, which seem to an important component of observed MRPK dispersion.

Finally, the last two columns of Table 3 show that the substantial degree of residual uncertainty implies significant losses in productivity and output. Compared to the full-information benchmark, in case 2, steady state TFP is lower in the US by 4% (more precisely, 0.04 log-points[41]) and 10% in India (the corresponding numbers for output are 5% to 14%). Estimated losses are significantly higher in case 1, ranging from 40% to about 80 in productivity and 60% to over 1 in output. In both cases, the US exhibits the smallest losses, reflecting the fact that US firms exhibit the smallest degree of ex-post uncertainty, and India the largest, with China falling in the middle. The differential impact of informational frictions leads to significant differences between the US and the two emerging markets in productivity and output, ranging from 3% to almost 40% for the former (across the two cases) and from 5% to over 50% for the latter (subtract $a^* - a$ and $y^* - y$ for the US from the corresponding values for China and India). These are somewhat modest, however, in comparison to standard measures of cross-country differences in aggregate TFP and income per-capita, but very much in line

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[41] In what follows, we adopt the convention of referring to log points as percentages.

with other estimates based on particular frictions - for example, Midrigan and Xu (2013). In Section 4.5, we will use the structural parameters of our model to investigate in more detail the differences in informational frictions across countries; specifically, we perform a number of counterfactual experiments exploring the potential gains in the emerging markets from having access to a US-quality information structure.

Case 1 vs Case 2. Table 3 shows that under either scenario, informational frictions have a significant detrimental impact on aggregate performance, both when comparing the parameterized economies to their full information benchmarks and when comparing the consequences of these frictions in emerging markets to the US. The magnitude of the effects depend to a great extent on the nature of the firm's input decisions, that is, whether all inputs are chosen subject to the friction or only traditionally quasi-fixed inputs like capital.[42] The intuition for these differences comes directly from equations (11) and (12): in case 1, given the level of uncertainty $\mathbb{V}$, the aggregate consequences depend only on the aggregate degree of curvature α; in case 2, the relative shares of capital and labor matter, captured by the term $(1 - \alpha_2)\alpha_1$. The greater is labor's share α_2, the greater the ability of firms to mitigate the effects of capital misallocation due to the informational friction by reallocating labor in a compensating fashion. Any such flexibility is absent in case 1, in which both inputs are subject to the same friction. To dig a bit deeper into this issue, note from equations (7) and (6) that in case 2, the marginal revenue product of labor (MRPL) is equalized across firms so that there should be no dispersion, while in case 1, dispersion in the MRPL exactly equals that in the MRPK. This is a direct result of our assumptions that in the former, all information is revealed before the labor choice, while in the latter, labor is chosen under the same limited information as capital. This implies a tight link between the cases and the ratio of the dispersion in the MRPL to that in the MRPK ($\sigma^2_{mrpl}/\sigma^2_{mrpk} = 0$ in case 2 and $= 1$ in case 1), which suggests that the empirical counterpart of this ratio might be useful in assessing how close we are to the two cases. We compute this ratio for the US firms in our sample, using total employment as an admittedly rough measure of labor inputs.[43] The ratio is equal to 0.57, suggesting that reality is indeed in the middle of the two extremes we have analyzed.[44] In this sense, we view our results as providing bounds on the adverse consequences of imperfect information for aggregate performance. Certainly, an important direction for future work is a more in-depth analysis of the precise nature of the

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[42]It is straightforward to extend our methodology to labor moments. The main hurdle is availability of data on labor input, especially in India and China. Even for the US, the coverage and quality of data on labor variables is lower than for capital.

[43]σ^2_{mrpl} is computed analogously to σ^2_{mrpk}.

[44]Data on the wage bill is available only for a small set of the US firms in our dataset; however, using wage bill to measure labor input for this small set of firms gives a very similar ratio of 0.53. Reliable employment data is not available in the emerging markets.

firm's input choices.

Transitory MRPK dispersion. Table 3 shows that uncertainty can account for a significant portion of the total MRPK dispersion observed in the data, ranging from 20-60% across countries and cases. Note, however, that the dispersion induced by uncertainty is short-lived. In the data, on the other hand, there is some evidence of permanent deviations in MRPK - in other words, measured MRPK deviations seem to comprise both a firm-specific fixed effect (which we cannot speak to with our theory) and a transitory component. In a sense, a more appropriate gauge of the contribution of information frictions would be a comparison to this latter piece. To address this question, we separate the two components for the US firms in our sample.[45] To do so, we restrict the sample to firms with at least 15 years of data and extend the data as far back as 1985, so that we can construct at least 5 and as many as 10 3-year observations for each firm. We compute the MRPK for each firm in each period and regress the result on a firm fixed-effect. The residuals from this regression capture the purely transitory component of MRPK deviations. We then compute the variance of this object, i.e., the dispersion in the transitory component of MRPK deviations, and ask how much of this dispersion do informational frictions account for? This exercise reveals, first, that dispersion in the transitory component is substantial, representing about one-third of the total. Moreover, our estimated $\mathbb{V}$ for the US accounts for about 60% in case 2, and just about the entirety in case 1, again pointing to the empirical relevance of informational frictions.[46]

The effect of curvature. The impact of informational frictions is sensitive to the degree of curvature, captured here by the elasticity of substitution θ, set equal to 6 in our baseline computations. Table 4 reports results for case 2 under two alternative values: $\theta = 4$, which is on the low end of the commonly used range, and $\theta = 10$, which is on the high end. Changes in θ lead to significant changes in the effects of the friction. Both the estimates of $\mathbb{V}$ and their impact on TFP are lower with a smaller θ, with TFP losses ranging from 2% in the US to 6% in India and higher, but similarly ordered, output losses. In contrast, the higher value of θ shows a more severe aggregate impact, ranging from 8% in the US to 16% in India for TFP and from 11% to almost 25% for output.[47] The impact of the friction varies with θ for 2 reasons: first, as can be seen in (12), for a given $\mathbb{V}$ the aggregate consequences are larger for higher values of θ (i.e., higher α). Second, the estimated $\mathbb{V}$ itself increases with θ, due to the fact the ρ_{pa}

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[45]Sufficient data to perform this analysis are not available in the other two countries.

[46]We again compute the average of within-industry dispersion.

[47]The sensitivity of our results to θ is not particular to our framework, but rather is common when using this class of model to study the aggregate implications of misallocation. See, for example, Hsieh and Klenow (2009), who find that gains from marginal product equalization approximately double in China and India when moving from $\theta = 3$ to $\theta = 5$.

we measure from the data falls as θ increases but the other two moments (importantly, ρ_{pi}) are unchanged. Moving between cases or changing θ both influence the degree of curvature, the former through the information structure, the latter through the curvature in demand. However, the effects are distinct: for a given change in curvature, the effect on $\mathbb{V}$ is the same, but the aggregate implications are quite different.[48]

Table 4: The Impact of Informational Frictions - Alternative Values of θ

	$\theta = 4$					$\theta = 10$				
	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_\mu^2}$	$\frac{\mathbb{V}}{\sigma_{mrpk}^2}$	$a^* - a$	$y^* - y$	$\mathbb{V}$	$\frac{\mathbb{V}}{\sigma_\mu^2}$	$\frac{\mathbb{V}}{\sigma_{mrpk}^2}$	$a^* - a$	$y^* - y$
US	0.05	0.25	0.14	0.02	0.03	0.12	0.57	0.31	0.08	0.11
China	0.13	0.52	0.28	0.04	0.06	0.17	0.63	0.36	0.11	0.17
India	0.17	0.60	0.37	0.06	0.08	0.24	0.83	0.53	0.16	0.24

4.4 The Sources of Learning

Decomposing $\mathbb{V}$. Table 3 shows that firm learning can be quite significant and potentially mitigates a substantial portion of the underlying fundamental uncertainty. Here, we explore the relative importance of the two sources of learning present in our model, i.e., private information versus market prices. We begin by reporting in the left-hand panel of Table 5 the total extent of learning and its aggregate consequences. To do so, we compute the reduction in $\mathbb{V}$ both in absolute and percentage terms due to learning from both channels, i.e., $\Delta\mathbb{V} = \mathbb{V} - \sigma_\mu^2$, and the resulting effects on aggregate productivity and output. The table shows that total learning can be quite important and translates into significant improvements in TFP and output: in case 2 ranging from 3% in India to 5% in the US for the former and from 4% to 8% for the latter, with substantially higher gains in case 1.[49] Interestingly, the contribution of learning in China in case 1 appears comparable to that in the US. Note that this does not imply that Chinese firms are necessarily as well-informed as US firms: due to the convexity of $\mathbb{V}$, it is possible that a noisier signal leads to a greater reduction in uncertainty. Intuitively, if there is simply a greater amount of underlying uncertainty, as is the case in China, even a signal with identical precision is in a sense more valuable.

To break down the sources of learning, it is easier to work with the inverse of $\mathbb{V}$, i.e., the

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[48]This latter point can be seen easily from equations (11) and (12), where θ and α (and its components α_1 and α_2 in case 2) enter in different ways.

[49]Note the difference between these calculations and those in Table 3. There, we compute losses compared to a full-information benchmark. Here, we are computing gains versus a no-information (about innovations to fundamentals) benchmark.

Table 5: The Importance and Sources of Learning

	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	Share from source	
					Private	Market
Case 2						
US	-0.12	-0.59	0.05	0.08	0.92	0.08
China	-0.10	-0.37	0.04	0.06	0.96	0.04
India	-0.06	-0.23	0.03	0.04	0.89	0.11
Case 1						
US	-0.08	-0.37	0.23	0.35	0.91	0.09
China	-0.10	-0.35	0.30	0.45	0.96	0.04
India	-0.04	-0.14	0.12	0.19	0.96	0.04

Notes: The left hand panel shows the total effect of learning, i.e., from all sources: the reduction in $\mathbb{V}$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The right hand panel shows the relative shares of private and market sources in total learning.

total precision of the firm's information, which lends itself to a simple linear decomposition:

$$\frac{1}{\mathbb{V}} = \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}$$

We focus on the latter two terms, which capture the contributions of private and market learning to overall precision and compute their relative import as $\frac{1/\sigma_e^2}{1/\sigma_e^2+1/\sigma_v^2\sigma_z^2}$ and $\frac{1/\sigma_v^2\sigma_z^2}{1/\sigma_e^2+1/\sigma_v^2\sigma_z^2}$, respectively. In this way, we calculate the fraction of the total increase in precision due to each source. We report the results in the right hand panel of Table 5. Strikingly, learning is due overwhelmingly to private sources: at best, markets account for about 10% of the increase in precision in the US and India, and about half of this in China. Clearly, these results point to firm private sources as the dominant channel for learning, with markets making only a small additional contribution. As we will see, the limited informational role of markets will prove to be a robust finding throughout our analysis.

The role of market information. To explore in greater depth the importance of new information in stock prices, we recompute $\mathbb{V}$ under the assumption that firms learn nothing from these prices, i.e., that they contain no information. This simply entails sending the noise in markets, $\sigma_v^2\sigma_z^2$, to infinity. The change in $\mathbb{V}$ is a measure of the contribution of stock market information to firm learning and satisfies

$$\Delta\mathbb{V} = \mathbb{V} - \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2}\right)^{-1}$$

We report this figure in the first columns of Table 6 along with the associated aggregate consequences. Even for the US, which has the highest degree of investment-return correlation, market information reduces uncertainty only modestly, eliminating about 2% of the fundamental uncertainty, with associated output gains as small as 0.3% and at a maximum, 2%. This also serves to reinforce our earlier point - the need to analyze these moments jointly through the lens of a structural model. For China and India, the contribution of market-produced information is even smaller, reducing uncertainty by between 0.5% and 2% and representing a maximum gain in output of about 1%.

Table 6: The Contribution of Market Information

	With both sources				With only market learning			
	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy
Case 2								
US	-0.004	-0.021	0.002	0.003	-0.021	-0.107	0.009	0.014
China	-0.003	-0.010	0.001	0.002	-0.007	-0.025	0.003	0.004
India	-0.005	-0.019	0.002	0.004	-0.009	-0.030	0.004	0.006
Case 1								
US	-0.004	-0.021	0.013	0.020	-0.010	-0.051	0.031	0.046
China	-0.003	-0.009	0.008	0.011	-0.006	-0.022	0.017	0.026
India	-0.001	-0.005	0.004	0.006	-0.002	-0.006	0.005	0.008

Notes: The table shows the reduction in $\mathbb{V}$ due to market information and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The left hand panel shows the effects of market learning in the presence of private learning and the right hand panel when markets are the only source of information.

Next, we ask whether the limited informational role of markets is due to the level of noise in prices or to the fact that firms already have considerable information about fundamentals, mitigating the incremental contribution of market information. This distinction is related to the concepts of forecasting price efficiency (FPE) versus revelatory price efficiency (RPE) as put forth in Bond et al. (2012). The former captures the extent to which prices reflect and predict fundamentals, i.e., the absolute level of information in prices; the latter, the extent to which prices promote real efficiency by revealing new information to the firm. In our framework, $\Delta\mathbb{V}$ in the first columns of Table 6 is the natural measure of RPE, since it is the marginal impact of the information in prices on uncertainty, given the other sources of firm information. As the table shows, at the estimated parameter values, markets in all 3 countries exhibit relatively low RPE. To measure FPE, we compute the reduction in $\mathbb{V}$ from the information in stock prices alone, i.e.,

$$\Delta\mathbb{V} = \left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)^{-1} - \sigma_\mu^2$$

This measures the contribution of market information assuming it is the only source of learning for firms. We report the results in the right hand panel of Table 6. In general, stock markets are rather weak predictors of fundamentals - even as the only source of learning, the information they provide lead to only modest reductions in uncertainty. For the US, which has the most informative prices, market generated information reduces fundamental uncertainty by between 5% and 10%. Compare this to the US values in Table 5, where a total of between 35% and 60% of uncertainty is eliminated. The associated TFP and output effects of markets as a standalone source of information in the US are, at highest, 3% and 5%, respectively, in case 1. The aggregate impact of markets is even more modest for the US in case 2, and is lower in the two emerging markets than in the US in both cases. Even in a forecasting sense then, the efficiency of stock markets is fairly low, suggesting that the limited role of market information is in large part due to its poor quality; in other words, the poor RPE of stock markets can be largely attributed to their low FPE. Comparing these values to the corresponding ones in the left hand panel shows that the already modest impact of the information in prices is even further diluted by the presence of alternative information available to firms, leading in net to the low levels of RPE shown.

While the limited informational role of markets is a robust pattern across countries and cases, these results should be interpreted somewhat cautiously. Our analysis focuses exclusively on information and decisions over the medium term. It is silent on the role of stock markets in guiding longer-term, strategic decisions (e.g., the decision to enter a new market or acquire another company). In fact, one possible source of 'noise' in stock returns is information about the firm's prospects over a much longer horizon. So long as it is orthogonal to fundamentals over the medium term, the relevant object for the firm's current decisions, our strategy will still lead us to the right measure of uncertainty and the associated misallocation - our estimates for the specific informational parameters and the informational role of financial markets, however, is likely to be sensitive to the assumptions about the nature of this 'noise' term.

Our analysis also abstracts from the contribution of financial market information in mitigating uncertainty about aggregate (or industry) conditions, particularly for outsiders (e.g., potential entrants, creditors, regulators, etc.). Explicitly modeling these features in a fully-fledged general equilibrium environment is a challenging task, but may well be essential for a comprehensive evaluation of the informational role of well-functioning financial markets[50].

The role of private information. Next, we explore the contribution of firm private learning to aggregate performance by performing experiments analogous to those above: specifically, we calculate the marginal contribution of private information to allocative efficiency, both in the

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[50]See section 5 in Bond et al. (2012) for a related discussion.

presence of market information and when it is the only source of learning for firms.

We report the results in Table 7. In contrast to the effect of market learning, firm private information plays a much larger role in reducing uncertainty and improving aggregate performance. Turning to the first set of columns, private information eliminates between about 30% and 50% of the fundamental uncertainty in the US. The associated aggregate gains are substantial, ranging from 4-20% for TFP and from 6-30% for output. The corresponding results are smaller for China and India in case 2 (although only slightly so in China), reflecting a worse quality of firm private information in those countries, and are smaller for India in case 1. Having said that, they are still quite significant across countries and are noticeably larger than the effects of market information. Similar to what we saw in Table 5, the contribution of private learning in China in case 1 is on par with that in US, again because even a less precise signal can have more value if the extent of uncertainty is greater.

Table 7: The Contribution of Private Information

	With both sources				With only private learning			
	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	$\Delta\mathbb{V}$	$\frac{\Delta\mathbb{V}}{\sigma_\mu^2}$	Δa	Δy
Case 2								
US	-0.10	-0.48	0.04	0.06	-0.11	-0.57	0.05	0.07
China	-0.09	-0.35	0.04	0.06	-0.09	-0.36	0.04	0.06
India	-0.06	-0.20	0.02	0.04	-0.06	-0.21	0.03	0.04
Case 1								
US	-0.07	-0.32	0.20	0.30	-0.07	-0.37	0.22	0.33
China	-0.09	-0.33	0.28	0.42	-0.10	-0.38	0.29	0.44
India	-0.04	-0.13	0.12	0.18	-0.04	-0.14	0.12	0.18

Notes: The table shows the reduction in $\mathbb{V}$ due to private information and the corresponding gains in the aggregates (hence the pattern of negatives and positives). The left hand panel shows the effects of private learning in the presence of market learning and the right hand panel when private sources are the only source of information.

The right hand panel of Table 7 reports the contribution of firm private information were it the only source of learning. A comparison of the two panels shows that the values are quite similar - the presence of market information does not significantly alter the importance of private information for overall learning.

4.5   Cross-country counterfactuals

Next, we use our estimates to run a couple of instructive counterfactual experiments. Specifically, we assess the potential gains to China and India from having access to US quality information, whether through more informative financial markets or from better firm-level private

information. In the first experiment, we compute the change in $\mathbb{V}$ in China and India under the assumption that the informativeness of prices - summarized by $\sigma_\nu^2 \sigma_z^2$ - is equal to that in the US, leaving all other country-specific parameters fixed. This quantifies the improvements in aggregate performance that would accrue in China and India from having financial markets as informative as that in the US. Second, we perform the same exercise for firm private information, that is, compute the change in $\mathbb{V}$ and aggregate improvements under the assumption that firms in China and India have the same σ_e^2 as their US counterparts (again leaving the other country-specific parameters fixed). In a final experiment, we turn away from learning and study the role of fundamentals in leading to a differential impact of informational frictions across countries; to do so, we recompute $\mathbb{V}$ in China and India assuming that firms in these countries face the same fundamental shocks as do US firms.

We report the results from these experiments in Table 8. The top panel shows that delivering US-quality markets to emerging economies results in potentially significant yet modest reductions in uncertainty. As a percent of total fundamental uncertainty, these range from 2% to 7% (across cases) with a corresponding aggregate impact ranging, for example, from 1% to 6% in output. The middle panel shows that access to US-quality private information would have a much larger impact, reducing $\mathbb{V}$ by as much as 40% of the underlying uncertainty in the most optimistic case, and, excepting China in case 1 which is an outlier here, more than 25% in the other cases. The resulting aggregate gains can be substantial, ranging, for example, from 4% to almost 40% in output. Across the board, the gains from accessing US-quality private information are much larger than from US-quality market information. To the extent that differences in learning lead to cross-country variation in economic aggregates, these disparities appear to be largely due to a lack of high quality firm private information, rather than to a lack of well-functioning (in an informational sense) financial markets in emerging markets. This is yet another instance of a result we have now seen several times: financial markets play only a modest informational role in promoting aggregate efficiency.

Finally, we recompute $\mathbb{V}$ in China and India under the assumption that firms in those countries face shocks of the same volatility as those in the US, i.e., we place the σ_μ^2 in the two countries with that of the US. We report the results in the bottom panel of Table 8, which shows that disparities in the volatility of shocks also contribute significantly to differences in the impact of informational frictions. Exposure to a fundamental process such as that in the US reduces $\mathbb{V}$ between about 10% and 25% of the underlying uncertainty, leading to TFP and output gains[51] that are potentially substantial, ranging from 1-20% and 2-30% respectively. [52]

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[51] These numbers correspond to $\Delta (a - a^*)$ and $\Delta (y - y^*)$. Recall that a^*, the full information productivity level, is also affected by the size of fundamental shocks, σ_μ^2.

[52] Asker et al. (2012) also highlight the role of different firm-specific shock processes in generating misallocation in a model with capital adjustment costs.

Table 8: The Effects of a US Information Structure

	Case 2				Case 1			
	$\Delta \mathbb{V}$	$\frac{\Delta \mathbb{V}}{\sigma_\mu^2}$	Δa	Δy	$\Delta \mathbb{V}$	$\frac{\Delta \mathbb{V}}{\sigma_\mu^2}$	Δa	Δy
Market Information								
China	-0.01	-0.05	0.01	0.01	-0.01	-0.02	0.02	0.02
India	-0.02	-0.07	0.01	0.01	-0.01	-0.05	0.04	0.06
Private Information								
China	-0.07	-0.26	0.03	0.04	-0.02	-0.08	0.06	0.09
India	-0.12	-0.43	0.05	0.08	-0.09	-0.31	0.26	0.39
Shocks								
China	-0.02	-0.09	0.01	0.02	-0.03	-0.13	0.10	0.15
India	-0.05	-0.19	0.02	0.03	-0.07	-0.24	0.20	0.31

Notes: The table shows the reduction in $\mathbb{V}$ and the corresponding gains in the aggregates (hence the pattern of negatives and positives) in India and China from having access to US-quality financial market information (top panel), private information (middle panel) and facing US fundamental shocks (bottom panel).

4.6 Capital Adjustment Costs

Section 3 demonstrated that our identification strategy exhibits a certain robustness to other factors potentially affecting the firm's capital choice. The analysis in that section, while reassuring, was done in a fairly stylized setting, primarily for analytical tractability. To explore the extent to which that intuition carries over to our more general setting, we now conduct a very similar numerical experiment. Specifically, we ask how the presence of a specific friction, namely capital adjustment costs, affects the empirical moments we use for inference and through them, our assessment of the severity of informational frictions.

We make two modifications to the baseline model. First, following Bloom (2009), Cooper and Haltiwanger (2006), and Asker et al. (2012), we subject firms to quadratic costs of adjusting capital $\zeta K_{t-1} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right)^2$.[53] Second, we assume that firms have full information ($\sigma_e^2 = 0$) but that stock markets are noisy ($\sigma_v^2 \neq 0, \sigma_z^2 \neq 0$). We then ask, if this were the true data generating process, to what extent would our empirical strategy lead us to infer firm-level uncertainty, when, in fact, there is none. More precisely, the exercise is as follows: we first parameterize and solve the full-information adjustment cost model in general equilibrium.[54] We next simulate

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[53]We focus on convex costs and abstract from the fixed-type costs also considered in the literature, as the former are most capable of delivering a configuration of moments that resemble those under informational frictions. Intuitively, convex costs imply that investment decisions respond to innovations in fundamentals only in part contemporaneously and then with a lag. This latter feature resembles the pattern implied by informational frictions.

[54]Since the firm's capital choice problem is now a dynamic one, we can no longer analytically characterize the joint distribution of fundamentals and capital across firms. We have to solve numerically for this distribution

firm-level data and generate the same moments that we use in our analysis above. We then use the intuitive mapping between these moments and the informational parameters to assess the effect on our inference. In particular, recall that the correlation of returns with investment relative to that of returns with fundamentals played a key role in our estimate of $\mathbb{V}$. Comparing this relative correlation in the simulated data with its empirical counterpart is a simple and intuitive way to assess the effect on our inference.

We parameterize the model as follows: we choose values of $(\sigma_v^2, \sigma_z^2, \zeta)$ to match 3 moments in the data - the volatility of prices and their correlation with fundamentals, σ_p^2 and ρ_{pa}, and the cross-sectional variation in firm-level investment. The first two are exactly those we used in our baseline analysis, while the last provides information on the adjustment cost parameter ζ.[55] We then examine the implications of this model for the correlations between investment and investment growth and returns, ρ_{pk} and ρ_{pi}, respectively;[56] these are moments not directly targeted here, but as we have seen, are closely linked to our measurement of firm-level uncertainty.[57]

We report the results in Table 9, which compares the values of ρ_{pk} and ρ_{pi} generated from the full-information adjustment cost model with their empirical counterparts.[58] The correlation of returns with investment choices implied by the full-information adjustment cost model, whether measured as ρ_{pk} or ρ_{pi}, is significantly lower than that observed in the data. In other words, holding fixed the volatility of returns and their correlation with fundamentals, the relationship between investment decisions and stock returns of fully-informed firms subject to adjustment costs is noticeably weaker than the empirical one. For example, the value of ρ_{pi} falls dramatically across the board, from levels well exceeding ρ_{pa} to levels below. Given the intuition underlying our identification strategy, this pattern suggests that applying that strategy to these simulated moments would result in substantially lower estimates of $\mathbb{V}$ than in our baseline analysis. This provides some additional reassurance that what we have uncovered is indeed a valid measure of firm-level uncertainty.

While this analysis has the admittedly somewhat limited goal of confirming the robustness of our estimates of uncertainty, it also suggests that both types of frictions (adjustment costs and imperfect information) might be necessary for a more comprehensive interpretation of

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(and the associated general equilibrium implications) in steady state. We refer the reader to the Appendix for details.

[55]This is similar in spirit to the estimation strategy is employed in Bloom (2009), Cooper and Haltiwanger (2006), and Asker et al. (2012).

[56]We report both correlations for completeness - our analytic expressions are in terms of ρ_{pk} while our numerical analysis, following the finance literature, uses ρ_{pi}. The patterns we document hold for both these measures.

[57]To be clear, what matters most for our estimate of $\mathbb{V}$ is the relative correlation, i.e., the difference between ρ_{pk} (or ρ_{pi}) and ρ_{pa}. Since the latter is directly targeted both in the baseline model and in the version with adjustment costs, we can simply compare ρ_{pk} (or ρ_{pi}) to see how the relative correlations change.

[58]We show in the Appendix the full set of target moments and resulting parameter values.

Table 9: A Full-Information Adjustment Cost Model

	Data			Model	
	ρ_{pa}	ρ_{pi}	ρ_{pk}	ρ_{pi}	ρ_{pk}
US	0.18	0.23	0.35	0.15	0.15
China	0.06	0.16	0.22	-0.02	0.16
India	0.08	0.25	0.20	0.06	0.10

Notes: The table shows the correlations of returns with fundamentals (ρ_{pa}), returns with investment growth (ρ_{pi}), and returns with investment (ρ_{pk}), both as computed from the data and as predicted by the full-information adjustment cost model. ρ_{pa} is a targeted moment and so is the same in the data and model.

the data. Such a unified framework would aid us in understanding the interactions between these frictions and in disentangling their effects on aggregate outcomes. In fact, it is not even clear that they should be thought of as two independent forces: informational frictions lead to sluggish responses to fundamentals and in this sense, provide a deeper theory of adjustment costs. Exploring these issues, both theoretically and quantitatively, would be a fruitful, if challenging, direction for future research.

5   Conclusion

We have laid out a theory of informational frictions that distort the allocation of factors across heterogeneous firms, leading to reduced aggregate productivity and output. A central element of our framework is a rich information structure in which firms learn from both private sources and imperfectly revealing stock market prices, a feature which allows us to jointly analyze moments of firm-level production variables and stock market returns to infer the severity of informational frictions.

Our approach reveals substantial micro-level uncertainty, particularly in China and India. Perhaps more interestingly, we find that to the extent firm learning serves to mitigate some portion of this uncertainty, firms turn overwhelmingly to private sources to obtain such information. Further, we find significant cross-country variation in the impact of informational frictions, much of which is attributable to differences in the quality of firms' own information sources (and to some extent, the volatility of the underlying shocks). Learning from financial markets seems to contribute little to aggregate allocative efficiency, even in a relatively developed market such as that in the US.

Our results suggest then that policies aimed at directly improving firm-level information may be more fruitful in improving emerging economy performance than those intended to de-

velop financial markets to a level closer to that in the US. Because our modeling of information is rather abstract, we are left to speculate on the exact form of such policies. As an example, one interpretation of cross-country differences in private information is better information collection/processing systems within firms, and/or the skill of managers. A thorough investigation of these issues is an important direction for future research.

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Appendix

A   Detailed Derivations

A.1   Case 1: Both factors chosen under imperfect information

As we show in equation (5) in the text, the firm's capital choice problem can be written as

$$\max_{K_{it}} \quad \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}\left[A_{it}\right] K_{it}^{\alpha} - \left(1 + \frac{\alpha_2}{\alpha_1}\right) R_t K_{it}$$

and optimality requires

$$\alpha \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} K_{it}^{\alpha-1} \mathbb{E}_{it}\left[A_{it}\right] = R_t$$

$$\Rightarrow \quad \left[\alpha \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \frac{\mathbb{E}_{it}\left[A_{it}\right]}{R_t}\right]^{\frac{1}{1-\alpha}} = K_{it}$$

Capital market clearing then implies

$$\int K_{it} di = \left[\left(\frac{N}{K_t} \right)^{\alpha_2} \alpha \left(\frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \frac{Y_t^{\frac{1}{\theta}}}{R_t} \right]^{\frac{1}{1-\alpha}} \int \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di = K_t$$

$$\Rightarrow \quad \left[\left(\frac{N}{K_t} \right)^{\alpha_2} \alpha \left(\frac{\alpha_1}{\alpha_1 + \alpha_2} \right) \frac{Y_t^{\frac{1}{\theta}}}{R_t} \right]^{\frac{1}{1-\alpha}} = \frac{K_t}{\int \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di}$$

from which we can solve for

$$K_{it} = \frac{\left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} K_t$$

From here, it is straightforward to express firm revenue as

$$P_{it} Y_{it} = K_t^{\alpha_1} N^{\alpha_2} Y_t^{\frac{1}{\theta}} A_{it} \left(\frac{\left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}}}{\int \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di} \right)^{\alpha}$$

and noting that aggregate revenue must equal aggregate output, we have

$$Y_t = \int P_{it} Y_{it} di = K_t^{\alpha_1} N^{\alpha_2} Y_t^{\frac{1}{\theta}} \frac{\int A_{it} \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{\alpha}{1-\alpha}} di}{\left(\int \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di \right)^{\alpha}}$$

or in logs,

$$y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \log \int A_{it} \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{\alpha}{1-\alpha}} di - \alpha \log \int \left(\mathbb{E}_{it} [A_{it}] \right)^{\frac{1}{1-\alpha}} di$$

Now, note that under conditional log-normality,

$$a_{it} | \mathcal{I}_{it} \sim \mathcal{N} \left(\mathbb{E}_{it} [a_{it}], \mathbb{V} \right) \quad \Rightarrow \quad \mathbb{E}_{it} [A_{it}] = \exp \left(\mathbb{E}_{it} a_{it} + \frac{1}{2} \mathbb{V} \right)$$

The true fundamental a_{it} and its conditional expectation $\mathbb{E}_{it} a_{it}$ are also jointly normal, i.e.

$$\begin{bmatrix} a_{it} \\ \mathbb{E}_{it} a_{it} \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \bar{a} \\ \bar{a} \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_a^2 - \mathbb{V} \\ \sigma_a^2 - \mathbb{V} & \sigma_a^2 - \mathbb{V} \end{bmatrix} \right)$$

We then have that

$$
\begin{aligned}
\log \int A_{it} \left(\mathbb{E}_{it} \left[A_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} di &= \log \int \exp \left(a_{it} + \frac{\alpha}{1-\alpha} \mathbb{E}_{it} a_{it} + \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V} \right) di \\
&= \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \left(\frac{\alpha}{1-\alpha} \right)^2 \left(\sigma_a^2 - \mathbb{V} \right) \\
&\quad + \frac{\alpha}{1-\alpha} \left(\sigma_a^2 - \mathbb{V} \right) + \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V} \\
&= \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \sigma_a^2 + \frac{1}{2} \alpha \frac{2-\alpha}{(1-\alpha)^2} \left(\sigma_a^2 - \mathbb{V} \right) \\
&\quad + \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V}
\end{aligned}
$$

and

$$
\log \int \left(\mathbb{E}_{it} \left[A_{it} \right] \right)^{\frac{1}{1-\alpha}} di = \frac{1}{1-\alpha} \bar{a} + \frac{1}{2} \left(\frac{1}{1-\alpha} \right)^2 \left(\sigma_a^2 - \mathbb{V} \right) + \frac{1}{2} \frac{1}{1-\alpha} \mathbb{V}
$$

Combining these,

$$
\log \int A_{it} \left(\mathbb{E}_{it} \left[A_{it} \right] \right)^{\frac{\alpha}{1-\alpha}} di - \alpha \log \int \left(\mathbb{E}_{it} \left[A_{it} \right] \right)^{\frac{1}{1-\alpha}} di = \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{1-\alpha} - \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V}
$$

Substituting and rearranging, we obtain the expressions in (10) and (11) in the text,

$$
\begin{aligned}
y_t &= \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n + \bar{a} + \frac{1}{2} \frac{\sigma_a^2}{1-\alpha} - \frac{1}{2} \frac{\alpha}{1-\alpha} \mathbb{V} \\
&= \hat{\alpha}_1 k_t + \hat{\alpha}_2 n + \frac{\theta}{\theta-1} \bar{a} + \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \frac{\sigma_a^2}{1-\alpha} - \frac{1}{2} \frac{\alpha}{1-\alpha} \left(\frac{\theta}{\theta-1} \right) \mathbb{V}
\end{aligned}
$$

with aggregate productivity given by

$$
a = \underbrace{\frac{\theta}{\theta-1} \bar{a} + \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \frac{\sigma_a^2}{1-\alpha}}_{a^*} - \frac{1}{2} \frac{\alpha}{1-\alpha} \left(\frac{\theta}{\theta-1} \right) \mathbb{V}
$$

It now remains to endogenize K_t. The rental rate in steady state satisfies

$$
R = \frac{1}{\beta} - 1 + \delta
$$

Then, from the optimality and market clearing conditions, we have from (4) that

$$
\frac{\alpha_1 N}{\alpha_2 K} = \frac{R}{W} \quad \Rightarrow \quad K \propto W
$$

i.e., the aggregate capital stock is proportional to the wage. To characterize wages, we return to the firm's profit maximization problem

$$\max_{K_{it}, N_{it}} \quad Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}[A_{it}] K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W N_{it} - R K_{it}$$

which, after maximizing over capital, can be written as

$$\max_{N_{it}} \quad (1 - \alpha_1) \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1}} \left(Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}(A_{it}) N_{it}^{\alpha_2}\right)^{\frac{1}{1-\alpha_1}} - W N_{it}$$

Optimality and labor market clearing imply

$$\left(\frac{\alpha_2}{W}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \left(Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}[A_{it}]\right)^{\frac{1}{1-\alpha_1-\alpha_2}} = N_{it}$$

$$\left(\frac{\alpha_2}{W}\right)^{\frac{1-\alpha_1}{1-\alpha_1-\alpha_2}} \left(\frac{\alpha_1}{R}\right)^{\frac{\alpha_1}{1-\alpha_1-\alpha_2}} \int \left(Y_t^{\frac{1}{\theta}} \mathbb{E}_{it}[A_{it}]\right)^{\frac{1}{1-\alpha_1-\alpha_2}} di = N$$

As before, letting $\alpha = \alpha_1 + \alpha_2$, we see that

$$
\begin{aligned}
W \quad &\propto \quad \left(\int \mathbb{E}_{it}[A_{it}]^{\frac{1}{1-\alpha}} di\right)^{\frac{1-\alpha}{1-\alpha_1}} Y_t^{\frac{1}{\theta} \frac{1}{1-\alpha_1}} \\
&= \quad \left[\left(\int \left(\exp\left(\mathbb{E}_{it} a_{it} + \frac{1}{2}\mathbb{V}\right)\right)^{\frac{1}{1-\alpha}} di\right)^{1-\alpha} Y_t^{\frac{1}{\theta}}\right]^{\frac{1}{1-\alpha_1}} = \left[\exp\left(\bar{a} + \frac{1}{2}\left(\frac{\sigma_a^2 - \mathbb{V}}{1-\alpha}\right) + \frac{1}{2}\mathbb{V}\right) Y_t^{\frac{1}{\theta}}\right]^{\frac{1}{1-\alpha_1}} \\
&= \quad \left[\exp\left(\bar{a} + \frac{1}{2}\left(\frac{\sigma_a^2 - \alpha\mathbb{V}}{1-\alpha}\right)\right) Y_t^{\frac{1}{\theta}}\right]^{\frac{1}{1-\alpha_1}}
\end{aligned}
$$

or in logs,

$$w \propto \left(\frac{1}{1-\alpha_1}\right)\bar{a} + \frac{1}{1-\alpha_1}\frac{1}{2}\left(\frac{\sigma_a^2 - \alpha\mathbb{V}}{1-\alpha}\right) + \frac{1}{\theta}\frac{1}{1-\alpha_1}y_t$$

Recalling that

$$K \propto W \quad \Rightarrow \quad \frac{dk}{d\mathbb{V}} = \frac{dw}{d\mathbb{V}}$$

which, in conjunction with (10) and (11), implies

$$
\begin{aligned}
\frac{dy}{d\mathbb{V}} &= \hat{\alpha}_1 \left(\frac{dk}{d\mathbb{V}}\right) - \frac{1}{2}\left(\frac{\theta}{\theta - 1}\right)\frac{\alpha}{1-\alpha} \\
&= \frac{\hat{\alpha}_1}{1-\alpha_1}\left[-\frac{1}{2}\frac{\alpha}{1-\alpha} + \frac{1}{\theta}\frac{dy}{d\mathbb{V}}\right] - \frac{1}{2}\left(\frac{\theta}{\theta - 1}\right)\frac{\alpha}{1-\alpha}
\end{aligned}
$$

and finally, collecting terms and rearranging, and using the fact that $\widehat{\alpha}_1 = \frac{\theta}{\theta-1}\alpha_1$, we obtain

$$\frac{dy}{d\mathbb{V}} = -\frac{1}{2}\frac{\alpha}{1-\alpha}\left(\frac{\theta}{\theta-1}\right)\frac{1}{1-\widehat{\alpha}_1} = \frac{da}{d\mathbb{V}}\frac{1}{1-\widehat{\alpha}_1}$$

A.2   Case 2: Only capital chosen under imperfect information

The firm's labor choice problem can be written as

$$\max_{N_{it}} \quad Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha_1} N_{it}^{\alpha_2} - W_t N_{it}$$

and optimality requires

$$N_{it} = \left(\frac{\alpha_2}{W}Y_t^{\frac{1}{\theta}}A_{it}K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}$$

Labor market clearing then implies

$$\int N_{it}\ di \ = \ \int\left(\frac{\alpha_2}{W}Y_t^{\frac{1}{\theta}}A_{it}K_{it}^{\alpha_1}\right)^{\frac{1}{1-\alpha_2}}di = N$$

$$\Rightarrow \quad N_{it} \ = \ \frac{(A_{it}K_{it}^{\alpha_1})^{\frac{1}{1-\alpha_2}}}{\int(A_{it}K_{it}^{\alpha_1})^{\frac{1}{1-\alpha_2}}di}N$$

Letting $\tilde{A}_{it} = A_{it}^{\frac{1}{1-\alpha_2}}$ and $\tilde{\alpha} = \frac{\alpha_1}{1-\alpha_2}$, we have

$$N_{it} = \frac{\tilde{A}_{it}K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}}di}N$$

which implies

$$W_t \ = \ Y_t^{\frac{1}{\theta}}A_{it}K_{it}^{\alpha_1}\alpha_2\left(\frac{\tilde{A}_{it}K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}}di}N\right)^{\alpha_2-1}$$

$$= \ \frac{\alpha_2}{N^{1-\alpha_2}}Y_t^{\frac{1}{\theta}}\left(\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}}\ di\right)^{1-\alpha_2}A_{it}K_{it}^{\alpha_1}\left(A_{it}^{\frac{1}{1-\alpha_2}}K_{it}^{\frac{\alpha_1}{1-\alpha_2}}\right)^{\alpha_2-1}$$

$$= \ \frac{\alpha_2}{N^{1-\alpha_2}}Y_t^{\frac{1}{\theta}}\left(\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}}\ di\right)^{1-\alpha_2}$$

From here, it is straightforward to express the firm's capital choice problem as in (8):

$$\max_{K_{it}} \quad (1-\alpha_2)\left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}}Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\mathbb{E}_{it}\left[\tilde{A}_{it}\right]K_{it}^{\tilde{\alpha}} - R_t K_{it}$$

Optimality requires

$$R_t = (1 - \alpha_2) \left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}} Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}} \mathbb{E}_{it}\left[\tilde{A}_{it}\right] \tilde{\alpha} K_{it}^{\tilde{\alpha}-1}$$

$$\Rightarrow \quad K_{it} = \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R}\right]^{\frac{1}{1-\tilde{\alpha}}} \left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}\frac{1}{1-\tilde{\alpha}}} Y_t^{\frac{1}{\theta(1-\alpha_2)(1-\tilde{\alpha})}} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}$$

Capital market clearing then implies

$$\int K_{it} di = \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R}\right]^{\frac{1}{1-\tilde{\alpha}}} \left(\frac{\alpha_2}{W_t}\right)^{\frac{\alpha_2}{1-\alpha_2}\frac{1}{1-\tilde{\alpha}}} Y_t^{\frac{1}{\theta(1-\alpha_2)(1-\tilde{\alpha})}} \int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di = K_t$$

$$\Rightarrow \quad K_{it} = \frac{\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}}{\int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di} K_t$$

and we can rewrite the labor choice as

$$N_{it} = \frac{\tilde{A}_{it} K_{it}^{\tilde{\alpha}}}{\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di} N = \frac{\tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di} N$$

Combining the solutions for capital and labor, we can express firm revenue as

$$P_{it} Y_{it} = Y_t^{\frac{1}{\theta}} A_{it} \left\{ \frac{\left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}}}{\int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di} K_t \right\}^{\alpha_1} \left\{ \frac{\tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\int \tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di} N \right\}^{\alpha_2}$$

$$= Y_t^{\frac{1}{\theta}} K_t^{\alpha_1} N^{\alpha_2} \frac{A_{it} \tilde{A}_{it}^{\alpha_2} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\alpha_1+\alpha_2\tilde{\alpha}}{1-\tilde{\alpha}}}}{\left\{\int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di\right\}^{\alpha_1} \left\{\int \tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di\right\}^{\alpha_2}}$$

$$= Y_t^{\frac{1}{\theta}} K_t^{\alpha_1} N^{\alpha_2} \frac{\tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}}}{\left\{\int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di\right\}^{\alpha_1} \left\{\int \tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di\right\}^{\alpha_2}}$$

and again using the fact that $y_t = \log \int P_{it} Y_{it} di$, we can write

$$y_t = \frac{1}{\theta} y_t + \alpha_1 k_t + \alpha_2 n - \alpha_1 \log \int \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{1}{1-\tilde{\alpha}}} di + (1-\alpha_2) \log \int \tilde{A}_{it} \left(\mathbb{E}_{it}\left[\tilde{A}_{it}\right]\right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di$$

Again, we exploit log-normality to obtain

$$\log \int \tilde{A}_{it} \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di = \log \int \exp \left(\tilde{a}_{it} + \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \mathbb{E}_{it} \tilde{a}_{it} + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\mathbb{V}} \right) di$$

$$= \frac{1}{1-\tilde{\alpha}} \tilde{a} + \frac{1}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \tilde{\alpha} \frac{2-\tilde{\alpha}}{(1-\tilde{\alpha})^2} \left(\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}} \right) + \frac{1}{2} \frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\mathbb{V}}$$

and similarly,

$$\log \int \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{1}{1-\tilde{\alpha}}} di = \log \int \left(\exp \left(\mathbb{E}_{it} \tilde{a}_{it} + \frac{1}{2} \tilde{\mathbb{V}} \right) \right)^{\frac{1}{1-\tilde{\alpha}}} di$$

$$= \frac{1}{1-\tilde{\alpha}} \tilde{a} + \frac{1}{2} \frac{\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}}}{(1-\tilde{\alpha})^2} + \frac{1}{2} \frac{1}{1-\tilde{\alpha}} \tilde{\mathbb{V}}$$

Combining, and using the fact that $\tilde{a}_{it} = \frac{a_{it}}{1-\alpha_2}$:

$$-\alpha_1 \log \int \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{1}{1-\tilde{\alpha}}} di + (1-\alpha_2) \log \int \tilde{A}_{it} \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di$$

$$= \frac{1-\alpha_2-\alpha_1}{1-\tilde{\alpha}} \tilde{a} + \frac{(1-\alpha_2)}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \left[\frac{(1-\alpha_2)(2\tilde{\alpha}-\tilde{\alpha}^2)}{(1-\tilde{\alpha})^2} \right] \left(\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}} \right)$$

$$= \frac{1-\alpha}{1-\tilde{\alpha}} \frac{\bar{a}}{1-\alpha_2} + \frac{(1-\alpha_2)}{2} \sigma_{\tilde{a}}^2 + \frac{1}{2} \frac{\alpha_1}{1-\tilde{\alpha}} \left(\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}} \right)$$

$$= \bar{a} + \frac{1}{2} \frac{1}{1-\alpha} \sigma_a^2 - \frac{1}{2} \frac{\alpha_1}{1-\alpha} (1-\alpha_2) \tilde{\mathbb{V}}$$

Substituting and collecting terms, we obtain expressions (10) and (12) in the text,

$$y_t = \widehat{\alpha}_1 k_t + \widehat{\alpha}_2 n + \frac{\theta}{\theta-1} \bar{a} + \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \frac{\sigma_a^2}{1-\alpha} - \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \frac{\alpha_1(1-\alpha_2)}{1-\alpha} \tilde{\mathbb{V}}$$

with aggregate productivity given by

$$a = \underbrace{\frac{\theta}{\theta-1} \bar{a} + \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \frac{\sigma_a^2}{1-\alpha}}_{a^*} - \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \left(\frac{\alpha_1}{1-\alpha} \right) (1-\alpha_2) \tilde{\mathbb{V}}$$

To endogenize K_t, we begin by characterizing the steady state wage W in terms of $\int \tilde{A}_{it} \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di$

and $Y^{\frac{1}{\theta}}$:

$$
\begin{aligned}
W &= \frac{\alpha_2}{N^{1-\alpha_2}} Y^{\frac{1}{\theta}} \left(\int \tilde{A}_{it} K_{it}^{\tilde{\alpha}} di \right)^{1-\alpha_2} \\
&= \frac{\alpha_2}{N^{1-\alpha_2}} Y^{\frac{1}{\theta}} \left\{ \int \tilde{A}_{it} \left(\frac{(1-\alpha_2)\tilde{\alpha}}{R} \left(\frac{\alpha_2}{W} \right)^{\frac{\alpha_2}{1-\alpha_2}} Y^{\frac{1}{\theta(1-\alpha_2)}} \mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \right\}^{1-\alpha_2} \\
&= \frac{\alpha_2}{N^{1-\alpha_2}} Y^{\frac{1}{\theta}} \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R} \right]^{\frac{\alpha_1}{1-\tilde{\alpha}}} \left(\frac{\alpha_2}{W} \right)^{\frac{\alpha_2\tilde{\alpha}}{1-\tilde{\alpha}}} Y^{\frac{1}{\theta}\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} \left\{ \int \tilde{A}_{it} \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \right\}^{1-\alpha_2}
\end{aligned}
$$

and rearranging,

$$
\begin{aligned}
W &= \left(\frac{\alpha_2}{N^{1-\alpha_2}} \right)^{\frac{1-\tilde{\alpha}}{1-\alpha_1}} \left[\frac{(1-\alpha_2)\tilde{\alpha}}{R} \right]^{\frac{\alpha_1}{1-\alpha_1}} \alpha_2^{\frac{\alpha_2\tilde{\alpha}}{1-\alpha_1}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_1}} \left\{ \int \tilde{A}_{it} \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \right\}^{\frac{(1-\alpha_2)(1-\tilde{\alpha})}{1-\alpha_1}} \\
&\propto \left\{ \int \tilde{A}_{it} \left(\mathbb{E}_{it} \left[\tilde{A}_{it} \right] \right)^{\frac{\tilde{\alpha}}{1-\tilde{\alpha}}} di \right\}^{\frac{1-\alpha}{1-\alpha_1}} Y^{\frac{1}{\theta}\frac{1}{1-\alpha_1}} \\
&= \left\{ \left[\exp \left(\frac{1}{1-\tilde{\alpha}} \bar{a} + \frac{1}{2}\sigma_{\tilde{a}}^2 + \frac{1}{2}\frac{2\tilde{\alpha}-\tilde{\alpha}^2}{(1-\tilde{\alpha})^2} \left(\sigma_{\tilde{a}}^2 - \tilde{\mathbb{V}} \right) + \frac{1}{2}\frac{\tilde{\alpha}}{1-\tilde{\alpha}} \tilde{\mathbb{V}} \right) \right]^{1-\alpha} Y^{\frac{1}{\theta}} \right\}^{\frac{1}{1-\alpha_1}} \\
&= \left\{ \exp \left[\bar{a} + \frac{1}{2}\frac{1}{(1-\alpha_2)(1-\tilde{\alpha})}\sigma_a^2 - \frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{1-\tilde{\alpha}} \tilde{\mathbb{V}} \right] Y^{\frac{1}{\theta}} \right\}^{\frac{1}{1-\alpha_1}}
\end{aligned}
$$

or in logs,

$$
w \propto \left(\frac{1}{1-\alpha_1} \right) \bar{a} + \frac{1}{2}\frac{1}{(1-\alpha_1)}\frac{1}{(1-\alpha)}\sigma_a^2 - \frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})}\tilde{\mathbb{V}} + \frac{1}{\theta}\frac{1}{1-\alpha_1} y_t
$$

As before,

$$
K \propto W \quad \Rightarrow \quad \frac{dk}{d\tilde{\mathbb{V}}} = \frac{dw}{d\tilde{\mathbb{V}}} = -\frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})} + \frac{1}{\theta}\frac{1}{1-\alpha_1}\frac{dy}{d\tilde{\mathbb{V}}}
$$

and substituting into the derivative of aggregate output,

$$
\begin{aligned}
\frac{dy}{d\tilde{\mathbb{V}}} &= \hat{\alpha}_1 \left(\frac{dk}{d\tilde{\mathbb{V}}} \right) - \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \left(\frac{\alpha_1}{1-\alpha} \right) (1-\alpha_2) \\
&= \hat{\alpha}_1 \left[-\frac{1}{2}\frac{\tilde{\alpha}(1-\alpha_2)}{(1-\alpha_1)(1-\tilde{\alpha})} + \frac{1}{\theta}\frac{1}{1-\alpha_1}\frac{dy}{d\tilde{\mathbb{V}}} \right] - \frac{1}{2} \left(\frac{\theta}{\theta-1} \right) \left(\frac{\alpha_1}{1-\alpha} \right) (1-\alpha_2)
\end{aligned}
$$

Finally, collecting terms and rearranging, and using the facts that $1 - \alpha = (1 - \tilde{\alpha})(1 - \alpha_2)$

and $\left(\frac{\theta}{\theta-1}\right)\alpha_1 = \widehat{\alpha}_1$, we obtain

$$\frac{dy}{d\widetilde{\mathbb{V}}} = -\frac{1}{2}(1-\alpha_2)\left(\frac{\theta}{\theta-1}\right)\left(\frac{\alpha_1}{1-\alpha}\right)\frac{1}{1-\widehat{\alpha}_1} = \frac{da}{d\widetilde{\mathbb{V}}}\left(\frac{1}{1-\widehat{\alpha}_1}\right)$$

A.3   The stock market

Here, we connect the expected profit, $\pi(\cdot)$, in the price function to the firm's problem. For brevity, we only show the algebra for case 1 (case 2 is very similar). The firm's profit is

$$
\begin{aligned}
&= \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} A_{it} K_{it}^{\alpha} - \left(1+\frac{\alpha_2}{\alpha_1}\right) RK_{it} \\
&= \left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} A_{it} \left(\frac{(\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}}}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di} K_t\right)^{\alpha} - \left(1+\frac{\alpha_2}{\alpha_1}\right)\frac{(\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}}}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di} RK_t \\
&= \underbrace{\left(\frac{N}{K_t}\right)^{\alpha_2} Y_t^{\frac{1}{\theta}} \left(\frac{K_t}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di}\right)^{\alpha}}_{\Gamma_1} A_{it} (\mathbb{E}_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} - \underbrace{\frac{\left(1+\frac{\alpha_2}{\alpha_1}\right) RK_t}{\int (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} di}}_{\Gamma_2} (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}} \\
&= \Gamma_1 A_{it} (\mathbb{E}_{it}[A_{it}])^{\frac{\alpha}{1-\alpha}} - \Gamma_2 (\mathbb{E}_{it}[A_{it}])^{\frac{1}{1-\alpha}}
\end{aligned}
$$

From the definition of the firm's information set,

$$\mathbb{E}_{it}[A_{it}] = \mathbb{E}\left(A_{it}|a_{it-1}, a_{it}+e_{it}, a_{it}+\sigma_v z_{it}\right)$$

The profit function $\pi(\cdot)$ is obtained after integrating out the noise term in the firm's signal

$$
\begin{aligned}
\pi(a_{it-1}, a_{it}, a_{it}+\sigma_v z_{it}) &= \Gamma_1 \int A_{it}\left[\mathbb{E}\left(A_{it}|a_{it-1}, a_{it}+e_{it}, a_{it}+\sigma_v z_{it}\right)\right]^{\frac{\alpha}{1-\alpha}} d\Phi\left(\frac{e_{it}}{\sigma_e}\right) \\
&\quad -\Gamma_2 \int \left(\left[\mathbb{E}\left(A_{it}|a_{it-1}, a_{it}+e_{it}, a_{it}+\sigma_v z_{it}\right)\right]\right)^{\frac{1}{1-\alpha}} d\Phi\left(\frac{e_{it}}{\sigma_e}\right)
\end{aligned}
$$

A.4 Special case: Transitory shocks

Identification. A log-linear approximation of prices (around the deterministic case, ignoring constants):[59]

$$
\begin{aligned}
P \exp(p_{it}) &= \tilde{\mathbb{E}}_{it} A K^\alpha \exp(a_{it} + \alpha k_{it}) - \tilde{\mathbb{E}}_{it} R K \exp(k_{it}) + \beta \tilde{\mathbb{E}}_{it} P_{it+1} \\
\Rightarrow P + P p_{it} &\approx A K^\alpha - R K + \beta P + A K^\alpha \tilde{\mathbb{E}}_{it}(a_{it} + \alpha k_{it}) - R K \tilde{\mathbb{E}}_{it} k_{it} + \beta \tilde{\mathbb{E}}_{it} P p_{it+1} \\
P p_{it} &\approx A K^\alpha \tilde{\mathbb{E}}_{it} a_{it} + (\alpha A K^\alpha - R K) \frac{\tilde{\mathbb{E}}[\mathbb{E} a_{it}]}{1 - \alpha} + \beta \tilde{\mathbb{E}}_{it} P p_{it+1} \\
&= A K^\alpha \tilde{\mathbb{E}}_{it} a_{it} + \beta \tilde{\mathbb{E}}_{it} P p_{it+1} \\
p_{it} &= \frac{A K^\alpha}{P} \tilde{\mathbb{E}}_{it} a_{it} + \beta \tilde{\mathbb{E}}_{it} p_{it+1} + \text{Const.}
\end{aligned}
$$

where $\tilde{\mathbb{E}}$ denotes the marginal investor's expectations. Guess-and-verify

$$
p_{it} = \xi \tilde{\mathbb{E}}_{it} a_{it}
$$

Substituting,

$$
\xi \tilde{\mathbb{E}}_{it} a_{it} = \frac{A K^\alpha}{P} \tilde{\mathbb{E}}_{it} a_{it} + \beta \xi \tilde{\mathbb{E}}_{it} a_{it+1} \qquad \Rightarrow \qquad \xi = \frac{Y}{P} = \frac{1 - \beta}{1 - \alpha}
$$

Variances (of growth rates):

$$
\begin{aligned}
\sigma_p^2 &\equiv \operatorname{Var}(p_{it} - p_{it-1}) = 2\psi^2 \xi^2 (\sigma_\mu^2 + \sigma_v^2 \sigma_z^2) = 2\psi^2 \xi^2 \left(1 + \frac{\sigma_v^2 \sigma_z^2}{\sigma_\mu^2}\right) \sigma_u^2 \\
\sigma_k^2 &\equiv \operatorname{Var}(k_{it} - k_{it-1}) = 2(\phi_1 + \phi_2)^2 \sigma_\mu^2 + \phi_1^2 \sigma_e^2 + \phi_2^2 \sigma_v^2 \sigma_z^2 \\
&= 2 \frac{\left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2 \sigma_z^2}\right)^2 \sigma_u^2 + \left(\frac{1}{\sigma_e^2}\right)^2 \sigma_e^2 + \left(\frac{1}{\sigma_v^2 \sigma_z^2}\right)^2 \sigma_v^2 \sigma_z^2}{\left(\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2 \sigma_z^2}\right)^2} \\
&= 2 \left(\frac{\frac{1}{\sigma_v^2 \sigma_z^2} + \frac{1}{\sigma_e^2}}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2 \sigma_z^2}}\right) \sigma_\mu^2 = 2(\sigma_\mu^2 - \mathbb{V}) \\
\sigma_a^2 &\equiv \operatorname{Var}(a_{it} - a_{it-1}) = 2\sigma_\mu^2
\end{aligned}
$$

Covariances (of growth rates):

---

[59]The aggregate constants multiplying revenues are normalized to 1.

$$
\begin{aligned}
\text{Cov}\,(p,k) &\equiv \text{Cov}\,(p_{it} - p_{it-1}, k_{it} - k_{it-1}) \\
&= 2\xi\psi\left(\phi_1 + \phi_2)\sigma_\mu^2 + \phi_3\sigma_v^2\sigma_z^2\right) \\
&= 2\xi\psi\left(\frac{\left(\frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}\right)\sigma_\mu^2 + \left(\frac{1}{\sigma_v^2\sigma_z^2}\right)\sigma_v^2\sigma_z^2}{\frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_e^2} + \frac{1}{\sigma_v^2\sigma_z^2}}\right) = 2\xi\psi\sigma_\mu^2 \\
\text{Cov}\,(p,a) &= 2\xi\psi\sigma_\mu^2
\end{aligned}
$$

The correlations:

$$
\begin{aligned}
\rho_{pa} &= \frac{\text{Cov}\,(p,a)}{\sigma_p\sigma_a} = \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2 + \sigma_v^2\sigma_z^2\right)}} = \frac{1}{\sqrt{1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}} \\
\rho_{pk} &= \frac{\text{Cov}\,(p,k)}{\sigma_p\sigma_k} = \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2 + \sigma_v^2\sigma_z^2\right)\left(\sigma_u^2 - \mathbb{V}\right)}} \\
&= \frac{1}{\sqrt{\left(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\left(1 - \frac{\mathbb{V}}{\sigma_\mu^2}\right)}}
\end{aligned}
$$

The volatility of returns:

$$
\begin{aligned}
\sigma_p^2 &= \xi^2\left(\frac{\sigma_z^2 + 1}{\sigma_z^2 + 1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}\right)^2\left(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\sigma_u^2 \\
&= \xi^2\left(\frac{\sigma_z^2 + 1}{\sigma_z^2 + \frac{1}{\rho_{pa}^2}}\right)^2\frac{1}{\rho_{pa}^2}\sigma_\mu^2
\end{aligned}
$$

Other distortions. Since distortions have only a second order effect on profits, the only moment that changes is ρ_{pk}. With correlated distortions, $k_{it} = \frac{1+\gamma}{1-\alpha}\mathbb{E}a_{it}$, which implies

$$
\begin{aligned}
\rho_{pk} &= \frac{\text{Cov}\,(p,k)}{\sigma_p\sigma_k} = \frac{2\xi\psi\sigma_\mu^2\left(1 + \gamma\right)}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2 + \sigma_v^2\sigma_z^2\right)\left(\sigma_u^2 - \mathbb{V}\right)\left(1 + \gamma\right)^2}} \\
&= \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2 + \sigma_v^2\sigma_z^2\right)\left(\sigma_u^2 - \mathbb{V}\right)}}
\end{aligned}
$$

the same as before. In the uncorrelated case,

$$
\begin{aligned}
\rho_{pk} &= \frac{\text{Cov}\,(p,k)}{\sigma_p \sigma_k} = \frac{2\xi\psi\sigma_\mu^2}{\sqrt{4\psi^2\xi^2\left(\sigma_\mu^2+\sigma_v^2\sigma_z^2\right)\left(\sigma_u^2+\sigma_\varepsilon^2-\mathbb{V}\right)}} \\
&= \frac{1}{\sqrt{\left(1+\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)\left(1-\frac{\mathbb{V}}{\sigma_\mu^2}+\frac{\sigma_\varepsilon^2}{\sigma_\mu^2}\right)}}
\end{aligned}
$$

Re-arranging yields the expression in the text.

Correlated signals. Here, we present a alternative, more direct derivation of equation (20), which does not rely on the details of the information structure. Since all shocks are iid, p_{it} is an iid random variable, uncorrelated with past and future realizations of a_{it}.

$$
\begin{aligned}
\text{Cov}\,(p_{it}-p_{it-1},a_{it}-a_{it-1}) &= \text{Cov}\,(p_{it},a_{it})+\text{Cov}\,(p_{it-1},a_{it-1}) \\
&= 2\text{Cov}\,(p_{it},a_{it}) \\
&= 2\text{Cov}\,(p_{it},\mathbb{E}a_{it}+\varpi_{it}) \\
&= 2\text{Cov}\,(p_{it},\mathbb{E}a_{it})+2\text{Cov}\,(p_{it},\varpi_{it}) \\
&= 2\text{Cov}\,(p_{it},\mathbb{E}a_{it})
\end{aligned}
$$

where ϖ_{it} denotes the firm's forecast error, which is uncorrelated with firm information - in particular, with any element in the firm information set. Therefore, $\text{Cov}(p_{it},\varpi_{it})=0$. Note that this is true independent of the correlation structure between p_{it} and the other elements of that information set. Now, we use $\text{Cov}(p_{it},k_{it})=\frac{1}{1-\alpha}\text{Cov}(p_{it},\mathbb{E}a_{it})$ and divide both sides of the equation by $\sigma\,(p_{it}-p_{it-1})\,\sigma\,(a_{it}-a_{it-1})$

$$
\begin{aligned}
\frac{\text{Cov}\,(p_{it}-p_{it-1},a_{it}-a_{it-1})}{\sigma\,(p_{it}-p_{it-1})\,\sigma\,(a_{it}-a_{it-1})} &= \frac{2\,(1-\alpha)\,\text{Cov}\,(p_{it},k_{it})}{\sigma\,(p_{it}-p_{it-1})\,\sigma\,(a_{it}-a_{it-1})}\frac{\sigma\,(k_{it}-k_{it-1})}{\sigma\,(k_{it}-k_{it-1})} \\
&= \frac{\text{Cov}\,(p_{it}-p_{it-1},k_{it}-k_{it-1})}{\sigma\,(p_{it}-p_{it-1})\,\sigma\,(k_{it}-k_{it-1})}\frac{(1-\alpha)\,\sigma\,(k_{it}-k_{it-1})}{\sigma\,(a_{it}-a_{it-1})} \\
\Rightarrow \rho_{pa} &= \rho_{pk}\frac{(1-\alpha)\,\sigma\,(k_{it}-k_{it-1})}{\sigma\,(a_{it}-a_{it-1})}
\end{aligned}
$$

Since

$$
\begin{aligned}
\sigma^2 \left(k_{it} - k_{it-1} \right) &= \left(\frac{1}{1-\alpha} \right)^2 2\sigma^2 \left(\mathbb{E}a_{it} \right) = \left(\frac{1}{1-\alpha} \right)^2 2 \left(\sigma_\mu^2 - \mathbb{V} \right) \\
&= \left(\frac{1}{1-\alpha} \right)^2 2\sigma_\mu^2 \left(1 - \frac{\mathbb{V}}{\sigma_\mu^2} \right) \\
&= \left(\frac{1}{1-\alpha} \right)^2 \sigma^2 \left(a_{it} - a_{it-1} \right) \left(1 - \frac{\mathbb{V}}{\sigma_\mu^2} \right) \\
\Rightarrow \quad & \frac{(1-\alpha)\,\sigma\,(k_{it} - k_{it-1})}{\sigma\,(a_{it} - a_{it-1})} = \sqrt{1 - \frac{\mathbb{V}}{\sigma_\mu^2}}
\end{aligned}
$$

Combining yields (20).

$$
\rho_{pa} = \rho_{pk} \sqrt{1 - \frac{\mathbb{V}}{\sigma_\mu^2}}
$$

A.5   Special case: Permanent shocks

Profits in period t are

$$
\pi_{it} = HA_{it}K_{it}^\alpha - RK_{it}
$$

where H is a constant term, summarizing general equilibrium interactions. This can be written as

$$
\begin{aligned}
\pi_{it} &= He^{\rho a_{it-1} + u_{it}} e^{(\rho a_{it-1} + \mathbb{E}_{it} u_{it} + \text{Const}) \frac{\alpha}{1-\alpha}} - Re^{(\rho a_{it-1} + \mathbb{E}_{it} u_{it} + \text{Const}) \frac{1}{1-\alpha}} \\
&= e^{\frac{\rho a_{it-1}}{1-\alpha}} \left[He^{u_{it}} e^{(\mathbb{E}_{it} u_{it} + \text{Const}) \frac{\alpha}{1-\alpha}} - Re^{(\mathbb{E}_{it} u_{it} + \text{Const}) \frac{1}{1-\alpha}} \right]
\end{aligned}
$$

When $\rho = 1$, i.e., shocks are permanent, we guess that the ex-dividend component[60] of price function (??) takes the form

$$
\mathcal{P}\left(a_{-1}, a, z \right) = e^{\frac{a_{-1}}{1-\alpha}} \hat{\mathcal{P}}\left(a - a_{-1}, z \right) = e^{\frac{a_{-1}}{1-\alpha}} \hat{\mathcal{P}}\left(u, z \right)
$$

Direct substitution into equation (??) verifies the guess (the term $e^{\frac{a_{-1}}{1-\alpha}}$ multiplies all the terms from both sides). In logs, this can be written as

$$
\begin{aligned}
p_{it-1} &= \log \left[\tilde{\mathbb{E}}_{it-1} e^{\frac{a_{it-1}}{1-\alpha}} \hat{\mathcal{P}}\left(u, z \right) \right] \\
&= \log \left[\tilde{\mathbb{E}}_{it-1} \left(e^{\frac{a_{it-1}}{1-\alpha}} \right) \tilde{\mathbb{E}}_{it-1} \hat{\mathcal{P}}\left(u, z \right) \right] \\
&= \frac{1}{1-\alpha} \tilde{\mathbb{E}}_{it-1} a_{it-1} + \text{const.}
\end{aligned}
$$

---

[60]The ex-dividend price in $t-1$ is the marginal investor's expectation of the price in t.

where the last two lines follow from the fact that (u, z) is i.i.d. This is the affine representation of the ex-dividend price used in the text.

To derive the moments of interest, note that with permanent shocks

$$\tilde{\mathbb{E}}a_{it} = a_{it-1} + \tilde{\mathbb{E}}\mu_{it}, \qquad \mathbb{E}a_{it} = a_{it-1} + \mathbb{E}\mu_{it}$$

which leads to

$$
\begin{aligned}
p_{it} &= \frac{1}{1-\alpha}a_{it-1} + \frac{1}{1-\alpha}\psi\left(u_{it} + \sigma_v z_{it}\right) + \text{Const}\\
(1-\alpha)\,k_{it} &= a_{it-1} + \phi_1\mu_{it} + \phi_2 e_{it} + \phi_3\sigma_v z_{it} + \text{Const.}
\end{aligned}
$$

where the coefficients ψ, ϕ_1 and ϕ_2 are the same as in the i.i.d. case. In growth rates,

$$
\begin{aligned}
\Delta a_{it} &= \mu_{it}\\
\Delta p_{it} &= \frac{1}{1-\alpha}\left(a_{it-1} - a_{it-2}\right) + \frac{1}{1-\alpha}\psi\left(\mu_{it} + \sigma_v z_{it} - \mu_{it-1} - \sigma_v z_{it-1}\right)\\
&= \frac{1}{1-\alpha}\left(1-\psi\right)\mu_{it-1} + \frac{1}{1-\alpha}\psi\mu_{it} + \xi\psi\sigma_v\left(z_{it} - z_{it-1}\right)\\
(1-\alpha)\,\Delta k_{it} &= \left(a_{it-1} - a_{it-2}\right) + \phi_1\left(\mu_{it} - \mu_{it-1}\right) + \phi_2\left(e_{it} - e_{it-1}\right) + \phi_3\sigma_v\left(z_{it} - z_{it-1}\right)\\
&= \left(1-\phi_1\right)\mu_{it-1} + \phi_1\mu_{it} + \phi_2\left(e_{it} - e_{it-1}\right) + \phi_3\sigma_v\left(z_{it} - z_{it-1}\right)
\end{aligned}
$$

Second moments (of growth rates):

$$
\begin{aligned}
\sigma_p^2 &= \left(\frac{1}{1-\alpha}\right)^2(1-\psi)^2\sigma_\mu^2 + \left(\frac{1}{1-\alpha}\right)^2\psi^2\sigma_\mu^2 + 2\left(\frac{1}{1-\alpha}\right)^2\psi^2\sigma_v^2\sigma_z^2\\
&= \left(\frac{1}{1-\alpha}\right)^2\sigma_\mu^2\left[1 - 2\psi + 2\psi^2(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2})\right] & (24)\\
\sigma_k^2 &= \left(1 - \phi_1 - \phi_2\right)^2\sigma_\mu^2 + \left(\phi_1 + \phi_2\right)\sigma_\mu^2 + 2\phi_1^2\sigma_e^2 + 2\phi_2^2\sigma_v^2\sigma_z^2 & (25)\\
&= \sigma_\mu^2 - 2\left(\sigma_\mu^2 - \mathbb{V}\right) + 2\sigma_\mu^2 - 4\mathbb{V} + 2\frac{\mathbb{V}^2}{\sigma_\mu^2} + 2\frac{\mathbb{V}^2}{\sigma_e^2} + 2\frac{\mathbb{V}^2}{\sigma_v^2\sigma_z^2} & (26)\\
&= \sigma_\mu^2 & (27)\\
\text{Cov}(p, a) &= \frac{1}{1-\alpha}\psi\sigma_\mu^2 & (28)
\end{aligned}
$$

55

$$\text{Cov}(p,k) = \frac{1}{1-\alpha}\left[(1-\psi)(1-\phi_1-\phi_2)\sigma_\mu^2 + \psi(\phi_1+\phi_2)\sigma_\mu^2 + 2\psi\phi_2\sigma_v^2\sigma_z^2\right]$$

$$= \frac{1}{1-\alpha}\left[(1-\psi-\phi_1-\phi_2)\sigma_\mu^2 + 2\psi(\phi_1+\phi_2)\sigma_\mu^2 + 2\psi\phi_2\sigma_v^2\sigma_z^2\right]$$

$$= \frac{1}{1-\alpha}\left[(1-\psi-\phi_1-\phi_2)\sigma_\mu^2 + 2\psi(\sigma_\mu^2 - \mathbb{V}) + 2\psi\mathbb{V}\right]$$

$$= \frac{1}{1-\alpha}\left(\mathbb{V} + \psi\sigma_\mu^2\right)$$

Directly,

$$\text{Cov}(p,k) - \text{Cov}(p,a) = \frac{1}{1-\alpha}\mathbb{V}$$

$$\rho_{pk}\sigma_p\sigma_k - \rho_{pa}\sigma_p\sigma_a =$$

$$\left(\rho_{pk} - \rho_{pa}\right)\sigma_p\sigma_\mu =$$

Re-arranging yields the expression in the text.

$$\rho_{pk} = \frac{\psi + \frac{\mathbb{V}}{\sigma_\mu^2}}{\sqrt{1 - 2\psi + 2\psi^2(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2})}}$$

$$\rho_{pa} = \frac{\psi}{\sqrt{1 - 2\psi + 2\psi^2\left(1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}\right)}} = \frac{1}{\sqrt{\left(\frac{1}{\psi} - 1\right)^2 + 1 + 2\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}}$$

Then,

$$\psi = \frac{\frac{\mathbb{V}}{\sigma_\mu^2}}{\frac{\rho_{pk}}{\rho_{pa}} - 1} = \frac{\left(\rho_{pk} - \rho_{pa}\right)\sigma_p}{\frac{1}{1-\alpha}\sigma_\mu\left(\frac{\rho_{pk}}{\rho_{pa}} - 1\right)} = \frac{\rho_{pa}\sigma_p}{\frac{1}{1-\alpha}\sigma_\mu} = \frac{\rho_{pa}}{\eta}$$

Given ψ, we then use the expression for ρ_{pa} to solve for $\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}$:

$$\rho_{pa}^2 = \frac{1}{\left(\frac{\eta}{\rho_{pa}} - 1\right)^2 + 1 + 2\frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}$$

$$\Rightarrow \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2} = \frac{1}{2\rho_{pa}^2} - \frac{1}{2}\left(\frac{\eta}{\rho_{pa}} - 1\right)^2 - \frac{1}{2}$$

$$= \frac{(1-\eta^2)}{2\rho_{pa}^2} + \frac{\eta}{\rho_{pa}} - 1$$

Finally, combining this with the definition of $\psi \equiv \frac{\sigma_z^2 + 1}{\sigma_z^2 + 1 + \frac{\sigma_v^2\sigma_z^2}{\sigma_\mu^2}}$, we can disentangle σ_z^2 and σ_v^2.

B Adjustment Costs

When firms are fully informed but subject to capital adjustment costs, their capital choice is a dynamic decision, and the value function takes the form:

$$V\left(\tilde{A}_{it}, K_{it-1}\right) = \max_{K_{it}, N_{it}} G\tilde{A}_{it}K_{it}^{\tilde{\alpha}} - K_{it} - H\left(K_{it}, K_{it-1}\right) + \beta \mathbb{E}V\left(\tilde{A}_{it+1}, (1-\delta)K_{it}\right)$$

where

$$H\left(K_{it}, K_{it-1}\right) = \zeta K_{it-1}\left(\frac{K_{it}}{K_{it-1}} - 1\right)^2$$

To characterize G and $\tilde{A}_{it}$, we begin similarly to case 2 in the text and examine the firm's labor choice problem

$$Y_t^{\frac{1}{\theta}}A_{it}K_{it}^{\alpha_1}N_{it}^{\alpha_2} - WN_{it}$$

where α's are as defined in (2). Optimizing over N_{it} and substituting back into the objective gives

$$P_{it}Y_{it} - WN_{it} = (1-\alpha_2)\left(\frac{\alpha_2}{W}\right)^{\frac{\alpha_2}{1-\alpha_2}}Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\tilde{A}_{it}K_{it}^{\tilde{\alpha}}$$

where $\tilde{\alpha}$ and $\tilde{A}_{it}$ are defined as in case 2 in the text. We can then solve for

$$G = (1-\alpha_2)\left[\left(\frac{\alpha_2}{W}\right)^{\alpha_2}Y_t^{\frac{1}{\theta}}\right]^{\frac{1}{1-\alpha_2}}$$

Next, using the firm's labor optimality condition, labor market clearing implies

$$\left(\frac{\alpha_2}{W}\right)^{\frac{1}{1-\alpha_2}}Y_t^{\frac{1}{\theta}\frac{1}{1-\alpha_2}}\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}}di = N$$

from which we can solve for

$$\left(\frac{\alpha_2}{W}\right)^{\alpha_2}Y_t^{\frac{1}{\theta}} = \left[\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}} \ di\right]^{\frac{1-\alpha_2}{\theta-1}(1-\theta\alpha_2)}N^{\frac{\theta}{\theta-1}\alpha_2(1-\alpha_2)}$$

and so

$$G = (1-\alpha_2)\left[\int \tilde{A}_{it}K_{it}^{\tilde{\alpha}} \ di\right]^{\frac{1-\theta\alpha_2}{\theta-1}}N^{\frac{\theta}{\theta-1}\alpha_2}$$

Note that under full information, the production side of the economy is completely decoupled from stock markets, so we have all we need to solve the model laid out above. Starting with a guess for the general equilibrium term G, we solve for the value functions (using a standard iterative procedure and a discretized grid for capital), simulate to obtain the steady state

distributions and verify that our initial guess for G is consistent with that distribution. If not, we update the guess and iterate until convergence.

We then solve for stock prices, again using the conjecture that the informed investors follow a threshold rule. Proceeding exactly as in the baseline model, we can then derive the following functional equation for the price function

$$
\begin{aligned}
P\left(a_{it-1}, k_{it-1}, a_{it}, z_{it}\right) &= \int \pi\left(a_{it}, k_{it-1}\right) H\left(a_{it} | a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it}\right) \\
&\quad + \beta \int \bar{P}\left(a_{it}, k^*\left(a_{it}, k_{it-1}\right)\right) H\left(a_{it} | a_{it-1}, a_{it} + \sigma_v z_{it}, P_{it}\right)
\end{aligned}
$$

The model is parameterized as described in the text. Table 10 reports the full set of target moments and parameter estimates. Notice that the first two columns are identical to those in Table 2 (case 2). σ_k^2 denotes the variance of investment rates, which is used to pin down the size of the adjustment cost ζ.

Table 10: Targets and Parameters - Adjustment Cost Model

	Target moments			Parameters		
	ρ_{pa}	σ_p^2	σ_k^2	ζ	σ_v	σ_z
US	0.18	0.23	0.10	0.45	0.37	4.25
China	0.06	0.14	0.15	0.18	1.00	4.00
India	0.08	0.23	0.10	0.52	0.69	6.36

C Data

We use annual data on firm-level production variables and stock market returns from Compustat North America (for the US) and Compustat Global (for China and India). For each country, we exclude duplicate observations (firms with multiple observations within a single year), firms not incorporated within that country, and firms not reporting in local currency. We build three year production periods as the average of firm sales and capital stock over non-overlapping 3-year horizons (i.e., $K_{2012} = \frac{K_{2010} + K_{2011} + K_{2012}}{3}$, and analogously for sales). We measure the capital stock using gross property, plant and equipment (PPEGT in Compustat terminology), defined as the "valuation of tangible fixed assets used in the production of revenue." We then calculate investment as the change in the firm's capital stock relative to the preceding period. We construct the firm fundamental a_{it} as the log of revenue less the relevant α (which depends on the case) multiplied by the log of the capital stock. Finally, we first-difference the investment and fundamental series to compute investment growth and changes in fundamentals.

Stock returns are constructed as the change in the firm's stock price over the three year period, adjusted for splits and dividend distributions. We follow the procedure outlined in the Compustat manual. In Compustat terminology, and as in the text, using p_{it} as shorthand for log returns, returns for the US are computed as

$$p_{it} = \log\left(\frac{\text{PRCCM}_{it} * \text{TRFM}_{it}}{\text{AJEXM}_{it}}\right) - \log\left(\frac{\text{PRCCM}_{it-1} * \text{TRFM}_{it-1}}{\text{AJEXM}_{it-1}}\right)$$

where periods denote 3 year spans (i.e., returns for 2012 are calculated as the adjusted change in price between 2009 and 2012), PRCCM is the firm's stock price, and TRFM and AJEXM adjustment factors needed to translate prices to returns from the Compustat monthly securities file. Data are for the last trading day of the firm's fiscal year so that the timing lines up with the production variables just described. The calculation is analogous for China and India, with the small caveat that global securities data come daily, so that the Compustat variables are PRCCD, TRFD, and AJEXDI, where "D" denotes days. Again, the data are for the last trading day of the firm's fiscal year.

To extract the firm-specific variation in our variables, we regress each on a time fixed-effect and work with the residual. This eliminates the component of each series common to all firms in a time period and leaves only the idiosyncratic variation. As described in the text, we limit our sample to a single cross-section, namely 2012, and finally, we trim the 2% tails of each series. It is then straightforward to compute our calibration targets, i.e., σ_p^2, ρ_{pi}, and ρ_{pa}. As described in the text, we lag returns by one period, so that, e.g., ρ_{pi} is the correlation of 2006-09 returns with investment growth from 2009-12.

To estimate the parameters governing the evolution of firm fundamentals, i.e., the persistence ρ and variance of the innovations σ_μ^2, we perform the autoregression implied by (1). Here we use annual observations on a_{it} at a 3-year frequency in order to simplify issues of time aggregation. We estimate the process using our data from 2012 and 2009. We include a time fixed-effect in order to isolate the idiosyncratic component of the innovations in a_{it}. Differences in firm fiscal years means that different firms within the same calendar year are reporting data over different time periods, and so the time fixed-effect incorporates both the reporting year and month. The results from this regression deliver an estimate for ρ and σ_μ^2, from which we trim the 1% tails.

Our leverage adjustment is as follows: we assume that claims to firm profits are sold to investors in the form of both debt and equity in a constant proportion (within each country). This implies that the payoff from an equity claim is $S_{it} = V_{it} - D_{it-1}$, where V_{it} is the value of the unlevered firm and $D_{it-1} = d\mathbb{E}_t[V_{it}]$, where $d \in (0, 1)$ represents the share of expected firm value in the hands of debt-holders. In other words, firm value is allocated to investors as a debt claim

that pays off a constant fraction of its ex-ante expected value and as a residual claim to equity holders. The change in value of an equity claim is then equal to $\Delta S_{it} = \Delta V_{it}$ and dividing both sides by the ex-ante expected value of the claim (i.e., the price) $\overline{S} = \overline{V} - d\overline{V}$, where $\overline{V} = \mathbb{E}_t[V_{it}]$, gives returns as $\frac{\Delta S_{it}}{\overline{S}} = \frac{\Delta V_{it}}{(1-d)\overline{V}}$. Taking logs and computing variances shows $\sigma_{v_{it}}^2 = (1-d)^2\,\sigma_{s_{it}}^2$, i.e., the volatility in (unlevered) firm value is a fraction $(1-d)^2$ of the volatility in (levered) equity returns. To assign values to d in each country, we examine the debt-asset and debt-equity ratios of the set of firms in Compustat over the period 2006-2009. Because these vary to some degree from year to year and depend to some extent on the precise approach taken (i.e., whether we use debt-assets or debt-equity and whether we compute average ratios or totals), we simply take the approximate midpoints of the ranges for each country, which are about 0.30 for the US and India and 0.16 for China, leading to adjustment factors of about 0.5 and 0.7, respectively.