Choosing Not To Grow: How Bad Policies Can Be Outcomes of Dynamic Voting Equilibria

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Abstract

Some economic policies and regulations seem to have only one purpose: to prevent technological development and economic growth from occurring. In this paper, we attempt to rationalize such policies as outcomes of voting equilibria. In our environment, some agents will be worse off if the economy grows, since their skills are complementary to resources that can be allocated to growth-stimulating activities. In the absence of arrangements where votes are traded, we show that for some initial skill distributions, the economy may stagnate due to growth-preventing policies. Different initial skill distributions, however, lead to voting outcomes and policies in support of technological development, and to persistent economic growth. In making our argument formally, we use a dynamic model with induced heterogeneity in agents' skills. In their voting decisions, agents compare how they will be affected under each policy alternative, and then vote for the policy that maximizes their welfare.

1 Introduction

Perú is one example of a country where opening a new business has been very costly (De Soto (1989)). In the popular as well as in the scientific debate, the explicit and implicit taxes associated with the adoption of new technologies have been suggested as main deterrents of economic growth. The recent theoretical literature on economic growth in particular has provided many examples of how high tax rates on activities associated with capital accumulation can drastically worsen an economy's growth performance. From these perspectives, the Peruvian policy seems outright silly. The present work is an attempt to explain how such policies can be implemented in societies populated by rational economic agents, including (and in particular) those societies where political decisions are democratic.[1]

The obvious explanation that comes to mind is that it may be in the interest of some group of agents to maintain a no-growth situation. In particular, agents whose skills or capital are substitutes for the competing new technologies would tend to prefer high tax rates on the adoption of such technologies. If the political power of this group is strong enough, it would then seem possible that the status quo is maintained. There have indeed been many circumstances in which groups with apparent vested interests have been able, at least temporarily, to stall adoption of new technologies. One much-publicized example of this is the episode when computerized typesetting became available to the typographic industries across the industrial-

[1]Another example is Spain in the 40's and 50's where it was practically impossible to import capital goods. In general, we could also think this way of restrictive trade policies.

ized countries. There, we observed that the workers with skills specialized in the old type-setting technologies were quite successful at delaying the adoption of the new technologies.[2]

Our suspicion is that vested interests on different levels in society may well be quantitatively important impediments to the adoption of growth-stimulating policies. We use a stylized model to capture an aggregate version of this phenomenon. In the context of a simple dynamic model, we try to capture the conflicting interests inherent in the political decisions involving tax rates on adoption of new technologies, and we then go on to formalize the economic and political behavior of agents in such an environment.

Our framework of study builds on the vintage human capital model of Chari and Hopenhayn (1991). Individuals live for three periods and are characterized by their age and their human capital. Human capital is vintage specific. It takes two periods as a worker in a given vintage technology to acquire vintage-specific skills that enable the worker to become a manager in her third period of life. Managerial skills are complementary to raw labor and pay higher spot wage rates. Stationary equilibria without government intervention have a fraction of agents choosing the managerial career path, and the rest working as laborers throughout their lives. As in Chari and Hopenhayn, young agents are indifferent between the different career paths, since the present value of the higher-paying labor wages in the old technologies exactly match the lifetime income of the managers: the latter

[2] Again, other examples come to mind at the micro level. They include phenomena like regulatory capture, and a variety of arbitrary barriers to entry such as CPA and bar exams.

initially work as unskilled in new, low-paying vintages, but later earn more as managers.

We consider the possibility of tax policies that effectively make it too expensive to open up businesses, i.e. to adapt new technologies. Old and middle-aged agents on the managerial path have skills specialized in old, but still usable, technologies. Their skills are complementary to the young workers who could choose to work for them; the larger the number of workers they can attract, the higher their managerial salaries. So agents on the managerial path therefore tend to dislike innovations as these lower the value of their acquired capital, and the vested interests lead them to back policies for high taxes on the adoption of new technologies, even if these taxes have no role in funding public expenditures. On the other hand, young agents have no vested interests and prefer growth-oriented policies since growth means that more productive technologies are adapted and these imply higher life-time utilities. Finally, unskilled old workers also tend to prefer growth, since it makes them a more select group if some young workers choose the managerial career within younger vintages instead of working as competing laborers: older workers are perfect substitutes with younger workers. These considerations lead to a non-trivial voting outcome. This outcome is a function of the current distribution of skills in the economy, and it may or may not imply growth orientation.

Although the interplay of the economic forces and political behavior are non-trivial in our setup, a natural conjecture emerges: it seems possible that two very different kinds of stationary outcomes could result for the same economy. One of these is a steady state with growth and the other one without. In the growth outcome, there would be a sufficient number of agents among the middle-aged and old who do not have vested interests and who join the young in preferring and voting in favor of continuing growth. In the no-growth outcome the number of middle-aged and old agents with vested interests in the old technologies is high enough that majority voting implies maintained taxes on the adoption of new technologies.

To properly describe voting equilibria in our framework is not a trivial exercise. The vintage capital model necessarily has non-linear dynamics, and any voting considerations would involve predictions of how different tax choices would influence the future law of motion of the skill distribution, and hence the present as well as future wage rates that are of direct concern to the voting agent. The general considerations about outcomes of votes as conjectured above therefore have to be substantially refined before definite conclusions can be drawn about the characteristics of dynamic voting equilibria.

Given a recursive law of motion for the tax policy parameter, an equilibrium is defined by maximization of agents and a law of motion for the state variable — the distribution of agents over vintages — that is consistent with individual optimizing behavior. As noted above, given a sequence of taxes, the economic equilibrium in the vintage capital model has non-linear dynamics, and the effects of changing the current tax rate can only be analyzed by finding the future law of motion of the distribution of skills implied by different tax policy choices. Our definition of equilibrium with voting puts no restrictions on agents' abilities to forecast or calculate the dynamic effect of different policies. In particular, it is an essential element of the voting equilibrium that the voters, who have to view their voting decision as "what I would do if I were a dictator", need to consider the economic outcomes of tax policies which will never be realized.

We are able, for simple example economies, to verify the above conjectures. Indeed, for these economies, any initial skill distribution leads to either one of two long-term outcomes: growth (continual adoption of new technologies) or no growth (where new technologies are not allowed). Which steady state will be reached depends critically on the initial distribution of skills. The transition paths leading to the steady states may or may not exhibit switching voting patterns, and involves nontrivial changes in the skill distribution. As conjectured, the current composition of skills among middle-aged is the most important determinant for whether or not growth will occur. However, the distribution of skills among old agents may, in some situations, also play a key role.

The literature on voting in the framework of dynamic general equilibrium models with optimizing agents is to date limited, but growing. Among the papers about which we are currently informed, Alesina and Tabellini (1988), Tabellini and Alesina (1989), Glazer (1989), Perotti (1990), Persson and Tabellini (1991), and Saint-Paul and Verdier (1992) are all examples of successful attempts to deal with this issue. A general approach is hard to implement since it involves analyzing dynamics of economies with heterogeneous agents. Typically, each one of the papers in the literature has provided some kind of short-cut that limits the complexity of the dynamics in order to allow manageable analysis, yet maintaining non-triviality of the main issue. We take a different approach in our work: we provide a general definition of recursive voting equilibria, and use recursive methods for computing these equilibria for as rich economic primitives as the present computer technology allows. We take this route for two reasons. First, there does not seem to be a convenient and powerful enough simplification for the problem at hand. Second, our recursive equilibrium definition is general enough to apply to a large set of economies, and can hence be used for other applications, subject to current computational limitations. We believe this generality is valuable because we envision quantitative-theoretic work as an important part of the research program on political economics, and such work is difficult to carry out if the theory is based on very restrictive assumptions about the economic or political environment.

Naturally, all the considerations here are made assuming that votes cannot be traded. Equilibria therefore may be inefficient. This is an unavoidable property of democratic societies: the voting activity, which is a decision with economic implications, is private and as such cannot be subject to trade or used as a contingency in contracts between agents.

In section 2 we lay out our version of the Chari and Hopenhayn framework, and in section 3 we discuss some properties of stationary equilibria when the tax policy is taken as exogenous. These stationary equilibria suggest how votes of different groups might turn out, and also hints why a more sophisticated analysis is needed. Section 4 makes this point clear, and then goes on to define equilibria with voting, and section 5 applies the equilibrium definition to compute equilibria for simple example economies.

2 The Model

The Chari and Hopenhayn vintage human capital model conveniently captures the tensions within a heterogeneous distribution of agents over capital types: some agents' specialization is complementary to the adaption of new technologies, whereas others' substitute for new technologies. We use the original model with some minor alterations. The economy has infinite time horizon, and is inhabited by overlapping generations of agents each of whom lives for three periods. We assume that each cohort of agents is of size one. The description of preferences and technology is as follows.

Preferences: The utility function has the form

$$c_1 + \beta c_2 + \beta^2 c_3, \tag{1}$$

where c_i is the consumption of an agent at age i.

Technology: The consumption good can be produced from different technologies. Each one of these technologies is constant returns to scale in labor inputs. Labor is of two kinds: managers and workers. For the purpose of illustration, we have chosen a simple production function:

$$f_\kappa(m,n) = \gamma^\kappa m^\alpha n^{1-\alpha}, \qquad 0 < \alpha < 1, \gamma > 1, \tag{2}$$

where κ is the generation of technology with productivity index κ, m is the input of managers and n is the input of workers. A new generation of technology is available for adoption every time period. Each new technology improves by a factor γ on the best existing technology. We assume that technologies have to be adapted in order: if in a given period the best technology in use is of type κ, then the next technology adopted is by necessity $\kappa + 1$, independently of when it is adopted. Chari and Hopenhayn have a slightly different assumption in this regard; they assume that technologies arrive exogenously, one every period, whether previous technologies were adopted or not. Our assumption means that the technologies arrive at a rate that is endogenously determined in the sense explained above, but we follow Chari and Hopenhayn in assuming that any new technology improves by an exogenous factor γ on the best existing technology.

We assume that it takes an agent two consecutive periods of working in a technology to achieve managerial skills for this technology. This implies, for the particular production function we choose, that there cannot be any production in a new technology κ until two periods after its adoption, since managers are a necessary input to produce. Agents on the managerial career path can hence either choose to become managers for an already existing technology, and earn positive wages throughout their career, or choose to adopt a new technology and earn no wage during the periods of learning but achieve higher productivity in the third period. We label managers who adopt new technologies students because of this lack of productivity; in particular we refer to them as 'undergraduates' after their first year working year, and 'graduates' after their second.

Formally, we let x denote the type of an agent at the beginning of a given period. The variable x is a member of X, which contains all combinations of age and work experience. The distribution of agents can hence be described by the measure $\mu : \sigma(X) \to \mathrm{R}_+$, where $\sigma(X)$ is a sigma-algebra on X. We will not list the elements of X here; we will, however, indicate its most important components.

We label the age component of x with $i \in \{0, 1, 2, 3\}$ referring to 'inactive', 'young', 'middle-aged', and 'old' agents, respectively. The experience component of x indicates the technology, and we use the variable τ to label technologies. As of the beginning of the current period, there is a number of existing technologies, some of which have been in production in previous periods, and some under development. There is also a new innovation technology that can be adapted in the current period by the current young. We refer to this innovation technology as technology -1. Furthermore, there can at most be two technologies under development as of the beginning of the period: the technology adapted and studied last period by some currently middle-aged agents, which we refer to as technology 0; and the technology adapted the period before that by the current old, which we call technology 1. Finally, technologies $\tau \in \{2, 3, \ldots\}$ then refer to successively older vintages, all of which were previously used in production.

Since young agents are born without productive knowledge, and the size of cohorts in our simple version of the model is constant, the part of the distribution of agents that is relevant as a state variable of the economy is the skill distribution, i.e. the specification of experience of middle-aged and old agents. The assumption that agents live for three periods implies that a given agent can work in three different technologies during the lifetime. If an agent switches technologies after the first period, then this agent cannot become a manager since it requires two periods of work in the same technology to qualify as a manager. There can hence be five types of agents in the distribution of skills: old who switched; old who did not switch – those experienced in new and those in old technologies; and middle-aged – those in old and those in new technologies. Old who switched have no productive experience and hence need no label. For this type, we first define

$\mu_s = \mu(\text{old agents who switched technologies}).$

Furthermore, old who did not switch as well as middle-aged also carry the label of their specialization, i.e. their technology generation. Let $\mu_{i,\tau}$ be the measure of agents of age $i \in \{2,3\}$ with experience in technology τ, and use the matrix μ_{ns} to organize these agents:

$$\mu_{ns} = \begin{pmatrix} \cdots & \mu_{3,\tau} & \cdots & \mu_{3,2} & \mu_{3,1} & \mu_{3,0} \\ \cdots & \mu_{2,\tau} & \cdots & \mu_{2,2} & \mu_{2,1} & \mu_{2,0} \end{pmatrix} \tag{3}$$

We also introduce the notation

$\mu_g = \mu_{3,1} = \mu(\text{graduates})$

$\mu_{u,\tau} = \mu_{2,\tau} = \mu(\text{undergraduates in technology } \tau \in \{0,1\}).$

The matrix therefore becomes

$$\mu_{ns} = \begin{pmatrix} \cdots & \mu_{3,\tau} & \cdots & \mu_{3,2} & \mu_g & 0 \\ \cdots & \mu_{2,\tau} & \cdots & \mu_{2,2} & \mu_{u,1} & \mu_{u,0} \end{pmatrix} \tag{4}$$

where we note that $\mu_{3,0}$ has to equal 0.

Government Power: We assume that it is in the power of the electorate to impose a 100% tax on opening up new businesses. More precisely, we mean by such a tax to be that young agents are not allowed to study the innovation technology, or, equivalently, that if they do, they are not allowed to operate this technology two periods hence. Retroactive laws forbidding new technologies are not considered, i.e. we assume that any technology that was allowed to be studied in the past can be operated in the present. We use π to denote our tax, with $\pi = 0$ meaning that new technologies can be studied, and $\pi = 1$ meaning they cannot.

The simple form of policy we consider is chosen mainly for analytical and computational convenience. It is important, however, to recognize that richer policy alternatives, particularly with respect to whether transfer schemes can be implemented, might fundamentally alter the predictions of the model. Any political equilibrium model should ideally consider all feasible policies as potential outcomes of the political process. For example, it seems reasonable to require that political competition should imply that no Pareto dominated outcome can be supported as a political equilibrium, provided that rich enough transfer schemes are available: a new party/candidate could then suggest a superior allocation and propose appropriate transfers to make sure to get more votes than the Pareto dominated policy proposal.

All existing dynamic general equilibrium voting models are subject to the potential critique above: they simply postulate given policy choices, and do not explain within the context of the model why alternative policies are not considered. Our approach can be interpreted in two ways. One is the standard way: we simply postulate two policies, which can be thought of as parties, representing two (polar) views on the policy in question. Another, and perhaps more appealing interpretation, is that the only feasible government policy in our environment is to close down or open the possibility of education. With this interpretation, transfers are simply not viewed as feasible. Particularly, transfers contingent on voting decisions are not feasible. This assumption is crude, but not necessarily unrealistic: it captures the imperfect inability of governments to commit to carrying out transfer programs.

We now need to specify the choices of agents.

Choice Variables: An agent's choice is denoted $y = (y_a, y_\tau; \tau \in \{-1, 0, 1, \ldots\})$, where $y_a \in \{\textit{manager, worker}\}$ refers to the activity undertaken in the current period. The variable $y_\tau \in \{0, 1\}$ is an indicator function for the technology chosen. As above, τ is the vintage as of the beginning of the current period: $\tau = -1$ refers to the innovation technology, $\tau = 0$ to the newest one already under development etc. Of course, y_τ can only be non-zero for one value of τ. Moreover, y is obviously constrained by the agent's state, x; for example, if x says that the agent is an old switcher, y_a cannot equal *manager*. The current tax policy π also constrains the agent's choice: if $\pi = 1$, then $y_1 = 0$. We use $y \in \Gamma(x, \pi)$ to summarize the constraints.

It is also useful to define aggregate measures of agents currently undertaking different activities:

m_τ = number of managers in τ, $\tau \in \{1, 2, \ldots\}$

$s_{1,-1}$ = number of young students in the innovation technology

$s_{1,0}$ = number of young students in technology 0

s_2 = number of middle-aged students in technology 0

$n_{1,\tau}$ = number of young workers in τ, $\tau \in \{1, 2, \ldots\}$

$n_{2,\tau,s}$ = number of switching middle-aged workers in τ, $\tau \in \{1, 2, \ldots\}$

$n_{2,\tau,ns}$ = number of non-switching middle-aged workers in τ, $\tau \in \{1, 2, \ldots\}$

$n_{3,\tau}$ = number of old workers in τ, $\tau \in \{1, 2, \ldots\}$

$n_\tau = n_{1,\tau} + n_{2,\tau,s} + n_{2,\tau,ns} + n_{3,\tau}$.

We denote a complete list of all these measures $\mathcal{Y}$, with $Y \in \mathcal{Y}$. Note that the individual constraints on choices implies that the aggregate choices have to satisfy:

$$m_1 \leq \mu_{3,1} \tag{5}$$

$$m_\tau \leq \mu_{3,\tau},\ \tau \in \{2, 3, \ldots\} \tag{6}$$

$$n_{2,1,ns} \leq \mu_{u,1} \tag{7}$$

$$n_{2,\tau,ns} \leq \mu_{u,\tau},\ \tau \in \{2, 3, \ldots\} \tag{8}$$

Laws of Motion: An agent's state x and current choice y determine the agent's future state x'. We write $x' = \xi(x, y)$ to describe this updating. Likewise, the current state of the economy and agents' choices together determine the future distribution μ'. The aggregate law of motion of the skill distribution (μ_s, μ_{ns}) satisfies

$$\mu_s' = \sum_\tau n_{2,\tau,s} \tag{9}$$

$$\mu'_g = s_2 \tag{10}$$

$$\mu'_{u,0} = s_{1,-1} \tag{11}$$

$$\mu'_{u,1} = s_{1,0} \tag{12}$$

$$\mu'_{3,\tau} = \begin{cases} n_{2,\tau-1,ns} & \text{if there is growth, i.e. if } m_1 > 0 \\ n_{2,\tau,ns} & \text{otherwise, i.e. if } m_1 = 0 \end{cases} \tag{13}$$

$$\mu'_{2,\tau} = \begin{cases} n_{1,\tau-1} & \text{if there is growth, i.e. if } m_1 > 0 \\ n_{1,\tau} & \text{otherwise, i.e. if } m_1 = 0 \end{cases} \tag{14}$$

The distinction between the cases of growth and no growth in the updating of μ is necessary because our technologies are labeled relative to the newest technology.

With this the description of the environment is complete.

3 Stationary Equilibria With and Without Growth

We now turn to describing equilibria for this environment. We adopt a recursive equilibrium definition: equilibrium prices and quantities are functions of the current state of the economy. In this section we treat taxes as exogenously set at either 0 or 1, and we furthermore focus on stationary equilibria, i.e. equilibria in which the skill distribution is constant. We postpone a detailed definition of equilibria to section 4, where we also model the political process determining the tax rates.

The decentralization we employ is parallel to that in Chari and Hopenhayn. There are competitive firms operating each technology, and managers and workers are hired at wage rates specific to this technology. Agents choose their activities based on the available wage prospects. The current wage rates associated with different activities are treated parametrically by the agent and depend on the aggregate choices of agents. The aggregate choices of agents in turn depend on the current skill distribution. The equilibrium is defined by agents choosing activities optimally during their life-time, by firms maximizing profits, and by agents' choices reproducing the price and quantity functions as well as the aggregate law of motion that they take as given in their maximization problems.

Some special attention needs to be paid to the possibility of an agent managing a technology that is currently not operated by other agents. We assume in this case that the agent can hire labor at the highest current wage rate, and that she becomes residual claimant to any profits accruing to the operated technology.

3.1 The No-Growth Equilibrium

We first treat the case of $\pi \doteq 1$, i.e. the case when the adaption of new technologies is not allowed. We conjecture the form of a stationary equilibrium and proceed to verify the conjecture and that the conjectured stationary equilibrium is unique.

The conjecture is as follows. There is only one technology in operation and all agents hence make the same choices in the first two periods. For values of $\alpha \geq \frac{1}{3}$ all agents become managers, for $\alpha < \frac{1}{3}$ a fraction $\phi < 1$

become managers and the rest workers. The skill matrix therefore becomes

$$\mu_{ns} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{15}$$

The wage rates associated with the conjectured equilibrium are

$$w_m = \alpha\gamma^\kappa \left(\frac{3-\phi}{\phi}\right)^{1-\alpha} \tag{16}$$

$$w_n = (1-\alpha)\gamma^\kappa \left(\frac{\phi}{3-\phi}\right)^{\alpha} \tag{17}$$

and it follows that

$$\frac{w_m}{w_n} = \frac{\alpha}{1-\alpha}\frac{3-\phi}{\phi} \tag{18}$$

so that $\phi \leq 1$ implies that $w_m \geq w_n$ if and only if $\alpha \leq \frac{1}{3}$. We hence find that the fraction ϕ has to equal 1 if $\alpha \geq \frac{1}{3}$ and that if $\alpha < \frac{1}{3}$, then $\phi = 3\alpha$ and the wage rates are equal.

Do agents optimize at these prices? Since the option of opening up new technologies is not available, there is only one technology to go to. The manager/worker decision is trivial in the third period and our prices guarantee that the proposed ϕ is consistent with optimization.

The proposed equilibrium is also unique. To see this, suppose to the contrary that there are more than one technology in operation. First, assume that this alternative equilibrium would not have agents switching technologies. Then, because of stationarity, it would have to be true that $m_\tau = n_\tau/2$

for all τ so that the ratio of inputs would be the same in all technologies. This, however, would imply that wage rates are different in different technologies, since total-factor productivity differs across technologies. In particular, the highest productivity technology would have uniformly higher wage rates, violating the assumption that not all agents are in one technology. Second, assume that there is switching between technologies. Agents who switch always pick the highest spot wage rate, so if there is switching between two technologies, these technologies have to have the same worker wage rate. But if they do, then managers' wage rates also have to be equal: if not, then a managerial career in one of the technologies would give higher total payoff than in the other. And with both wage rates equal in the two technologies, the input ratios also have to be equal, which contradicts the equal wage rates since total-factor productivities differ across the technologies. Hence we conclude that only one technology can be operated in a stationary equilibrium without growth.

3.2 The Growth Equilibrium

When $\pi = 0$, there is no government restriction on the activities chosen by agents. We will focus on the simplest kind of stationary equilibrium with growth, namely one where there is only one technology currently in operation, and two under development. We believe that the result from Chari and Hopenhayn that stationary equilibria are unique also characterizes this environment, but we have not verified this belief. We describe our proposed equilibrium and show the conditions on primitives under which all equilibrium conditions are met. We have not characterized or proved existence of stationary equilibria for other parameter values. Such values are likely to imply stationary distributions with more than one technology under operation.

The stationary equilibrium we consider has a fraction ϕ of the agents choosing a managerial career. Managers moreover develop new technologies, as opposed to learning how to manage existing technologies. The remainder of the agents work in the only technology under operation, $\tau = 1$, and hence switch twice during their life-time. We therefore have the following skill matrix:

$$\mu_{ns} = \begin{pmatrix} 0 & \phi & 0 \\ 1-\phi & 0 & \phi \end{pmatrix}, \tag{19}$$

with $\mu_s = 1 - \phi$. It also follows that $m_1 \equiv m = \phi$, that $n_1 \equiv n = 3(1-\phi)$, that $s_{1,-1} = s_2 \equiv s = \phi$, and that $s_{1,0} = 0$.

Wage rates have to be such that young agents are indifferent between managerial and working careers, so that

$$w_n + \beta\gamma w_n + (\beta\gamma)^2 w_n = (\beta\gamma)^2 w_m. \tag{20}$$

Wages are, moreover, marginal productivities, hence satisfying

$$w_m = \alpha\left(\frac{n}{m}\right)^{1-\alpha} \tag{21}$$

$$w_n = (1-\alpha)\left(\frac{m}{n}\right)^{\alpha}, \tag{22}$$

where the current technology is normalized to have total-factor productivity one. The restrictions on wage rates and activities now imply that we can solve for ϕ:

$$\phi = \frac{1}{1 + \frac{1+\beta\gamma+(\beta\gamma)^2}{\beta\gamma}\frac{1-\alpha}{3\alpha}}. \tag{23}$$

To verify that all agents maximize is a little more involved than in the no-growth steady state. First, old workers, who all switched technologies the previous period, cannot do better than what our equilibrium dictates, i.e. they switch again. Second, middle-aged workers could decide not to switch, and instead study their technology for one period on their own and then attract workers and manage this technology the next period. This would give a life-time income which has to satisfy

$$w_n + \beta^2 \max_n \left\{n^{1-\alpha} - \gamma^2 w_n n\right\} \geq w_n \left(1 + \beta\gamma + (\beta\gamma)^2\right) \tag{24}$$

in order that it does not pay to switch to a managerial path. This restriction can be rewritten as

$$\gamma^{\frac{2}{\alpha}} \frac{1+\beta\gamma}{1+\beta\gamma+(\beta\gamma)^2} > 1. \tag{25}$$

Third, middle-aged students obviously do not want to switch to working given the wage equality above on which the equilibrium is based. Fourth, young future managers could choose to specialize in some other technology than the newest. The inequality derived above ensures that it does not pay to go into technology 1, and technologies 2, 3, etc. are clearly worse still. What about technology 0, which is already under study by the current middle-aged? Using it would pay in the second period, because then the agent would receive compensation for the learning – this technology will be managed that period by the current middle-aged. It would, however, give lower return in the last period, because the technology has lower productivity than the newer on and because workers have to be paid the higher wage. To rule out that the alternative strategy gives higher life-time utility, we have to require that

$$\beta\gamma w_n + \beta^2 \max_n \left\{\gamma n^{1-\alpha} - \gamma^2 w_n n\right\} \geq w_n \left(1 + \beta\gamma + (\beta\gamma)^2\right). \qquad (26)$$

This inequality amounts to

$$\frac{1}{\beta\gamma}\gamma^{\frac{1}{\alpha}}\frac{1+(\beta\gamma)^2}{1+\beta\gamma+(\beta\gamma)^2} > 1. \qquad (27)$$

If inequalities (25) and (27) are met, we are assured that the proposed behavior is indeed optimal. These inequalities are non-trivial restrictions on the parameter space. However, they define a set of parameter values which is of full measure.

4 Equilibrium With Dynamic Voting

We now begin our discussion of voting behavior in the model of the previous sections. We first informally discuss the possibility that the stationary equilibria described in the previous section can be self-generating, i.e. that they can be supported by dynamic voting equilibria. This discussion makes clear a need for a formal definition of voting equilibria, which we then provide in the following subsection.

4.1 Could the Stationary Equilibria Be Supported By Voting?

With the stationary equilibria with and without growth characterized, we are ready to discuss how different agents might vote over tax policies. We have in mind, primarily, a political system in which one agent can cast one vote, for or against the tax on letting the current young adopt a new technology. As in all voting models with atomistic agents, any voting behavior in this model is, strictly speaking, individually rational; even the question why agents vote is fundamentally unexplained. Typically it is assumed, however, that voters view their votes as having an impact in their voting decision. We take the view here that voters contemplate the equilibrium effects of the different possible policies on their welfare, and then vote for the policy that gives them the highest lifetime welfare. Specifically, the voter views herself as "dictator for a day": she ponders the effects of each vote today, as if she could dictate current policy (or as if her current vote were pivotal), evaluates her utility in each case, and votes to maximize this utility. Note that this behavior allows the agent to imagine an impact on future votes: although she is not directly choosing future policy in her thought policy alternatives, different policies today would impact the current behavior of agents and thus the future skill distribution, which in turn implies a future voting pattern.

We start with discussing what different agents might prefer when there is growth, and then turn to the no-growth situation. Throughout this discussion we will be rather loose about the equilibrium concept we have in mind; later we will lay out such definitions more carefully.

When there is growth, there are five groups in distinct situations: young agents, and middle-aged and old with managerial or working careers. First, young agents will always prefer growth, since it makes higher-productivity technologies available. Second, agents on a managerial career path either study (the middle-aged) or are ready to manage a firm (the old). The former as well as the latter group will be against growth, because with young workers are complementary to their capital: old agents' productivities increase in the number of current young workers, and the productivity of the middle-aged one period hence also increases in the number of young who choose a working career in the current period. Third, working agents, both middle-aged and old, prefer growth: these agents' productivities decrease in the number of young who choose a working career, since workers are perfect substitutes.

Given the sizes of the groups, the size of the vote in favor of growth will hence be $1 + (1 - \phi) + (1 - \phi) = 3 - 2\phi$, and the vote against $\phi + \phi = 2\phi$. If the primitives are such that $\phi < \frac{3}{4}$, then growth seems possible as a self-generating outcome of dynamic voting.

Turning to the no-growth situation, there are really only three distinct groups: young, middle-aged, and old. As before, young prefer growth. Old prefer not to have growth, since they have specialized capital which is complementary with working labor, so the fewer the students the better for them. Finally, middle-aged agents in this case have a harder voting decision. If some young become students, their current wage is higher, since there is less competition in the work force. At the same time, however, the same agents face lower returns as managers tomorrow if the young agents choose to study, since the managerial salary increases in the number of workers. How should, then, the middle-aged vote?

The voting decision of the middle-aged in the no-growth situation illustrates the difficult aspect of analyzing dynamic voting models. We can conclude, first, that if the agent knew the current and next-period future wage rate with and without a current tax on technology adoption, then it would be an easy task to figure out the right vote. Second, we know that the wage rate in each case is a simple function of the number of managers and the number of workers. The number of managers is known, but the number of workers is not. For if there is growth, we do not know how many of the young agents would choose to study. This choice, in turn, is not at all straightforward to calculate, and it is even conceptually non-trivial to formulate: young workers decide on their career based on current and future wage rates, all of which depend on aggregate behavior in the current and future periods. Moreover, and very importantly, how future generations will behave with respect to career decisions of course depends on future tax policies. The latter point forces the current young to ask themselves not only how future young would behave given tax rates, but also how the future generations will vote over the same tax rates.

The difficulty in figuring out the voting behavior of the middle-aged in the no-growth steady state also to some extent applies to the voting behavior of the other agents both in the stationary equilibria without and with adoption of new technologies. In the discussion above we merely asserted on an intuitive basis how the different categories of agents would vote. To verify the assertions it would seem necessary to be explicit about the equilibrium response of the current and future generations from the change in the tax rate, which involves the difficult issues raised above. We confront all of these issues in the next subsection, where the main focus is conceptual: how should our dynamic equilibrium with voting be defined? What are the problems solved by the agents, both in their role as consumers/workers and in their role as voters?

4.2 A General Definition

We utilize a recursive definition of the dynamic equilibrium. The state variable of the economy is the distribution of skills. Wage rates, as well as the aggregate choice variables, are then given as functions of the state. The law of motion of the aggregate state gives tomorrow's state as a function of today's. Moreover, the voting outcome is also a function of the state of the economy. The focus on recursive equilibria is not restrictive per se. However, it may be that the restriction on the state only to include current givens (and not past endogenous variables) rules out equilibria. This is particularly likely if there are multiple sequential equilibria.

We model agents in their two roles: as consumers and as voters. The consumer agent takes the wage, aggregate choice, and voting functions as given, as well as the aggregate law of motion, when solving her dynamic utility maximization problem. The voting agent views herself as "dictator for a day" and figures out the equilibrium effect of making the current vote differ from the vote implied by the aggregate voting function. In this analysis the "dictator for a day" takes next period's actions and votes to be given by those implied by the equilibrium functions – hence our label on the voting agent. The voter then compares the utility resulting from the two votes, and votes to maximize utility.

We define our equilibrium for a much wider class of models than the one described in the previous section. In particular, infinitely lived agent economies are a special case. Our definition leads to a natural procedure for characterizing equilibria numerically, and we apply it to our model in the next section.

The Consumer's Problem: The consumer's problem can be cast as follows.

$$\begin{aligned} v(x,\mu) &= \max_{y,x'} \{R(x,y,W)+\beta v(x',\mu')\} \text{ s.t.} \\ y &\in \Gamma(x,\pi) \\ x' &= \xi(x,y) \\ \mu' &= h_\mu(\mu) \\ Y &= h_Y(\mu) \\ W &= W(Y) \\ \pi &= \Psi(\mu). \end{aligned} \tag{28}$$

v is the value function in this dynamic programming problem, and y is the agent's current choice given a value of her state, x, which includes age and skill, and the aggregate state μ. The variable Y is a vector of aggregate choices ($m_\tau, s_{1,-1}$ etc.), and W is the vector of current prices. As announced, the agent is fully aware of the way the future skill distributions, aggregate choices, wages, and votes depend on the current distribution. This is captured by the functions $h \equiv (h_\mu, h_Y)$, W, and Ψ. We use the function R to indicate the current return. The finite live of the agent is captured by R being zero for agents with $i = 0$. We will not spell out R fully here; it simply combines the choice y and the wage rates W to select the appropriate payoff.

Note the generality that lies in the formulation: the variable x here captures any aspect of the individual's state, which for example could include current stocks of other types of capital. Further, in a model with infinitely-lived agents, the age component of x simply loses the significance it has in our context; the problem is otherwise unchanged.

We let decision rules be denoted by functions $g(x, \mu)$. In so doing, we implicitly assume that there is a unique solution to the agent's problem in every state. Our economy of the previous sections does not produce unique choices generally (young agents may be indifferent between careers). We use functions here only for notational convenience, and in the next section on computations we allow for multiple maximizing choices.

It is clear that an equilibrium with policy updating according to Ψ could be defined by the problem (28), and a condition that individual choices are consistent with aggregate choices, without reference to how Ψ is chosen endogenously. Here, however, focus is on Ψ. We now need to specify how the "dictator for a day" thinks about the equilibrium effects of different policy choices.

First, if the voter chooses the policy $\pi = \Psi(\mu)$, she will know how to evaluate her utility: she solves (28), and the sought utility is then simply given by $v(x, \mu)$.

Second, if she chooses another policy, $\pi \neq \Psi(\mu)$, the calculation is more difficult: she needs to find the equilibrium response to a changed tax rate. This alternative tax rate will not be chosen by the electorate for the current state μ; neither will the implied equilibrium response of agents' aggregate behavior ever be realized. Nevertheless, it is a crucial element of our equilibrium definition, and certainly also of "real world" agents' voting considerations, that the effects of different votes are calculated in a sensible way. The only sensible way we know is to let agents form rational expectations of the effects of the votes, and in this case this amounts to figuring out the equilibrium response to the different tax rates. Therefore, let the function $H(\mu, \pi) \equiv (H_\mu(\mu, \pi), H_Y(\mu, \pi))$ report the law of motion of the aggregate state and the aggregate choices for the equilibrium where today's policy is an arbitrary π (which then does not have to equal $\Psi(\mu)$), and tomorrow's aggregate choices and vote are given by the original functions h and Ψ. The equilibrium with this "one day" deviation of the policy also needs to include the agent's maximization problem:

$$\begin{aligned} V(x, \mu, \pi) &= \max_{y, x'} \{R(x, y, W) + \beta v(x', \mu')\} \text{ s.t.} \\ y &\in \Gamma(x, \pi) \\ x' &= \xi(x, y) \\ \mu' &= H_\mu(\mu, \pi) \\ Y &= H_Y(\mu, \pi) \\ W &= W(Y) \end{aligned} \tag{29}$$

The policy corresponding to this problem is $G(x, \mu, \pi)$. It is key to note in this maximization problem that tomorrow's value is given by v, i.e. the "one-day" deviation is reflected in the agent's evaluation of attaining the different possible future states.

We note that the utility attained for the alternative policy is given by $V(x, \mu, \pi)$. Consistency now requires $g(\mu) = G(\mu, \Psi(\mu)), v(x, \mu, \Psi(\mu)) = V(x, \mu, \Psi(\mu))$, and $h(\mu) = H(\mu, \Psi(\mu))$. We denote by $\psi(x, \mu)$ the chosen vote for an agent with individual state x when the aggregate state is μ:

$$\psi(x, \mu) = \arg\max_{\pi} V(x, \mu, \pi) \tag{30}$$

Here as well, we assume for expositional simplicity that there are no ties.

Finally, the outcome of the vote given the current state μ is determined by a voting aggregator function A:

$$\Psi(\mu) = A(\psi, \mu). \tag{31}$$

In the context of our simple example with only two possible values of π, the obvious choice of an aggregator function is the majority rule:

$$\Psi(\mu) = \arg\max_{\pi \in \{0,1\}} \int_{x:\psi(x,\mu)=\pi} \mu(dx). \tag{32}$$

If the policy had been more complicated, for example by having more possible π values, or by including other taxes, the choice would be less trivial. Note, however, that our methods here are to derive indirect utilities for each agent over the different policies, and any voting theory derived in a static framework could then be applied here.

The equilibrium with voting can now be defined as follows:

Definition: An equilibrium with dynamic voting is a set of functions:

$v, V, g, G, h, H, W, \psi$, and Ψ

with the following properties

(i) (v, g) and (V, G) solve (28) and (29), respectively.

(ii) h_Y and H_Y generate the laws of motion h_μ and H_μ.

(iii) The consumers' choices are consistent with h_Y and H_Y.

(iv) The price function, W, is competitively determined.

(v) Ψ is given by (31).

(vi) $g(x,\mu) = G(x,\mu,\Psi(\mu)), v(x,\mu) = V(x,\mu,\Psi(\mu)),\ h(\mu) = H(\mu,\Psi(\mu))$.

Note that environments with additional market clearing conditions would need corresponding requirements in the definition of equilibrium. Note also that property (iii) is somewhat loosely expressed; wages are marginal productivities in our framework, with the exception of wages for managers and workers in technologies where there is currently no manager. In such technologies we need to use the highest current working wage as wage, since potential managers may want to contemplate going into this technology as the only person with this skill and attract workers at their alternative wage. The corresponding wage for the manager would be the maximum profits given the working wage.

5 A Numerical Example

In this section we study a simple example — one with three agents per generation. The restriction to a small number of agents per generation is made for analytical/computational reasons, and we devote parts of the Appendix to a discussion of the numerical issues involved in calculating dynamic voting equilibria more generally. In the Appendix, we also describe in some detail the computational procedures we used to construct the equilibrium discussed in the present section, as well as some more fundamental theoretical issues that arise in the context of our model with finite-agent generations and nonconvex activity choices.

The equilibrium we construct assumes particular values for the three parameters of the model; we used $\alpha = .8, \beta = 1.2$, and $\gamma = 2.2$. At least for these parameter values, a key feature of the equilibrium is that only one technology is employed in production at any point in time. Given this fact, it turns out that the number of relevant states in this economy is 28. Table 1 lists possible values of the relevant state variable: the set of distributions of skills for middle-aged and old agents:

The index s in the table represents the state; *eo* stands for 'experienced old' (but not graduate), *io* for 'inexperienced old' (switcher), *g* for 'graduate', *im* for 'non-studying middle-aged', and *u* for 'undergraduate'. The distributions that have been left out of this list first of all include those with skills in more than the three most recent technologies. Second, some properties of the equilibrium behavior of agents imply that certain distributions cannot arise. In particular, agents who study never change their mind and switch technologies. Moreover, middle-aged agents with the same skills always make the same choices; only young agents can display indifference among their alternative activities. These facts rule out the distributions $(1,1,1;\cdot,\cdot), (2,1,0;\cdot,\cdot)$, and $(1,2,0;\cdot,\cdot)$.

s	$\mu_s(eo)$	$\mu_s(io)$	$\mu_s(g)$	$\mu_s(im)$	$\mu_s(u)$
1	3	0	0	3	0
2	3	0	0	0	3
3	3	0	0	2	1
4	3	0	0	1	2
5	0	3	0	3	0
6	0	3	0	0	3
7	0	3	0	2	1
8	0	3	0	1	2
9	0	0	3	3	0
10	0	0	3	0	3
11	0	0	3	2	1
12	0	0	3	1	2
13	2	0	1	3	0
14	2	0	1	0	3
15	2	0	1	2	1
16	2	0	1	1	2
17	1	0	2	3	0
18	1	0	2	0	3
19	1	0	2	2	1
20	1	0	2	1	2
21	0	2	1	3	0
22	0	2	1	0	3
23	0	2	1	2	1
24	0	2	1	1	2
25	0	1	2	3	0
26	0	1	2	0	3
27	0	1	2	2	1
28	0	1	2	1	2

Table 1: Set of Possible Distributions

If the current state is μ_s and next period's state is $\mu_{s'}$, then it is clear what today's choices of the currently middle-aged and young must be. In order to have a complete description of the current actions, it is therefore necessary to also list the actions of the currently old agents — their choices cannot be read off tomorrow's state. Table 2 lists the possible actions of the currently old. The index l is used for the possible aggregate choices of the old.[3]

l	m_1	m_2	n
1	0	0	3
2	0	3	0
3	3	0	0
4	0	1	2
5	1	0	2
6	0	2	1
7	2	0	1

Table 2: List of Old Agents' Choices

Now the equilibrium can be described by means of two tables. The first table, Table 3, describes the functions h_μ, H_μ, the current choices of the old, and the voting function Ψ. This table, by listing h_μ and Ψ, enables us to completely describe the law of motion of the skill distribution and the aggregate vote for any initial skill distribution in the set of 28. It also shows how a one period deviation from the realized policy would affect the system over time: this is described by H_μ.[4]

[3]Note that $l = 4$ and $l = 6$ have been included in the table even though they, as asserted above, will not occur in equilibrium. Also, the possibility $m_2 = m_1 = n = 1$ is not listed.

[4]In the table, h_Y and H_Y refer to the actions of the currently old, as listed in Table 2.

s	$h_\mu(\mu_s)$	$h_Y(\mu_s)$	$H_\mu(\mu_s, \Psi^c(\mu_s))$	$H_Y(\mu_s, \Psi^c(\mu_s))$	$\Psi(\mu_s)$
1	1	2	4	2	1
2	9	2	12	2	1
3	16	2	13	2	0
4	17	2	20	2	1
5	4	1	1	1	0
6	12	1	9	1	0
7	16	1	13	1	0
8	20	1	17	1	0
9	1	3	4	3	1
10	9	3	12	3	1
11	24	3	21	3	0
12	25	3	28	3	1
13	4	5	1	5	0
14	12	5	9	5	0
15	24	5	21	5	0
16	28	5	25	5	0
17	1	7	4	7	1
18	9	7	12	7	1
19	24	7	21	7	0
20	28	7	25	7	0
21	4	5	1	5	0
22	12	5	9	5	0
23	24	5	21	5	0
24	28	5	25	5	0
25	1	7	4	7	1
26	9	7	12	7	1
27	24	7	21	7	0
28	28	7	25	7	0

Table 3: Equilibrium Laws of Motions and Voting Outcomes

Table 3 immediately reveals that there are two attracting, self-perpetuating states, namely states 1 and 28. State 1 is the no-growth steady state, whereas state 28 has growth. State 1 furthermore has a majority against allowing growth, whereas the majority in state 28 is in favor of growth.

It is moreover possible to see from the table that 16 out of the 28 states lead to state 1, i.e. to stagnation. Along these paths toward stagnation, it is possible that growth is initially allowed, but eventually there is a large enough vested interest in the old technologies that growth policies will be voted down. The longest transition to stagnation in our example spans three periods, for example from state 14, via 12 and 25, to 1.

The remaining 12 states lead to state 28 and prosperity. Along all these paths growth is allowed. Hence, in our example, a no-vote once is a sufficient, but not necessary signal, that the economy will end up in stagnation. The transition to prosperity is at most 2 periods long.

The voting considerations can be read off the table as follows. In, say, state 1, the voting outcome is $\Psi = 1$ (i.e. no to growth), and next period's state is 1 as well. If an agent were to consider an alternative policy (yes to growth), then H tells us that tomorrow's state would be state 4. It also follows, from reading off h for states 4 and 17, that state 4 would be followed by state 17, and that thereafter state 1 would reoccur: a one-period policy deviation in this case would lead to a deviation from the steady state, but the change in the distribution of agents is not radical enough to carry the economy to state 28, i.e. to the growth steady state. On the other hand, a similar deviation from state 28 would in fact lead to stagnation.

We also note that the composition of the middle-aged is key to where the economy is heading. If either all or none among the middle-aged are students, then the economy will go to state 1. If there is a large number of students, who all are against growth, then growth will be voted down, and stagnation result.

Also in the case where there are no students, the number of middle-aged workers is large enough that these agents' voting calculation lead them to be against growth. This calculation was hinted at in Section 4: on the one hand, these agents want the young to study this period to avoid competition on the labor market. On the other, they will become managers one period later, and will then want a large number of students. It turns out that the second aspect is more important to them: their vote reflects the aim to effectively maximize the number of workers they, as managers, can attract next period. It is interesting to note that this is accomplished in two ways: first, they vote down growth today, which makes today's young become workers tomorrow. Second, by voting no today, they also influence tomorrow's distribution of agents so that tomorrow's vote will be against growth as well. This way the currently middle-aged gets a very large number of workers tomorrow.

When there are 1 or 2 students among the currently middle-aged, growth may or may not result; the middle-aged workers are now sure that they will not be managers next period (since there are better managers around), and as workers they prefer growth, because growth keeps the labor force smaller and they get higher wages that way. In these cases, the votes of the currently old agents are pivotal, depending on the exact distribution, the economy goes to state 1 or to state 28.

The voting behavior for each group is described in Table 4. Simply note that the results in this table supports the intuition suggested above, with the qualification that all old voters are indifferent in states 5-8, because in these states there are no managers, and hence no production can take place, which means that the situation of the old cannot be influenced at all in the vote (the program is written so that agents agree with the majority in the event of a tie).

It should be pointed out that all the properties asserted in this section that lead to the state description we employ (only one technology in operation at any point in time etc.) are verified in our program routine. Note also that the outcome of the computations, although constructed by a computer, can be regarded as theorems: all calculations by agents in this economy lead to corner solutions, so there is no sense in which computer approximations are critical to the analysis. It would be entirely straightforward, but tedious, to rewrite the computer output as a theorem and verify the asserted properties one by one as simple inequalities. The computer was, however, very helpful in its clever construction of the equilibrium, which is complicated in nature, and hence made it possible for us to formulate the theorem.

s	$\psi(eo,\mu_s)$	$\psi(io,\mu_s)$	$\psi(g,\mu_s)$	$\psi(im,\mu_s)$	$\psi(u,\mu_s)$	$\psi(y,\mu_s)$
1	1	0	1	1	1	0
2	1	0	1	0	1	0
3	1	0	1	0	1	0
4	1	0	1	0	1	0
5	0	0	0	1	1	0
6	0	0	0	0	1	0
7	0	0	0	0	1	0
8	0	0	0	0	1	0
9	0	0	1	1	1	0
10	0	0	1	0	1	0
11	0	0	1	0	1	0
12	0	0	1	0	1	0
13	0	0	1	1	1	0
14	0	0	1	0	1	0
15	0	0	1	0	1	0
16	0	0	1	0	1	0
17	0	0	1	1	1	0
18	0	0	1	0	1	0
19	0	0	1	0	1	0
20	0	0	1	0	1	0
21	0	0	1	1	1	0
22	0	0	1	0	1	0
23	0	0	1	0	1	0
24	0	0	1	0	1	0
25	0	0	1	1	1	0
26	0	0	1	0	1	0
27	0	0	1	0	1	0
28	0	0	1	0	1	0

Table 4: Individual Voting Choices

APPENDIX

Computing dynamic voting equilibria for economies with an infinite number of agents is a formidable task when heterogeneity is allowed among these agents. The problem is that our voting equilibrium requires a fully dynamic analysis, and it is well known that the computational complexity for dynamic models increases very rapidly in the number of agents in the economy. In our case, it would be necessary to find a specific mapping (a fixed point) from the set of possible distributions of types into itself. These sets are large; in particular, in our example, they are the product set of a two-dimensional unit simplex and a three dimensional unit simplex. Moreover, the voting features imply that for every iteration in the voting function we have to solve a fixed point problem of the type described above. For these reasons, we have chosen to compute equilibria for a model with a finite number of agents in each generation.

The example we are considering has three agents born each perod. This is the smallest number that makes ties impossible, as the total number of people alive at any point in time, nine, is an odd number. The rest of the features of the example are the same as in the model above. Now the problem remains at a manageable dimension, as there is only a finite number of possible distributions of agents. However, the introduction of this small number of agents does not come for free: there are a number of small issues that arise and that have to be taken care of. In the next few paragraphs, we describe these issues.

With a small number of agents, the behavior of each agent has aggregate implications. In our present context, we are not interested in strategic behavior, and we consequently try to abstract as much as possible from issues of market power. This means that we consider price taking behavior: consumers and firms optimize treating prices as beyond their control.

In our model, the only choice of an agent is which technology to join. This is a nonconvex choice. If identical agents take different actions in equilibrium, then it has to be because these actions lead to the same level of utility. In environments with a continuum of agents this can be achieved with lotteries, (see Rogerson (1988) for a general implementation of these market institutions), or by using the fact that the utility of the different choices is a continuous function of the fraction of agents that choose each activity, with the equilibrium having a partition of the population that delivers equal utility to all types (for an example see Ríos-Rull (1992); for an example of another paper in economics see Krusell (1991)). In environments with a finite number of agents, the fractions route cannot be followed, but lotteries can be implemented. In our context, then, we use lotteries as a decentralizing device.

The lottery equilibrium has the decisions made by the young agents taken collectively. A planner representing the young agents chooses the alternative that provides the maximum total utility to the generation as a whole. The solution is implemented by solving a generation-wide planning problem where agents act collectively **only** when young; in later periods agents act on their own (we consider commitment not to be possible, both with respect to future choices of technology and with respect to future votes). We assume that the planner representing the young agents solves a maximization problem in which full account is taken of the impact of the choices of the young on the future distribution of agents. Lotteries are then used to decide on whose lot it is to work, and on whose lot it is to study.

The computational procedure that we use is straightforward, and it is described in detail below. One important, and normally unfortunate, feature of our methods is that the procedures themselves do not guarantee convergence to an equilibrium. This makes intelligent guesses about equilibrium properties crucial as inputs in the computational algorithm. We therefore make certain guesses initially, impose the implications of these guesses on the state space and behavior of agents, run the algorithm, and verify the guesses at the end.

The guesses are: (i) only one technology — the most recent one for which there are skilled old agents — is operated at each point in time; (ii) only skills in the two most recent technologies are possible; (iii) agents on a managerial career path always choose to continue this career; and (iv) middle-aged and old agents with the same skill behave the same way (no lotteries are needed for these agents).

Working through the implications of these guesses, one arrives to the conclusion that there are 28 possible distributions μ, listed in Table 1 in Section 5. A first look at the problem at hand shows that the set of possible laws of motions is the set of all mappings from 28-element-sets into itself. There are 28^{28} = 33145523113253374862572728253364605812736 of them. Associated to this set is the set of possible voting functions: this set is exactly the set of all possible ordered combinations of 0's and 1's of size 28: a set with $2^{28} = 268435456$ elements. The size of these numbers gives an idea of the size of the computations involved in a general procedure for finding the equilibrium fixed point, and hence of the importance of good guesses.

Numerical algorithm:

We now describe the iterative procedure used to compute the equilibria. For this, we recall some notation.

Let X be the set of all possible types, specifying properties like age, managerial abilities, wealth or anything additional that makes people different.

Let $\mu \in \mathcal{M}$ be the set of all possible distributions. μ is a measure on X.

Let $\Psi : \mathcal{M} \rightarrow \{0, 1\}$ be an aggregate voting policy.

Let π be any voting policy.

Let $h : \mathcal{M} \rightarrow \mathcal{M} \times \mathcal{Y}$ jointly denote a specification of the aggregate law of motion of the economy h_μ, and of aggregate actions of all agents h_Y under voting policy Ψ every period. Note that the law of motion of the economy, h_μ, is completely determined by the projection of h_Y over the agents that will be alive the next period.

Let $H : \mathcal{M} \rightarrow \mathcal{M} \times \mathcal{Y}$ denote a specification of the aggregate law of motion of the economy, H_μ, together with a specification of aggregate actions of all agents H_Y under voting policy π today, and voting policy Ψ from tomorrow on. As was the case for h_μ, H_μ is completely determined by those components of H_Y that refer to agents who are alive next period.

Let $v(x, \mu; h, \Psi)$ be the value function of an agent of type x when the distribution of types (the aggregate state of the economy)) is μ, when the economy's law of motion is given by h, and its voting policy is Ψ.

Let $g(x, \mu; h, \Psi)$ be the optimal decision rule of type x agent. This agent is restricted in his choices by his type, and by the policy implemented. Specifically, agents do not choose their age, they can only manage a technology if they have the skill to do so, and they can only start the project that leads to an innovation if it is legal to do so.

Let $V(x, \mu, \pi; H, h, \Psi)$ be the value function of an agent of type x when the distribution of types (the aggregate state of the economy) is μ, when the economy's law of motion is given by H today, and h from tomorrow on, and its voting policies are π today and Ψ from tomorrow on.

Let $G(x, \mu, \pi; H, h, \Psi)$ the optimal decision rule associated to value function V.

Let $\psi(x, \mu; H, h, \Psi)$ denote the preferred policy option, when the equilibrium laws of motion and voting policy are H, h, and Ψ, of a type x agent when the state of the economy is given by μ.

Finally, we describe the algorithm used in the computation of equilibria.

Step 1: Initialize Ψ.

Step 2: Initialize h.

Step 3: Compute $v(x, \mu; h, \Psi)$ and $g(x, \mu; h, \Psi)$. This is solved by backward induction, noting that old agents have no future to worry about.

Step 4: Compute the law of motion of the economy generated by decision rules $g(x, \mu; h, \Psi)$. Check whether this law of motion coincides with the assumed law of motion h. If it does, proceed to step 5, if it does not, update the law of motion h taking into account decisions g and go back to Step 3.

Step 5: Initialize H.

Step 6: Compute $V(x, \mu, \pi; H, h, \Psi)$, and $G(x, \mu, \pi; H, h, \Psi)$. These objects are computed in one step. They arise in a one step iteration where agents evaluate the future using function v, not V, but take into account that tomorrow's distribution μ' is given by H as opposed to h, and that today's vote is given by Ψ^c, and not by Ψ. (If $\pi = \Psi$, then $V = v, G = g$, and $H = h$.)

Step 7: Compute the law of motion of the economy generated by decision rules G. Check whether this law of motion coincides with the assumed law of motion H. If it does, proceed to step 8. If it does not, update the law of motion H taking into account decisions G and go back to Step 6.

Step 8: Compute $\psi(x, \mu; H, h, \Psi)$ by letting agents choose what policy they prefer. This is done by strict comparison of the values of v and $V(\neq v)$. If the first is higher then $\psi(x, \mu; H, h, \Psi) = \Psi(\mu)$; otherwise $\psi(x, \mu; H, h, \Psi) = \pi$.

Step 9: Compute the winners choice. This is done by computing $\int_X \psi(x, \mu)\mu(dx)$, for every μ. We use democracy (one agent one vote), so if this expression has a value of more than half the amount of agents, then the winner is to choose the policy equal to 1. Otherwise the policy chosen is 0. Next, compare the election winner with policy Ψ. If they differ, update policy Ψ using the winner's information; if they coincide, we are done.

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