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Money, Trade Credit, and Asset Prices

by

Jeffrey M. Lacker and Stacey L. Schreft
Research Department, Federal Reserve Bank of Richmond,
P.O. Box 27622, Richmond, VA 23261

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ABSTRACT: We describe a stochastic economic environment in which the mix of money and trade credit used as means of payment is endogenous. The economy has an infinite horizon, spatial separation and a credit-related transaction cost, but no capital. We find that the equilibrium prices of arbitrary contingent claims to future currency differ from those from one-good cash-in-advance models. This anomaly is directly related to the endogeneity of the mix of media of exchange used. In particular, nominal interest rates affect the risk-free real rate of return. The model also has implications for some long-standing issues in monetary policy and for time series analysis using money and trade credit.

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I. Introduction

This paper studies economies in which the use of money and trade credit as means of payment is endogenous and can vary in response to both real and policy shocks. It extends the standard cash-in-advance model by allowing agents to choose which goods to purchase with money and which with trade credit. Clower called for this type of extension in 1971:

> To come to grips with any practical problem, we clearly must be willing to contemplate more realistic situations in which money consists of currency, demand deposits, and trade credit--the last item being especially important since it is used as means of payment in virtually all business transactions.

In the third quarter of 1990, all U.S. manufacturing firms' accounts receivable totalled \$369 billion, or 39 percent of total current assets, while accounts payable were \$189 billion, or 30 percent of total liabilities.[1]

In our model economies with multiple means of payment, fluctuations in the opportunity cost of money will alter the mix of exchange media used. The equilibrium mix depends on the relative costs of cash (i.e. fiat currency) and trade credit. Cash use results in no real resource costs but an opportunity cost of foregone interest when the nominal interest rate is positive. In contrast, trade credit use permits individuals to avoid the opportunity cost of holding cash but instead imposes a real resource cost. Individuals balance the resource cost of trade credit against the opportunity cost of cash at the margin to determine the use of both transaction media.

Like most models of multiple means of payment, the model used here is a derivative of that in Lucas and Stokey's (1983) paper. In their paper, they allow for two types of consumption goods: "cash goods," which must be purchased with currency, and "credit goods," for which securities can be issued. Although the model generally has a positive opportunity cost associated with currency usage, it assumes costless trade credit. In contrast, the model here generalizes Lucas and Stokey's work by formally specifying the transaction costs associated with trade credit and endogenizing the choice of payment methods.

Because it formally incorporates transaction costs, this model generates asset-pricing anomalies not present in nonmonetary models or in one-good cash-in-advance models. In particular, expressions for the equilibrium prices of state-contingent claims in securities markets contain terms for the average cost (the sum of resource and opportunity costs) of monetary and trade credit exchange at different dates, and these terms vary directly with nominal interest rates. An increase in the current nominal interest rate, for example, raises the opportunity cost of a cash purchase by the same amount, but the average resource cost of a trade credit purchase increases less. Consequently, the average transaction cost associated with cash and credit use varies less than one-for-one with the nominal rate. Thus, real returns are determined in part by the path of nominal interest rates, and the usual Fisherian independence of real from nominal returns fails to hold.

The asset-pricing anomalies identified here are related to those displayed by Townsend (1987). In Townsend's model, as in Lucas and Stokey (1983), the asset-pricing formula takes the standard form when written in terms of the marginal utility of the "cash good." In contrast, the separability of intratemporal preferences in our model allows us to find asset-pricing formula expressed in terms of the marginal utility of a composite consumption good. This is important because empirical tests of asset-pricing formulas have used measures of consumption that include both cash and credit purchases (see e.g. Singleton (1990)). Moreover, the anomaly we describe here would appear, albeit in a simpler form, in the models of Townsend and Lucas and

Stokey if the assumptions necessary to aggregate intratemporal consumption over cash and credit goods were made.[2]

In our model transaction costs associated with trade credit purchases increase with a shopper's distance from home, as in Schreft (1987). This is perhaps the simplest way of endogenizing the choice between cash and trade credit purchases. However, asset-pricing anomalies like those found here are likely to be present in any model with a nontrivial trade-off between currency and trade credit in exchange. To focus on the substitution between the use of cash and credit in exchange, we assume that preferences are Leontieff across goods at a given date, effectively requiring that they be consumed in equal amounts. This allows a particularly simple aggregation over consumption of goods within periods. Our results would carry over to environments with more general preferences that allow substitution among consumption of different goods as well as substitution in the means of payment, although deriving the results in such environments would be considerably more complex.

Because payment system use is endogenous in our model, our results have serious empirical implications. Ramey (1988) has suggested that time series observations on money, trade credit, output and nominal interest rates might allow us to disentangle real versus monetary sources of business cycle shocks. Our model provides a microfoundation for her work in that money and trade credit respond in the same direction to real shocks, but in opposite directions to policy shocks. However, our research shows that a stringent, and implausible, identifying assumption is needed to obtain this result: monetary policy must make nominal interest rates independent of all current and past real disturbances.

Moreover, the implications of the availability of money substitutes like trade credit go beyond asset-pricing anomalies. This paper illustrates the impact of money substitutes on the monetary authority's ability to control the money base and thus the price level and nominal GNP. The effects of trade credit use figured prominently into the Radcliffe Committee's 1959 report on the workings of the British monetary system. The Committee criticized the monetarists, as the banking school had criticized the currency school a century earlier, for appealing to the quantity theory of money, which postulates a tight relationship between the money supply and the price level.[3] In keeping with this age-old debate, we examine the response of trade credit use to a policy shock and find that trade credit weakens the effects of money growth on interest rates. We also find that trade credit increases the sensitivity of velocity to interest rates. Thus, our work supports the view of the banking school and Radcliffe report that trade credit use "frustrates" monetary policy and so should be considered by policymakers.

II. The Economic Environment

We study a discrete-time, infinite-horizon economy with many goods and many agents. Time is indexed by $t=1,2,\ldots$. Uncertainty is embodied in a state vector s_t that evolves according to a first-order Markov process. The contents of the state vector will be described later.

There is a continuum of locations, arranged in a circle and indexed by $z\in[0,1]$. At each location, there are a large and equal number of households, each consisting of two agents. Each household is endowed at each date with an amount of labor time that can be devoted to leisure or to producing a non-storable good. Households at location z are capable of producing only type-z good. Each household desires consumption of goods of all types.

Within each period, exchange takes place in two distinct stages. First a securities market opens at each location. Agents can trade in any securities market. Once in the securities markets, they can buy or sell arbirary contingent claims, payable in next period's securities markets, and buy or redeem one-period state-contingent government bonds. Goods cannot be traded at this time because agents can produce only during the second part of the period and goods produced in previous periods cannot be stored across periods. After securities trading concludes, goods production and exchange begin. At this time, an agent from each household--the "shopper"--travels around the circle once, acquiring goods from various locations. The other agent in the household--the "worker"--stays at home to produce goods and sell them to visiting shoppers from other locations. Shoppers are unable to carry their home-produced good with them and thus cannot barter. Sales are made on trade credit or for cash.

The economy's assets have the following features. Currency is the usual portable, durable, infinitely-divisible, counterfeit-proof object. In principle, any two agents can exchange contingent claims, at any future period in any location, payable in objects--goods or currency--that are available. Only two simple claims are necessary, however, to achieve equilibrium allocations: trade credit and cash loans. Trade credit is a promise, issued by a shopper in the current period's goods market, to pay a certain quantity of currency at the beginning of the next date. Thus trade credit is an alternative to paying in currency. A cash loan is a promise, issued in one of the current period's securities markets, to pay a certain quantity of currency in next period's securities market in the same location. Although these two claims are all we require to construct equilibria, we will, using now standard techniques, derive expressions for the prices at which arbitrary contingent claims would sell if they were traded.

Whenever two agents exchange a contingent claim, a real resource cost of kDx units of output is incurred, where k is a strictly positive constant, D is the shortest distance between the two agents' home locations, and x is the total real present value of the claim (units of the good purchased in the case of a trade credit purchase or real balances lent in the case of a cash loan). Equivalently, this transaction cost could be modelled as a cost of labor time, as in McCallum (1983). In what follows, however, it is measured in terms of foregone output.

The transaction cost can, perhaps, be viewed as the cost of verifying either the borrower's identity using the services of a third party (e.g. a notary or credit-rating agency) or the value of collateral (explicit or implicit). Such verification might require information-gathering costs that are related to distance. The proportionality of costs to transaction size might stem from the greater use of the notary's services because a more valuable contract exposes the seller to greater potential liability or a more valuable quantity of collateral must be assessed.[4]

In principle, an agent from one location could attend the securities markets at other locations and issue claims there. No agents do so in equilibrium. The presence of a large and equal number of identical agents at each location means that the securities markets are identical and perfectly competitive. This, along with the assumption that the transaction cost increases with distance, implies that optimizing agents only trade securities in the market at their home location.

Notice that for any given purchase, an agent can choose to use either cash or trade credit. This contrasts with standard cash-in-advance models. In those models, a given good is available either for cash or for credit, but not both, and the choice is given exogenously (see Lucas and Stokey (1983) and

(1987)). Here, cash purchases involve no transaction costs (only opportunity costs), and trade credit purchases involve a cost proportional to distance. Agents compare these costs in choosing whether to use cash or credit; thus, the division of expenditures between cash and trade credit purchases is endogenous.

Throughout this paper, we compare our economy with cash and trade credit to an economy that is identical except that all purchases of goods must be made using cash. The cash-only economy can be obtained as a special case in which k, the transaction cost parameter, approaches positive infinity.

To formally specify the model, assume that all households at a given location are identical. Define $c_{ht}(z,s_t)$ as the consumption at date t of type-z (i.e. location z, where $z\epsilon[0,1]$) good by a representative household with home location h, given state s_t. Each household is endowed with e_t units of labor time at time t. The representative household from location h devotes n_{ht} units of labor time to production, resulting in n_{ht} units of type-h good. To capture uncertainty concerning productivity, assume that e_t is random and thus is one component of the state vector s_t.

Household preferences are represented by

$$E_0 \sum_{t=1}^{\infty} \beta^t U(W(c_{ht}(\cdot,s_t)),e_t-n_{ht}), \tag{2.1}$$

where $W(c_{ht}(\cdot,s_t)) = \inf_{z\epsilon[0,1]} \{c_{ht}(z,s_t)\}$. $U(\cdot,\cdot)$ is twice continuously differentiable, strictly increasing and strictly concave. We will eventually assume that U() is logarithmic. Our assumption about W() eliminates substitution among consumption of goods of various types and allows us to focus attention on the choice between the use of cash and trade credit. Specifically, if the opportunity cost of acquiring consumption goods is positive for all goods, as is true in all equilibria considered, then households will buy the same amount of each good. We have, then, that $c_{ht}(z,s_t) = c_{ht}(s_t)$ for all $z \in [0,1]$.[5]

In each location, perfect competition implies that the cash price of a unit of goods is identical for all shoppers. Let p_{ht} be the price in fiat currency of a unit of location h good at date t. Any currency that a household receives from making cash sales must be held until the next period's securities markets open. A trade credit sale of one unit of the good to a shopper whose home location is a distance D away involves a real transaction cost of kD units of the good. The interest rate charged on the purchase of one unit of the good on trade credit must provide the worker with an amount of currency in next period's securities market equal to $p_{ht}(1+kD)$, the proceeds of selling 1+kD units of the consumption good for cash. Thus a credit shopper from a location at distance D is charged a gross nominal interest rate 1+kD.

From the perspective of a shopper at a distance D from home, the opportunity cost of using cash rather than trade credit to make a purchase is the cost of borrowing (or the cost in foregone interest of not lending) the cash in the period t securities market at the net nominal interest rate i_t. An optimizing agent chooses the least costly method of financing a purchase and, therefore, buys on credit if D satisfies

$$1+kD \leq 1+i_t. \tag{2.2}$$

Let D_t^* represent the distance from home at which a shopper is indifferent between the use of cash and trade credit at date t. Accordingly,

$$D_t^* = \min\,[i_t/k,\ 1/2]; \tag{2.3}$$

trade credit is used for purchases at or within a distance of D_t^* from home in either direction, and cash is used elsewhere. If $i_t \geq k/2$, all purchases are made on credit, and none are made with cash. For finite nominal interest rates, D_t^* approaches zero as k approaches $+\infty$; i.e. only cash purchases are made.

At the beginning of each period the state, s_t, is revealed to all households. Because agents at each location are identical each period, we can focus on a representative household at a representative location and thus drop the notation for the household's location. The household begins period t with m_t' units of currency held over from the previous date, b_{t-1} units of nominal one-period government bonds that mature in period t, and a_{t-1} units of outstanding one-period cash loans. The government bonds and the cash loans both pay a nominal interest rate i_{t-1}. In addition, the household has trade credit receivables of $(1+kD)p_{t-1}\tau_{t-1}$ from purchases by shoppers with homes a distance D away, where τ_{t-1} is the amount of the consumption good sold during the previous period to each customer (net of real transaction costs) and p_{t-1} is the common cash price of the goods.[6] Sales to each customer from location h are identical because of the symmetry in the economy; thus, $c_{ht}(s_t) = c_t(s_t)$ for all h. Because only shoppers located at or within a distance D_t^* in either direction use credit, total trade credit receivable is twice the integral of $(1+kD)p_{t-1}\tau_{t-1}$ from 0 to D_{t-1}^*, or $(2D_{t-1}^*+kD_{t-1}^{*2})p_{t-1}\tau_{t-1}$. Similarly, the household's total trade credit payable is $(2D_{t-1}^*+kD_{t-1}^{*2})p_{t-1}c_{t-1}$, due in the period t securities market. During securities trading, the household uses currency to purchase b_t government bonds and a_t cash loans. The remaining currency, m_t, is held for use in the period t goods markets. Summarizing then, the household faces the following constraint on the sources and uses of currency in the securities market:

$$m_t + b_t + a_t = m_t' + (1+i_{t-1})(b_{t-1}+a_{t-1}) - (2D_{t-1}^*+kD_{t-1}^{*2})p_{t-1}c_{t-1}$$

$$+ (2D^*_{t-1}+kD^{*2}_{t-1})p_{t-1}\tau_{t-1}. \tag{2.4}$$

In the goods markets, output is sold for cash or trade credit or devoted to transaction costs. Because τ_t is sold to each shopper and transactions costs of $kD\tau_t$ are incurred for all shoppers from less than a distance D^*_t away, a total of $(2D^*_t+kD_t^{*2})\tau_t$ units of output is exhausted on trade credit sales and associated costs. The remaining output is sold for cash, so the worker receives $[n_t-(2D^*_t+kD_t^{*2})\tau_t]p_t$ units of currency during the goods trading session. The shopper makes a fraction $1-2D^*_t$ of purchases using cash; this requires $(1-2D^*_t)p_t$ units of currency. Thus, the quantity of currency held until the next period is determined by

$$m'_{t+1} = m_t + [n_t-(2D^*_t+kD_t^{*2})\tau_t]p_t - (1-2D^*_t)p_tc_t. \tag{2.5}$$

Note that in the cash-only economy (i.e. the economy with $k = +\infty$), $D^*_t = 0$ for all t, simplifying (2.4) and (2.5) significantly.

Currency acquired from goods market cash sales during period t cannot be used for purchases at date t because these occur simultaneously at spatially separated locations. The household thus faces an endogenous "cash-in-advance constraint":

$$(1-2D^*_t)p_tc_t \leq m_t. \tag{2.6}$$

For the cash-only economy this reduces to its standard form: $p_tc_t \leq m_t$.

The government has at the securities market at each location an agent who conducts open market operations. Identical operations occur in each market. A positive quantity B_t of nominal one-period bonds is outstanding from period t to period t+1, bearing interest at rate i_t. At date t+1 the maturing obligation, $(1+i_t)B_t$, is funded by a combination of new bond issue, B_{t+1}, and new money issue, $M_{t+1}-M_t$.[7] There are no government taxes or transfers. Therefore, open market operations must satisfy the following government budget constraint:

$$M_{t+1} + B_{t+1} = M_t + (1+i_t)B_t. \tag{2.7}$$

For convenience, parameterize policy by assuming that the government randomly sets $x_{t+1} = M_t/M_{t+1}$, the inverse of the money growth rate.

The randomness in the economy is summarized in the state vector s_t that includes the labor endowment, e_t, and the policy variable, x_t. The state s_t is an element of the bounded and strictly positive state space $S \subset \Re^n_+$, where $n \geq 2$. The state evolves according to a stationary, first-order Markov process with transition function F; for each s_t, $F(\cdot, s_t)$ is a probability measure on the Borel sets of S governing s_{t+1}. We assume that if $g(s_{t+1})$ is a bounded, continuous, measurable function, then $\int g(s_{t+1})F(ds_{t+1}, s_t)$ is a bounded, continuous, measurable function of s_t, and that $F(\cdot, s_t)$ has support S for all $s_t \epsilon S$.

III. Equilibrium

A stationary symmetric monetary equilibrium is a set of functions $\{i(s_t), c(s_t), n(s_t), D^*(s_t)\}$ that govern real variables and relative prices; stochastic sequences for nominal variables, $\{p_t\}$, $\{M_t\}$ and $\{B_t\}$, that depend upon the entire sequence of states s_{t-j}, $j=1,2,\ldots t$, and the initial conditions; and initial conditions M_0 and $B_0(1+i_0)$, such that (i) households maximize expected utility, (2.1), subject to (2.4)-(2.6), (ii) the government budget constraint, (2.7), is satisfied, and (iii) the markets for money, bonds, loans and goods clear. The goods market clearing condition (and feasibility constraint) is

$$n_t = (1+kD_t^{*2})c_t, \text{ for all } t. \tag{3.1}$$

We restrict attention in what follows to equilibria with finite expected utility, strictly positive money supplies, strictly positive and finite price levels and strictly positive interest rates.

The first order conditions for the household's problem are

$$\beta^t U_1(c_t, e_t - n_t) - (2D_t^* + kD_t^{*2}) p_t E_t \lambda_{1t+1} - (\lambda_{2t} + \lambda_{3t})(1 - 2D_t^*) p_t = 0 \tag{3.2}$$

$$-\beta_t U_2(c_t, e_t - n_t) + \lambda_{2t} p_t = 0 \tag{3.3}$$

$$-\lambda_{1t} + (1 + i_t) E_t \lambda_{1t+1} = 0 \tag{3.4}$$

$$-\lambda_{1t} + \lambda_{2t} + \lambda_{3t} \leq 0, = \text{ if } m_t > 0 \tag{3.5}$$

$$-\lambda_{2t} + E_t \lambda_{1t+1} \leq 0, = \text{ if } m'_{t+1} > 0, \tag{3.6}$$

where λ_{1t}, λ_{2t} and λ_{3t} are the (stochastic) multipliers on constraints (2.4), (2.5) and (2.6), respectively, at date t. E_t denotes the expectations operator, conditional on s_t, and $U_j(c_t, e_t - n_t)$ is the derivative of U() with respect to its jth argument.

IV. Trade Credit Economies

For now we restrict attention to economies with both cash and trade credit used in exchange (i.e. economies with finite and positive values of k); later we will compare them to a cash-only economy in which k is taken to be $+\infty$. We can immediately verify that in equilibrium $D_t^* = i_t/k < 1/2$. For currency to be valued, (3.5) and (3.6) must hold with equality. Together with (3.4), they imply that $\lambda_{1t} i_t/(1+i_t) = \lambda_{2t} i_t = \lambda_{3t}$. From (3.2) and (3.3), λ_{1t} and λ_{2t} are positive, so the sign of λ_{3t} depends on the sign of i. Thus, $\lambda_{3t} > 0$, and (2.6) holds with equality when the nominal interest rate is positive. With (2.6) satisfied at equality, m_t/p_t is strictly positive, and thus $1 - 2D_t^* >$ 0.

Equilibria may be characterized by using the fact that most endogenous variables can be expressed as functions of the nominal interest rate. Thus, begin by deriving a stochastic difference equation in i_t. Solving this equation yields the function $i(s_t)$ describing the equilibrium behavior of the interest rate. Given $i(s_t)$, the behavior of all other endogenous variables can be determined.

To derive a stochastic difference equation in i_t, use (3.5) to eliminate $\lambda_{2t} + \lambda_{3t}$ from (3.2). This yields

$$\beta^t U_1(c_t, e_t - n_t) = \phi(i_t) p_t \lambda_{1t}, \tag{4.1}$$

where $\phi(i_t) \equiv [2D_t^*(1+kD_t^*/2) + (1-2D_t^*)(1+i_t)]/(1+i_t) = 1 - \frac{i_t^2}{(1+i_t)k}$.

The term inside the brackets in the definition of $\phi(i_t)$ is the effective gross nominal interest rate paid on the purchase of one unit of consumption at date t. It reflects the fact that a fraction $2D_t^*$ of each unit of consumption is bought with trade credit at an average gross interest rate of $1+kD_t^*/2$, and a fraction $1-2D_t^*$ is bought with cash that instead could have been lent in the securities markets at gross interest rate $1+i_t$. Thus, the bracketed term is the average cost (i.e. the sum of gross transaction costs and opportunity costs) in terms of date t+1 currency of a unit of goods at t. $\phi(i_t)$ is the value of this cost in terms of date t currency; that is, the cost is discounted at the rate i_t. The second expression for $\phi(i_t)$ makes use of the fact that $D_t^* = i_t/k$. $\phi(i_t)$ is less than one when the net nominal interest rate is positive, reflecting the fact that some trade credit bears a nominal interest rate less than i_t. It plays a crucial role in the anomalous asset-pricing formula derived below.

Equation (4.1) can be combined with the date t+1 version of the same equation and (3.4) to relate the rate of return on currency to the marginal rate of intertemporal substitution:

$$\frac{1}{\phi(i_t)p_t} = E_t\left[\frac{\beta U_1(c_{t+1},e_{t+1}-n_{t+1})(1+i_t)}{U_1(c_t,e_t-n_t)\phi(i_{t+1})p_{t+1}}\right]. \tag{4.2}$$

The left side of (4.2) is the value, in units of consumption good at date t, of one unit of date t currency. Lending that unit of currency provides $(1+i_t)$ units of currency at date t+1, which has a value of $(1+i_t)/\phi(i_{t+1})p_{t+1}$ in units of consumption good at date t+1. Thus (4.2) states that the current real value of money is the expected value of the real return on money loans weighted by the marginal rate of substitution between current and future consumption.

Continuing, eliminate p_t using the fact that (2.6) holds with equality and that the money market clears. Making this substitution and rearranging yields

$$\frac{(1-2i_t/k)}{(1+i_t)\phi(i_t)} = E_t\left[\frac{\beta U_1(c_{t+1},e_{t+1}-n_{t+1})c_{t+1}M_t(1-2i_{t+1}/k)}{U_1(c_t,e_t-n_t)c_tM_{t+1}\phi(i_{t+1})}\right]. \tag{4.3}$$

We prove the existence of an equilibrium for the special case of logarithmic utility. Substituting x_{t+1} for M_t/M_{t+1}, (4.3) becomes

$$\frac{(1-2i_t/k)}{(1+i_t)\phi(i_t)} = E_t[\beta x_{t+1}(1-2i_{t+1}/k)(\phi(i_{t+1}))^{-1}]. \tag{4.4}$$

The left side of (4.4) depends only on i_t, while the right side depends only on x_{t+1} and i_{t+1}. If we substitute $i(s_t)$ for i_t and $i(s_{t+1})$ for i_{t+1}, then (4.4) becomes a functional equation in i(). Establishing the existence of equilibrium requires first the proof of the existence of a solution to (4.4). The proof, which appears in the Appendix, verifies our guess about the sign of i() and depends on conditions summarized in the following proposition:

Proposition: If $\beta^{-1}(1+k/2)^{-1} < E[x_{t+1}|s_t] \le \beta^{-1}$ for all $s_t \epsilon S$, then there is a unique strictly positive function i() that solves (4.4).

Given the function $i(s_t)$ that determines the nominal interest rate each period as a function of the state, the remaining endogenous variables can be readily calculated. First, $D_t^* = D^*(s_t) = i(s_t)/k$. Next, combine the first order conditions (3.2) and (3.3), substituting for D_t^*, to get

$$\frac{U_1(c_t, e_t - n_t)}{U_2(c_t, e_t - n_t)} = 1 + i(s_t) \cdot (i(s_t))^2/k. \tag{4.5}$$

Together with the feasibility condition $n_t = (1+(i(s_t))^2/k)c_t$, (4.5) can be solved for c_t and n_t to obtain the functions $c(s_t)$ and $n(s_t)$. Note that the slope of the feasibility condition differs, in general, from the marginal rate of substitution in (4.5). The latter reflects the trade-off in the household's budget constraint and includes the (intangible) opportunity cost of money balances. Real money balances are

$$\frac{M_t}{P_t} = (1-2i(s_t)/k)c(s_t), \tag{4.6}$$

and real trade credit balances are

$$r_t = 2i(s_t)c(s_t)/k. \tag{4.7}$$

When utility is not logarithmic, the solution is more tedious to obtain. First, one solves (4.5) and the feasibility condition to obtain c_t and n_t as implicit functions of e_t and i_t. These are then substituted for c_t, n_t, c_{t+1} and n_{t+1} in (4.5), yielding, again, a functional equation in i(). Unlike the logarithmic case, however, the real shocks e_t and e_{t+1} appear on the right side of (4.3), so that even if money growth is independent of real shocks, the latter affect the nominal rate.

V. Cash-Only Economies

In cash-only economies, the equilibrium conditions are more simple. For any positive finite nominal interest rate, $D_t^* = i_t/k = 0$, and thus all purchases are made using cash. The first order conditions from the household's problem are identical to those for trade credit economies, except that (3.2) can now be written

$$\beta_t U_1(c_t, e_t - n_t) - (\lambda_{2t} + \lambda_{3t}) p_t = 0. \tag{5.1}$$

Combining (5.1) with (3.4) and (3.5) yields an equation analogous to (4.2):

$$\frac{1}{(1+i_t)p_t} = E_t \left[\frac{\beta U_1(c_{t+1}, e_{t+1} - n_{t+1})}{U_1(c_t, e_t - n_t) p_{t+1}} \right]. \tag{5.2}$$

This is a standard expression for one-good cash-in-advance models. The endogenous "cash-in-advance constraint" $p_t c_t = m_t$ can be used to eliminate p_t. Again, specializing to the case of logarithmic utility and substituting x_{t+1} for M_t/M_{t+1}, (5.2) becomes

$$\frac{1}{1+i_t} = E_t[\beta x_{t+1}]. \tag{5.3}$$

A comparison of (5.3) to (4.4) indicates that the nominal interest rate depends solely on the anticipated real rate of return on money in the cash-only economy, while anticipated future nominal rates also affect the current nominal rate in trade credit economies. The equilibrium function for the nominal interest rate, $i(s_t)$, can be obtained directly from (5.3):

$$i(s_t) = \frac{1}{E_t[\beta x_{t+1} | s_t]} - 1. \tag{5.4}$$

Given $i(s_t)$, equilibrium functions for c_t and n_t can be obtained exactly as for economies with trade credit by solving (4.5) and the goods market clearing, now $n_t = c_t$, simultaneously. This amounts to solving for c_t in

$$\frac{U_1(c_t, e_t - c_t)}{U_2(c_t, e_t - c_t)} = 1 + i(s_t). \tag{5.5}$$

As in trade credit economies, unless x_t is serially correlated, unanticipated realizations of money growth have no effect other than a strictly proportional change in the price level.

VI. Asset Prices in Securities Markets

In any period's securities markets agents could, in principle, exchange any arbitrary contingent claims payable in next period's securities markets. These were safely neglected above because the symmetry among households and locations implies that net demand will be identically equal to zero. Using now standard techniques, first described in Lucas (1982), one-period-ahead Arrow-Debreu securities can be introduced, expressions derived for their equilibrium prices, and the prices used to derive prices for any arbitrary contingent claim. In particular, we can obtain an expression for the real rate of return in the <u>securities</u> markets. This real rate has an interesting anomalous form.

Formally, let $\chi(s_{t+1})$ be an arbitrary (bounded and measurable) function of next period's state, and let $q_\chi(s_t)$ be the price in a securities market in period t if the current state is s_t of a claim that provides $\chi(s_{t+1})$ units of currency in the same securities market in period t+1 if the state is s_{t+1}. Denote the number of units of this claim purchased by a household by v_t. Then the security market constraint (2.4) can be written

$$m_t + b_t + q_\chi(s_t)v_t = m_t' + (1+i_{t-1})b_{t-1} + v_{t-1}\chi(s_t)$$
$$- (2D^*_{t-1}+kD^{*\,2}_{t-1})p_{t-1}c_{t-1} + (2D^*_{t-1}+kD^{*\,2}_{t-1})p_{t-1}y_{t-1}. \tag{6.1}$$

In an equilibrium, v_t must be zero because households are identical. The resulting first order condition for v_t is

$$-\lambda_{1t}q_{\chi}(s_t) + E_t\chi(s_{t+1})\lambda_{1t+1} = 0. \tag{6.2}$$

(Recall that λ_{1t} and λ_{1t+1} depend on s_t and s_{t+1}, respectively.) Substituting from (4.1) yields

$$q_{\chi}(s_t) = E_t\left[\frac{\beta U_1(c_{t+1},e_{t+1}-n_{t+1})\phi(i_t)p_t\chi(s_{t+1})}{U_1(c_t,e_t-n_t)\phi(i_{t+1})p_{t+1}}\right]. \tag{6.3}$$

Compare this expression to the asset-pricing formula from monetary models with costless credit for "credit goods" (e.g. Lucas and Stokey (1987) or Townsend (1987)). The analogue of the standard formula for our environment is

$$E_t\left[\frac{\beta U_1(c_{t+1},e_{t+1}-n_{t+1})p_t\chi(s_{t+1})}{U_1(c_t,e_t-n_t)p_{t+1}}\right], \tag{6.4}$$

where $U_1()$ is the marginal utility of the "cash good." Verification that this expression is the appropriate asset-pricing formula for the cash-only economy is straightforward. The difference between (6.3) and (6.4) is the presence of the term $\phi(i_t)/\phi(i_{t+1})$ in the former. Expression (6.4) is also the asset-pricing formula in one-good cash-in-advance models such as Cooley and Hansen (1989).

To understand the term involving the function $\phi()$, consider a claim, issued in the period t securities market at a given location, that provides one unit of the composite consumption good in the same period at the same location <u>after</u> goods market trading but before consumption takes place. The price of such a claim would be $\phi(i_t)p_t$ in our trade credit economies but would be simply p_t in the cash-only economy (or in the Lucas-Stokey economy without trade credit costs). Recall that $\phi(i_t)$ is the average cost of a unit of consumption, discounted by the factor $1+i_t$, and takes into account the fact that inframarginal trade credit (at distances less than D_t^*) costs less than the opportunity cost of currency (the nominal rate). As a result, $\phi(i_t)$ varies negatively with the nominal rate because average trade credit costs vary less than the nominal rate.

The marginal utility in (6.3) is with respect to a composite good, essentially the marginal utility of varying the (identical) consumption of every good by an identical amount. With more general preferences that allow substitution across goods, the marginal utilities to a given household of various goods at a given date will be related via their relative transaction costs based on the household's location. If preferences still satisfy the necessary conditions for consumption to be treated as a composite commodity, then something very much like (6.3) would hold for these more general preferences. Similarly, aggregating across different consumption goods in the Townsend and Lucas and Stokey models would yield an asset-pricing formula identical to (6.3), but with $\phi(i_t)$ taking a different form. Although $\phi(i_t)$ would depend on the nominal interest rate, the dependence would be simpler because of the absence of trade credit costs from their models.

One interesting implication of (6.3) is that the nominal rate affects the risk-free real rate of return in the securities markets. The real rate of return, r_t, can be calculated from a contingent claim that yields $\chi(s_{t+1}) = p_{t+1}/p_t$ in all states s_{t+1}. The result is

$$\frac{1}{1+r_t} = E_t\left[\frac{\beta U_1(c_{t+1},e_{t+1}-n_{t+1})\phi(i_t)}{U_1(c_t,e_t-n_t)\phi(i_{t+1})}\right]. \tag{6.5}$$

The cash-only economy yields the standard formula for the real rate: the same expression but without the $\phi(i_t)/\phi(i_{t+1})$ term. Because $\phi(i_t)$ is decreasing in i_t, (6.5) implies that, <u>ceteris paribus</u>, the real rate is increasing in the nominal rate. Intuitively, an increase in the nominal rate increases the wedge between consumption and currency, as would an increase in a proportional tax on consumption, and $U_1(c_t,e_t-n_t)/\phi(i_t)$ goes up for any given (c_t,n_t). This effect is not present in standard cash-in-advance models.

Notice that this anomaly is not an artifact of the sequence of securities and goods exchange. If agents could meet in their own locations after goods market trading ends and exchange claims, currency or consumption, identical allocations and asset-pricing formulas would result.[8] The same anomaly, as well as others, would arise if securities and goods market trading were concurrent, as in Lucas (1990).

VII. Other Properties of Equilibria

In general, the properties of the equilibrium time series of the model depend on the transition function, F, governing real and policy variables, and, as in Lucas and Stokey (1987), virtually anything is possible. A few simple special cases, however, generate sharp results and yield insights into the economics of the model. First, suppose money growth is an i.i.d. random variable, independent of the real shocks. In this case the current state contains no information about the distribution of x_{t+1}. The solution to (4.4) is a constant nominal interest rate, i, that satisfies

$$\frac{1}{1+i} = E_t[\beta x_{t+1}], \tag{7.1}$$

a simple version of the Fisher equation. Because the nominal rate is constant, the asset-pricing formula is as denoted in (6.4), and because of the logarithmic preferences, the real rate is β^{-1}-1. Consumption and employment are jointly determined by (3.1) and (4.5). Clearly, both c_t and n_t respond positively to real shocks. Real money balances and real trade credit outstanding also respond positively, as can be seen in (4.6) and (4.7). Note, however, that the ratio of the two is a constant that depends on i.

Now suppose that money growth is independent of current and past real shocks, so that $F(dx'|s)$ is independent of e, but that expected future money growth fluctuates. Under this condition, real shocks can be eliminated from the right side of the functional equation (4.4) because they do not help forecast x_{t+1}. This implies that the solution for the nominal rate depends only on information about future money growth and, thus, is independent of real shocks. The nominal rate can be viewed, then, as responding to fluctuations in expected money growth, or, alternatively, as set by policy, with (4.4) determining future money growth. In the latter case money growth is actually being constrained by the government budget constraint, as in Leeper's (1990) Region II. As is standard in cash-in-advance models, realizations of the money growth process have a strictly proportional effect on the price level and have no other effects unless they are correlated with future money growth.

Policy shocks that alter the nominal interest rate have novel real effects. A policy shock that raises the nominal interest rate causes households to use trade credit for a larger fraction of their purchases and money for a smaller fraction. This causes an increase in the share of output devoted to the real resource costs of trade credit, driving a larger wedge between employment and consumption in the feasibility condition (3.1). In addition, the marginal rate of substitution between consumption and leisure--the right side of (4.5)--is larger. Given that leisure and consumption are normal goods, both effects of an increase in i_t cause consumption to decrease. Thus real money balances unambiguously decrease with increases in i_t. The effect on employment is ambiguous; the nominal rate has different effects on the marginal rate of substitution and the slope of the feasibility condition. The response of trade credit outstanding to a blip in the nominal rate is also ambiguous; the higher nominal rate causes households to substitute trade credit for money balances, but the decline in consumption has an offsetting effect. However, the ratio of trade credit to money balances unambiguously falls.

Policy shocks that affect the nominal rate will have implications for the real rate that depend on the serial correlation of those shocks. These implications can be understood by noting that, under logarithmic preferences, equation (6.5) for the real rate can be written as

$$\frac{1}{1+r_t} = E_t\left[\frac{\beta c_t \phi(i_t)}{c_{t+1}\phi(i_{t+1})}\right]. \tag{7.2}$$

Because c_t and $\phi(i_t)$ are both decreasing in i_t, the right side of (7.2) is decreasing in i_t, unless the correlation between i_t and i_{t+1} is sufficiently strong and positive. If policy is such that the nominal rate is i.i.d., for example, policy shocks that increase the nominal rate also increase the real rate. On the other hand, policy that makes the nominal rate highly persistent reduces the variation in the real rate. Thus, smoothing nominal interest rates reduces the distortion in intertemporal choice implied by the impediments to exchange.

The behavior of velocity depends on how it is calculated. One reasonable measure is the ratio of nominal expenditures (including both cash and trade credit goods) to the stock of currency.[9] By this measure, velocity is equal to $p_t c_t/m_t = (1-2i_t/k)^{-1}$, which varies positively with the nominal interest rate. This measure corresponds closely to empirical measures of velocity that fail to distinguish between cash and credit transactions.[10]

The case in which money growth fluctuates but is independent of real shocks produces a striking result: the ratio of trade credit to money balances varies with policy shocks that affect nominal interest rates, but is invariant with respect to real shocks. This raises the intriguing possibility, first proposed by Valerie Ramey (1988), that time series observations on money, trade credit, output and nominal interest rates might allow the disentangling of real versus monetary sources of business cycle shocks. Her approach is to specify a structural model based on the arguments in King and Plosser (1984) that "inside money" (equivalently, "credit") ought to respond to current and anticipated real shocks rather than to monetary policy. Our model provides an underpinning for her idea in that money and trade credit respond in the same direction to real shocks, but in opposite directions to policy shocks. Our model also demonstrates, however, that stringent identifying assumptions are required to obtain this result. Specifically, monetary policy must be independent of all current and past real disturbances because otherwise current real shocks will help forecast future money growth and thus will affect current nominal rates. This assumption is quite implausible, as argued in Lacker (1988, 1990), and makes the interpretation of Ramey's empirical results ambiguous. Furthermore, with logarithmic preferences, current and expected future real shocks in general will affect nominal rates through their effect on the volume of trade.

VIII. Comparison of Trade Credit and Cash-Only Economies

As noted in Section I, a tight linkage between money and prices is central to the quantity theory. The idea that the creation of inside money weakens this linkage is not new; the antibullionists made this argument at the end of the eighteenth century. The banking school took up the battle cry in its debate with the currency school in the mid-1800s. More recently, in 1959, the Radcliffe Committee continued the attack, claiming that the use of money substitutes such as trade credit interferes with a monetary authority's ability to control the monetary base and thus the price level and nominal GNP. (See Humphrey (1974).)

These arguments suggest comparing the response to a policy shock of a cash-only economy (one with $k \rightarrow +\infty$) to that of an economy with both cash and trade credit used in exchange (one with k positive and finite). Trade credit attenuates the effect of money growth on the nominal interest rate. Suppose expected money growth is i.i.d., and consider a policy innovation that raises the expected growth rate of money (i.e. a downward shift in the distribution governing x_{t+1}, the inverse money growth rate). In both trade credit and cash-only economies, the nominal interest rate is higher because of a higher inflation premium. The response of the nominal interest rate is smaller, however, the <u>smaller</u> is k, and thus the response is largest in the cash-only economy. Put another way, in trade credit economies, both the nominal interest rate and the real rate rise in response to an increase in expected money growth, while in the cash-only economy only the nominal rate is affected. The reason is that in trade credit economies the fraction of goods bought with money, $1-2i_t/k$, can vary in response to changes in the rate of return on money. When increases in the latter make market transactions more costly, the increased use of trade credit allows agents to reduce the effect on current consumption. If monetary policy is thought of as choices of money growth rates, there is thus a sense in which trade credit "frustrates" the interest rate effects of policy.

Trade credit increases the sensitivity of velocity to interest rates. In the cash-only economy, velocity, as measured by the ratio of nominal expenditures to the stock of currency, is always unity, while as noted above, velocity varies positively with the nominal rate in trade credit economies. The derivative of velocity with respect to the nominal rate is decreasing in k. A "tight" monetary policy that raises the current nominal rate has a smaller effect on the current price level in trade credit economies because velocity can vary in the opposite direction. Thus, if one thinks of policy innovations as nominal rate innovations, there is a sense in which trade credit "frustrates" policy by allowing offsetting variations in velocity.[11] Consequently, trade credit use, and the use of money substitutes more generally, must be considered in designing monetary policy.

IX. Conclusion

The relationship between money and credit has been central to the effort to capture monetary phenomena in well-articulated economic environments. In this paper, we explored one aspect of this relationship by focusing on the interaction between money and one particular, though ubiquitous, form of credit. Our primary finding was that, when the transaction costs associated with trade credit use are formally modelled, asset-pricing anomalies arise despite the existence of perfectly competitive and frictionless securities markets. We are persuaded that this result will obtain in other monetary model in which payment system usage is costly. Our result suggests more generally that models with impediments to exchange carefully specified can have empirical implications that differ significantly from models of frictionless environments. This, in turn, suggests that a more rigorous modelling of the payment system is imperative in future research.

Appendix

Proof of the Proposition:

Preliminaries. Define a variable ψ by

$$\psi = \frac{1-2i/k}{(1+i)\phi} = \frac{1-2i/k}{1+i-i^2/k}. \tag{A.1}$$

Solving (A.1) for 1+i yields

$$1 + i = 1 + k/2 + \psi^{-1} - \sqrt{\psi^{-2}+k+k^2/4}, \tag{A.2}$$

where we have used the requirement that $0 \leq i < k/2$ to eliminate one quadratic root. It can be easily verified that ψ in (A.1) is a strictly decreasing function of i, and that $0 \leq i < k/2$ if and only if $0 < \psi \leq 1$. Define a function $\Gamma(\psi)$ by

$$\Gamma(\psi) \equiv 1 + (1+k/2)\psi - \sqrt{1+(k+k^2/4)\psi^2}.$$

$\Gamma(\psi)$ is equal to $(1+i)\psi$ if 1+i is given by (A.2).

Equation (4.4) can be rewritten as

$$\psi(s) = \int \beta x' \Gamma(\psi(s'))F(ds',s), \tag{A.3}$$

where we write s for s_t, s′ for s_{t+1}, and x′ for x_{t+1}. Define the function space Ψ as the set of continuous, measurable functions $\psi : S \to (0,1]$. Define the operator A on Ψ as the right side of (A.3), i.e.

$$(A\psi)(s) = \int \beta x' \Gamma(\psi(s'))F(ds',s), \text{ for } s \in S. \tag{A.4}$$

We seek a unique fixed point for the operator A, a unique function ψ^* in Ψ that satisfies $\psi^* = A\psi^*$. Because Ψ is not compact, and A is not necessarily a contraction, the usual fixed point theorems based on contraction mappings or compactness do not apply. Instead, we will apply a theorem on monotone operators to prove the existence of at least one fixed point, and then adapt a theorem on concave operators to prove uniqueness.

Existence. We will say that "$\psi_2 \geq \psi_1$" if $\psi_2(s) \geq \psi_1(s)$ for all $s \epsilon S$. Then $(\Psi, \geq)$ is a partially ordered set. We prove a set of facts about A and $(\Psi, \geq)$.

$A: \Psi \to \Psi$. It is easily verified that $\Gamma: (0,1] \to (0,1]$ is continuous, strictly increasing and that $\Gamma(1) = 1$. Therefore, $A\psi$ is bounded, continuous, measurable, and strictly positive for all $\psi \epsilon \Psi$. It remains to show that $(A\psi)(s) \leq 1$ for all $s \epsilon S$.

$$\begin{aligned}(A\psi)(s) &= \int \beta x' \Gamma(\psi(s')) F(ds', s) \\ &\leq \int \beta x' \Gamma(1) F(ds', s) \\ &= \int \beta x' F(ds', s) \leq 1,\end{aligned}$$

where the last inequality follows from the condition in the proposition that $E[x_{t+1} | s_t] \leq \beta^{-1}$. Therefore, $A\psi \epsilon \Psi$ for all $\psi \epsilon \Psi$.

Define an operator T on (Ψ, ψ_i) to be continuous if for each countable chain $\{\geq\}$ having a supremum, $T\bar{\psi} = \sup \{T\psi_i\}$, where $\bar{\psi} = \sup \{\psi_i\}$. It is easy to verify that A is continuous.

Next we show that there exists a function $\psi_0 \epsilon \Psi$ such that $A\psi_0 \geq \psi_0$. Define a function $a(z,s)$ by

$$\begin{aligned}a(z,s) &= \int \beta x' \Gamma(z) F(ds', s) \\ &= \beta \Gamma(z) E[x' | s].\end{aligned}$$

Note that

$$\begin{aligned}a(z,s) &\geq \beta \Gamma(z) \inf_s \{E[x' | s]\} \\ &= \beta z \Gamma'(\hat{z}) \inf_s \{E[x' | s]\},\end{aligned}$$

for some $\hat{z} \epsilon (0,z)$, by the Mean Value Theorem and the fact that $\Gamma(0) = 0$. The derivative, $\Gamma'(z)$, is continuous and equal to $1+k/2$ when $z = 0$. The condition of the proposition that $E[x_{t+1} | s_t] > \beta^{-1}(1+k/2)^{-1}$ implies that $\beta \Gamma'(0) \inf\{E[x' | s]\}$

> 1. By the continuity of $\Gamma'(z)$, there exists a $z_0 > 0$ such that

$$a(z_0,s) \geq \beta z_0 \Gamma'(\hat{z}_0)\inf_s\{E[x'|s]\}$$
$$> z_0$$

for some $\hat{z}_0 \epsilon (0,z_0)$. Setting $\psi_0(s) = z_0$ for all s, implies therefore that $A\psi_0 \geq \psi_0$.

Note finally that for every countable sequence $\{\psi_i\}$ in the set $\{\psi|\psi \geq \psi_0\}$, a supremum exists. We have now demonstrated that $(\Psi,\geq)$ and A satisfy the conditions of the Tarski-Kantorovich Theorem (see Dugundji and Granas (1982, p. 15)), which is restated here for reference:

Theorem: Let $(\Psi,\geq)$ be a partially ordered set, and let $A:\Psi\rightarrow\Psi$ be continuous. Assume that there is a $\psi_0 \epsilon \Psi$ such that (i) $A\psi_0 \geq \psi_0$ and (ii) every countable sequence in $\{\psi|\psi\geq\psi_0\}$ has a supremum. Then the set of fixed points of A is nonempty. Moreover, $\psi^* = \sup_n\{A^n(\psi_0)\}$ is a fixed point, and ψ^* is the supremum of the set of fixed points of A in $\{\psi|\psi\geq\psi_0\}$.

Thus we have proven the existence of a fixed point.

Uniqueness. First note that with the sup norm, Ψ is a Banach space. An operator $T:\Psi\rightarrow\Psi$ on a Banach space Ψ is concave if (i) for each nonzero $\psi\epsilon\Psi$, there exists a scalar $\alpha>0$ such that $(T\psi)(s) \geq \alpha$ for all $s\epsilon S$, and (ii) for each $\psi\epsilon\Psi$ such that for some $\alpha_1 > 0$, $\psi(s) \geq \alpha_1$ for all $s\epsilon S$, we have that

$$(T\lambda\psi)(s) \geq [1+\eta(\psi,\lambda)]\lambda(T\psi)(s) \text{ for all } s\epsilon S,$$

where $\eta(\psi,\lambda) > 0$ and $0 < \lambda < 1$.

We now prove that A is a concave operator. (i) Suppose ψ is nonzero. Then for each $s\epsilon S$, $\Gamma(\psi(s'))$ is positive on a set of positive measure.

Because x' is strictly positive, it follows that there exists an $\alpha > 0$ such that

$$\int \beta x' \Gamma(\psi(s')) F(ds', s) \geq \alpha \text{ for all } s \epsilon S.$$

(ii) Note that Γ is strictly concave on $(0,1]$. Suppose that $\psi(s) \geq \alpha$ for some $\alpha > 0$. Then

$$\begin{aligned}(A\lambda\psi)(s) &= \int \beta x' \Gamma(\lambda\psi(s')) F(ds', s) \\ &= \lambda \int \beta x' \Gamma(\psi(s')) F(ds', s) + \int \beta x' [\Gamma(\lambda\psi(s')) - \lambda\Gamma(\psi(s'))] F(ds', s) \\ &= (1+\eta(\psi,\lambda))\lambda(A\psi)(s),\end{aligned}$$

where

$$\eta(\psi,\lambda) = \frac{\int \beta x' [\Gamma(\lambda\psi(s')) - \lambda\Gamma(\psi(s'))] F(ds', s)}{\int \beta x' \Gamma(\psi(s')) F(ds', s)}.$$

The strict concavity of Γ implies $\eta(\psi,\lambda) > 0$.

We now prove the following Theorem, adapted from Krasnosel'skiǐ and Zabreĭko (1984, Theorem 46.1, p. 290).

<u>Theorem</u>: Let T be a concave monotone operator on Ψ. Then T has at most one fixed point in Ψ.

<u>Proof</u>: Suppose $\psi_1 = T\psi_1$ and $\psi_2 = T\psi_2$, $\psi_1 \neq \psi_2$, and, without loss of generality, $\psi_1 \not> \psi_2$. From (i) of the definition of concave operators, $\psi_1 = T\psi_1 \geq \alpha$. Because $\psi_2 \in \Psi$, we have $\bar{\psi} \geq \psi_2 = T\psi_2$, where $\bar{\psi}(s) = 1$, for all $s \epsilon S$. Therefore $\psi_1 \geq \lambda\psi_2$ for small positive λ. Then there is a $\lambda_0 \in (0,1)$ such that $\psi \geq \lambda_0\psi_2$ and $\psi_1 \not\geq \lambda\psi_2$ for all $\lambda > \lambda_0$. From (ii) of the definition of concave operators,

$$(T\lambda_0\psi_2)(s) \geq (1+\eta(\psi_2,\lambda_0))\lambda_0(T\psi_2)(s) \text{ for all } s \epsilon S,$$

where $\eta(\psi_2,\lambda_0) > 0$. Then we have

$$\psi_1 = T\psi_1 \geq T\lambda_0\psi_2 \geq (1+\eta(\psi_2,\lambda_0))\lambda_0 T\psi_2$$
$$= (1+\eta(\psi_2,\lambda_0))\lambda_0\psi_2.$$

Thus $\psi_1 \geq \lambda_1\psi_2$ where $\lambda_1 = (1+\eta(\psi_2,\lambda_0))\lambda_0 > \lambda_0$, a contradiction.■

Because A is a concave operator on Ψ, it has at most one fixed point in Ψ, and because we have already proved that A has at least one fixed point, we know that A has exactly one fixed point. If $\psi^*(s)$ is the unique fixed point, then i(s) can be found by substituting $\psi^*(s)$ for ψ in (A.2).

Footnotes

1. Data are from the U.S. Commerce Department, Bureau of the Census, Quarterly Financial Report for the third quarter of 1990, p. 4.

2. This is also true in McCallum's (1983) shopping-time model.

3. See Humphrey (1974).

4. Kroncke, Nemmers and Grunewald (1978, pp. 126-127) describe the information gathering activities firms undertake before approving trade credit sales.

5. Any consumption bundle $c_{ht}(z,s_t)$ maximizing utility satisfies $c_{ht}(z,s_t) = c_{ht}(s_t)$ almost everywhere and $c_{ht}(z,s_t) > c_{ht}(s_t)$ otherwise. Because the set of z for which this inequality is satisfied is of measure zero, $c_{ht}(z,s_t)$ is assumed, without loss of generality, to equal c_t for all z.

6. Recall, as stated earlier, that we only consider equilibria in which each good sells for the same cash price. This is common in the cash-in-advance literature.

7. Government bonds are nonnegotiable and thus cannot perform the same role as currency in exchange patterns. Issuing negotiable claims backed by bonds is legally prohibited.

8. Note that the price of (composite) consumption in this market would be $p'_t = \phi(i_t)(1+i_t)p_t$. The standard asset-pricing formula would apply in this market relative to p'_t; that is, (6.4) would hold with p_t replaced by p'_t. However, all observable goods market exchange takes place at p_t, not at p'_t. Thus empirical work on asset pricing based on (6.4) arguably employs the counterpart of p_t, not p'_t.

9. Alternatively, velocity may be defined as the number of times per period that currency is exchanged directly for consumption. This measure is always one in our model because all currency is spent on goods each period when the nominal interest rate is positive. Yet another measure of velocity is nominal output divided by the currency stock, $p_t n_t/m_t = (1+i_t^2/k)(1-2i_t/k)^{-1}$, which also varies positively with i_t.

10. In our model, all trade credit transactions are eventually settled in cash. Nevertheless, velocity is altered by trade credit use because the same currency can be used to settle a cash purchase and a trade credit purchase made on the same day. For example, A spends one dollar today with B, who then uses it tomorrow to settle today's credit transaction with C. In this sense, currency changes hands more than once per period because it is used to settle both credit and cash transactions.

11. A "tight" policy that raises the nominal rate is associated with higher money growth because of the unpleasant arithmetic of the government budget constraint. See Leeper (1990) or Sargent and Wallace (1981).

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