Exercise 8: A Steady State Distribution Model
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People

$$\max E\{\sum_t \beta^t \theta_t u(c_t)\}$$

subject to

$$c_t + m_{t+1} \leq y + m_t$$

where

i. real cash balances $m \in \{0,1,2,3,\ldots,m_{max}\}=M$
ii. u is strictly increasing and strictly concave;
iii. $0<\beta<1$;
iv. y is a lot bigger that 1 and a lot smaller than m_{max};
v. a person's θ_t are i.i.d. over time and belong to a finite set of strictly positive real numbers Θ. The biggest θ is denoted θ_{max} ;
vi. the probability that $\theta_t=\theta$ is $\pi(\theta)$ for $\theta \in \Theta$;
vii. the $\{\theta_t\}$ processes are "independent" across individuals and there are a large number of individuals so laws of large numbers hold.

a. Write down the dynamic programming optimality equation.
Let $m_{t+1}=g(m_t,\theta_t)$ be the optimal policy function if the price level is constant.

b. Prove that for any m_0 in M, the sequence $\{m_{t+1}=g(m_t,\theta_{max})\}$ converges to 0.

c. Let x(m) be the fraction of people with real cash balances m belonging to M. Write down the system of equations which specify what next period's fractions x'(m') would be if everyone uses the policy function g(m,θ).

d. Prove that the distribution of people as indexed by m converge to a limiting distribution x* which is independent of x_0. By prove I mean state a theorem and verify that the conditions for the theorem to hold are satisfied for this problem.

e. Let M^s be the money supply and P the price level. They are constant over time. State the equilibrium condition that determines P given M^s.