Contract Multiplier Revisited: Solving the Persistence Problem in a Model with Staggered Contracts

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Abstract

In a standard dynamic stochastic general equilibrium model with deterministic price staggering, Chari, Kehoe, and McGrattan (1998) find that staggered price contracts in the spirit of Taylor (1979,1980) cannot generate persistence in real effects of monetary disturbances. This paper reconsiders the ability of staggered contracts to generate persistent effects of monetary disturbances. In a model with price and wage contracts in the spirit of Calvo (1983), I demonstrat that the "contract multiplier" is generated by nominal rigidities in both labor and goods markets. Other features of business cycles, such as the hump-shaped responses of output, real wage acyclicality, and the persistence in inflation rate are also well explained by the model. Calibration exercises and analytical solutions of stripped-down versions of the model suggest that wage stickiness is more effective in generating persistence, since it directly controls the marginal cost of firms and thereby dampens the incentive for firms to raise prices after expansionary monetary shocks. Comparing stochastic and deterministic staggered contracts, I find that the oscillatory responses (hence, no persistence) in output in CKM is due to a counterintuitive nuisance feature of deterministic staggered price contracts (i.e., the initial overshooting of prices reset after monetary disturbances), and that, free of such nuisance feature, stochastic staggering is in principle capable of generating persistence even if marginal costs are highly procyclical.

1 Introduction

In this paper, I construct a stochastic general equilibrium model in which nominal disturbances generate considerable business cycle fluctuations, and use the model to address the following questions: *i)* staggered nominal contracts can be a propagation channel through which monetary shocks can generate persistent output variability; *ii)* two alternative sources of nominal rigidities

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(i.e., wage stickiness and price stickiness) have equivalent implications to the real consequences of nominal disturbances; and *iii)* two alternative schemes of staggered contracts (i.e., deteministic and stochastic staggering) work in a similar way to generate persistence in real variability after nominal shocks.

This paper builds on the long tradition of incorporating staggered nominal contract to induce persistence into rational expectation models. Dating back two decades, the early New Keynesian rational expectation models pioneered in providing a promising channel for generating persistence via staggered nominal prices and/or wages. For example, Taylor (1979, 1980) describes an economy in which *deterministic* (or time-dependent) staggered wage contracts as short as one year generate movements in aggregate variables with patterns of persistence similar to those observed in the postwar US business cycles. Blanchard (1983) shows that similar results also obtain when prices rather than wages are staggered by firms. A different strand of staggering scheme, one known as *stochastic* staggering, has also been used by some authors. For example, Calvo (1983) and Rotemberg (1982) develop dynamic price setting rules, which show how a rational firm would select its price in the current period given that it will have to keep its price fixed for an interval of stochastic length.

Recently, the prototype business cycle models have been enriched by New Keynesian features nature and sources of fluctuations other than technology shocks. Such expansion of the research arena results in models aiming toward the reconciliation of the real business cycles framework with New Keynesian features of market imperfections.[1]

Having nominal rigidities as inherent elements, models in the NNS literature can easily produce real effects of monetary shocks in a predictable way, and some are highly successful in replicating essential empirical features of data.[2] More recently, however, the challenging work of Chari, Kehoe, and McGrattan (1998, henceforth CKM) put into question the ability of staggered price contracts, possibly one of the most promising transmission channels for nominal shocks, to generate persistent real effects of monetary disturbances. Their surprising conclusion is that, once cast into a business cycle model, staggered prices in the spirit of Taylor *cannot* deliver an *endogenous persistence in* output. The authors attribute the inability of their model to account

[1]The tendency toward the intellectual fusion has been conspiuous enough for Goodfriend and King (1997) to coin out the terminology "New Neoclassical Synthesis (NNS)", the key elements of which are summarized as: *i)* intertemporal optimization, *ii)* rational expectations, *iii)* imperfect competition, and *iv*) costly price adjustments.

[2]For example, in a "menu cost"framework Kim(2000) succeeds in getting persistence in output.

for persistence in output the high procyclicality of marginal costs(especially labor costs) implied by standard assumptions about preference and factor market clearing.

The gauntlet thrown down by CKM brought forth a voluminous literature incorporating various New Keynesian features to help generate the persistence in output. As emphasized by Ball and Romer (1990), price stickiness only occurs if the "cost push" effects of marginal costs are moderate after the economy is perturbed by an expansionary nominal shock.[3]Motivated by this penetration, several recent papers investigate the role of labor market frictions in accounting for contract multiplier without having to incorporate a large exogenous component of price stickiness. For eample, Erceg (1997) replaces the assumption of spot labor market-clearing as in CKM with the wage-setting process a la Taylor, and obtains a modest increase in the degree of persistence in output. Jeanne (1997) follows Calvo in specifying nominal rigidity in the goods market, but introduces *real rigidity* in the labor market through an *ad hoc* real wage function in terms of aggregate hours and output. Erceg et al. (2000) formulate a model with both staggered wages and prices follwoing Calvo (1983), and examines optimal monetary policy.

Among the proponents of staggered wages, general consensus has been reached once Taylor deterministic staggering is granted : *i)* while the staggered prices necessarily produce dampened oscillation in output after nominal shocks, models with staggered wages produce monotonically dampening responses, and *ii)* the reason for the poor performance of price staggering is the highly procyclical real marginal costs, as argued by CKM.

This paper reconsiders staggered wage contracts as a transmission mechanism either complementary or supplementary to staggered price contracts. My approach is to introduce nominal rigidities in the spirit of Calvo (1983) into both goods and labor markets. The intuition behind this dual staggering is: by introducing staggered wage contracts, the marginal cost curve of individual firms will be "flattened" and thereby the incentive of firms to raise prices after expansionary monetary shock will decrease, and this mechanism will be further reinforced by staggered price

[3]An inexhaustive list of the literature includes the following: Dotsey, King, and Wolman (1999) mitigate the rise in marginal cost by assuming a high degree of labor supply elasticity. Kiley (1997) also considers an infinite labor supply elasticity, and increasing returns to scale in addition. Bergin and Feenstra (1998) move away from the CES specification of preferences and consider a translog functional form instead. Using this form, they develop an endogenous pricing rule of a firm which is not only a simple markup over marginal costs but directly influenced by competitors' prices as well. Edge (2000) moderates the increase in marginal costs after an expansionary nominal shock assuming firm-specific labor input. More specifically, she argues that the increase in price by a firm reduces demand for its output, causing reduced demand for its specific labor input, which in turn lowers the wage rate and marginal costs the firm pays.

contracts.

I find that results from the full and stripped-down versions of the model square with the intuition and the general consensus described above: in the presence of both wage and price staggering, the magnitude of the contract multiplier increases significantly, and wage staggering is a better and possibly dominant device for persistence, generating typical hump-shaped responses in output unlike price staggering. I interpret the latter finding as running counter to the seeming equivalence of wage and price stickiness.

I also find Taylor and Calvo *price staggering* have qualitatively different implications for the responses of output after a nominal shock: while the former necessarily leads to oscillatory responses of output, the latter produces monotonically dampening responses even if the real marginal costs are highly procyclical. This finding implies that the "lack of persistence" and the "oscillation in output", which are interpreted as equivalent by CKM, are not necessarily the same phenomenon, and that the procyclicality of real marginal cost is not the only force behind the lack of persistence in CKM.

To further examine the different implications of Taylor and Calvo staggering, I construct both a full and stripped down versions of a DSGE model with Taylor staggered contracts. Based on calibration exercises and analytical solutions, I argue the lack of persistence in CKM is due to a nuisance feature *inherent* in *deterministic price* staggering: after an expansionary monetary shock there occurs an initial overshooting in prices set by the optimizing firms, and hence the "catch-up" behavior of aggregate price level cannot be replicated. In contrast, this anomaly does not appear under staggered wage contracts, whether they are of stochastic or deterministic nature. I interpret this "robustness" across different staggering scheme as another sense in which wage stickiness is favorable.

The outline of the paper is as follows. Section 2 is devoted to the description of the model with nominal rigidities. A stationary rational expectation equilibrium is defined, and the solution of log-linearized model is obtained by the method of Sims (2002). In Section 3, the economy is calibrated and the time series properties of the model-generated variables are investigated. I address the main question of whether staggered nominal contracts can generate persistence in output, using different versions of the theoretical model developed in section 2. I also examine which version better replicates some important empirical features of data. Section 4 provides the intuition for the results obtained from the calibration exercises using stripped-down versions of the model. Section 5 reconsiders the finding of CKM in the context of stripped-down models with deterministic staggering. Section 6 concludes the paper. The appendix contains the derivation of key equations and some nontrivial steady state values of system variables.

2 The Model

This section presents the model used as the framework of analysis that follows later. There are four types of agents in the economy: households, firms, government, and the aggregator. The aggregator performs two roles: in the labor market, it transforms heterogeneous labor (supplied by households) into a single composite labor, used as an input for production by firms. In the goods market, it transforms differentiated goods (supplied by producers) into a single composite good which households use for consumption and investment. Since each household and firm have monopoly power over their own labor and products, respectively, they face individual demands by the aggregator. Firms are monopolistic competitors producing differentiated goods with capital (rented from households) and a bundle of labor service (purchased from the aggregator), and they satisfy all individual demand at posted prices. Households purchase output from the aggregator and supply capital (to firms) and differentiated labor (to the aggregator). They are also assumed to satisfy all individual demands for labor at posted wages. The government manages the supply of nominal money by making lump-sum transfers.

Key features of the model are the rigidities in aggregate price and wage. Following the spirit of Calvo (1983), price and wage settings are staggered and overlapped in a stochastic fashion: at any given period, randomly selected fractions of producers and households optimally determine their nominal prices and wages, respectively, considering the stochastic length of the time over which prices and wages are fixed. Since the two sources of nominal rigidities will interact with each other in dampening the incentive of firms and households to raise prices or wages after expansionary monetary shocks, I expect a considerable degree of contract multiplier even with standard specification of production and utility functions.

2.1 Aggregator

The function of the aggregator is to collect and transform all the differentiated goods and labor service into a single composite good and labor service, respectively, and supply them to their ultimate demanders. The transform technologies are

$$Y = (\int Y_j^{\theta_Y} dj)^{\frac{1}{\theta_Y}}, \text{ and} \tag{2.1}$$

$$L = (\int L_i^{\theta_L} di)^{\frac{1}{\theta_L}} \tag{2.2}$$

where Y_j and L_j denote differentiated goods and labor service, respectively. I assume $0 < \theta_Y, \theta_L < 1$.[4]The aggregator maximizes its profits in goods and labor markets by solving

$$\max \; PY - \int P_j Y_j dj, \text{ and} \tag{2.3}$$

$$\max \; WL - \int W_i L_i di \tag{2.4}$$

where P_j is the output price of firm j, Y_j is the output supply of firm j, and P is the aggregate price index defined below. W_i, L_i, and W are defined in a similar way.

>From the above maximization problems of the aggregator, I obtain the following demand functions individual firms and households are faced with:

$$Y_{jt} = (\frac{P_{jt}}{P_t})^{\frac{1}{\theta_Y - 1}} Y_t, \text{ and} \tag{2.5}$$

$$L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t \tag{2.6}$$

where the aggregate price level P and wage rate W defined respectively as

$$P_t = (\int P_{jt}^{\frac{\theta_Y}{\theta_Y - 1}} dj)^{\frac{\theta_Y - 1}{\theta_Y}}, \text{ and} \tag{2.7}$$

$$W_t = (\int W_{it}^{\frac{\theta_L}{\theta_L - 1}} di)^{\frac{\theta_L - 1}{\theta_L}} \tag{2.8}$$

2.2 Firms

2.2.1 Technology and Costs

It is useful to assume a firm maximizes its profit through two steps of decision making: First, the *production unit* minimizes production costs given the factor prices and the level of output it is directed to produce by the pricing unit. Second, the *pricing unit* determines the price of firm's

[4] $\frac{1}{1-\theta_Y}$ and $\frac{1}{1-\theta_L}$ measure the elasticity of substitution between different goods and labor service, respectively. For example, $\theta_Y \to 1$ implies all goods become perfect subsitutes.

output, taking into account demand and cost conditions in the present and future periods. This subsection is devoted to studying the firms' decisions concerning production and factor demand. The problem of optimal pricing is deferred to the next subsection.

There is a continuum of firms, indexed by j and distributed on the unit interval [0,1] . They have access to the identical production technology

$$Y_{jt} = K_{jt}^{\alpha} {L_{jt}}^{1-\alpha}, \quad \alpha \in (0,1) \tag{2.9}$$

where K_{jt} is the capital input and L_{jt} is the quantity of composite labor used by firm j.

The production unit solves the static problem

$$min \;\; \Gamma(Y_{jt}) = Z_t K_{jt} + W_t L_{jt} \;\; s.t. \; Y_{jt} = K_{jt}^{\alpha} {L_{jt}}^{1-\alpha} \tag{2.10}$$

where Z_t is the nominal rental price of capital determined by the demand supply of capital good, and W_t is the nominal price of composite labor determined by equation (2.8). Since firms are assumed to be price takers in the factor market, standard microeconomic conditions for cost minimization yield the following conditional factor demands and cost functions:

$$K_{jt}^{d} = Z_t^{\alpha-1} W_t^{1-\alpha} (1-\alpha)^{\alpha-1} \alpha^{1-\alpha} Y_{jt} \tag{2.11}$$

$$L_{jt}^{d} = Z_t^{\alpha} W_t^{-\alpha} (1-\alpha)^{\alpha} \alpha^{-\alpha} Y_{jt} \tag{2.12}$$

$$\Gamma(Y_{jt}) = Z_t^{\alpha} W_t^{1-\alpha} (1-\alpha)^{\alpha-1} \alpha^{-\alpha} Y_{jt} \tag{2.13}$$

$$MC_{jt} = Z_t^{\alpha} W_t^{1-\alpha} (1-\alpha)^{\alpha-1} \alpha^{-\alpha} \tag{2.14}$$

where $\Gamma(Y_{jt})$ and MC_{jt} are the total and marginal costs for producing Y_{jt} units of output, respectively, both in nominal terms.

Since equation (2.11) and (2.12) hold for all firms in the economy, the economy-wide capital to labor ratio is also determined as

$$\frac{L_t}{K_t} = \frac{Z_t}{W_t} \frac{1-\alpha}{\alpha}. \tag{2.15}$$

Equation (2.15) will be used later when I impose factor market equilibrium conditions.

2.2.2 Price Setting and Nominal Rigidity

The structure of price setting employed in this paper follows the legacy of Calvo (1983): at each period t, a randomly selected fraction ϕ_Y of firms area allowed to revise their prices according to the simple indexation rule:[5]

$$P_{jt} = \overline{\Pi} P_{j,t-1}, \quad j \in rev(t) \tag{2.16}$$

where $\overline{\Pi}$ is the steady state gross inflation rate, and $rev(t)$ denotes the set of producers revising their prices at t. The remaining 1-ϕ_Y fraction of producers, denoted by $j \in flex(t)$, choose P_{jt} so as to maximize their expected present discounted stream of real profits by solving

$$\max \ E_t \left[\sum_{\tau=t}^{\infty} \frac{\rho_{j\tau}}{\rho_{jt}} \phi_Y^{\tau-t} \frac{P_{j\tau} Y_{j\tau} - \Gamma(Y_{j\tau})}{P_\tau} \right] \tag{2.17}$$

where $\rho_{j\tau}/\rho_{jt}$ is the stochastic discounting factor for firm j's real profit in the period τ.

Since firms are not allowed to reoptimize on their prices every period, the optimal pricing problem of a firm is inherently dynamic. Suppose that a firm $j \in flex(t)$ is chosen to optimize on its price in the current period t. Then with probability $\phi_Y^{\tau-t}$, the demand it faces from the period t on evolves as

$$Y_{i\tau} = \left[\overline{\Pi}^{\tau-t} P_{jt}/P_\tau \right]^{\frac{1}{\theta_Y - 1}} Y_\tau, \quad \tau \geq t \tag{2.18}$$

Using equation (2.18) and equating dynamic marginal revenue and marginal cost (with respect to P_{jt}), I obtain an optimal price for firm $j \in flex(t)$:

$$P_{jt} = \frac{1}{\theta_Y} \frac{E_t \left[\sum\limits_{\tau=t}^{\infty} \rho_{j\tau} (\phi_Y \overline{\Pi}^{\frac{1}{\theta_Y - 1}})^{\tau-t} MC_\tau P_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau \right]}{E_t \left[\sum\limits_{\tau=t}^{\infty} \rho_{j\tau} \phi_Y^{\tau-t} \overline{\Pi}^{\frac{\theta_Y(\tau-t)}{\theta_Y - 1}} P_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau \right]} \tag{2.19}$$

where

$$MC_\tau = Z_\tau^\alpha W_\tau^{1-\alpha} (1-\alpha)^{\alpha-1} \alpha^{-\alpha}, \quad \tau \geq t. \tag{2.20}$$

Equation (2.19) has an interpretation that firm $j \in flex(t)$ determines its nominal price as a weighted average of its future expected marginal costs, scaled up by the constant markup factor $1/\theta_Y$ which is greater than one. In fact, putting $\phi_Y = 0$ in equation (2.19) leads to the optimization condition that would hold when all prices are flexible: $P_{jt} = \frac{1}{\theta_Y} MC_{jt}$.

[5] I follow Yun (1996) in assuming indexation, since it helps simplify mathematics. In equation (2.18), for example, by assuming indexation I can avoid dealing with infinitely many future steady state relative price of firm j.

2.3 Households

There is a continuum of identical households indexed by i and distributed on the unit interval $[0,1]$. I also assume households optimize on their nominal wages at infrequent intervals in an isomorphic way to price setting in the goods market: at each period, a randomly selected fraction ϕ_L of households revise their wages according to a simple indexation rule, and the other 1-ϕ_L fraction of households choose their nominal wages optimally. Other than its wage income, a household receives capital rental income, interest income from bond holding, a lump-sum transfer from the government, and a constant share of profits.

2.3.1 Preference

A typical household $i \in [0,1]$ solves

$$\max E_t[\sum_{\tau=t}^{\infty} \beta^{\tau} U(C_{i\tau}, 1 - L_{i\tau}, M_{i\tau}/P_{\tau})] \tag{2.21}$$

subject to the budget constraint

$$\begin{aligned} & C_{i\tau} + K_{i,\tau+1} - (1-\delta)K_{i,\tau} + \frac{M_{i\tau}}{P_\tau} - \frac{M_{i,\tau-1}}{P_\tau} + \frac{B_{i\tau}}{P_\tau} - \frac{B_{i,\tau-1}}{P_\tau} \\ \leq \ & \frac{W_{i\tau}L_{i\tau}}{P_\tau} + \frac{Z_\tau K_{i\tau}}{P_\tau} + T_{i\tau} + \frac{\int s_{ij}\Pi_{j\tau}dj}{P_\tau} + r_{\tau-1}\frac{B_{i,\tau-1}}{P_\tau} \end{aligned} \tag{2.22}$$

and a borrowing constraint $B_{i\tau} \geq -\overline{B}$ for a large positive number $\overline{B}$. In equation (2.22) above, the uses of the household's date t wealth are consumption C_{it}, net investment $K_{i,\tau+1} - (1-\delta)K_{i,\tau}$, net acquisition of money balance $\frac{M_{i\tau}}{P_\tau} - \frac{M_{i,\tau-1}}{P_\tau}$, and net acquisition of one-period nominal bonds $\frac{B_{i\tau}}{P_\tau} - \frac{B_{i,\tau-1}}{P_\tau}$. The sources of wealth are current labor income $\frac{W_{i\tau}L_{i\tau}}{P_\tau}$, return on capital rental $\frac{Z_\tau K_{i\tau}}{P_\tau}$, the net lump-sum government transfer T_{it}, dividend income $\frac{\int s_{ij}\Pi j\tau dj}{P_\tau}$, and the interest income from the previous period's bond holding $r_{\tau-1}\frac{B_{i,\tau-1}}{P_\tau}$ where $r_{\tau-1}$ is the net nominal interest rate between the period $\tau-1$ and τ. I assume s_{ij}, household i's share of the firm j, is fixed beyond the control of the household. The term δ denotes the depreciation rate of capital.

I follow CKM and specify the instantaneous utility function U as

$$\begin{aligned} U(C_t, 1-L_t, \frac{M_t}{P_t}) &= \left[C_t^*(1-L_t)^{\psi}\right]^{1-\sigma} / (1-\sigma) \\ &= \left[(C_t^{\nu} + b(\frac{M_t}{P_t})^{\nu})^{\frac{1}{\nu}}(1-L_t)^{\psi}\right]^{1-\sigma} / (1-\sigma) \end{aligned} \tag{2.23}$$

in which consumption C and real money balances $\frac{M}{P}$ interacts through a CES function, and the instantaneous utility is a CRRA transform of the "basket" of the consumption bundle C^* and leisure $1-L$.[6]

It is also convenient to break down the problem of households into two steps. First, a household either indexes or optimizes on its wage, depending on whether it is chosen reset its nominal wage in the current period. Second, given the level of its nominal W_{it} determined in the first step, the household chooses rules for consumption, capital investment, money and bonds holding.

I first consider the FOCs for choosing (C, M, K, B) in that order:

$$\frac{\partial U_t}{\partial C_{it}} = \Lambda_{it} \tag{2.24}$$

$$\frac{\partial U_{it}}{\partial m_{it}} \frac{1}{P_t} = \frac{\Lambda_{it}}{P_t} - \beta E_t \frac{\Lambda_{i,t+1}}{P_{t+1}} \tag{2.25}$$

$$\Lambda_{it} - \beta E_t \left[\Lambda_{i,t+1}(1-\delta)\right] = \beta E_t \left[\Lambda_{i,t+1} \frac{Z_{t+1}}{P_{t+1}}\right] \tag{2.26}$$

$$0 = \frac{\Lambda_{it}}{P_t} - \beta R_t E_t \left[\frac{\Lambda_{i,t+1}}{P_{t+1}}\right] \tag{2.27}$$

where $\Lambda_{i\tau}$ is the Lagrangian multiplier on the household i's budget constraint (2.22), and $R_t = 1 + r_t$ is the gross nominal interest rate. As in Kim (2000), equations (2.24), (2.25) and (2.27) yield the following money demand function:

$$1 - R_t^{-1} = b(P_t C_{it}/M_{it})^{1-\nu} \tag{2.28}$$

from which it is deduced that two households need the same quantity of money if they consume the same amount of final good . This fact will be used in aggregating households' equations later.

2.3.2 Wage setting

Now I address the problem of wage setting by households. As with producers, I denote by $rev(t)$ and $flex(t)$ the set of households revising and optimizing on their nominal wages in the current

[6] The instantaneous utility function is somewhat nonstandard in that the "basket" $C_t^*(1-L_t)^{\psi}$ is not of the usual Cobb-Douglas form appearing in the literature. I can transform U into

$$U = (1+\psi)\left[C_t^{*\frac{1}{1+\psi}}(1-L_t)^{\frac{\psi}{1+\psi}}\right]^{1-\Sigma} / (1-\Sigma)$$

where $\Sigma^{-1} = \frac{1+\psi}{\sigma+\psi} > 0$ is the intertemporal elasticity of substitution appropriate for the new Cobb-Douglas basket $C_t^{*\frac{1}{1+\psi}}(1-L_t)^{\frac{\psi}{1+\psi}}$.

period t, respectively[7]. Therefore, a household $i \in rev(t)$ revises its nominal wage following[8]

$$W_{it} = \overline{\Pi}_\omega W_{i,t-1}, \quad i \in rev(t) \tag{2.29}$$

where $\overline{\Pi}_\omega$ is the steady state growth rate of nominal wage rate.

Since households are not allowed to reoptimize on their wage rates every period, the optimal wage setting problem of a household is also inherently dynamic. Suppose that a household $i \in flex(t)$ is chosen to optimize on its wage rate in the current period t. Then with probability $\phi_L^{\tau-t}$, the demand it faces from the period t on evolves as

$$L_{i\tau} = \left[\overline{\Pi}_\omega^{\tau-t} W_{it}/W_\tau\right]^{\frac{1}{\theta_{L-1}}} L_\tau, \ \tau \geq t \tag{2.30}$$

Using equation (2.30), I get the first order condition for household $i \in flex(t)$ to determine its optimal wage:

$$-E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_L^{\tau-t} \frac{\partial U_{i\tau}}{\partial L_{i\tau}} \frac{dL_{i\tau}}{dW_{it}}\right] = E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_L^{\tau-t} \frac{\Lambda_{i\tau}}{P_\tau} \overline{\Pi}_\omega^{\tau-t}\left(L_{i\tau} + W_{it}\frac{dL_{i\tau}}{dW_{it}}\right)\right] \tag{2.31}$$

Equation (2.31) gives a straightforward interpretation. First note that -$\frac{\partial U_{it}}{\partial L_{it}}\frac{dL_{it}}{dW_{it}}$ is the marginal disutility of working (with respect to W_{it}), and $\frac{\Lambda_{it}}{P_t}\left(L_{it} + W_{it}\frac{dL_{it}}{dW_{it}}\right)$ is the marginal utility of labor income from working (with respect to W_{it}). Therefore, equation (2.31) means that the optimal wage is so determined as to equate the present discounted value of marginal disutility from work and that of real wage income measured in utility terms through the Lagrangian multiplier $\Lambda_{i\tau}, \forall \tau \geq t$.

Rearranging equation (2.31), I obtain the following formula to set optimal wage rate:

$$W_{it} = -\frac{1}{\theta_L}\frac{E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_L^{\tau-t}\frac{\partial U_{i\tau}}{\partial L_{i\tau}}\overline{\Pi}_\omega^{\frac{\tau-t}{\theta_L-1}} W_\tau^{\frac{1}{1-\theta_L}} L_\tau\right]}{E_t\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t}\phi_L^{\tau-t}\frac{\Lambda_{i\tau}}{P_\tau}\overline{\Pi}_\omega^{\frac{\theta_L(\tau-t)}{\theta_L-1}} W_\tau^{\frac{1}{1-\theta_L}} L_\tau\right]} \tag{2.32}$$

Equation (2.32) says that the optimal wage of household $i \in flex(t)$ is set as a weighted average of its future expected marginal disutility from working, scaled up by a "markup" factor $\frac{1}{\theta_L}$, which is greater than one. Putting $\phi_L = 0$ in equation (2.32) yields the same optimization

[7] At the risk of possible confusion, I use the same notation for both firms and households.

[8] Since I assume no growth in real variables, $\overline{\Pi}$ is equal to $\overline{\Pi}_\omega$, which in turn is equal to the rate of growth in money. In case real variables are assumed to follow a balanced growth path with the rate of growth G, $\overline{\Pi}$ is equal to $\overline{\Pi}_\omega/G$.

condition as in flexible wage models: $\Lambda_{it}\frac{W_t}{P_t} = -\frac{\partial U_{it}}{\partial L_{it}}$. It is worth noting that the structure of labor market stipulates households meet all demand for their labor at their respective nominal wage, determined either by equation (2.31) or by equation (2.32). This is an essential feature of this paper: by assuming so, I can break the link between the wage rate and marginal rate of substitution between leisure and consumption and discard the implausibly high elasticities of labor supply with respect to wage, often assumed to generate persistent effects of monetary policy.

2.4 Monetary Policy

Monetary policy is implemented by distributing new money balance to consumers in a lump-sum fashion:

$$\frac{M_t - M_{t-1}}{P_t} = T_t \tag{2.33}$$

where $M_t = \int M_{it} di$, and $T_t = \int T_{it} di$.

Nominal money stock grows following

$$\frac{M_t}{M_{t-1}} = \mu_t \tag{2.34}$$

where μ_t denotes the growth rate of the money stock. Its stochastic properties are given by

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + (1 - \rho_\mu)\log(\overline{\mu}) + \varepsilon_{\mu t} \tag{2.35}$$

where $\overline{\mu}$ is the steady state rate of money growth. I assume $\rho_\mu \in (-1, 1)$ and $\varepsilon_{\mu t} \sim WN(0, \sigma_\mu^2)$.

2.5 Intuition: Are the Two Rigidities Equivalent?

As described before, the model economy has two distinctive sources of nominal rigidities. A seemingly innocuous conjecture around the two sources of rigidities would be: they will have equivalent implications to the real effects of nominal shocks, especially when the production function is CRS and labor is an intensively used production factor. Despite their seeming equivalence, however, some recent work has emphasized their non-equivalence. For example, in a simple two period model, Andersen (1998) shows that staggered wage contracts a la Taylor can produce output persistence and inflation inertia, while staggered prices implies oscillatory responses of output and lack of inflation persistence. Huang and Liu (1998) also compares the implications of staggered price and wage contracts in the context of a dynamic general equilibrium model, and obtain similar results.

In this subsection, I check the validity of the above conjecture in a static version of the model with either one or the other rigidity. More specifically, I assume *i)* production function is CRS, *ii)* labor is the sole production factor, *iii)* budget constraint of household is $L_{it}W_{it} = P_tC_t$, and i*v)* the instantaneous utility function is of the form[9]

$$U(C,L) = \log\left[C - \frac{L^{1+\gamma}}{1+\gamma}\right], \qquad \gamma > 0$$

2.5.1 The Effects of Wage Stickiness

A monopolistically competitive firm maximizes its single period profit

$$\begin{aligned} \Pi_{jt} &= p_{jt}^{\frac{\theta_Y}{\theta_Y-1}} P_tY_t - p_{jt}^{\frac{1}{\theta_Y-1}} Y_tW_t &(2.36)\\ &= y_{jt}^{\theta_Y} P_tY_t - y_{jt}Y_tW_t &(2.37)\end{aligned}$$

where equation (2.36) gives the profit of a firm j in terms of its relative price $p_{jt} = P_{jt}/P_t$, while equation (2.37) does in terms of its relative output $y_{jt} = Y_{jt}/Y_t$.

Firm j maximizes its profit by equating marginal revenue and marginal cost. If its relative price is used as the decision variable, the profit maximization condition is given by equating[10]

$$MR = \frac{\theta_Y}{\theta_Y-1} p_{jt}^{\frac{1}{\theta_Y-1}} P_tY_t \qquad (2.38)$$

and

$$MC = \frac{1}{\theta_Y-1} p_{jt}^{\frac{2-\theta_Y}{\theta_Y-1}} Y_tW_t. \qquad (2.39)$$

Equivalently, firm j may use its relative output as the decision variable. In this case, the corresponding marginal revenue and marginal cost curves are given by, respectively,

$$mr = \theta_Y y_{jt}^{\theta_Y-1} P_tY_t \qquad (2.40)$$

and

$$mc = W_tY_t \qquad (2.41)$$

[9] This form of utility function is used by King and Wolman(1999).

[10] Note that the marginal revenues and marginal costs implied by equations (2.38) and (2.39) are negative because θ_Y is smaller than 1.

Figure I illustrates the optimal relative price is determined at the intersection of MR and MC curves in panel (a), and the optimal relative output is determined at the intersection of mr and mc curves in panel (b).[11]

To see the impact of wage stickiness on firms' price and output decisions, suppose there occurs an disturbance in the form of the increase in money stock. Panel (a) illustrates the changes in the firm's optimal price decision after the disturbance: the increase in money stock shifts down the marginal revenue (in terms of relative price) curve from MR_0 to MR_1 in proportion to the rate of increase in money. If wages are flexible, the marginal cost (also in terms of relative price) curve also shifts down from MC_0 to $MC_1$by the same proportion, leading to the same optimal relative price of firm j as before the disturbance. Now suppose that the aggregate wage is somehow sticky and that the increase in aggregate wage after the disturbance is half of what under wage flexibility. The lower aggregate wage level after the shock is reflected by the shift of MC_0 to MC_2, which lies halfway between MC_0 and MC_1. The resulting new optimal price at E_2 under wage stickiness is lower than that under wage flexibility.

Panel (b) describes the same phenomenon in terms of firm j's relative output: the increase in money stock shifts up the marginal revenue (in terms of relative output) curve from mr_0 to mr_1 in proportion to the rate of increase in money. If wages are flexible, the marginal cost (also in terms of relative output) curve also shifts up from mc_0 to $mc_1$by the same proportion, leading to the same optimal relative output of firm j as before the disturbance. However, if the aggregate wage somehow sticky and the increase in aggregate wage after the disturbance is half of what under wage flexibility, the mc_0 curve shifts up to mc_2, by half as much as under wage flexibility. The resulting new optimal output under wage stickiness is higher than that under wage flexibility. In summary, panels (a) and (b) shows the relative price of firm j is 14% lower and its relative output is 40% higher as a result of wage stickiness.

[11] In those panels, I assume $\theta_Y = 0.5, Y_t = 1$, and $\theta_Y P_t = W_t$.

2.5.2 The Effects of Price Stickiness

Now I consider the problem of a household. Eliminating the budget constraint, I can show the problem of a household i is to maximize

$$U_{it} = \log[w_{it}^{\frac{\theta_L}{\theta_L-1}} W_t L_t - \frac{w_{it}^{\frac{1+\gamma}{\theta_L-1}}}{1+\gamma} L_t^{1+\gamma} P_t] - \log P_t \tag{2.42}$$

$$= \log[l_{it}^{\theta_L} W_t L_t - \frac{l_{it}^{1+\gamma}}{1+\gamma} L_t^{1+\gamma} P_t] - \log P_t \tag{2.43}$$

where (2.42) formulates the utility of household i in terms of its relative wage $w_{it} = W_{it}/W_t$, while (2.43) does in terms of its relative hours worked $l_{it} = L_{it}/L_t$.

Household i maximizes its utility by equating marginal utility of consumption and marginal disutility from working. If its relative wage is used as the decision variable, the utility maximization condition is given by equating[12]

$$MU = \frac{\theta_L}{\theta_L - 1} w_{it}^{\frac{1}{\theta_L-1}} W_t L_t \tag{2.44}$$

and

$$MD = \frac{1}{\theta_L - 1} w_{it}^{\frac{2-\theta_L+\gamma}{\theta_L-1}} L_t^{1+\gamma} P_t \tag{2.45}$$

Equivalently, household i can use its relative hours as the decision variable. In this case, the corresponding marginal utility and marginal disutility curves are given by, respectively,

$$mu = \theta_L l_{it}^{\theta_L-1} W_t L_t \tag{2.46}$$

and

$$md = l_{it}^{\gamma} L_t^{1+\gamma} P_t \tag{2.47}$$

Figure I illustrates the optimal relative wage is determined at the intersection of MU and MD curves in panel (c), and the optimal relative hours are determined at the intersection of mu and md curves in panel (d).[13] To see the impact of price stickiness on households' wage and hours decisions, suppose there occurs an disturbance in the form of the increase in money stock. Panel (c) illustrates the changes in the households' optimal wage decision after the disturbance: suppose the aggregate wage rate has increase due to an expansionary nominal disturbance. Then

[12] Note that the MU and MD in equations (2.44) and (2.45) are negative because θ_L is smaller than 1.

[13] To make the situation in the labor market isomorphic to that in the goods market, I put $\theta_L = 0.5, L_t = 1$, and $\theta_L W_t = P_t$. γ is set to be 0.1, which implies a highly elastic labor supply. W_0 is also normalized to one.

the marginal utility (in terms of relative wage) curve shifts down from MU_0 to MU_1 in proportion to the rate of increase in aggregate wage. If prices are flexible, the marginal disutility (also in terms of relative wage) curve shifts down from MC_0 to $MC_1$by the same proportion, leading to the same optimal relative wage rate of household i as before. Now suppose that the aggregate price is somehow sticky and that the increase in aggregate price after the disturbance is half of what under price flexibility. The lower aggregate price level after the shock is reflected by the shift of MC_0 to MC_2, which lies halfway between MC_0 and MC_1. The resulting new optimal wage rate under price stickiness is lower than that under price flexibility.

Panel (d) describes the same phenomenon in terms of household i's relative hours: the nominal disturbance shifts up the marginal utility (in terms of relative hours) curve from mu_0 to mu_1 in proportion to the rate of increase in aggregate wage rate. If wages are flexible, the marginal disutility (also in terms of relative hours) curve shifts up from md_0 to $md_1$by the same proportion, leading to the same optimal relative hours of household i as before. However, if the aggregate price level is somehow sticky and therefore it increases by half of its increase under price flexibility, the md_0 curve shifts up to mc_1, by half as much as under price flexibility. The resulting new optimal hours under price stickiness is higher than that under price flexibility. In summary, panels (c) and (d) shows the relative wage of household i is 12% lower and its relative hours are about 30% higher as a result of price stickiness.

2.5.3 Equivalence?

So far, I have examined how the two sources of nominal rigidities work in generating real effects of nominal disturbances. Despite the apparent symmetry between the two propagation channels, Figure I implies generally wage stickiness is more potent: with the same degree of rigidities, the increase in output due to wage stickiness is 33% higher and the increase in price is 14% lower than under price stickiness.

The reason for this non-equivalence of the two rigidities is simple: while the marginal cost optimizing firms face is $flat$ in panel (b), the marginal disutility of optimizing households is *upward sloping* in panel (d), due to the convexity of the utility function in terms of hours worked. The key parameter is γ, which measures how fast the marginal disutility from working increase as a households increases its labor supply. As γ increases the MD curves in panel (c) and (d) become steeper than the MC curves in panel (a) and (b), and hence price stickiness cannot be passed on to wage stickiness in one-to-one fashion.[14] At least up to these simple static versions of the model, the two rigidities are equivalent only when either *i*) $\gamma = 0$, i.e., marginal disutility of labor is constant, or *ii*) marginal costs of individual producers are increasing (or decreasing) in their own output (or prices).[15]

2.6 The Equilibrium

To make the system complete, I need a relation between the stochastic discount factors for households and those for firms. Although not specified in the budget constraints of households, I assume that every agent in the economy has access to complete markets for contingent claims, so that for all possible states s^t and s^τ in period t and $\tau \geq t$, respectively,

$$\frac{\beta\Lambda_i(s^\tau)}{\Lambda_i(s^t)} = \frac{\beta\Lambda_{i'}(s^\tau)}{\Lambda_i'(s^t)}, \ \forall i, i' \in [0,1], \tau \geq t \tag{2.48}$$

i.e., all households share the same market rate of discount between the period t and $\tau \geq t$. I further assume that $s_{ij} = s_{j,} \ \forall i, i' \in [0,1]$ all households have the same share of profits of the firm j.[16] Then the unique market discount factor implies the following equation holds at all states:[17]

$$\frac{\rho_j(s^\tau)}{\rho_j(s^t)} = \frac{\beta\Lambda(s^\tau)}{\Lambda(s^t)}, \forall j \in [0,1], \tau \geq t \tag{2.49}$$

where $\Lambda(s^t)$is the "average" marginal utility of consumption in the state s^t, defined as $\int \Lambda_i(s^t)di$.

An *equilibrium* for this economy is a collection of allocations for consumers $C_{it}, K_{i,t+1}, M_{it}, B_{it}$ and L_{it} for $i \in [0,1]$; allocations for producers K_{jt}^d and L_{jt}^d,for $j \in [0,1]$; allocations for the

[14] When I use more general utility function $U = \left[C^a(1-L)^{1-a}\right]^{1-\sigma}/(1-\sigma)$, the optimal relative wage and hours worked are constants independent of the degree of price stickiness.

[15] Other than the trivial assumption of decreasing returns to scale in the production function, one possible way to have increasing marginal costs at firm level is to assume firm-specific production factors with a CRS production function. Edge (2000) argues that both rigidities are equivalent when labor service is firm-specific, which is a mechanism to generate decreasing returns.

[16] This implies implies $s_{ij} = 1, \forall i, j \in [0,1]$,

[17] If state s^τoccurs in the period $\tau \geq t$, the present discount value of the firm j's profit in the period *t+1* in terms of its shareholders' utilities is

$$\begin{aligned} & \int \beta\frac{\Lambda_i(s^\tau)}{\Lambda_i(s^t)}s_{ij}\pi_j(s^\tau)di \\ = & \ \beta\pi_j(s^\tau)\int\frac{\Lambda_i(s^\tau)}{\Lambda_i(s^t)}di \\ = & \ \beta\frac{\Lambda(s^\tau)}{\Lambda(s^t)}\pi_j(s^\tau) \end{aligned}$$

aggregator Y_t, L_t, Y_{jt}, and L_{it} for $i, j \in [0,1]$; together with prices $\{P_{jt}, W_{it}, Z_t, r_t : i, j \in [0,1]\}$ that satisfy the following conditions: *(i)* given all prices but its own, the wage W_{it} set by a household $i \in flex(t)$ solves (2.32); *(ii)* given all prices, each household solves the utility maximization problem (2.21) - (2.22); *(iii)* given all prices but its own, the price P_{jt} set by a firm $j \in flex(t)$ solves (2.19); *(iv)* taking as given all prices, each firm solves its profit maximization problem (2.10); *(v)* given all prices, the allocations of the aggregator solves (2.3) and (2.4); *(vi)* the goods market, capital market, labor market, bonds market, and money market clears; *(vii)* monetary policy is implemented by (2.33) and (2.34).

In what follows, I focus on a *symmetric* equilibrium in which producers (or households) j (or i) $\in flex(t)$ make identical decisions. Equation (2.49) justifies this kind of symmetry among the optimizing firms, since the optimal price is the same for all $j \in flex(t)$ if equation (2.19) has a unique solution P_t^*. To impose the symmetry across households $i \in flex(t)$, I use rather strong assumption that consumption of all households, regardless of whether they belong to $flex(t)$ or not, are completely pooled and therefore they choose the same amount of consumption.[18] That assumed, I can use the single C_t and M_t for all households in $[0,1]$, and further impose the symmetry in the nominal wages set by all households in $flex(t)$.

In this economy, the aggregate output Y defined by equation (2.1) is different from the simple sum of individual output $\int Y_{jt} dj$. To simplify aggregation, I use an auxiliary price index defined as

$$P_t^R = (\int P_{jt}^{\frac{1}{\theta_Y - 1}} dj)^{\theta_Y - 1} \tag{2.50}$$

The new price index is useful in representing aggregate market clearing conditions in goods and capital market. For example, the nonstandard output aggregate Y and the simple sum of all firms' output are related by:

$$\int Y_{jt} dj = Y_t \left(\frac{P_t^R}{P_t} \right)^{\frac{1}{\theta_Y - 1}} \tag{2.51}$$

In this paper, I will focus on a stationary equilibrium. Before going on further to the description of it, however, note that nominal variables in the model grow over time in the steady state,

[18] In fact, I resort to the assumption that state contingent claims allow households to insure themselves against the idiosyncratic consumption risk they suffer because they change their respective wages at different dates. Note that, even in a symmetric equilibrium of this sort, the cross-household differential in wages and labor hours requires one to trace out individual Λ_i's. As will be shown in the appendix, however, I can circumvent this problem resorting to the "random sampling" nature of Calvo staggering.

due to the positive rate of money growth. Therefore, in order to impose stationarity, I transform a nominal variable X_t into $x_t = X_t/\overline{\mu}^t$, where $\overline{\mu}$ is the steady state growth rate of money. After variables are transformed appropriately, the whole system is log-linearized around the steady state and cast into the form[19]

$$\Gamma_0 \widehat{x}_t = \Gamma_1 \widehat{x}_{t-1} + \Gamma_2 \varepsilon_t + \Gamma_3(\widehat{x}_t - E_{t-1}\widehat{x}_t) \tag{2.52}$$

where $\widehat{x}_t = \log x_t - \log \overline{x}$ is the percentage deviation of x_t from its steady state value, Γ_i's are matrix functions of deep parameters in the model, and ε_t is the vector of exogenous disturbances.

I obtain linear decision rules by solving equation (2.52) using the method developed by Sims (2002). The resulting solution will have the form

$$\widehat{x}_t = \Psi_1 \widehat{x}_{t-1} + \Psi_2 \varepsilon_t \tag{2.53}$$

which allows one to simulate the model and compute impulse responses.

3 Findings from Model Simulation

3.1 Contract Multiplier: the Resurrection

Table I reports the parameter values used for calibration purpose, many of which are obtained from CKM with a few exceptions: *i)* to impose "symmetry", I set the value of θ_Y and θ_L, which measure the elasticity of individual output and labor demand, respectively, to be 0.9, *ii)* ψ, the share parameter for leisure in the utility function, is determined so that the steady state value of aggregate labor L will be 1/3 given the values of all relevant parameters, *iii)* b, the coefficient on the real balance in the utility function, is calibrated by using money demand equation (2.28) and the corresponding actual dataset,[20] and *iv)* when needed, I set ϕ_Y and ϕ_L, the proportion of non-optimizing firms and households, respectively, to be 3/4 to make the time structure of the model comparable to CKM.[21]

Table 1: Parameter Values

[19] The log-linearized version of the stationary-transformed system is given in the appendix.

[20] I use per capita real M2 balance, Federal Funds rate, per capita consumption, and GDP deflator, spanning the period 1959: I - 1999:III. b is determined to match the RHS and LHS of equation (2.28), constructed from the variables above.

[21] $\phi_Y = \phi_L = 3/4$ implies firms and households are chosen to optimize every four quarters on average. Especially when $\overline{\Pi} = 0$, the average duration of price fixity is $\frac{1}{1-\phi_Y} = 4$ quarters.

Preference	$\nu = -17.52$	$\psi = 1.3472$	$\sigma = 5$	$b = 0.00298$
Technology	$\alpha = 0.333$	$\delta = 1 - 0.9^{\frac{1}{4}}$	$\theta_Y = 0.9$	$\theta_L = 0.9$
Stickiness	$\phi_Y = 3/4$	$\phi_L = 3/4$		
Money Growth	$\overline{\mu} = 1.06^{\frac{1}{4}}$	$\rho_\mu = 0.57$		
Discount Factor	$\beta = 0.96^{\frac{1}{4}}$			

Now I perform the experiment with a shock raising the growth rate of money one percent above the steady state level. As a benchmark case, I first examine how much output changes following a monetary shock in a rigidity-free version of the model with $\phi_L = \phi_Y = 0$. Figure 2 displays the percentage deviations of some key variables from their steady state values, the plots of which verify that money is neutral when all prices are flexible: there are virtually no changes in real variables such as output, real wage, and real marginal cost, whereas all nominal variables including price and wage adjust toward the new steady state levels almost all at once toward the new steady state level $1/(1-\rho_\mu)$=2.3256.[22]

Then I consider the case with $\phi_L = 0$ and $\phi_Y = 3/4$, where nominal rigidity exists only in goods market[23]. Figure 3(a) shows output rises about 1.77% initially as a result of monetary expansion. One year after the shock output is still above its long-run level, but only by 0.26%. Therefore, the model with price stickiness alone generates a rather weak degree of contract multiplier. In fact, the "boom-bust" responses of output to monetary shock is reminiscent of CKM, who argue that price staggering fails to produce significant persistence due to the high procyclicality of marginal costs[24]. Other panels of Figure 3 help explain what is behind the weak persistence. The increase in output require a large increase in labor demand, which in turn causes the aggregate wage rate to rise significantly. As a result, the wage rate increases by 4.5% initially, as displayed in panel (e). The resulting rapid increase in real marginal cost displayed in panel (d) induces optimizing firms to raise their prices much. Hence, the aggregate price level rises about 1% initially and increases rapidly toward new steady state level as panel (c) displays. Panel (c) also shows that most of the adjustment in price level is completed within a year after the shock.

However, when I incorporate wage stickiness as an additional source of nominal rigidity, the story is quite different. Figure 4 is drawn for $\phi_L = \phi_Y = 3/4$ to allow both sources of nominal stickiness in the model. In panel (a), output rises by about 1.6% initially and still stays 1% above

[22] The subsequent adjustments in price and wageare due to the AR(1) specification of the monetary disturbance.

[23] This case is interesting because it corresponds the the experiment of CKM, who consider only the effect of price stickiness.

[24] But note that we observe monotonically dampening rather than oscillation, unlike CKM. I will come back to this issue later.

the long-run level after two years. Panels (c) and (e) show that aggregate price and wage rise much more slowly in the presence of wage stickiness than in its absence. Panels (d) and (e) illustrate the reason introducing wage additionally brings forth such qualitatively different results. Unlike in Figure 3, nominal wage increases only gradually after the shock due to wage stickiness. But the price setting rule depends on the CES bundle of individual wage rates, only one quarter of which are newly set in any period. Therefore, real marginal cost in panel (d) increases just 1.03% initially, dampening the incentive of optimizing firms to raise their prices.

Some other features of Figure 4 are worth noting as well: First, as displayed in panel (b), real wage is almost invariant over the typical business cycle frequencies unlike in the previous two figures. Therefore, the model with both sources of stickiness better captures another stylized fact of business cycle, i.e., the real wage acyclicality, without having to assume either implausibly high elasticity of labor supply or *ad hoc* real wage functions[25]. Second, panel (a) of Figure 4 shows that the model reproduces the hump-shaped responses in the cyclical component of output, which has been frequently documented by many authors.

Other than the criticism by CKM on staggered-price DSGE models regarding their "lack of output persistence", Fuhrer and Moore (1995) raise another issue about the *inflation persistence.* Using a highly stylized model with wage staggering a la Taylor, they argue that such a model *cannot* explain inflation inertia which is a prominent feature of actual data[26]. They also point out the lack of inflation persistence leads to the counterfactual implication that a credible disinflation policy can achieve its goal without inflicting any output cost to the economy.

Panel (f) of Figure 4 displays the impulse responses of inflation rates, where inflation rates exhibit a higher degree of persistence than under pure price stickiness: after showing a hike by 0.4 % following the monetary shock, the inflation rate returns gradually to its long run rate.

Now the question "which rigidity better explains persistence and other empirical features?' in the context of a full dynamic general equilibrium model. For that purpose, I do another experiment using model with wage rigidity alone, i.e., $\phi_Y = 0$ and $\phi_L = 3/4$. The conclusion is: wage rigidity is a better device to replicate various empirical features of data. Panel (a) of Figure 6 shows that with wage rigidity alone the impulse responses of output are qualitatively the same as what was obtained in the presence of both rigidities: the hump-shaped and persistent responses

[25] When I decrease ϕ_L to 0.6, the real wage shows moderate procyclicality as shown in Figure 5.

[26] This is because the equation for inflation rate has no lagged inflation terms in their model.

of output is generated under pure wage rigidity, though with a slightly smaller magnitude. This result suggests that, in terms of output persistence, wage rigidity is a better channel for the propagation of nominal shocks than price rigidity. A rather surprising fact is that the price level increases more slowly under pure wage rigidity than under pure price rigidity: compared with Figure 3(c) drawn for the pure price rigidity case, Figure 4(c) shows price level display smaller initial response and more gradual adjustment toward its steady state level. Also, in Figure 6(b), real wage shows weakly countercyclical behavior over the business cycle frequency. Compared with Figure 3 showing too procyclical real wage, wage rigidity performs better, although not much, than price stickiness does.

My interpretation of these results is that they square with those obtained earlier in the context a static model of rigidities: as can be seen from Figure 3 and 6, wage stickiness is *per se* better than price stickiness in generating persistence. Another criterion against which the two rigidities are compared is the increase in persistence when one rigidity is introduced additionally in the presence of the other. Figure 3 and 4 illustrate the reinforcement by wage stickiness wins the losing battle to achieve persistence, while Figure 4 and 6 show wage stickiness does not need the backup of price stickiness to achieve persistence. Hence, wage stickiness is a better channel in this sense as well.

I now summarize the results obtained from calibration exercises so far. Experiments with various different versions of the model suggest: *i)* the contract multiplier is strongly reinforced if wage rigidity is introduced as an additional source of nominal rigidity, *ii)* wage rigidity is possibly the dominant source of generating persistence in output and capturing the empirical features of the economy, *iii)* acyclical real wage is well explained by considering two sources of nominal rigidity together, *iv)* inflation stickiness is relatively well explained by allowing both rigidities.

4 Some Intuition

In this section, I develop some intuition behind the dynamics in the previous section. To do so, I consider stripped down(but inherently dynamic) versions of the model economy in which the equilibrium can be solved for analytically. To be more specific, I assume *i)* there is no capital stock in the economy, *ii)* the production function is CRS in labor, *iii)* the instantaneous utility function is given by $U(C, L) = \log C + \eta \log(1 - L)$, for some $\eta > 0$, *iv)* money demand is given by a simple quantity equation, *v)* β is set to be 1, and *vi)* money follows random walk in logarithmic form.

4.1 Staggered Price Contracts

Suppose that staggered price contracts are the only source of nominal rigidities in the economy. I first define the log deviations of some variables:

$$x_t = d\log P_t^*, \quad p_t = d\log P_t, \quad w_t = d\log W_t, \; m_t = d\log M_t, \quad y_t = d\log Y_t$$

Then the price equation (2.19) is log-linearized into

$$x_t = (1-\phi_Y)E_t\left[\sum_{\tau=t}^{\infty}\phi_Y^{\tau-t}w_\tau\right] \tag{4.1}$$

which in turn implies

$$x_t - \phi_Y E_t x_{t+1} = (1-\phi_Y)w_t \tag{4.2}$$

Log-linearization of the wage equation (2.32) with $\phi_L = 0$ gives

$$w_t = (\frac{L}{1-L}+1)y_t + p_t = (s+1)y_t + p_t, \quad \text{where } s = L/(1-L). \tag{4.3}$$

I impose a static money demand equation:

$$m_t = y_t + p_t \tag{4.4}$$

Finally, the price level p_t is an weighted average of p_{t-1} and x_t:

$$p_t = \phi_Y p_{t-1} + (1-\phi_Y)x_t \tag{4.5}$$

The system of equations (4.1) -(4.5) can be solved to determine how money shocks affect prices and output. Substituting for y_t and w_t into (4.2) and using equation (4.5), I obtain

$$E_t x_{t+1} - \Psi_Y x_t + x_{t-1} = -\frac{1}{\phi_Y}(1-\phi_Y)(1+s)(m_t - \phi_Y m_{t-1}) \tag{4.6}$$

where $\Psi_Y = (1+\phi_Y^2 + (1-\phi_Y)^2 s)/\phi_Y$.

Applying standard methods for solving second order stochastic difference equations I can write x_t as

$$\begin{aligned} x_t &= \lambda_y x_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)(1+s)E_t\left[\sum_{i=0}^{\infty}\lambda_y^i(m_{t+i}-\phi_Y m_{t-1+i})\right] \\ &= \lambda_y x_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)(1+s)\left[\frac{1-\phi_Y}{1-\lambda_y}m_t + \phi_Y \Delta m_t\right] \end{aligned} \tag{4.7}$$

where λ_y is the root with absolute value less than one which solves the quadratic equation $\lambda^2 - \Psi_Y\lambda + 1 = 0$. This root is given by

$$\lambda_y = \frac{1}{2}\left[\Psi_Y - \sqrt{\Psi_Y^2 - 4}\right] > 0$$

Then the price level and output are shown to follow

$$\begin{aligned} p_t &= \lambda_y p_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)^2(1+s)E_t\left[\sum_{i=0}^{\infty}\lambda_y^i m_{t+i}\right] \\ &= \lambda_y p_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)^2(1+s)\frac{m_t}{1-\lambda_y} \end{aligned} \tag{4.8}$$

and

$$y_t = \lambda_y y_{t-1} + m_t - \lambda_y m_{t-1} - \frac{\lambda_y(1-\phi_Y)^2(1+s)}{\phi_Y(1-\lambda_y)}m_t \tag{4.9}$$

It should be noted in 4.9) that the parameter λ_y determines endogenous persistence and the amplitude of the responses of output to monetary shocks.

Figure 7(a) plots the responses of (x_t, p_t, y_t) for the values of $(\phi_Y, \theta_Y, s) = (3/4, 0.9, 1/2)$. There are several things worth noting.

The first thing is the persistence in output after the unit monetary shock. This obtains because both the aggregate price adjust toward the new higher steady state only in a sluggish manner due to the staggered nature of price contracts: in fact, the loci of p_t and x_t show that the former catches up the latter only with a lag after the monetary expansion. This is in contrast to CKM, in which deterministic price staggering necessarily leads to oscillatory responses of output and hence no persistence.

The second thing that should not be overlooked is that I get persistence in output in spite of the fact that the elasticity of real marginal cost (with respect to output) is greater than one[27].

[27] In fact, I can show

$$mc_t - p_t = w_t - p_t = (1+s)y_t$$

Therefore, the elasticity of real marginal cost with respect to output is greater than one so long as $0 < L < 1$.

CKM (1996) argue that the reason why they cannot get persistence in various versions of their model with staggered price contracts is the high procyclicality of marginal cost(especially labor cost) implied by standard assumptions about preference and factor market clearing. But equation (4.9) and Figure 7(a) show I obtain persistence in output even with more than unit elastic real marginal costs. Combined with the first finding described above, this implies that the lack of persistence in CKM depends not on the procyclicality of real marginal costs but on the oscillation in output. I will discuss this issue in the next section in more detail.

The third is that this bare-bone version of staggered price contracts is in principle able to generate the *inflation persistence*. By simple manipulations, I get

$$p_t - p_{t-1} = \lambda_y(p_t - p_{t-1}) + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)^2(1+s)\frac{\Delta m_t}{1-\lambda_y} \tag{4.10}$$

Since $\{m_t\}$ is here assumed to follow random walk, the inertial behavior of inflation rates does not depend on the persistence in the driving disturbances themselves. Since the coefficient on the lagged inflation rate is equal to that on the lagged price level in equation (4.10), inflation rates are persistent if and only if the price level is. Hence, contrary to the argument of Fuhrer and Moore (1995), the lack of inflation persistence is not a ubiquitous phenomenon in staggered price contracts models.

4.2 Staggered Wage Contracts

Now I turn to the case where staggered wage contracts are the sole origin of nominal rigidities. For notational simplicity, I let

$$x_t = d\log W_t^*$$

Log-linearization of the wage equation (2.32) implies

$$\begin{aligned}(\frac{1-\theta_L+s}{1-\theta_L})\left[x_t - \phi_L E_t x_{t+1}\right] &= (1-\phi_L)\left[(1+s)m_t + s\frac{\theta_L}{1-\theta_L}w_t\right] \\ &= (1-\phi_L)(1+s)m_t + (1-\phi_L)^2\frac{s\theta_L}{1-\theta_L}(1-\phi_L B)^{-1}x_t\end{aligned} \tag{4.11}$$

where B is the lag operator. The second equality in equation (4.11) comes from the equation for aggregate wage rate:

$$w_t = \phi_L w_{t-1} + (1-\phi_L)x_t \tag{4.12}$$

Rearranging terms in equation (4.11), I get

$$E_t x_{t+1} - \Psi_L x_t + x_{t-1} = -\frac{1}{A\phi_Y}(1-\phi_L)(1+s)(1-\phi_L)m_t \tag{4.13}$$

where

$$\Psi_L = \frac{D}{A\phi_L}, \quad A = \frac{1-\theta_L+s}{1-\theta_L}, \quad D = A + A\phi_L^2 - (1-\phi_L)^2 \frac{s\theta_L}{1-\theta_L}$$

Applying standard methods for solving second order stochastic difference equations, I can write x_t as

$$\begin{aligned} x_t &= \lambda_l x_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)(1+s)(1-\phi_L L)E_t\left[\sum_{i=0}^{\infty}\lambda_l^i m_{t+i}\right] \\ &= \lambda_l x_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)(1+s)\left[\frac{1-\phi_L}{1-\lambda_l}m_t + \phi_L\mu_t\right] \end{aligned} \tag{4.14}$$

where λ_l is the root with absolute value less than one, solving the quadratic equation $\lambda^2 - \Psi_L\lambda + 1 = 0$. This root is given by

$$\lambda_l = \frac{1}{2}\left[\Psi_L - \sqrt{\Psi_L^2 - 4}\right] > 0$$

Then the price level and output are shown to follow

$$\begin{aligned} p_t &= \lambda_l p_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)^2(1+s)E_t\left[\sum_{i=0}^{\infty}\lambda_l^i m_{t+i}\right] \\ &= \lambda_l p_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)^2(1+s)\frac{m_t}{1-\lambda_l} \end{aligned} \tag{4.15}$$

and

$$y_t = \lambda_l y_{t-1} + m_t - \lambda_l m_{t-1} - \frac{\lambda_l(1-\phi_L)^2(1+s)}{A\phi_L(1-\lambda_l)}m_t \tag{4.16}$$

The parameter λ_l measures endogenous persistence and the amplitude of the responses of output to monetary shocks. Figure 7(b) plots the responses of (x_t, w_t, y_t) for the values of $(\phi_L, \theta_L, s) = (3/4, 0.9, 1/2)$. I get qualitatively similar impulse responses compared with those in panel (a), while both the persistence and amplitude in the responses of output is bigger under staggered wage contracts. This is due to the fact that $0 < \lambda_y < \lambda_l$.[28]

The intuition behind the better performance of staggered wage contracts explaining persistence is again clear: staggered wage contracts outperforms staggered price contracts because the

[28] In Figure 9, the values of λ_y and λ_l are 0.4313 and 0.7035, respectively.

elasticity of real marginal cost with respect to output is zero under staggered wage contracts[29]. In other words, the rigidity in the aggregate wage rate reduces the "cost push" effect of the increase in nominal demand by lowering the marginal cost (due to the higher marginal disutility from working) directly, whereas the staggered prices only work to reduce the degree by which the higher marginal costs are reflected in higher aggregate price level. Therefore, the former is the more "fundamental" apparatus in generating contract multiplier.

5 Why a la Calvo, not Taylor?

In the previous section, it is observed that staggered price contracts following Calvo produces monotonically dampening responses of output even with procyclical real marginal costs. In this section, I argue that the lack of persistence in CKM is not attributable only to the high incentive of firms to adjust prices: the oscillatory boom-bust behavior of output pervading in various versions of CKM is attributable to a puzzling artifact inherent to deterministic price staggering, i.e., the overshooting of the newly set price by optimizing firms.

I consider a version of the model in section 2 with two modifications. First, the instantaneous utility function is specified as

$$U(C_t, 1-L_t, \frac{M_t}{P_t}) = \frac{1}{\nu}\log(C_t^\nu + b(\frac{M_t}{P_t})^\nu) + \psi\log(1-L_t) \tag{5.1}$$

Second, since this version of model exhibits too high volatility of investment, I introduce a quadratic capital adjustment cost. Therefore, the budget constraint is given by

$$\begin{aligned} & C_{i\tau} + I_{it}(1+\frac{\phi_K}{2}\frac{I_{it}}{K_{it}}) + \frac{M_{i\tau}}{P_\tau} - \frac{M_{i,\tau-1}}{P_\tau} + \frac{B_{i\tau}}{P_\tau} - \frac{B_{i,\tau-1}}{P_\tau} \\ \leq \quad & \frac{W_{i\tau}L_{i\tau}}{P_\tau} + \frac{Z_\tau K_{i\tau}}{P_\tau} + T_{i\tau} + \frac{\int s_{ij}\Pi_{j\tau}dj}{P_\tau} + r_{\tau-1}\frac{B_{i,\tau-1}}{P_\tau} \end{aligned} \tag{5.2}$$

where

$$I_{it} = K_{i,t+1} - (1-\delta)K_{i,t} \tag{5.3}$$

In a similar manner to CKM, I assume the following nature of price and wage stickiness: households and firms are indexed so that $i, j \in [0, 1/4]$ set wages and prices at period 0,4,8,...,

[29] It can be shown

$$mc_t - p_t = w_t - p_t = 0 \quad .$$

Therefore, the elasticity of real marginal cost with respect to output is always zero.

and $i, j \in [1/4, 1/2]$ at period 1,5,9,..., etc. for four different cohorts of households and firms. All other features of the model in section 2 are inherited[30].

Figure 8 display the impulse responses of some key variables, which closely replicates the results of CKM. In panel (a), output rises as much as 5% initially as a results of monetary expansion, but there is no endogenous persistence: in the fourth quarter after the shock when all cohorts of households and firms have been able to adjust their wages and prices, output is below normal. Hence, even in the presence of capital adjustment costs and wage stickiness (which are expected to enhance the persistence in output), deterministic staggered nominal contracts generate oscillatory rather than monotonous dampening of output responses.

In panel (f), I plot two wage indices together, one for the aggregate wage rate (W) and the other for the wage rate set each period (W*). The plots show the typical "catch-up" pattern: the aggregate wage rate follows the newly set wages with some lags.

When it comes to the price indices, however, panel (e) shows that the prices reset every period (P*) display an overshooting and oscillatory pattern - jump up in the price of the first cohort, followed by a considerable decrease in those of the second and third cohorts, and ensuing recovery by the fourth cohorts. As a result, the relative magnitude of P* with respect to P is reversed in the third period after the shock.

Taylor (1980) gives an intuition for staggered contract: the idea behind staggered price (or wage) contracts is that smoothed-out adjustment of aggregate price (or wage) will be achieved when firms (or households) look both forward and backward in time to see what other firms (or households) charge during their own contract period, and this causes shocks to be passed on from one contract to another. But the plots in (e) and (f) show this intuition works in the labor market only. On this account, I interpret the oscillatory response of price and output in CKM as counterintuitive and deterministic staggered price contracts cast in a DSGE model as incapable of generating contract multiplier.

The next step is to see if the "nuisance feature" is inherent in the *deterministic* staggered *price* contracts and if wage staggering is free of the counter-intuitive anomaly. I will develop the intuition behind these issues in a similar setup used in section 4.

[30]Most of parameter values in Table I are used here, with only two exceptions : *i)* ϕ_K is set to be 2, and *ii)* ψ is set at 1.3884 to match $L = 1/3$. The form of adjustment cost implies the steady state marginal rate of transformation between investment goods and consumptions is 1.052.

5.1 Price Contracts

I assume that individual prices are set optimally every two periods while individual wages are reset every period. In the context of the model in section 2, this case is analogous to $(\phi_Y, \phi_L) = (1/2, 0)$.

I first log-linearize the pricing equation around the deterministic steady state. I let

$$x_t = d\log P_t^*, \quad p_t = d\log P_t, \quad w_t = d\log W_t, \quad mc_{jt} = d\log MC_{jt}, \quad \text{and} \quad y_t = d\log Y_t$$

Then the price equation is log-linearized into

$$x_t = \frac{1}{2}\left[mc_{jt} + E_t mc_{j,t+1}\right], \qquad j \in [0, 1/F] \tag{5.4}$$

Since marginal cost is equal to wage rate, the price equation becomes

$$x_t = \frac{1}{2}\left[w_t + E_t w_{t+1}\right] \tag{5.5}$$

Log-linearizing the wage equation, I get

$$w_t - p_t = \frac{L}{1-L} l_t + y_t = (s+1)y_t \tag{5.6}$$

I impose a static money demand equation:

$$m_t = y_t + p_t \tag{5.7}$$

The price level p_t is an average of the individual prices

$$p_t = \frac{1}{2}(x_t + x_{t-1}) \tag{5.8}$$

and $\{m_t\}$ is a random walk process.

The system of equations (5.4) -(5.8) can be solved to determine how money shocks affect prices and output. Substituting for y_t and p_t, I obtain

$$E_t x_{t+1} - 2\frac{1-\theta_Y+\Psi_Y}{1-\theta_Y-\Psi_Y} x_t + x_{t-1} = -\frac{2\Psi_Y}{1-\theta_Y-\Psi_Y} E_t(m_t + m_{t+1})$$

where $\Psi_Y = (s+1)(1-\theta_Y)$.

The above second order stochastic difference equations can be solved for x_t:

$$x_t = a_y x_{t-1} + (1-a_y)m_t$$

where a_y is the root with absolute value less than one which solves the quadratic equation $a^2 - \Omega_y a + 1 = 0$, with $\Omega_y = 2(1-\theta_Y+\Psi_Y)/(1-\theta_Y-\Psi_Y)$. This root is given by

$$a_y = \frac{\sqrt{1-\theta_Y}-\sqrt{\Psi_Y}}{\sqrt{1-\theta_Y}+\sqrt{\Psi_Y}} < 0 \tag{5.9}$$

Using (5.7) and (5.8), I obtain

$$p_t = a_y p_{t-1} + \frac{1}{2}(1-a_y)(m_t + m_{t-1}) \tag{5.10}$$

and

$$y_t = a_y y_{t-1} + \frac{1}{2}(1+a_y)(m_t + m_{t-1}) \tag{5.11}$$

In order for endogenous price stickiness to arise, a_y needs to be as close to one as possible. However, the specifications of the momentary utility function and parameter values yield negative values for a_y. Of course, this is due to the strong procyclicality of real marginal costs (i.e., 1+s > 0), and it is in this context that CKM conclude price staggering is incapable of generating persistence. However, recall that in the previous section Calvo staggered price contracts generate persistence even with highly procyclical marginal costs, with neither oscillation nor overshooting. This qualitative difference between the results here and those of CKM motivates one to see whether the oscillatory behavior of output is also to be seen under deterministic staggered wage contracts.

5.2 Wage contracts

Now I consider the opposite situation where individual wages are set optimally every two periods and individual prices are reset every period. I first log-linearize the wage equation around the deterministic steady state. Let

$$x_t = d\log W_t^*, \quad and \quad l_{1t} = d\log L_{1t}$$

where L_{1t} is the demand for labor of the cohort of households(denoted by "1") who set their wages in the current period t. Then the wage equation is log-linearized into

$$x_t = \frac{1}{2}s\left[l_{1t} + E_t l_{1,t+1}\right] + \frac{1}{2}\left[y_t + p_t + E_t\left(y_{t+1}+p_{t+1}\right)\right] \tag{5.12}$$

But the individual labor demand function implies

$$\begin{aligned} l_{1t} &= \frac{1}{\theta_L - 1}\left[x_t - w_t\right] + l_t \\ l_{1,t+1} &= \frac{1}{\theta_L - 1}\left[x_t - w_{t+1}\right] + l_{t+1} \end{aligned} \tag{5.13}$$

Therefore the wag equation becomes

$$\begin{aligned} & [1+\frac{1}{1-\theta_L}s]x_t \\ = \quad & \frac{1}{2}s\left[\frac{1}{1-\theta_L}\left(w_t+E_t w_{t+1}\right)+l_t+E_t l_{t+1}\right]+\frac{1}{2}\left[m_t+E_t m_{t+1}\right] \end{aligned} \tag{5.14}$$

Using the production function, money demand, and price equation, I have

$$l_t = y_t = m_t - w_t \tag{5.15}$$

The aggregate wage rate w_t is an average of the individual wage rates

$$w_t = \frac{1}{2}(x_t + x_{t-1}) \tag{5.16}$$

After some manipulations, it can be shown that the log-linearized version of the wage equation is

$$E_t x_{t+1} - 2\frac{\Psi_L + s\theta_L + \Psi_L}{\Psi_L + s\theta_L - \Psi_L}x_t + x_{t-1} = -\frac{2\Psi_L}{\Psi_L + s\theta_L - \Psi_L}E_t(m_t + m_{t+1}) \tag{5.17}$$

where $\Psi_L = (s+1)(1-\theta_L)$.

Applying standard methods for solving second order stochastic difference equations, I can write x_t as

$$x_t = a_l x_{t-1} + (1-a_l)m_t \tag{5.18}$$

where a_l is the root with absolute value less than one which solves the quadratic equation $a^2 - \Omega_l a + 1 = 0$, with $\Omega_l = 2(\Psi_L + s\theta_L + \Psi_L)/(\Psi_L + s\theta_L - \Psi_L)$. This root is given by

$$a_l = \frac{\sqrt{\Psi_L + s\theta_L} - \sqrt{\Psi_L}}{\sqrt{\Psi_L + s\theta_L} + \sqrt{\Psi L}} > 0$$

Finally, price and output are shown to follow

$$p_t = w_t = a_l w_{t-1} + \frac{1}{2}(1-a_l)(m_t + m_{t-1}) \tag{5.19}$$

and

$$y_t = a_l y_{t-1} + \frac{1+a_l}{2}(m_t - m_{t-1}) \tag{5.20}$$

Equation (5.20) is in line with literature favoring wage stickiness as a propagation channel for nominal shocks. Despite the apparent similarity between (5.10)-(5.11) and (5.19)-(5.20), there is an important qualitative difference: contrary to the staggered price case, the sign of a_l is always positive, resulting in persistence in output. and behind the persistence is the ‘catch-up” of newly set wages implied by positive a_l.

Panels (c) and (d) in Figure 7 illustrate the difference between deterministic wage and price contracts: panel (c) displays impulse responses under price staggering, where the newly set prices and output show the anomalous oscillation- no persistence. However, panel (d) shows that a reasonable degree of persistence obtains without oscillatory behavior in W* if the labor market is the only source of nominal rigidity.

6 Conclusion

The most vital challenge to monetary business cycle theory is to construct a model with an ability to generate persistence in output after monetary shocks. This paper develops a formal DSGE model with staggered contracts in goods and/or labor market, and succeeds in addressing the persistence problem. Specifically, Calvo staggered contracts proves to be an effective mechanism through which the monetary shock propagates in the real side of economy. I also find that staggered wage contracts are favorable in addressing the persistence problem compared with staggered price contracts.

I find the lack of persistence in various versions of CKM is due to a nuisance feature, i.e., the oscillatory behavior of newly set prices each period, rather than the procyclicality of real marginal cost. Results from calibration exercises and stripped down versions of model show such an anomaly does not occur under staggered wage contracts either stochastic a la Calvo or deterministic a la Taylor. Genral conclusion drawn from the above findings is that wage stickiness and stochastic staggering are favorable to their respective competitors in generating persistence.

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7 Appendix

7.1 Derivation of Equation (2.47)

Equation (2.47) expresses the relative factor demands for firm j in terms of its relative price with respect to the aggregate price level. Using equation (2.11), it is easy to show

$$\frac{K_{j't}}{K_{jt}} = \left(\frac{P_{j't}}{P_{jt}}\right)^{\frac{1}{\theta_Y - 1}}, \quad \forall\ j, j\prime \in [0,1]\ and\ \ j \neq j \tag{A1}$$

By aggregating (A1) over $j\prime \in [0,1]$, I get

$$\frac{K_t}{K_{jt}} = \left[\int\limits_{j' \in [0,1]} P_{j't}^{\frac{1}{\theta_Y - 1}} dj'\right] P_{jt}^{\frac{-1}{\theta_Y - 1}} \tag{A2}$$

7.2 Aggregation

The existence of heterogenous households and firms requires explicit aggregation over firms and households. From now on, when used, the subscript 0 denotes variables aggregated over the firms(or households) in *flex(t),* and the subscript 1 denotes variables aggregated over those in *rev(t).* To investigate a stationary equilibrium, I transform all nominal variables into stationary ones before aggregation. I use lowercase letters for transformed variables

7.2.1 Firms Equations

Equations (A3)- (A7) describing firms block of the system are transformed as follows:[31]

$$Y_{0t} = (1 - \phi_Y) K_t{}^{\alpha} L_t^{1-\alpha} \left(\frac{p_t^*}{p_t^R}\right)^{\frac{1}{\theta_Y - 1}} \tag{A3}$$

[31] I use β_Y for $\beta\phi_Y$ to save space. The last equality in the equation (A.4) holds due to the random sampling nature of Calvo staggering.

$$
\begin{aligned}
Y_{1t} &= (K_t/L_t)^{(\alpha-1)} \int\limits_{j\in rev(t)} K_{jt} dj \\
&= (K_t/L_t)^{(\alpha-1)} K_t \left[\int\limits_{j\in rev(t)} \left(\frac{p_{j,t-1}}{p_t^R}\right)^{\frac{1}{\theta_Y-1}} dj \right] \\
&= \phi_Y K_t^\alpha {L_t}^{1-\alpha} \left(\frac{p_{t-1}^R}{p_t^R}\right)^{\frac{1}{\theta_Y-1}}
\end{aligned} \tag{A4}
$$

$$
L_t = \frac{1-\alpha}{\alpha}\frac{z_t}{w_t}K_t \tag{A5}
$$

$$
p_t^* = \frac{E_t\left[\sum\limits_{\tau=t}^{\infty} {\beta_Y}^{\tau-t}\frac{\Lambda_\tau}{\Lambda_t} p_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau A_\tau^{-1/\gamma} z_\tau^\alpha w_\tau^{1-\alpha}(1-\alpha)^{\alpha-1}\alpha^{-\alpha}\right]}{\theta_Y E_t\left[\sum\limits_{\tau=t}^{\infty} {\beta_Y}^{\tau-t}\frac{\Lambda_\tau}{\Lambda_t} p_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau\right]} \tag{A6}
$$

7.2.2 Households Equations

Equations describing the households block of the system are follows[32]:

$$
\Lambda_t = [C_t^\nu + b\,(m_t/p_t)^\nu]^{\frac{1-\sigma-\nu}{\nu}} \left[\int\limits_{i\in[0,1]} (1-L_{it})^{\psi(1-\sigma)} di\right] C_t^{\nu-1} \tag{A7}
$$

$$
\Lambda_{0t} = (1-\phi_L)[C_t^\nu + b\,(m_t/p_t)^\nu]^{\frac{1-\sigma-\nu}{\nu}} (1-L_{0t})^{\psi(1-\sigma)} C_t^{\nu-1} \tag{A8}
$$

$$
1 - R_t^{-1} = b(P_t C_{it}/M_{it})^{1-\nu} \tag{A9}
$$

$$
\beta E_t\left[\Lambda_{t+1}\left\{(1-\delta) + \frac{z_{t+1}}{p_{t+1}}\right\}\right] = \Lambda_t \tag{A10}
$$

$$
0 = \frac{\Lambda_t}{p_t} - \beta\mu^{-1}(1+r_t)E_t\frac{\Lambda_{t+1}}{p_{t+1}} \tag{A11}
$$

$$
w_t^* = \frac{\psi E_t\left[\sum\limits_{\tau=t}^{\infty} {\beta_L}^{\tau-t}\psi[C_\tau^\nu + b_\tau\left(\frac{m_\tau}{p_\tau}\right)^\nu]^{\frac{1-\sigma}{\nu}}\{1-(\frac{w_t^*}{w_\tau})^{\frac{1}{\theta_L-1}} L_\tau)\}^{\psi-1-\psi\sigma} w_\tau^{\frac{1}{1-\theta_L}} L_\tau\right]}{\theta_L E_t\left[\sum\limits_{\tau=t}^{\infty} {\beta_L}^{\tau-t}\frac{\Lambda_{i\tau}}{p_\tau} w_\tau^{\frac{1}{1-\theta_L}} L_\tau\right]} \tag{A12}
$$

where (A12) holds for $i \in flex(t)$.

[32] Iuse β_L for $\beta\phi_L$ to save space.

7.2.3 Other Equations

By combining aggregate budget constraint of households, aggregate profit of firm, and the government budget constraint (2.33), I get the familiar resource constraint:

$$\begin{aligned} Y_t &= \left(\frac{p_t^R}{p_t}\right)^{\frac{1}{1-\theta_Y}} [Y_{0t} + Y_{1t}] \\ &= C_t + K_{t+1} - (1-\delta)K_t \end{aligned} \tag{A13}$$

Aggregate price and wage indices are also stationary transformed:

$$p_t^{\frac{\theta_Y}{\theta_Y-1}} = \phi_Y p_{t-1}^{\frac{\theta_Y}{\theta_Y-1}} + (1-\phi_Y) p_t^{*\frac{\theta_Y}{\theta_Y-1}} \tag{A14}$$

$$\left(p_t^R\right)^{\frac{1}{\theta_Y-1}} = \phi_Y \left(p_{t-1}^R\right)^{\frac{1}{\theta_Y-1}} + (1-\phi_Y) p_t^{*\frac{1}{\theta_Y-1}} \tag{A15}$$

$$w_t^{\frac{\theta_L}{\theta_L-1}} = \phi_L w_{t-1}^{\frac{\theta_L}{\theta_L-1}} + (1-\phi_L) w_t^{*\frac{\theta_L}{\theta_L-1}} \tag{A16}$$

The system is complete with the monetary policy rule:

$$\overline{\mu}\frac{m_t}{m_{t-1}} = \mu_t \tag{A17}$$

$$\log(\mu_t) = \rho_\mu \log(\mu_{t-1}) + (1-\rho_\mu)\log(\overline{\mu}) + \varepsilon_{\mu t} \tag{A18}$$

7.3 Log-linearization

I log-linearize the aggregated system comprising (A3)-(A18) around the steady state.

7.3.1 Firms Equations

Log-linearization of equations (A3)-(A5) is straightforward:[33]

$$\widehat{Y}_{0t} = \alpha \widehat{K}_t + (1-\alpha)\widehat{L}_t + \frac{1}{\theta_Y - 1}\left(\widehat{p}_t^* - \widehat{p}_t^R\right) \tag{A19}$$

[33] In (A19) and (A20), I use the following equality

$$d\log P = \log\left[\left(\int P_i^{\theta_Y-1} di\right)^{\theta_L}\right] = \int \log P_i di = \log\left[\int P_i di\right]$$

valid up to a first order log-linear approximation.

$$\widehat{Y}_{1t} = \alpha \widehat{K}_t + (1-\alpha)\widehat{L}_t + \frac{1}{\theta_Y - 1}\left(\widehat{p}^R_{t-1} - \widehat{p}^R_t\right) \tag{A20}$$

$$\widehat{L}_t = \widehat{z}_t - \widehat{w}_t + \widehat{K}_t \tag{A21}$$

I first log-linearize the numerator of the RHS of equation (A6):

$$Num = \frac{1-\gamma}{\gamma}\frac{Y}{Y+\Phi}\frac{1}{\theta_Y - 1}p^*_t + (1-\beta_Y)\widehat{\Theta}^Y_t \tag{A22}$$

where

$$\begin{aligned} & \widehat{\Theta}^Y_t - \beta_Y E_t \widehat{\Theta}^Y_{t+1} \\ = \; & \widehat{\Lambda}_t - \alpha \widehat{z}_t + (1-\alpha)\widehat{w}_t + \frac{\theta_Y}{1-\theta_Y}\widehat{p}_t + \widehat{Y}_t \end{aligned}$$

The denominator is log-linearized into

$$Den = (1-\beta_Y)\widehat{\Xi}^Y_t \tag{A23}$$

where

$$\widehat{\Xi}^Y_t - \beta_Y E_t \widehat{\Xi}^Y_{t+1} = \left[\widehat{\Lambda}_t + \frac{\theta_Y}{1-\theta_Y}\widehat{p}_t + \widehat{Y}_t\right]$$

Then, the pricing equation is finally log-linearized into

$$\widehat{p^*_t} = (1-\beta_Y)\left[\widehat{\Theta}^Y_t - \widehat{\Xi}^Y_t\right] \tag{A24}$$

Equations (A19) - (A24) constitute the firms' block of the log-linearized system.

7.3.2 Households

Equations (A7) and (A8) are log-linearized into[34]

$$\begin{aligned} \widehat{\Lambda}_t \;=\; & ((1-\sigma-\nu)D_1 + \nu - 1)\widehat{C}_t \\ & + (1-\sigma-\nu)D_2\widehat{m}_t - (1-\sigma-\nu)D_2\widehat{p}_t - \psi(1-\sigma)\frac{L}{1-L}\widehat{L}_t \end{aligned} \tag{A25}$$

[34] I use the following equality

$$\log L = \log\left[\left(\int L_i^{\theta_L} di\right)^{\theta_L}\right] = \int \log L_i di = \log\left[\int L_i di\right]$$

valid up to a first order log-linear approximation.

$$\begin{aligned}
\widehat{\Lambda}_{0t} &= ((1-\sigma-\nu)D_1+\nu-1)\widehat{C}_t+(1-\sigma-\nu)D_2\left(\widehat{m}_t-\widehat{p}_t\right) \\
&\quad -\psi(1-\sigma)\frac{L}{1-L}\left[\frac{1}{\theta_L-1}\left(\widehat{w}_t^*-\widehat{w}_t\right)+\widehat{L}_t\right] \\
&= \widehat{\Lambda}_t+\psi(1-\sigma)\frac{L}{1-L}\frac{1}{\theta_L-1}(\widehat{w}_t-\widehat{w}_t^*)
\end{aligned} \tag{A26}$$

where $D_1=(C/C^*)^{\nu}$ and $D_2=1-$ D_1, the values of which are given later in this appendix. In equation (A26), I use $L_{0t}=(\frac{W_t^*}{W_t})^{\frac{1}{\theta_L-1}}L_t$.

Note that (A26) greatly simplifies the log-linearization of the wage equation (A12), because (A26) eliminates the burden of tracing out household $i\in flex(\tau),\forall\tau\geq t$. In particular, using the assumption of completely pooled consumption, I get for all $\tau\geq t+1$

$$\begin{aligned}
E_t\widehat{\Lambda}_{0\tau} &= (1-\phi_L)^{-1}E_t\left[\int\limits_{i\in flex(t)}\widehat{\Lambda}_{i\tau}di\right] \\
&= ((1-\sigma-\nu)D_1+\nu-1)E_t\widehat{C}_\tau \\
&\quad +(1-\sigma-\nu)D_2E_t\left[\widehat{m}_\tau-\widehat{p}_\tau\right]-\psi(1-\sigma)\frac{L}{1-L}E_t\widehat{L}_\tau \\
&\quad -\psi(1-\sigma)\frac{L}{1-L}\frac{1}{\theta_L-1}(1-\phi_L)^{-1}E_t\left[\int\limits_{i\in flex(t)}\left(\widehat{w}_{i\tau}-\widehat{w}_\tau\right)di\right]
\end{aligned} \tag{A27}$$

But the "random sampling" nature of Calvo staggering makes the last term in (A27) disappears up to the first order approximation, thereby yielding $E_t\widehat{\Lambda}_{0\tau}=E_t\widehat{\Lambda}_\tau,\forall\tau\geq t+1$.

Log-linearization of equations (A9) - (A11) gives the following:

$$\frac{1}{1-R}\widehat{R}_t=\widehat{b}_t+(1-\nu)\widehat{C}_t+(1-\nu)\widehat{p}_t-(1-\nu)\widehat{m}_t \tag{A28}$$

$$\widehat{\Lambda}_t+\beta\delta E_t\widehat{\delta}_{t+1}=E_t\widehat{\Lambda}_{t+1}+\beta\frac{z}{p}\left[E_t\widehat{z}_{t+1}-E_t\widehat{p}_{t+1}\right] \tag{A29}$$

$$r_t=E_t\left[\widehat{\Lambda}_t-\widehat{\Lambda}_{t+1}+\widehat{p}_{t+1}-\widehat{p}_t\right] \tag{A30}$$

Log-linearizing the wage equation (A12) involves more complexity. I first log-linearize the numerator in (A12) into

$$Num=(1-\beta_L)\widehat{\Theta}_t^L-(\psi-1-\psi\sigma)\frac{L}{1-L}\frac{1}{\theta_L-1}\widehat{w}_t^* \tag{A31}$$

where

$$\begin{aligned}
& \widehat{\Theta}_t^L - \beta_L E_t \widehat{\Theta}_{t+1}^L \\
= \; & (1-\sigma) D_1 \widehat{C}_t + (1-\sigma) D_2 \left[\widehat{m}_t - \widehat{p}_t\right] + \frac{1}{1-\theta_L}\left(1 - \{\psi - 1 - \psi\sigma\}\frac{L}{1-L}\right)\widehat{w}_t \\
& + \left(1 - \{\psi - 1 - \psi\sigma\}\frac{L}{1-L}\right)\widehat{L}_t \qquad \text{(A32)}
\end{aligned}$$

The denominator in (A12) is log-linearized into

$$Den = (1-\beta_L)\widehat{\Xi}_{i,t}^L \qquad \text{(A33)}$$

where

$$\widehat{\Xi}_{it}^L - \beta_L E_t \widehat{\Xi}_{i,t+1}^L = \widehat{\Lambda}_{it} - \widehat{p}_t + \frac{1}{1-\theta_L}\widehat{w}_t + \widehat{L}_t, \qquad i \in flex(t) \qquad \text{(A34)}$$

Rearranging terms in (A31) and (A34), I get

$$\begin{aligned}
& \left(1 - \frac{L}{1-L}\frac{1}{\theta_L - 1}\left[a + (1-a)\sigma\right]\right)\widehat{w}_t^* \qquad \text{(A35)} \\
= \; & (1-\beta_L)\widehat{\Theta}_t^L - (1-\beta_L)\widehat{\Xi}_{it}^L
\end{aligned}$$

Then by aggregating (A35) over $i \in flex(t)$, I get

$$\begin{aligned}
& (1-\phi_L)\left(1 + \frac{L}{1-L}\frac{1}{\theta_L - 1}\left[\psi(1-\sigma) - 1\right]\right)\widehat{w}_t^* \qquad \text{(A36)} \\
= \; & (1-\phi_L)(1-\beta_L)\widehat{\Theta}_t^L - (1-\beta_L)\int_{i \in flex(t)} \widehat{\Xi}_{it}^L di
\end{aligned}$$

But using (A27), I can show

$$\int_{i \in flex(t)} \widehat{\Xi}_{it}^L di = (1-\phi_L)\widehat{\Psi}_t^L + \psi(1-\sigma)\frac{L}{1-L}\frac{1}{\theta_L - 1}(1-\phi_L)(\widehat{w}_t - \widehat{w}_t^*) \qquad \text{(A37)}$$

where

$$\widehat{\Psi}_t^L - \beta_L E_t \widehat{\Psi}_{t+1}^L = \widehat{\Lambda}_t - \widehat{p}_t + \frac{1}{1-\theta_L}\widehat{w}_t + \widehat{L}_t \qquad \text{(A38)}$$

Therefore, the final form of the log-linearized version of (A12) is given by

$$\begin{aligned}
& \left[1 + \frac{L}{1-L}\frac{1}{\theta_L - 1}\left\{\psi(1-\sigma)\beta_L - 1\right\}\right]\widehat{w}_t^* \qquad \text{(A39)} \\
= \; & (1-\beta_L)\left(\widehat{\Theta}_t^L - \widehat{\Psi}_t^L\right) - (1-\beta_L)\psi(1-\sigma)\frac{L}{1-L}\frac{1}{\theta_L - 1}\widehat{w}_t
\end{aligned}$$

Equations (A25)-(A26), (A28)-(A30), (A32), and (A38)-(A39) constitute the households' block of the log-linearized system.

7.3.3 Resource Constraint and Evolution of Price and Wage Indices

Resource constraint is log-linearized as

$$\widehat{Y}_t = \frac{1}{1-\theta_Y}\left(\widehat{p}_t^R - \widehat{p}_t\right) + (1-\phi_Y)\widehat{Y}_{0t} + \phi_Y \widehat{Y}_{1t} \tag{A40}$$

together with

$$\widehat{Y}_t = \frac{C}{Y}\widehat{C}_t + \frac{K}{Y}\widehat{K}_{t+1} - \frac{K}{Y}(1-\delta)K_t \tag{A41}$$

The log-linearized versions of other equations are as follows:

$$\widehat{p}_t = \phi_Y \widehat{p}_{t-1} + (1-\phi_Y)\widehat{p}_t^* \tag{A42}$$

$$\widehat{p}_t^R = \phi_Y \widehat{p}_{t-1}^R + (1-\phi_Y)\widehat{p}_t^* \tag{A43}$$

$$\widehat{w}_t = \phi_L \widehat{w}_{t-1} + (1-\phi_L)\widehat{w}_t^* \tag{A44}$$

$$\widehat{m}_t - \widehat{m}_{t-1} = \widehat{\mu}_t \tag{A45}$$

$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \varepsilon_t \tag{A46}$$

7.3.4 Steady State values

The steady-state values of some variables are given below:

$$R = 1 + r = \beta^{-1}\mu \tag{A47}$$

$$\frac{z}{p} = \beta^{-1} - 1 + \delta \tag{A48}$$

$$\frac{K}{Y} = \theta_y \alpha \left[\frac{z}{p}\right]^{-1} \tag{A49}$$

$$\frac{C}{Y} = 1 - \frac{K}{Y}\delta \tag{A50}$$

$$\left[\frac{C}{C^*}\right]^\nu = \frac{C^\nu}{C^\nu + b(m/p)^\nu} = \frac{1}{1 + b^{\frac{1}{1-\nu}}(1-\frac{\beta}{\mu})^{\frac{\nu}{\nu-1}}} \tag{A51}$$

$$\frac{L}{1-L} = \theta_Y \theta_L \frac{1-\alpha}{\psi}\left[\frac{C}{C^*}\right]^\nu \left[\frac{C}{Y}\right]^{-1} \tag{A52}$$

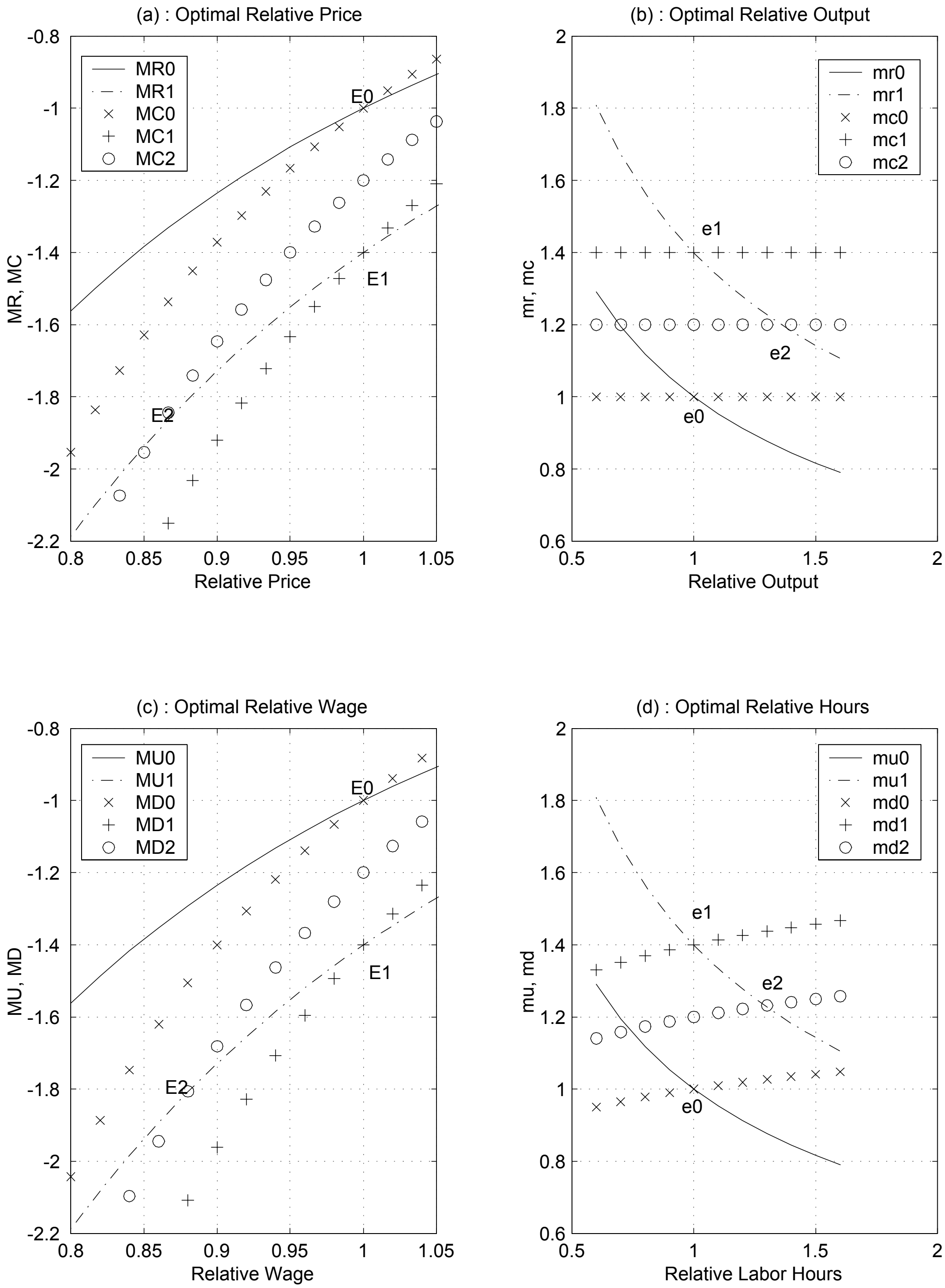

Figure 1: Effects of Wage and Price Rigidities

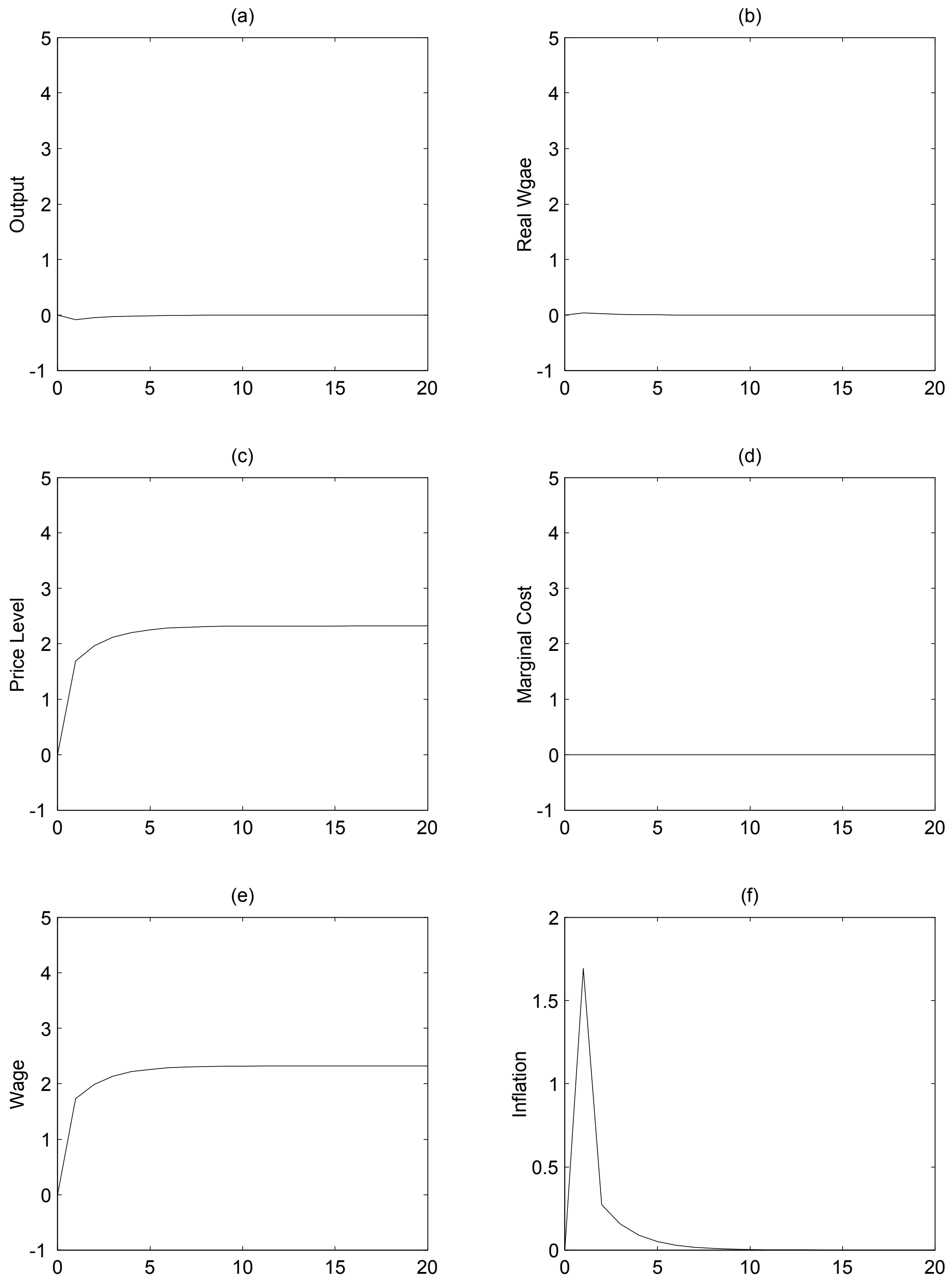

Figure 2: Impulse Responses - Flexible Prices and Wages

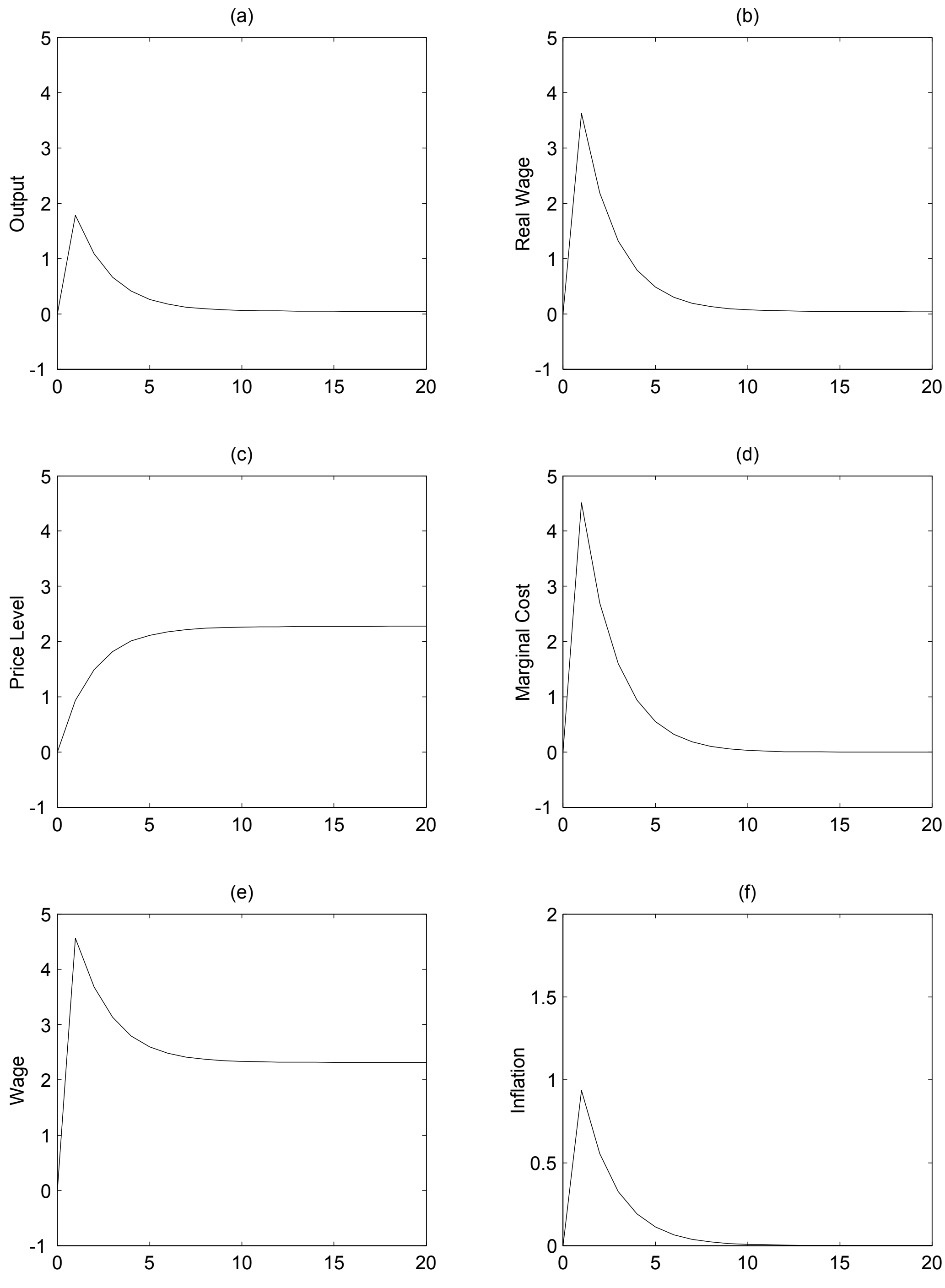

Figure 3: Impulse Responses - Staggered Prices

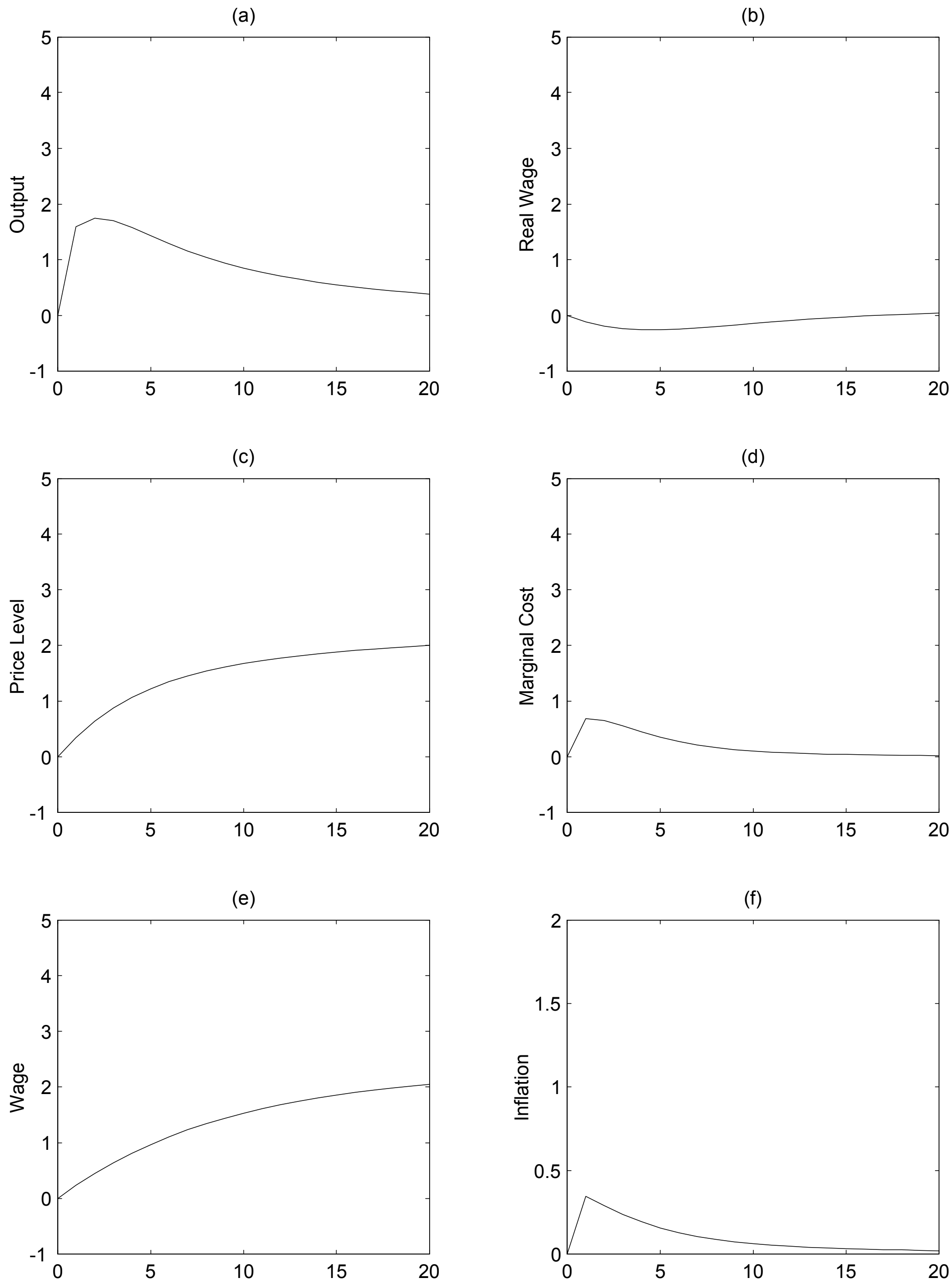

Figure 4: Impulse Responses - Staggered Prices and Wages for $(\phi_Y, \phi_L) = (3/4, 3/4)$

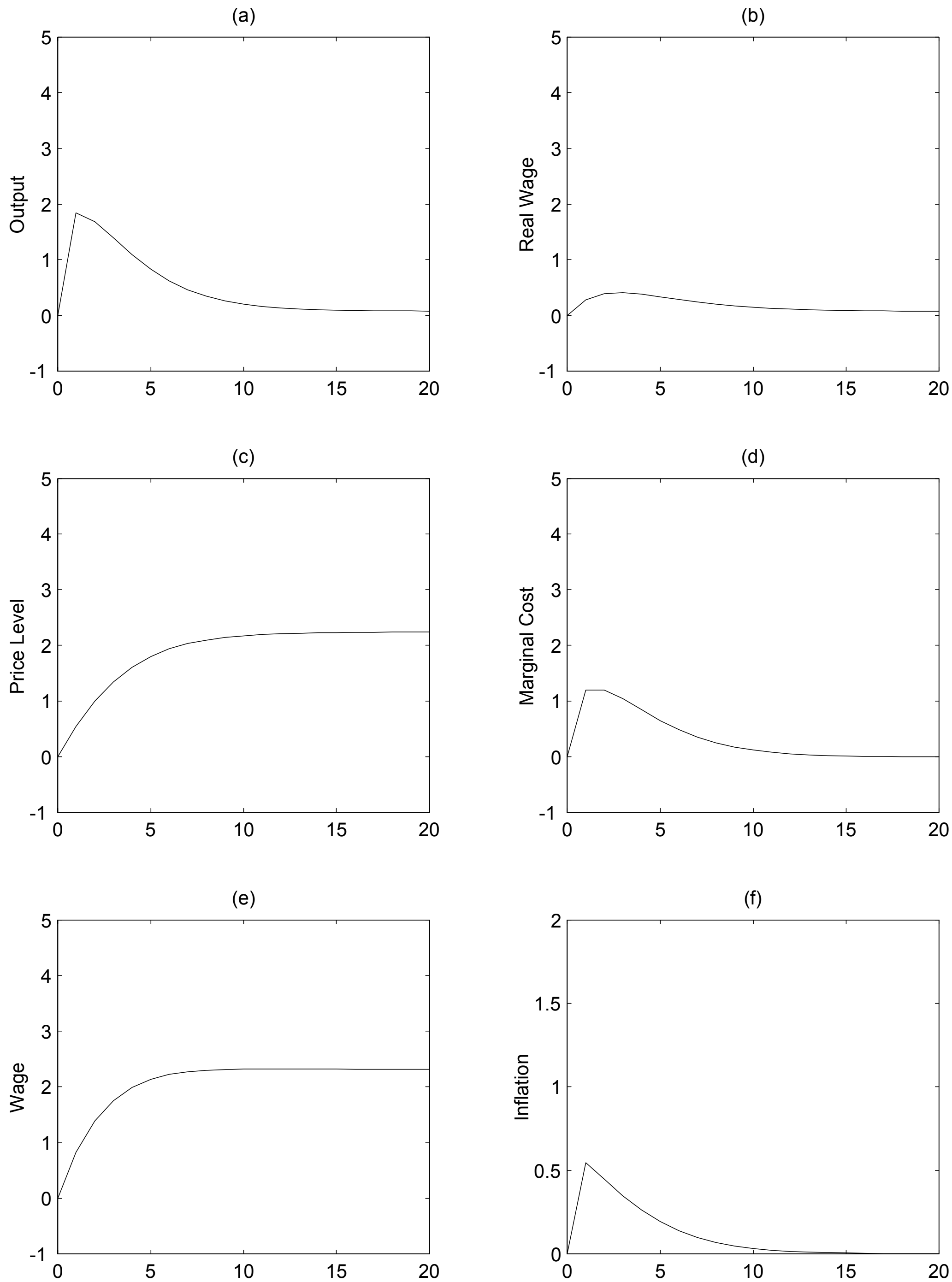

Figure 5: Impulse Responses - Staggered Prices and Wages for $(\phi_Y, \phi_L) = (3/4, 1/2)$

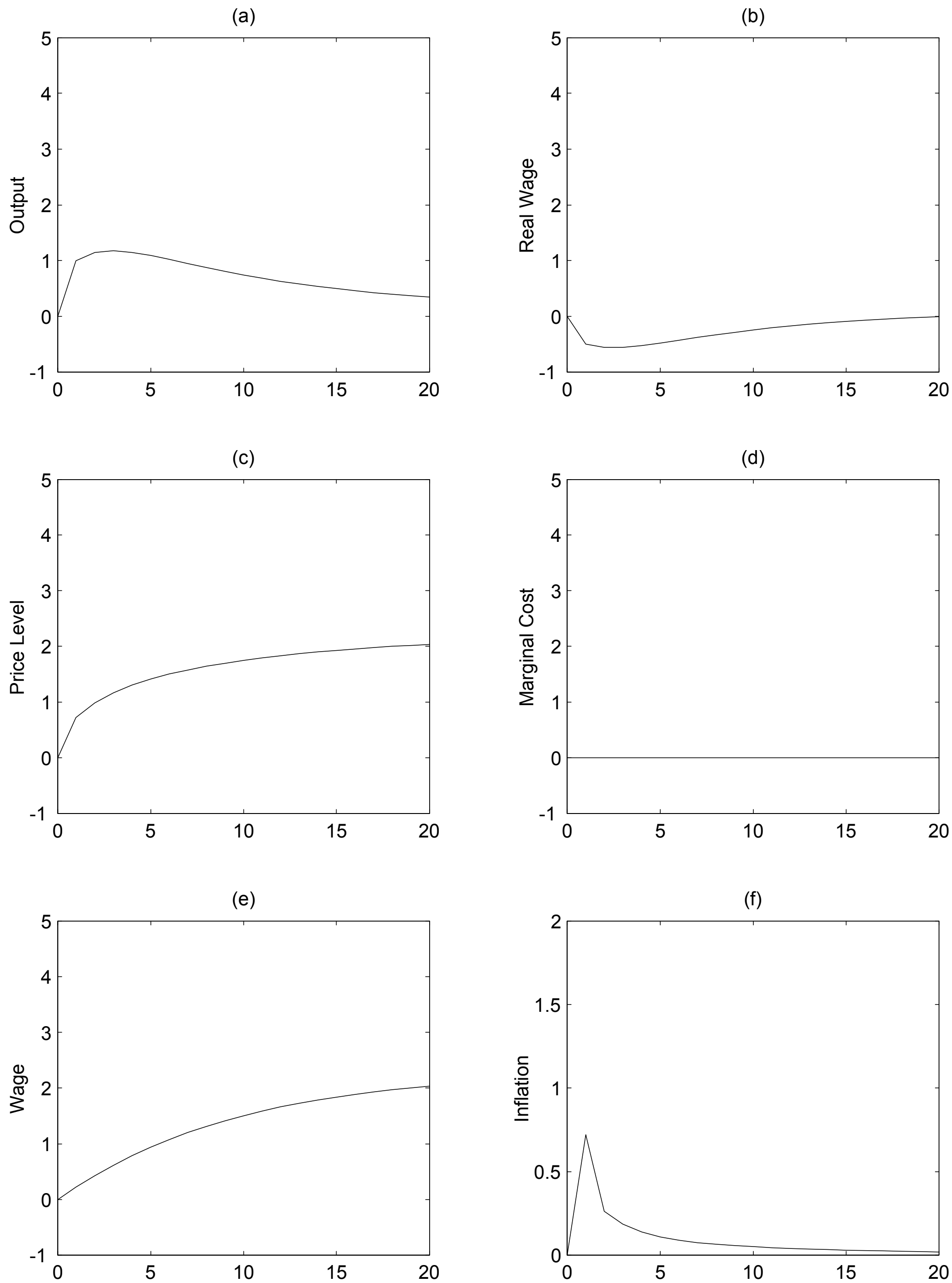

Figure 6: Impulse Responses - Staggered wages

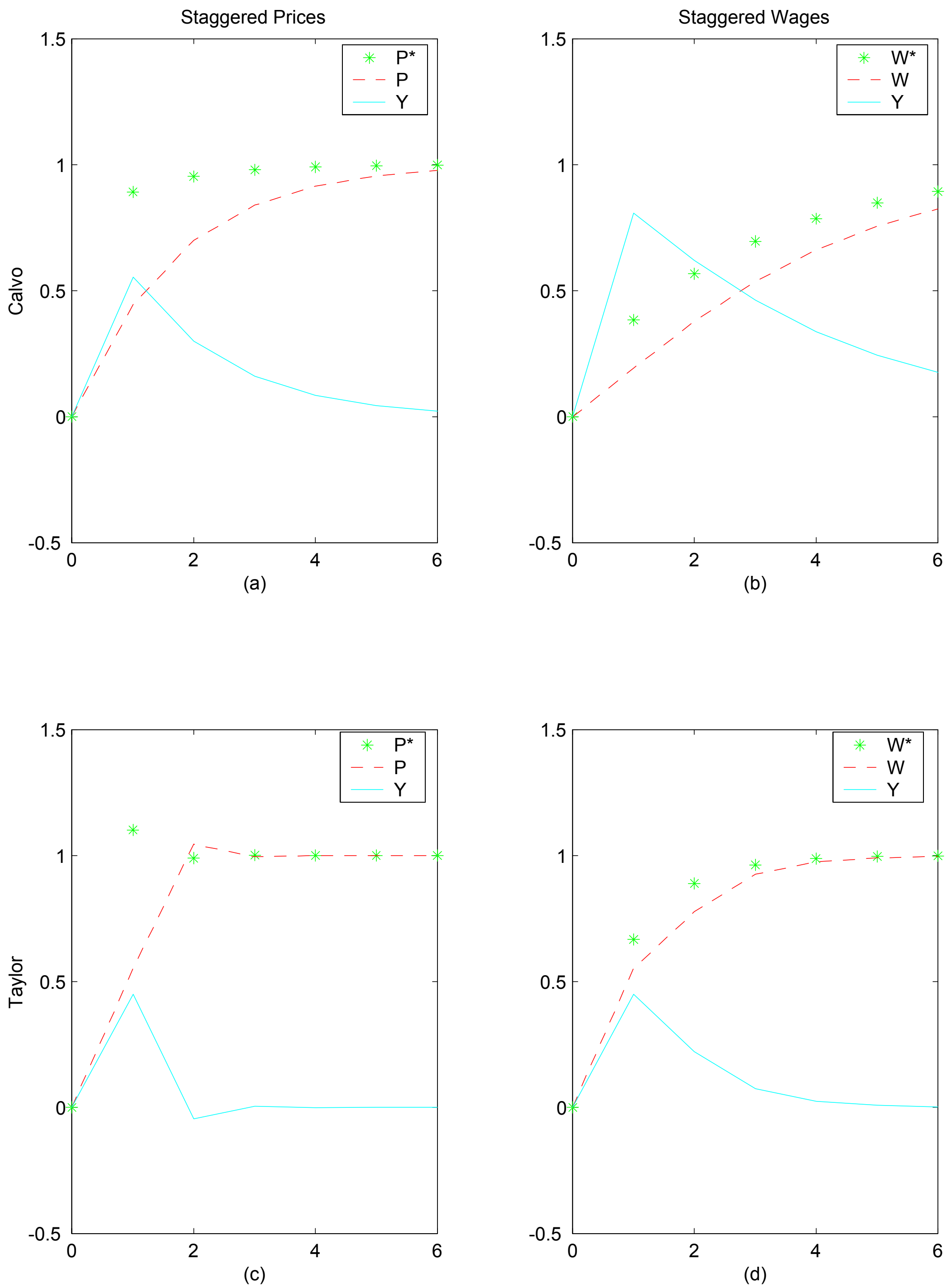

Figure 7: Calvo vs Taylor Staggered Contracts

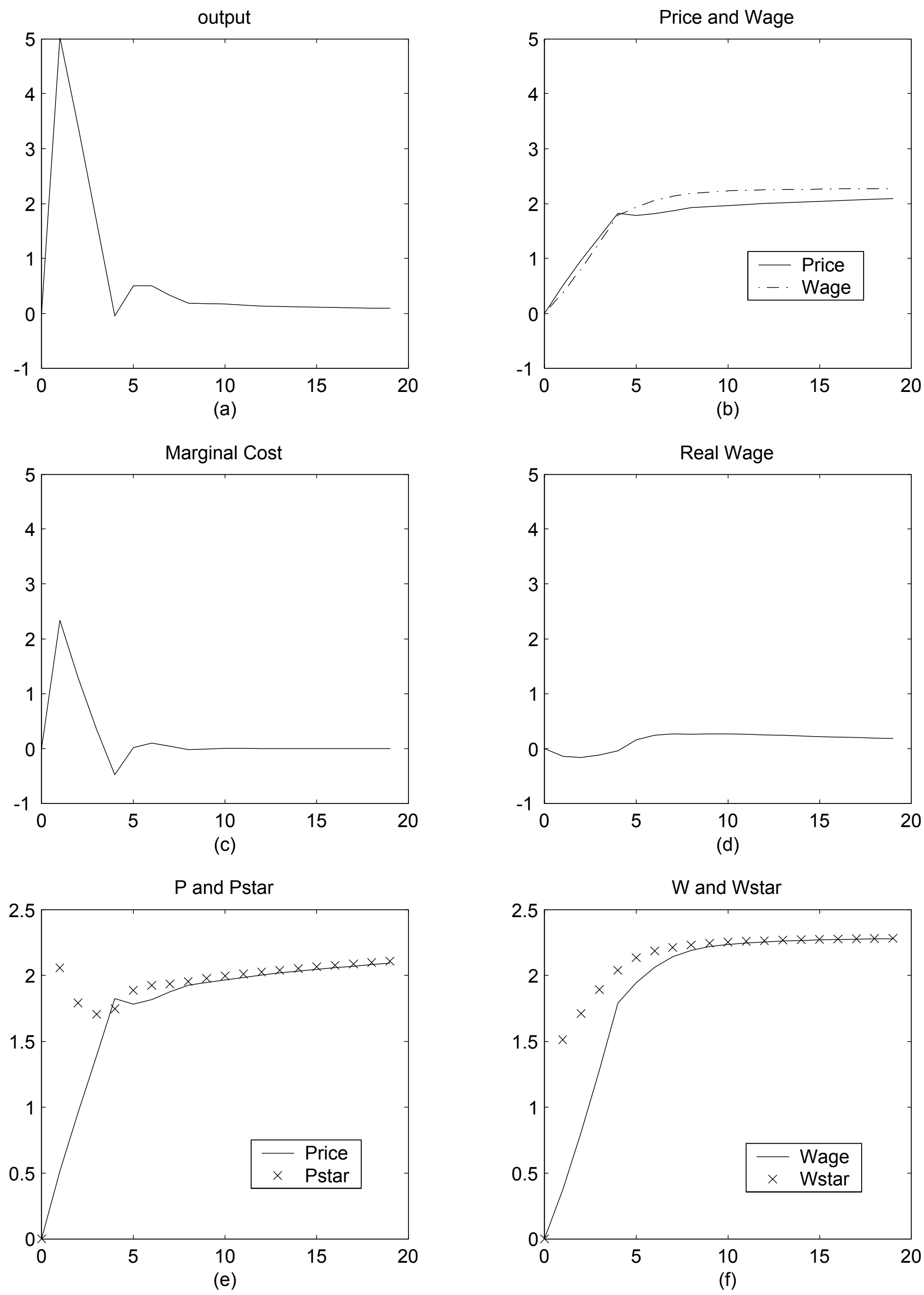

Figure 8: Impulse Responses in a Model with Taylor Price and Wage Contracts