Alternative Nominal Anchors: a Welfare Comparison

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Keywords: Alternative Nominal Anchors, Fixed Targeting Rules, Nominal Rigidities, Welfare Evaluations, Quadratic Approximate Solutions.

JEL Classification: E32, E52

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1. Introduction

The recent recognition of the possible time inconsistency problem has urged a majority of monetary economists to change their focus away from the merits of full discretion to respond to unforeseen disturbances in the economy. This very change of interest is also guided by an appreciation of the gains from imposing a credible (or dynamically consistent) precommitment or rule on monetary policy, thereby "tying the hands" of the monetary authority. In practice, conducting monetary policy via a credible precommitment generally involves choosing a particular nominal anchor, a nominal variable that the monetary authorities commits themselves to keep close to a predetermined path. Presumably, the rationales for signaling out such intermediate targets of monetary policy are *i)* the increased transparency in the conduct of monetary policy, and *ii)* the ease of monitoring and evaluation by the public what the central bank does.

The issue of choosing a nominal anchor dates back to the late 70's, when the monetarists favored a pre-announced path for M1 (or other broader monetary aggregates such as M2) as an intermediate target. However, the breakdown of demand for M1 and M2 in 80s and 90s, respectively, forced those who in favor of some degree of precommitment to quest for a new candidate for a nominal target. In the literature assuming a closed economy, two (of many) competing candidates for the nominal anchor have been considered as an intermediate target for policy. The first, advocated by McCallum (1997) and Frankel and Chinn (1995), is stabilizing nominal income. For example, the latter authors argue in a static model that nominal GNP targeting gives an outcome characterized by greater stability of output and the price level, compared with other nominal anchors such as money and price. The second, advocated by Barro (1986), Ball (1997), and Svensson (1997), focuses on stabilizing the aggregate price level or its rate of change. In particular, Svensson (1997) finds that it is optimal to target the forecasts of the inflation if price stabilization is the sole goal of monetary policy.

This paper seeks to evaluate the welfare implications of choosing alternative nominal anchors. In particular, the present paper constructs and estimates a monetary business cycle model with nominal rigidities, and use the estimated model to compare the welfare levels achieved by keeping preannounced fixed paths of three alternative anchors: inflation, money growth, and nominal income growth. For that aim, these anchors are compared in terms of a natural welfare metric derived from the representative agent's utility, and the welfare effects of non-linear dynamics of the model are captured by using a quadratic approximate solution method developed and extended by Sims (2000) and Kim et al. (2002).

The estimated model exhibits *i)* a considerable degree of nominal rigidities in both the goods and labor markets, and *ii)* a sizable amount of welfare losses from holding the non-interest bearing asset, i.e, nominal money. These features require the monetary authority to consider two welfare effects when choosing a nominal anchor: *i)* in terms of the short-run stabilization, as in Erceg et al. (2000), a good nominal anchor should provide the best compromise between stabilizing price inflation, wage inflation, and real output, and *ii)* for the sake of the long-run efficiency gains, the authority should choose an anchor that is compatible with low (possibly negative) rate of long-run inflation, subject to the zero bound on nominal interest rate.

Welfare analysis reveals that adopting strict inflation is not desirable unless the monetary authority has to run high rates of long-run inflation. It is because, in view of the first feature of the estimated model economy, putting an inertial anchor (i.e., the inflation rate) on a fixed path will necessarily require too frequent and excessive adjustments in the nominal interest rates, which is feasible only when the long-run inflation rate remains away from zero. When the (net) long-run inflation rate is lowered close to zero, fixed nominal income targeting and money growth targeting show comparable welfare performance and are superior to the feasible fixed inflation targeting. These findings show that there is a trade-off between welfare gains from the short-run stabilization of price inflation and the efficiency gains from lower long-run inflation, and that nominal income growth targeting may be a good firsthand strategy to strike a balance between these two welfare considerations.

The paper is organized as follows. Section 2 presents the model and its estimated snapshot. In section 3, I describe the utility-based metric for welfare evaluations. Section 4 compares the performance of the fixed targeting rules for alternative nominal anchors. Section 5 concludes the paper.

2. The Model

The economy consists of four types of agents: households, firms, government, and the aggregator. Firms produce differentiated products using capital and labor supplied by households and the aggregator, respectively. Households purchase output from the aggregator for consumption and investment purposes, and supply capital and differentiated labor to firms and the aggregator, respectively. The aggregator combines the differentiated goods (and labor service) into a homogenized output (and labor), and sells the resulting output (and labor) to households (and firms). The government manages monetary policy.

2.1 Firms and Price Setting

During period t, an individual firm $j \in [0, 1]$ hires K_{jt} units of physical capital (from households) and L_{jt} units of aggregate labor service (from the aggregator), and produce Y_{jt} units of its own product. All firms have the identical CRS production technology

$$Y_{jt} = A_t K_{jt}^{\alpha_t} (g^t L_{jt})^{1-\alpha_t}, \ g \geq 1 \tag{1}$$

where A_t and α_t are the *aggregate productivity shock* and the *capital share shock* α_t, respectively.

Each firm sells its differentiated output to the aggregator, who uses a CRS technology

$$Y_t = (\int Y_{jt}^{\theta_Y} dj)^{\frac{1}{\theta_Y}} \quad , \quad \theta_Y \in [0,1] \tag{2}$$

to transform the differentiated products into a single output Y_t. The implied demand function for the firm j's output Y_{jt} is

$$Y_{jt}^d = (\frac{P_{jt}}{P_t})^{\frac{1}{\theta_Y - 1}} Y_t \tag{3}$$

where the aggregate price level P_t is defined as

$$P_t = (\int P_{jt}^{\frac{\theta_Y}{\theta_Y - 1}} dj)^{\frac{\theta_Y - 1}{\theta_Y}} \quad . \tag{4}$$

Nominal rigidity in the goods market is formulated in the spirit of Calvo (1983): in each period t, a randomly chosen $1-\phi_Y$ fraction of firms, denoted by *opt(t)*, are able to reoptimize their individual nominal prices. The other ϕ_Y fraction of firms, denoted by *res(t)*, are assumed to reset their prices according to an index rule

$$P_{jt} = \Pi_{t-1} P_{j,t-1}, \qquad j \in rev(t) \tag{5}$$

where Π_{t-1} is the actual aggregate inflation rate in the last period.

In period t, the firm $j \in$ *opt(t)* solves the following profit maximization problem:

$$\max E_t \left[\sum_{\tau=t}^{\infty} \phi_Y^{\tau-t} \frac{\beta^{\tau-t}\Lambda_\tau}{\Lambda_t} \frac{1}{P_\tau} \left(Y_{j\tau} P_{j\tau} - Y_{j\tau} \frac{W_\tau}{MPL_\tau} \right) \right] \tag{6}$$

where $\frac{\beta^{\tau-t}\Lambda_\tau}{\Lambda_t}$ is the discount factor for its real profit between period t and τ, and $\Lambda_t = \int_{[0,1]} \Lambda_{it} di$ is the average marginal utility of consumption across all households. The term $\frac{W_t}{MPL_t}$ denotes the marginal cost, the nominal wage (W_t) divided by the marginal productivity of labor (MPL_t).

Profit maximization of the firm $j \in opt(t)$ requires the following first order conditions:

$$\frac{L_{jt}}{K_{jt}} = \frac{Q_t/P_t}{W_t/P_t} \frac{1-\alpha_t}{\alpha_t} \tag{7}$$

$$MPL_t = (1-\alpha_t) A_t K_{jt}^{\alpha_t} L_{jt}{}^{-\alpha_t} g^{t(1-\alpha_t)} \tag{8}$$

$$\frac{P_t^*}{P_t} = \frac{1}{\theta_Y} \frac{E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_Y)^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \left(\frac{\Pi_t}{\Pi_\tau}\right)^{\frac{1}{\theta_Y - 1}} \frac{W_\tau/P_\tau}{MPL_t} Y_\tau \right]}{E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_Y)^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \left(\frac{\Pi_t}{\Pi_\tau}\right)^{\frac{\theta_Y}{\theta_Y - 1}} Y_\tau \right]} \tag{9}$$

where Q_t is the rental price of capital and P_t^* is the common optimal price for all firms in $opt(t)$.

As in Yun (1996), an auxiliary price index $P_t^R = (\int P_{jt}^{\frac{1}{\theta_Y - 1}} dj)^{\theta_Y - 1}$ relates the aggregate output Y_t in (2) and the simple sum of the individual output $\int_{[0,1]} Y_j dj$:

$$Y_t = \left[\frac{P_t^R}{P_t}\right]^{\frac{1}{1-\theta_Y}} \int_{[0,1]} Y_j dj \tag{10}$$

The two price indices, P_t and P_t^R, evolve as

$$P_t^{\frac{\theta_Y}{\theta_Y - 1}} = \phi_Y (\Pi_{t-1} P_{t-1})^{\frac{\theta_Y}{\theta_Y - 1}} + (1 - \phi_Y) (P_t^*)^{\frac{\theta_Y}{\theta_Y - 1}} , \tag{11}$$

$$\left(P_t^R\right)^{\frac{1}{\theta_Y - 1}} = \phi_Y \left(\Pi_{t-1} P_{t-1}^R\right)^{\frac{1}{\theta_Y - 1}} + (1 - \phi_Y) (P_t^*)^{\frac{1}{\theta_Y - 1}} . \tag{12}$$

2.2 Households and Wage Setting

An individual household $i \in [0, 1]$ carries $M_{i,t-1}$ units of nominal money, $B_{i,t-1}$ units of government bond, and K_{it} units of physical capital from the previous period. In period t, the household i earns factor income $W_{it}L_{it} + Q_t K_{it}$ from renting capital K_{it} and labor service L_{it}, where W_{it} and Q_t denote the nominal wage rate and nominal rental rate for capital, respectively. The interest income from government bond holding is $(R_{t-1} - 1)B_{i,t-1}$ with R_{t-1} being the gross nominal interest rate between period t-1 and t, and the dividend income from firms is $\int s_{ij}\Gamma_{ijt} dj$ where s_{ij} and Γ_{ijt} are household i's fixed share of firm j and the profit of firm j, respectively. The household also receives a lump-sum nominal transfer payment T_{it} from the government.

The household i uses its funds to purchase the final good from the aggregator at the price of P_t, and divide its purchase into consumption C_{it} and investment I_{it}. In order to make new capital operational, the household needs to purchase additional materials in the amount

$$AC_{it}^k = \frac{\phi_K}{2} \left[\frac{I_{it}}{K_{it}} - \frac{\overline{I}}{\overline{K}}\right]^2 K_{it} \tag{13}$$

where $I_{it} = K_{i,t+1} - (1-\delta_t)K_{it}$ is the real investment spending, $\phi_K > 0$ is the scale parameter for the capital adjustment costs, and $\frac{\overline{I}}{\overline{K}}$ is the steady state ratio of investment to existing capital stock. Dubbed the *depreciation shock,* δ_t denotes the stochastic decay rate of capital stock. The household then carries M_{it} units of nominal money, B_{it} units of government bond, and $K_{i,t+1}$ units of capital into period $t+1$.

Therefore, the household i maximizes its lifetime utility

$$E_0[\sum_{t=0}^{\infty} \beta^t U(C_{it}, L_{it}, \frac{M_{it}}{P_t})], \quad 0 < \beta < 1 \tag{14}$$

subject to the budget constraint

$$\begin{aligned} & C_{it} + K_{i,t+1} - (1-\delta_t)K_{it} + \frac{M_{it}}{P_t} - \frac{M_{i,t-1}}{P_t} + \frac{B_{it}}{P_{it}} - \frac{B_{i,t-1}}{P_t} + AC_{it}^k \\ \leq \; & \frac{W_{it}L_{it}}{P_t} + \frac{Q_t K_{it}}{P_t} + T_{it} + \frac{\int s_{ij}\Gamma_{ijt}dj}{P_t} + \frac{(R_{t-1}-1)B_{i,t-1}}{P_t}, \quad t \geq 0 \end{aligned} \tag{15}$$

where the instantaneous utility function $U(\cdot)$ in (14) has the form

$$U(C_{it}, L_{it}, M_{it}/P_t) = \frac{\left[(C_{it}^{\nu} + b_t (M_{it}/P_t)^{\nu})^{\frac{a}{\nu}} (1 - h_t L_{it})^{1-a} \right]^{1-\sigma} - 1}{1-\sigma}, \quad 0 < a < 1, \; \nu < 0 \tag{16}$$

with b_t and h_t being the *money demand shock* and the *labor supply shock,* respectively.

As in the goods market, the individual demand for household i's labor service is

$$L_{it}^d = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t \quad , \quad \theta_L \in [0,1] \tag{17}$$

where the aggregate labor supply L_t and aggregate wage are defined as

$$L_t = (\int L_{it}^{\theta_L} di)^{\frac{1}{\theta_L}} \quad , \quad W_t = (\int W_{it}^{\frac{\theta_L}{\theta_L - 1}} di)^{\frac{\theta_L - 1}{\theta_L}}. \tag{18}$$

Nominal rigidities in the labor market is specified via adjustment costs: each household, when changing its nominal wage, pays the quadratic costs of the form

$$AC_{it}^w = \frac{\Phi_w}{2} \left(\frac{W_{it}}{W_{i,t-1}} - \Pi_{t-1}^w \right)^2 \frac{W_t}{P_t} \tag{19}$$

where $\Pi^w_{t-1} = W_{t-1}/W_{t-2}$ is the gross wage inflation rate at period $t-1$, and $\Phi_w > 0$ is the scale parameter for the degree of nominal rigidity in the labor market.[1]

Utility maximization requires the following first order conditions:

$$\frac{\partial U_{it}}{\partial C_{it}} = \Lambda_{it} \tag{20}$$

$$1 - {R_t}^{-1} = b_t (C_{it} P_t / M_{it})^{1-\nu} \tag{21}$$

$$\Lambda_{it}\left[1 + \frac{\partial AC^k_{it}}{\partial K_{i,t+1}}\right] = \beta E_t \left[\Lambda_{i,t+1}\left(Q_{t+1}/P_{t+1} + 1 - \delta_{t+1} - \frac{\partial AC^k_{i,t+1}}{\partial K_{i,t+1}}\right)\right] \tag{22}$$

$$\Lambda_{it} = \beta R_t E_t \left[\Lambda_{i,t+1} \frac{P_t}{P_{t+1}}\right] \tag{23}$$

$$\begin{aligned}
\left[\frac{W_{it}}{W_t}\right]^{\frac{1}{1-\theta_L}-1} MRS_{it} \quad = \quad & \theta_L \left[\frac{W_{it}}{W_t}\right]^{\frac{1}{1-\theta_L}} \frac{W_t}{P_t} + \frac{1}{L_t}(1-\theta_L)\phi_W \left[\frac{W_{it}}{W_{i,t-1}} - \Pi^w_{t-1}\right] \frac{W_t}{P_t} \\
& + \frac{\beta(1-\theta_L)\phi_W}{L_t} E_t \left[\frac{\Lambda_{i,t+1}}{\Lambda_{it}} \left(\frac{W_{i,t+1}}{W_{it}}\right)^2 \frac{W_{t+1}}{P_{t+1}} \frac{W_t}{W_{it}}\right] \\
& - \frac{\beta(1-\theta_L)\phi_W}{L_t} E_t \left[\frac{\Lambda_{i,t+1}}{\Lambda_{it}} \Pi^w_t \frac{W_{t+1}}{P_{t+1}} \frac{W_{i,t+1}}{W_{it}} \frac{W_t}{W_{it}}\right]
\end{aligned} \tag{24}$$

$$MRS_{it} = (1-a_t) C_{it}^{*a(1-\sigma)} (1 - L_{it})^{(1-a_t)(1-\sigma)-1} \Lambda_{it}^{-1} \tag{25}$$

where MRS_{it} is the household i's marginal rate of substitution between leisure and consumption.

2.4 Closing the Model

The government is assumed to maintain balanced budget every period by financing the total lump-sum payment to households with the seigniorage gain and issuance of net debt:

$$T_t = M_t - M_{t-1} + B_t - R_{t-1} B_{t-1} \tag{26}$$

where $T_t = \int_0^1 T_{it} di$, $M_t = \int_0^1 M_{it} di$, and $B_t = \int_0^1 B_{it} di$.

[1] Nominal rigidites in the labor market can alternatively be specified via Calvo-style wage contracts, under which the model has the same first order properties. In that case, however, the way labor service enters $U(\cdot)$ in (16) prevents the equlibrium conditions from being cast into what is suitable for the second order solution method used later for welfare calculations. Therefore, the welfare imlications in the present work are to be interpreted on the caveat that the inefficient distribution of work hours across households is ignored.

In the benchmark economy to be estimated, monetary policy is specified as a generalized feedback rule of Taylor (1993)[2]

$$\log\frac{R_t}{\overline{R}} = \rho_R \log\frac{R_{t-1}}{\overline{R}} + (1-\rho_R)\left[\gamma_\pi \log\frac{\Pi_t}{\overline{\Pi}} + \gamma_{y_1}\log\frac{Y_t}{\overline{Y}_t} + \gamma_{y_2}\log\frac{Y_{t-1}}{\overline{Y}_{t-1}} + \gamma_m \log\frac{MG_t}{\overline{MG}}\right] + \varepsilon_{Rt},\ 0<\rho_R<1 \tag{27}$$

where $\overline{R}$ is the gross nominal interest rate, $\overline{MG}$ is the growth rate of nominal money, $\overline{R}$ is the steady state gross nominal interest rate, all in the steady state. Π_t and MG_t are the rates of gross inflation and money growth, respectively, between period $t-1$ and t, and $\overline{Y}_t$ is the deterministic level of output at period t. $\overline{\Pi}$ is the long-run level of inflation rate the monetary authority maintains. The monetary policy disturbance ε_{Mt} is a white noise with mean 0 and variance σ_ε^2 and independent of all other disturbances in the model.

Beside the monetary policy disturbance ε_{Rt}, the model is driven by five structural shocks $(A_t, \alpha_t, \delta_t, b_t, h_t)$, each of which follows a stationary AR(1) in logarithmic form

$$\log\frac{\chi_t}{\overline{\chi}} = \rho_1 \log\frac{\chi_{t-1}}{\overline{\chi}} + \varepsilon_{\chi t} \tag{28}$$

where $\overline{\chi}$ is the steady state level of χ_t, and $\varepsilon_{\chi t}$ is a white noise with mean 0 and variance σ_χ^2. The six innovations are uncorrelated with one another, except that those in A_t and α_t are correlated because they appear jointly in the production function.

In what follows, I focus on a particular *symmetric* equilibrium in which i) all firms in $opt(t)$ set the same optimal price ; and ii) all households make identical decisions on (C, K, M, B, W). In an equilibrium, most of the model's real and nominal variables inherit deterministic trends due to the constant rate of labor-augmenting technical progress (g) and the long-run rate of inflation the monetary authority maintains. When required, I deflate such trend variables by their respective deterministic growth rates, and use lowercase letters for the resulting stationary-transformed

[2]The specification of monetary policy rule in (27) is partly motivated by Levine et al. (1999), who found fairly aggressive responses of nominal rates toward real output in both levels and growth rates over the period 1980:1-1996:4.

variables. For example, the output, price, and wage rate are transformed as:

$$y_t = Y_t/g^t,\ p_t = P_t/\overline{\Pi}^t,\ w_t = W_t/\left(\overline{\Pi}^{\omega}\right)^t\ .$$

Equations describing this stationary-transformed symmetric equilibrium of the model are given in the appendix.

2.5 Estimation

The raw data used for estimation, summarized in Table 1, are extracted from DRI BASIC economic series for the sample period 1959:Q1-1999:Q3. The following six series are used for the actual estimation purpose: per capita output (Y), per capita labor hours (L), rate of price inflation (Π), the growth rate of per capita money balance (MG), interest rates (R), and wage inflation rates (Π^w). To express the data series conformable to their model counterparts, output and money growth are suitably transformed via population size. Per capita labor hours are obtained by dividing weekly working hours by 120, under the assumption that each worker is endowed with 5 working days per week. The resulting series imply households devote 33.8% of their time endowment to working. Since federal funds rates are measured in annual percentage rates, I transform them into quarterly rates by dividing by 400 and adding one. Price and wage inflations are obtained by log-differencing the price and wage series.

A log-linear approximate solution of the model can be obtained using the method by Sims (2002), and the resulting solution takes the form

$$d\log z_t = g_1 d\log z_{t-1} + g_2 \varepsilon_t \tag{29}$$

where g_1 and g_2 are complicated matrix functions of the model parameters, and $d\log z_t$ is the log-deviation of the system variables from steady state. With a selection matrix H that singles out the observables from the state vector z_t, I have the following state-space representation:

$$\boxed{\begin{array}{ll} \text{transition equation :} & d\log z_t = g_1 d\log z_{t-1} + g_2\varepsilon_t, \quad \varepsilon_t \backsim iiN(0,\Sigma_\varepsilon) \\ \text{observation equation :} & d\log\omega_t = H d\log z_t \end{array}} \tag{30}$$

where ω_t denotes the variables corresponding to the observable data series and Σ_ε is the covariance matrix of the innovations ε_t. From (30), it is straightforward to construct a Gaussian likelihood function for the entire parameter vector θ:

$$\begin{aligned} L_T(\theta \quad | \quad z_1, ..., z_T) = & -\frac{1}{2}\sum_{t=1}^{T} \log |{}_{t-1}\Sigma_t^z(\theta)| \\ & -\frac{1}{2}\sum_{t=1}^{T} [d\log z_t -{}_{t-1} d\log z_t(\theta)]' [{}_{t-1}\Sigma_t^z(\theta)]^{-1} [d\log z_t -{}_{t-1} d\log z_t(\theta)] \end{aligned} \tag{31}$$

where ${}_{t-1}d\log z_t(\theta)$ and ${}_{t-1}\Sigma_t^z\ (\theta)$ are one-step ahead forecasts of mean and variance of $d\log z_t$, respectively, and T is the sample size. For $t = 1$, the unconditional mean and variance of $d\log z_t$ implied by the first order solution (29) are used for ${}_{t-1}d\log z_t(\theta)$ and ${}_{t-1}\Sigma_t^z(\theta)$, respectively.

When the likelihood function (31) is maximized, the estimates tend to move toward the region where the model-determined standard deviations of output and labor supply are higher their sample counterparts roughly by the factor of 2. To get more realistic estimates, therefore, I use a mixed estimation strategy to obtain an estimate of θ as in:

$$\widehat{\theta} = \max_{\{\theta\}} \left\{ L_T(\theta \mid z_1, ..., z_T) + \log f(\frac{1}{\kappa}\frac{SD_m(\theta) - SD_d}{SDd}) \right\}, \quad \kappa > 0 \tag{32}$$

where $SD_m(\theta)$ and SD_d are the model-determined and empirical standard deviations, respectively, of the observables, and $f(\cdot)$ is the density function of the standard normal variate. Intuitively, the quasi-likelihood function in (32) has the effect of pulling the estimator toward what makes $SD_m(\theta)$ and SD_d are comparable, and the degree of such prior belief about θ is controlled by κ. I treat the resulting quasi-Bayesian estimate of θ as asymptotically normally distributed with variance matrix determined by the numerically computed Hessian at the optimum.

Some structural parameters are fixed before estimation: steady state values of capital share $\overline{\alpha}$ and depreciation $\overline{\delta}$ are fixed at 1/3 and 0.025, respectively. The market power θ_Y in the goods market is fixed at the conventionally calibrated value of 0.9, because only two of $(\overline{A}, \theta_Y, \theta_L)$ are identified from the series on output and labor. Assuming the Fed has been successfully managed the inflation rate around its "long-run target" level, I fix the steady state inflation rate $\overline{\Pi}$ at its actual average 1.01005 over the sample period. The CRRA parameter σ is fixed at 1, which amounts to the logarithmic instantaneous utility function.

Table 2 summarizes the functional forms of the model equations and the estimates of parameters for $\kappa = 0.1$, with standard errors in parentheses. The estimate of growth rate g is 1.0049, which is very close to the actual average growth rate 1.0052 over the sample period. The estimate of discount factor β is 0.9984, falling between the estimate 0.9974 for post-79 era in Ireland (2001) and 0.9999 in Kim (2000) for 59:Q1- 95:Q1, although higher than the usually calibrated value of 0.99. The share a of consumption bundle C_t^* in the instantaneous utility function is 0.4069, close to the conventionally calibrated value of 0.4. The estimates of (ν, b) are (-22.8590, 0.0008). These estimates imply an interest-semi-elasticity of money demand is about 0.04, which is well below the usual empirical estimates.[3] In terms of substitutability, money is held as a complement to consumption to facilitate transactions.[4]

The estimate of θ_L of households' market power in the labor market is 0.8849, higher than the calibrated value 0.75 in Huang and Liu (1999). The capital adjustment cost parameter ϕ_K =14.2090 shows a sizable degree of real rigidity in the economy. When the economy is initially at the estimated steady state, transforming one unit of consumption good into the same unit of operational capital involves about 0.06 units of output additionally as adjustment costs.

The parameters for the monetary policy rule, used as the benchmark, show the systematic

[3] This seems to arise because the model makes the demand for money adjust instantaneously, whereas empirical work usually allows lags, or uses longer frequency data.

[4] In the estimated steady state, the indifference curves on the (C, RM) plain are highly convex to origin: one percent increase in $\frac{C}{M/P}$ ratio results in 23.7561 percent decrease in the marginal rate of substitutions between C and RM.

evolution of nominal interest rates in response to inflation and money growth, and real output growth over the sample period. The estimate $\rho_M = 0.2767$ implies a modest degree of policy inertia.

The estimated AR(1) coefficients for the structural disturbances show the economy has been subject to highly persistent shocks. Except for the labor supply shock, the half-lives of the aggregate shocks are around 6 years. The labor supply shock a_t exhibits a negligible degree of positive serial correlations. Finally, the innovations in the shocks A_t and α_t are negatively correlated with correlation coefficient of -0.9231.

Regarding the nominal rigidity parameters, the estimate of $\phi_Y = 0.6562$ implies that, every firm has the opportunity to reoptimize on their price every 1/(1-0.6989)=2.91 quarters on average. Resorting to the first order equivalence between the adjustment costs and Calvo staggering as in Rotemberg (1982), the estimate of $\phi_W = 19.0440$ is translated into the average wage fixity of 1/(1-0.6989)=3.32 quarters.[5]

3. Welfare Metrics

The following analysis uses a representative household's lifetime utility as a natural welfare metric to compare alternative nominal anchors against. Conditional on some initial state Ω_0, the expectation of a representative household's lifetime utility can be represented as

$$\begin{aligned} EW &= E\left[\sum\nolimits_{t=0}^{\infty} \beta_*^t U_t \mid \Omega_0\right] \\ &\simeq \frac{1}{1-\beta_*} U(\overline{\zeta}) + \left[\frac{dU_t(\overline{\zeta})}{d\zeta_t} \otimes \overline{\zeta}\right]' \sum\nolimits_{t=0}^{\infty} \left(\beta_*^t E\left[d\log\zeta_t \mid \Omega_0\right]\right) \\ &\quad + \frac{1}{2} tr \sum\nolimits_{t=0}^{\infty} \left(\beta_*^t Var\left[d\log\zeta_t \mid \Omega_0\right] \left[\frac{d^2U_t(\overline{\zeta})}{d\zeta_t^2} \otimes (\overline{\zeta\zeta'})\right]\right) \end{aligned} \tag{33}$$

[5] Suppose that, as in the goods market, a randomly chosen $1-\phi_L$ fraction of households are able to reoptimize their individual nominal wages. Then the first order functional equivalence gives the following relation:

$$\Phi_w^{-1} = \frac{1-\phi_L}{\phi_L}(1-\beta\phi_L g^{a(1-\sigma)})\frac{1-\theta_L}{\theta_L}\overline{\Pi}^{w2}\frac{1}{XL},\ X = 1 + \frac{1}{1-\theta_L}\frac{L}{1-L}.$$

where $\beta_* = \beta g^{a(1-\sigma)}$, $\zeta_t = (c_t, m_t/p_t, L_{t,} b_t, h_t)$, $tr(\cdot)$ is the trace of a square matrix, and the symbol $\otimes$ denotes the element-by-element multiplication.

As discussed in Sims (2000) and Kim et al. (2003), correct welfare evaluations in terms of utility levels require higher order accuracy of the model solutions beyond those by the conventional log-linear methods. I use the second order approximate solution method developed and extended by those authors in order to compute the paths of the first and second moments of $d\log z_t$ conditional on an state $\Omega_0 : \{E[d\log z_t \mid \Omega_0], Var[d\log z_t \mid \Omega_0], t = 1, 2, ...\}$. In the following analysis, the initial state Ω_0 comprises the unconditional first and second moments of $d\log z_t$, calculated by the same solution method given the estimated monetary policy rule (27). The details involved are in the appendix.

For interpretational convenience, the performances of alternative nominal anchors are also measured by *consumption compensations.* More specifically, suppose that the (discounted) utility in the estimated deterministic steady state with $\overline{\Pi} = 1.01005$is given by $\overline{EW}_0 = \frac{1}{1-\beta_*}U(\overline{c}_0, \overline{L}_0, \frac{\overline{m}_0}{\overline{p}_0})$. Then, if the monetary authority implements an alternative policy targeting a nominal anchor with the resulting welfare level EW_1, the amount of consumption dc_1 to be compensated every period to yield the same level of welfare as $\overline{EW}_0$ is determined by

$$\overline{EW}_0 = EW_1 + \frac{1}{1-\beta_*}\overline{\lambda}_1 dc_1 \tag{34}$$

where $\overline{\lambda}_1$ is the steady state marginal utility of consumption under the alternative monetary policy.

4. Alternative Targeting Strategies

4.1 Alternative Policy Rules[6] If the monetary authority sets nominal interest rate to keep inflation rate at fixed levels, the corresponding path of nominal interest rate can be found from the

[6] Note that targeting rules for inflation and nominal incoe growth considered in this section are not policy *recipes* in a strict sense. Instead of giving a functional form for a policy instrument to follow, they describes how the instrument evolves with other variables on the RHS if such anchors are *somehow* kept constant.

money demand function (21). Noting that $\frac{M_t/P_t}{M_{t-1}/P_{t-1}} = MG_t/\Pi_t$, strict inflation targeting requires

$$1 - R_t^{-1} = \left[1 - R_{t-1}^{-1}\right] \left[\frac{b_t}{b_{t-1}}\right] \left[\frac{c_t}{c_{t-1}} MG_t^{-1} \overline{\Pi}\right]^{1-\nu} \tag{35}$$

Therefore, the strict inflation targeting requires the change in interest rate be positively related with consumption growth, and negatively related with nominal money growth. This rule will be labelled **PHIT** for future references.

Since the rate of nominal income growth NIG_t is given by $NIG_t = \frac{P_t Y_t}{P_{t-1} Y_{t-1}} = g \frac{\Pi_t y_t}{y_{t-1}}$, fixed nominal income growth targeting requires

$$1 - R_t^{-1} = \left[1 - R_{t-1}^{-1}\right] \left[\frac{b_t}{b_{t-1}}\right] \left[\frac{c_t}{c_{t-1}} \frac{y_{t-1}}{g y_t} MG_t^{-1} \overline{NIG}\right]^{1-\nu} \tag{36}$$

where $\overline{NIG}$ is the rate of nominal income growth in a deterministic steady state. This rule is labelled as **NIGT** for future reference.

Finally, under strict money growth targeting, money growth rate is held constant:

$$\frac{MG_t}{MG_{t-1}} = \overline{MG} \tag{37}$$

which will be labelled as **MGT** for future reference.

4.2 Welfare Comparison Table 3 compares the alternative targeting strategies according to their effects on welfare, measured by EW and dc. This welfare comparison uses the benchmark estimates of parameters in Table 1 and $\overline{\Pi} = 1.01005$. The table's second column shows that **PHIT** yields the highest level of expected utility. **NIGT** comes in second, and **MGT** finishes last. In terms of consumption compensations in the third column, adopting **PHIT** instead of **NIGT** results in a welfare gains equivalent to 0.013 units of consumption every period. Adopting **MGT** instead of **PHIT** costs 0.0288 units of consumption per period.

Even if the parametrization in Table 1 gives a true description of the economy, announcing the welfare dominance of **PHIT** over the other targeting rules ignores one important issue around implementing a monetary policy rule: the occurrence of negative nominal interest rates. Negative nominal rates, although highly implausible in practice, are almost never excluded in policy rules analyses and this study is not an exception. The final column of Table 3 reports the ratio of the unconditional mean of the nominal interest rate to its standard deviation under alternative targeting rules. The ratio 1.9313 for **PHIT** implies that, if nominal interest rate were normally distributed, strict inflation targeting gives about 2.68% possibility of violating the zero bound on nominal interest rate each quarter. In a quarterly model like the present one, such ratio also implies that nominal rate falls below zero about once every $\frac{1}{4\times 0.0268} = 9.3284$ years, which is uncomfortably often. On the other hand, **NIGT** and **MGT** would not yield zero nominal rates, under which the mean to standard deviation ratios are greater than 10. Therefore, the dominance of **PHIT** is spurious in the sense that strict inflation targeting is not feasible at the rate of inflation of 1.01005.

One way to address the issue of zero bound is to impose a restriction that, for a reasonable $k > 0$, the k-standard deviation confidence interval of the nominal interest rate around its unconditional expectation should not contain zero (or equivalently, the mean-standard deviation ratio should be greater than k).[7]Once the level of k is determined, a natural way to evaluate alternative anchors are to compare their best welfare performances (achieved by varying the long-run inflation rate) not violating the zero bound constraint implied by such k. Intuitively, the higher k is, the more concerned the monetary authority is about the occurrence of zero nominal interest rate. Unless explicitly noted, the level of k in the following analysis is 2.0468, which is equal to the empirical mean-standard deviation ratio from the sample used for estimation.

With the consideration of the zero bound restriction, the welfare performances of the three nominal anchors are quite different from those without. Table 4 reports the best welfare performance of each nominal anchor, together with the corresponding optimal long-run inflation rate. When

[7]This kind of constraint on nominal interest rate variabiliy is frequently employed in the monetary policy literature, e.g., Rotemberg and Woodford (1999) and Williams (2003).

$\overline{\Pi} = 1.0111$, for which **PHIT** is barely feasible, **PHIT** still outperforms both **NIGT** and **MGT**. For example, adopting **PHIT** instead of **MGT** is tantamount to having 0.0287 units of additional consumption every quarter. However, when $\overline{\Pi}$ is lowered to 0.9967, for which now **NIGT** is barely feasible, the expected utility of **NIGT** and **MGT** increase to 411.9511 and 411.6910, respectively. In terms of consumption compensations, these increases in welfare from lowering the long-run inflation from $\overline{\Pi} = 1.0111$ amount to 0.052 and 0.0575 units of consumption per quarter. When when $\overline{\Pi}$ is further lowered to 0.9949, for which only **MG**T is barely feasible, **MGT** yield further increases in welfare equal to 0.0093 units of consumption per quarter. It is worth noting in Table 4 that the performance of **MGT** for $\overline{\Pi} = 0.9949$ is worse than that of **NIGT** for $\overline{\Pi} = 0.9967$, suggesting **NIGT** is a good firsthand compromise between welfare gains from short-run stabilization of inflation and those from lower long-run inflation,although the validity of this interpretation should be formally examined in view of the uncertainties of the estimated parameters in Table 1.

Figure 1 plots the welfare levels (measured by dc) under alternative targeting rules, calculated for various long-run inflation rates with the mean-to-standard deviation ratio greater than 2. In each panel, the left endpoint corresponds to the long-run inflation rate that yield with the mean-to-standard deviation ratio of 2. As can readily be seen, all three targeting rules show steady increase in welfare (or decrease in dc) as the long-run inflation rate is lowered. However, **PHIT** is likely to violate the zero bound even for quite high long-run inflation rate, while the other two rules can yield further increase in welfare by lowering long-run inflation rate. For example, when the long-run inflation rate is 6% per annum, the corresponding mean-to-standard deviation ratio of 2.5014 under **PHIT** implies the violation of the zero bound every 40.3 years, while **MGT** with long-run deflation of $\overline{\Pi}$ =0.9957 (or 1.7% per annum) has the same frequency of zero nominal rates.

The general picture emerging from the analysis so far is that *i)* strict inflation targeting is compatible and desirable only when the long-run inflation rate is high, while targeting of the other two anchors (especially, money growth) can be implemented with much lower rate of long-run inflation, and *ii)* given any feasible nominal anchor, keeping the long-run inflation low yields higher welfare levels. It is worthwhile to examine which features of the estimated model contribute to these welfare implications. One conspicuous feature of the model is the presence of nominal rigidities in the goods and labor markets, and the interaction of the two nominal rigidities yields a considerable degree of inertia in the price inflation. For example, after a monetary policy shock in (27), the half-life of the responses in price inflation is 5.93 quarters, while those of money growth and nominal income growth are less than one quarter. Since the monetary policy effects on inflation are smooth and delayed, strict inflation targeting necessarily requires excessive and too frequent adjustments in the nominal interest rate, in an attempt to keep so slowly responding an anchor on track every instance. As a result, the zero bound restriction is highly likely to be binding for **PHIT** when the long-run inflation rate is not far from zero.

Another feature of the model is the inclusion of nominal money balance in the utility function. Recall the dictum of Friedman (1969) that the optimal long-run inflation policy is that which makes the private cost of holding money (i.e, the nominal interest rate) equal to the social cost (i.e., zero). By lowering the long-run inflation rate and therefore the long-run nominal rate as well, subject to the zero bound, the monetary authority can achieve higher steady state welfare captured by the first term in (33).

One way to measure the gains from lower inflation is to compute consumption compensations, again relative to $\overline{EW}_0$. More specifically, if the may be the decrease in the steady-state transaction costs as the long-run inflation rate is lowered from the benchmark level of $\overline{\Pi} = 1.01005$. More specifically, if the discounted steady state welfare for a long-run inflation rate $\overline{\Pi}_1$ is given by $\overline{EW}_1$, I compute the consumption compensation dc_1determined analogously to (34). The results of this calculation are illustrated in Figure 2, which shows that a movement from $\overline{\Pi} = 1.01005$ to $\overline{\Pi} = 1$

(i.e., zero inflation) would lead to 0.0180 units of consumption every quarter. Adopting Friedman's rule with $\overline{\Pi} = 0.9934$ instead would be tantamount to 0.0307 units of additional quarterly consumptions.

A question that arises now is : of the two welfare gains, i.e., the steady state efficiency gains from keeping lower long-run inflation rate and the welfare gains from being able to targeting inflation in the short run, which is the dominant factor? Figure1 suggests that the former does, as frequently observed in the literature since Lucas (1987), e.g., Otrok (2001) and Kiley (2003): when the long-run inflation rate falls below 2% per annum, pursuing **MGT** or **NIGT** yield higher welfare levels than under **PHIT**.

5. Conclusion

This paper compares the welfare implications of strictly targeting three alternative nominal anchors, price inflation, money growth, and nominal income growth, in the context of an estimated full-pledged monetary business cycle model. The findings show that **PHIT** outperforms the others only when the monetary authority should maintain high long-run inflation rates, and that for low long-run inflation rates **MGT** and **NIGT** yield comparably higher welfare levels than **PHIT**.

It must be admitted that, however, the analysis in this paper resorts to a number of simplifying assumptions, made both explicitly and implicitly. In terms of the model structure, the price and wage adjustment scheme, degree of nominal rigidities in markets, and the way nominal money enters the present model are particularly critical features to which the sensitivity of welfare ranking of alternative anchors should be checked. In particular, the instability of money demand coupled with the advent of the evermore increasing interest bearing monetary assets is an important issue to be addressed before taking the results in this paper as warranted. One suggestion by Lucas (2000) of applying Divisia monetary index is a promising way for further research to resolve this difficulty.

Another possible direction of extension is to introduce gradualism in the behavior of the monetary authority. In fact, nearly the entire literature on inflation targeting emphasizes that policy should attempt to bring inflation to the target level only gradually. Extending the formulations of the targeting rules appropriately and finding the optimal degrees of inertia would be interesting topics for further research.

Finally, the present analysis assumes no lag structure that would make difficult hitting targets precisely every period. Allowing lags between the realizations of the target variables and their measurements, and even longer lags between changes in monetary policy and their effects on realized targets policy instrument should be included in the list of further extensions of research.

6. Appendix

6.1 Stationary Transform of the System Three different transform schemes are used to make the system stationary in a symmetric equilibrium. First, all occurrences of deflated nominal variables $(M_t/P_t, Q_t/P_t, W_t/P_t)$ are re-defined as real variables:

$$RM_t = M_t/P_t,\ RQ_t = Q_t/P_t,\quad RW_t = W_t/P_t \quad .$$

Second, real variables $(Y_t, C_t, K_t, \Lambda_t, MRS_t, RM_t, RQ_t, RW_t)$ are transformed using respective deterministic trend growth rates. For example:

$$y_t = Y_t/g^t,\ c_t = C_t/g^t,\ k_t = K_t/g^t,\quad \lambda_t = \Lambda_t/g^{[-1+a(1-\sigma)]t},\ rm_t = RM_t/g^t \quad .$$

Finally, occurrences of $(P_t/P_{t-1}, W_t/W_{t-1}, M_t/M_{t-1})$ are replaced by growth rates:

$$\Pi_t = P_t/P_{t-1},\quad \Pi_t^w = W_t/W_{t-1},\quad MG_t = M_t/M_{t-1} \quad .$$

For notational simplicity, I define

$$x_t = \left[\frac{gk_t - (1-\delta_{t-1})k_{t-1}}{k_{t-1}} - \delta_g\right],\quad \delta_g = g - 1 + \delta,\ \beta_g = \beta g^{a(1-\sigma)-1} \ .$$

6.1.1 Household Block The stationary-transformed version of the households' block of the system is given below.

$$\lambda_t = a[c_t^{\nu} + b_t (rm_t)^{\nu}]^{\frac{a-a\sigma-\nu}{\nu}} (1-L_t)^{(1-a_t)(1-\sigma)} c_t^{\nu-1} \tag{A1}$$

$$1 - 1/R_t = b_t (c_t/rm_t)^{1-\nu} \tag{A2}$$

$$\lambda_t [1 + \phi_K x_{t+1}] = \beta_g \mathcal{Z}_{1t} \tag{A3}$$

$$\mathcal{Z}_{1,t-1} = \lambda_t \left[1 - \delta_t + rq_t + \phi_K \frac{gk_{t+1}}{k_t} x_{t+1} - \frac{\phi_K}{2} x_{t+1}^2 \right] + \eta_{1t} \tag{A4}$$

$$\lambda_t = \beta_g R_t \mathcal{Z}_{2t} \tag{A5}$$

$$\mathcal{Z}_{2,t-1} = \lambda_t \Pi_t^{-1} + \eta_{2t} \tag{A6}$$

$$\begin{aligned} mrs_t &= \theta_L rw_t + (1-\theta_L)\Phi_w \left[\Pi_t^w - \Pi_{t-1}^w\right] rw_t \Pi_t^w L_t^{-1} \\ &-\beta g^{a(1-\sigma)} \Phi_w (1-\theta_L) \mathcal{Z}_{3t} + \beta g^{a(1-\sigma)} \Phi_w (1-\theta_L) \mathcal{Z}_{4t} \end{aligned} \tag{A7}$$

$$mrs_t = \lambda_t^{-1} c_t^{*a(1-\sigma)} (1-a_t)(1-L_t)^{(1-a)(1-\sigma)-1} \tag{A8}$$

$$\mathcal{Z}_{3,t-1} = \frac{\lambda_t}{\lambda_{t-1}} [\Pi_t^w]^2 rw_t L_{t-1}^{-1} + \eta_{3t} \tag{A9}$$

$$\mathcal{Z}_{4,t-1} = \frac{\lambda_t}{\lambda_{t-1}} \Pi_t^w \Pi_{t-1}^w rw_t L_{t-1}^{-1} + \eta_{4t} \tag{A10}$$

where η_t's are the martingale difference expectational errors.

6.1.2 Firms Block For the notational convenience, define $\psi_t = p_t^*/p_t$, $\varphi_t = p_t^R/p_t$, and $\beta_Y = \beta g^{a(1-\sigma)} \phi_Y$. Then the equations for the decision problems of firms are given by

$$\frac{L_t}{k_t} = \frac{rq_t}{rw_t} \frac{1-\alpha_t}{\alpha_t} \tag{A11}$$

$$\psi_t \Pi_t = \frac{1}{\theta_Y} \frac{\mathcal{Z}_{5,t}}{\mathcal{Z}_{6,t}} \tag{A12}$$

$$mpl_t = A_t (1-\alpha_t) k_t^{\alpha_t} L_t^{-\alpha_t} \tag{A13}$$

$$\mathcal{Z}_{5,t-1} - \beta_Y \mathcal{Z}_{5t} = \lambda_{t-1} y_{t-1} \Pi_{t-1}^{\frac{1}{1-\theta_Y}} \frac{rw_{t-1}}{mpl_{t-1}} + \eta_{5t} \tag{A14}$$

$$\mathcal{Z}_{6,t-1} - \beta_Y \mathcal{Z}_{6t} = \Pi_{t-1}^{\frac{\theta_Y}{1-\theta_Y}} \lambda_{t-1} y_{t-1} + \eta_{6t} \tag{A15}$$

where η_t's are again the martingale difference expectational errors.

6.1.3 Other Equations Total output in the economy is determined by

$$y_t = A_t k_t^{\alpha_t} L_t^{1-\alpha_t} \times \varphi_t^{\frac{1}{1-\theta_Y}} \tag{A16}$$

Combining the budget constraint of households, aggregate profit of firms, and the government budget constraint, we get the resource constraint :

$$c_t + g k_{t+1} - (1-\delta_t) k_t + \frac{\phi_K}{2} \left[\frac{g k_{t+1} - (1-\delta_t) k_t}{k_t} - \delta_g \right]^2 k_t = y_t. \tag{A17}$$

The aggregate real wage and real money stock evolve following

$$g \frac{rw_t}{rw_{t-1}} = \frac{\Pi_t^{\omega}}{\Pi_t}, \quad g \frac{rm_t}{rm_{t-1}} = \frac{MG_t}{\Pi_t} \ . \tag{A18}$$

For the benchmark economy, the monetary policy rule is transformed into

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1-\rho_R) \left[\gamma_\pi \widehat{\Pi}_t + \gamma_{y1} \widehat{y}_t + \gamma_{y2} \widehat{y}_{t-1} + \gamma_m \widehat{MG}_t \right] + \varepsilon_{Mt} \ . \tag{A19}$$

The evolutions of the two price indices p_t and p_t^R are combined into

$$\varphi_t^{\frac{1}{1-\theta_Y}} = \phi_Y \left(\varphi_{t-1}\right)^{\frac{1}{\theta_Y - 1}} \left(\frac{\Pi_t}{\Pi_{t-1}} \right)^{\frac{1}{1-\theta_Y}} + (1-\phi_Y) \left(\psi_t\right)^{\frac{1}{\theta_Y - 1}} . \tag{A20}$$

The evolution of the exogenous shocks, omitted here, do not need stationary-transformations.

6.2 Construction of *EW* The equilibrium system described above can be cast into

$$\begin{aligned} G_1(z_t, z_{t-1}, \varepsilon_t) &= 0_{N_1 \times 1} \\ G_2(z_t, z_{t-1}, \varepsilon_t) + \eta_t &= 0_{N_2 \times 1} \end{aligned} \tag{A21}$$

where ε_t is the vector of period t innovations in the exogenous shocks, and η_t is a vector of endogenous errors satisfying $E_{t-1}\eta_t = 0$. The N-dimensional system vector z_t is correspondingly decomposed as $z_t = (z'_{1t}, z'_{2t})'$, where z_{2t} denotes the N_2-dimensional auxiliary variables defined for conditional expectation terms in equations (A4), (A6), (A9) , (A10), (A14) and (A15), and z_{1t} is the $(N - N_2)$ dimensional vector of all other variables (including exogenous disturbances and endogenous state variables). Under a set of regularity conditions, a unique and stationary second order accurate solution to (24) is given by

$$\begin{aligned} \widehat{z}_{1it} &= F_{1ij}\widehat{z}_{1j,t-1} + F_{2ij}\varepsilon_{jt} + F_{3i} && \text{(A22a)} \\ &\quad +0.5\left(F_{11ijk}\widehat{z}_{1j,t-1}\widehat{z}_{1k,t-1} + 2F_{12ijk}\widehat{z}_{1j,t-1}\varepsilon_{kt} + F_{22ijk}\varepsilon_{jt}\varepsilon_{kt}\right), \\ \widehat{z}_{2it} &= S_i\widehat{z}_{1it} + T_iM_{11ijk}\widehat{z}_{1jt}\widehat{z}_{1kt} + T_iM_{2i}\ . && \text{(A22b)} \end{aligned}$$

where $\widehat{z}_t = \log z_t - \log \overline{z}$ denotes the % deviation of z_t from its deterministic steady state, and S, T, F's, and M's are matrix functions of the deep parameter of the model.[8] In particular, the terms F_3 and M_2 represent the degree of certainty non-equivalence. Note that equations in (A22) utilize the tensor notation for the simplicity of exposition. For example, the term $F_{11ijk}\widehat{z}_{1j,t-1}\widehat{z}_{1k,t-1}$ can be interpreted as the quadratic form in terms of lagged $\widehat{z}_t$ for the i^{th} equation, constructed by the lag of $\widehat{z}_{1t}$.

By using equation (A22a) recursively, I can compute $\{\mu_{1t}, \Sigma_{1t} : t \geq 0\}$, the conditional first and second moments of $\widehat{z}_{1t}$, from which the welfare measure EW is constructed. For the sake of second order accuracy, all terms of orders higher than two may be dropped out: accordingly, only the first two terms in equation (A22a) describe the evolution of the conditional variances of $\widehat{z}_{1t}$:

$$\Sigma_{1t} = F_1\Sigma_{1,t-1}F'_1 + F_2\Sigma_\varepsilon F'_2 \qquad \text{(A23)}$$

[8] The original codes of Sims (2000) give a slightly different (but essentially equivalent) solution, in which the solutions for the "jump" variables z_{2t} are indirectly given by linear combinations of the whole system vector in log-deviations. The routine for transforming the original solutions into those in (A21) is available upon request.

where F_1 and F_2 are the matrices of the coefficients on $\widehat{z}_{1,t-1}$ and ε_t, respectively, representing the first order parts of the solution.

Recursive calculations of μ_{1t} are more involved. The subsystem (A22a) may be re-written in an expanded form as

$$\begin{aligned}\widehat{z}_{1t} &= F_1\widehat{z}_{1,t-1} + F_2\varepsilon_t + F_3 \\ &+\frac{1}{2}\begin{bmatrix} \widehat{z}'_{1,t-1}F_{11}^{(1)}\widehat{z}'_{1,t-1} \\ \vdots \\ \widehat{z}'_{1,t-1}F_{11}^{(N_1)}\widehat{z}'_{1,t-1} \end{bmatrix} + \begin{bmatrix} \widehat{z}'_{1,t-1}F_{12}^{(1)}\varepsilon_t \\ \vdots \\ \widehat{z}'_{1,t-1}F_{12}^{(N_1)}\varepsilon_t \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \varepsilon_t F_{22}^{(1)}\varepsilon_t \\ \vdots \\ \varepsilon_t F_{22}^{(N_1)}\varepsilon_t \end{bmatrix}\end{aligned} \tag{A24}$$

where F_3 is a $N_1 \times 1$ column vector, and $F_{jk}^{(i)\prime}$s are the matrices constructing quadratic terms for the i^{th} equation in the second order solution (A21).

Taking expectation of (A24) conditional on Ω_0, I get

$$\begin{aligned}\mu_{1t} &= F_1\mu_{1,t-1} + F_3 \\ &+\frac{1}{2}\begin{bmatrix} tr\left(\Sigma_{1,t-1}F_{11}^{(1)}\right) \\ \vdots \\ tr\left(\Sigma_{1,t-1}F_{11}^{(N_1)}\right) \end{bmatrix} + \frac{1}{2}\begin{bmatrix} tr\left(\Sigma_\varepsilon F_{22}^{(1)}\right) \\ \vdots \\ tr\left(\Sigma_\varepsilon F_{22}^{(N_1)}\right) \end{bmatrix}\end{aligned} \tag{A25}$$

where $tr(\cdot)$ is the trace of a square matrix. One can calculate $\{\mu_{1t}, \Sigma_{1t} : t \geq 1\}$ recursively by using (A23) and (A25) jointly given some initial condition $\Omega_0 = (\mu_{10}, \Sigma_{10})$.

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Table 1: Raw Data Series

output :	gross domestic products, billions of 1992 dollars.
employment :	average weekly hours of production workers in manufacturing sector.
price :	implicit price deflator for gross national products.
money :	M2 stock, billions of current dollars.
interest rate :	federal funds rate, per annum.
wage :	index of compensation per hour in nonfarm business sector, 1982=100.
population	civilian population, in thousands.

Note : All series, except for interest rate and wage, are seasonally adjusted.

Table 2: Functions and Parameter Estimates ($\overline{\Pi} = 1.01005$)

$Y_t = A_t K_t^{\alpha_t} (g^t L_t)^{1-\alpha_t}$	A=5.5452 (0.0208), g=1.005(0.0010)
$\beta^t U(C_t, L_t, \frac{M_t}{P_t}) = \beta^t \log \left[C_t^{*a} (1 - L_t)^{1-a_t} \right]$	$\beta = 0.9984$ (0.0467), $a = 0.4069$ (0.0007)
$C_t^* = (C_t^\nu + b_t (M_t/P_t)^\nu)^{\frac{1}{\nu}}$	ν=-22.8590 (0.1897), b =0.0008 (5.5×10^{-5})
$L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t$	$\theta_L = 0.8849$ (0.0060)
$AC_t^k = \frac{\phi_K}{2} (\frac{I_t}{K_t} - \frac{I}{K})^2 K_t$	$\Phi_k = 14.2090$ (0.6961)
$\log \frac{R_t}{R} = \rho_R \log \frac{R_{t-1}}{R} + (1 - \rho_M) \times \left[\begin{array}{c} \gamma_\pi \log \frac{\Pi_t}{\overline{\Pi}} + \gamma_{y_1} \log \frac{Y_t}{\overline{Y}_t} \\ + \gamma_{y_2} \log \frac{Y_{t-1}}{\overline{Y}_{t-1}} + \gamma_m \log \frac{MG_t}{MG} \end{array} \right] + \varepsilon_{Mt}$	$\rho_R = 0.2767$ (0.0099), γ_π=0.6992 (0.0030) γ_{y_1}=0.3561 (0.0089), $\gamma_{y_2} = -0.3627$ (0.0040) γ_m=0.2333 (0.0020), $\sigma_M^2 = 2.9 \times 10^{-5}$ (1.4×10^{-6})
$P_t^{\frac{\theta_Y}{\theta_Y - 1}} = (1 - \phi_Y) P_t^{*\frac{\theta_Y}{\theta_Y - 1}} + \phi_Y \Pi_{t-1} P_{t-1}^{\frac{\theta_Y}{\theta_Y - 1}}$	$\phi_Y = 0.6562$ (0.0191)
$AC_{it}^w = \frac{\Phi_w}{2} \left(\frac{W_{it}}{W_{i,t-1}} - \Pi_{t-1}^w \right)^2 \frac{W_t}{P_t}$	$\phi_W = 19.0440$ (0.0051)
$\log \frac{A_t}{A} = \rho_A \log \frac{A_{t-1}}{A} + \varepsilon_{At}$	ρ_A=0.9755 (4.7×10^{-5}), σ_A^2=0.0019 (4.5×10^{-5})
$\log \frac{\alpha_t}{\alpha} = \rho_\alpha \log \frac{\alpha_{t-1}}{\alpha} + \varepsilon_{\alpha t}$	ρ_α=0.9684 (8.0×10^{-5}), σ_α^2=0.0005 (1.6×10^{-5})
$\log \frac{\delta_t}{\delta} = \rho_\delta \log \frac{\delta_{t-1}}{\delta} + \varepsilon_{\delta t}$	ρ_δ=0.9554 (0.0001), σ_δ^2=0.0064 (0.0003)
$\log \frac{b_t}{A} = \rho_b \log \frac{b_{t-1}}{b} + \varepsilon_{bt}$	ρ_b=0.9438 (8.7×10^{-5}), σ_b^2=0.0726 (0.0005)
$\log \frac{a_t}{a} = \rho_a \log \frac{a_{t-1}}{a} + \varepsilon_{at}$	ρ_a=0.0117 (0.0005), σ_a^2=0.0182 (0.0011) $cov(\varepsilon_{At}, \varepsilon_{\alpha t}) = -0.0009$ (7.50×10^{-9})

Table 3: Performances of Alternative Rules ($\overline{\Pi} = 1.01005$)

	EW	dC	$\frac{mean(R)}{sd(R)}$
PHIT	411.0662	0.0835	1.9313
NIGT	410.7408	0.0965	10.2708
MGT	410.3495	0.1123	10.4085

Table 4: Best Performances of Alternative Rules ($k = 2.0468$)

Π		**PHIT**	**NIGT**	**MGT**
1.0111	EW	410.9731	410.6525	410.2561
	dc	0.0873	0.1001	0.1159
0.9967	EW	*not feasible*	411.9511	411.6910
	dc	*not feasible*	0.0481	0.0584
0.9945	EW	*not feasible*	*not feasible*	411.9253
	dc	*not feasible*	*not feasible*	0.0491

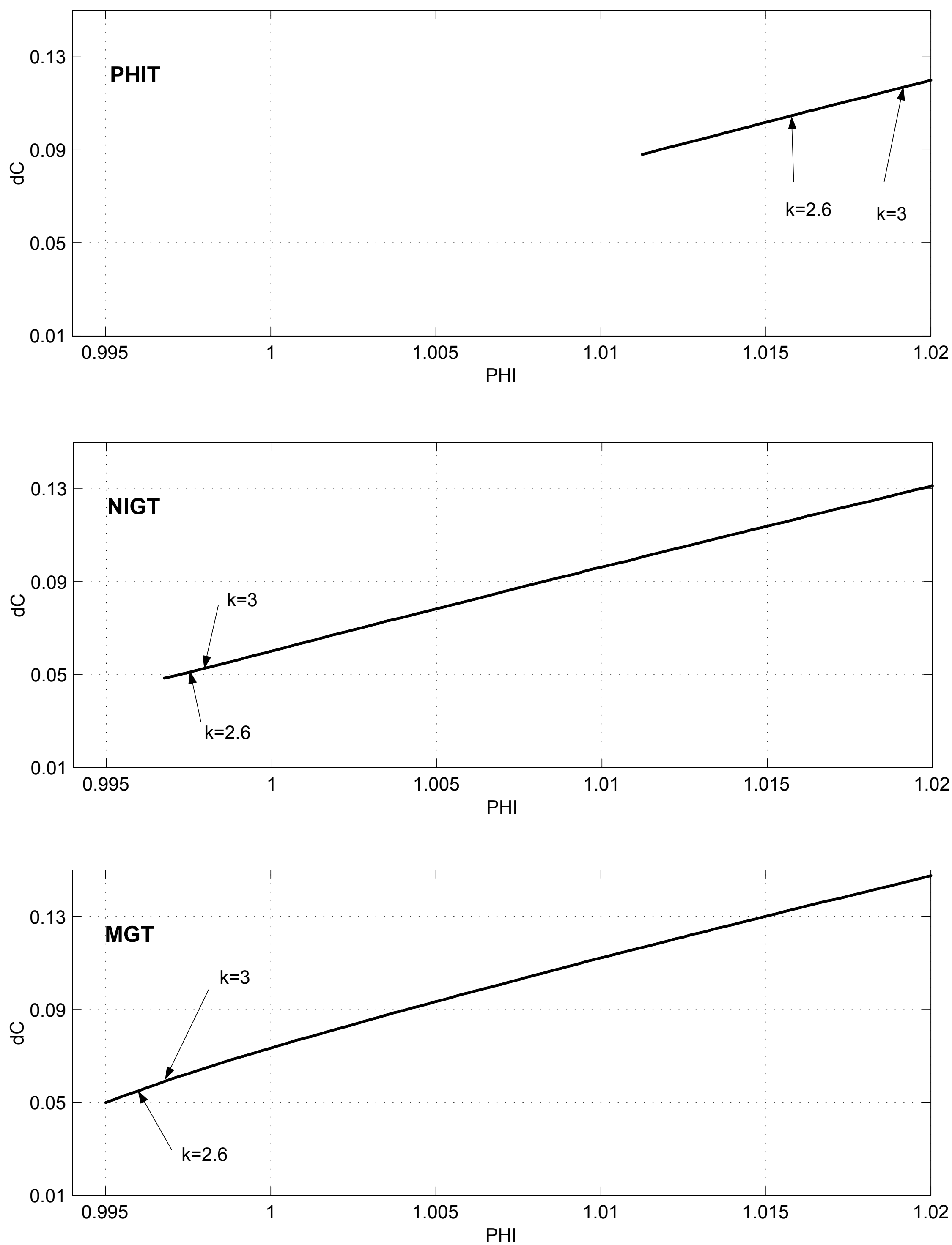

Figure 1: Welfare Plots Under Alternative Targeting Rules

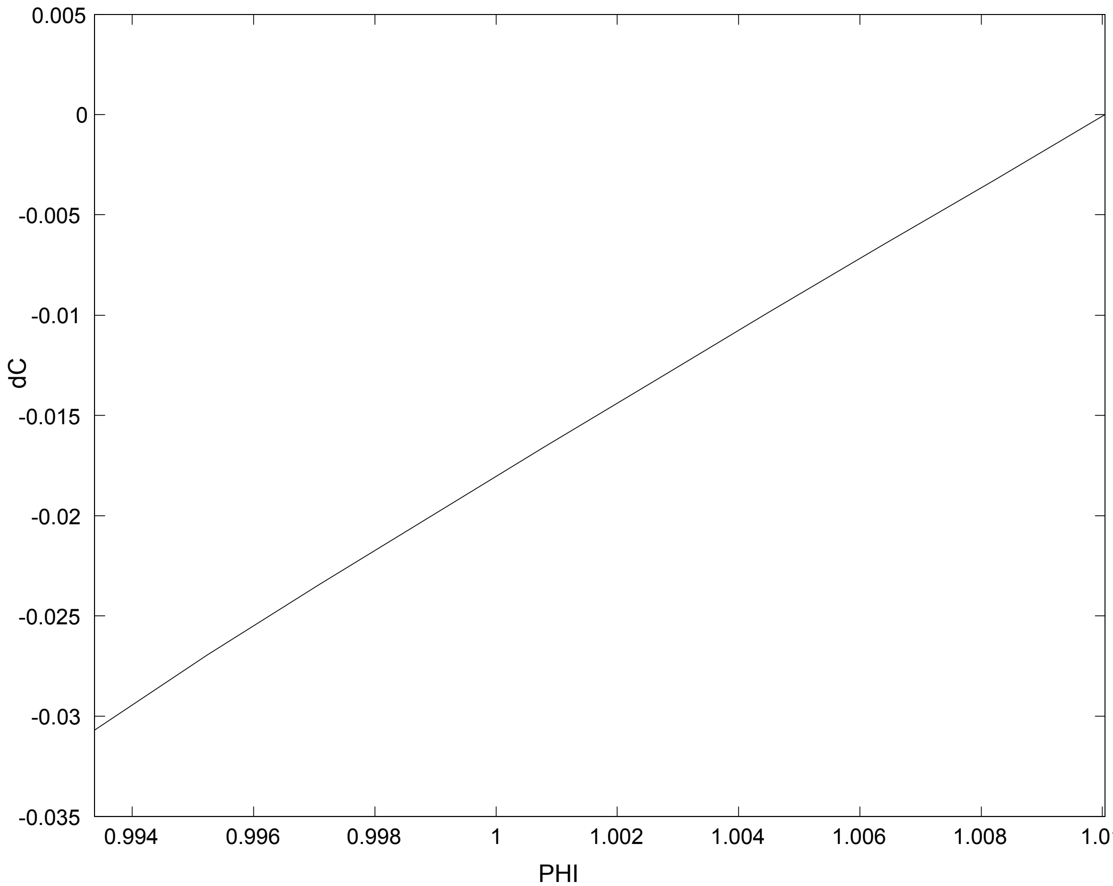

Figure 2: Steady State Consumption Compensations