Optimal Outlooks

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Disclaimer and Acknowledgements

Disclaimer: I am not speaking for others in the Federal Reserve System.

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Need for Outlooks

- A policymaker needs to make a decision today.

- The *current* decision results in random *future* net losses to society.

- Hence, the policymaker's decision depends on his or her outlook about those net losses.

Question

What's the appropriate notion of an outlook for this policymaker?

Answer

- The needed outlook is not a statistically motivated **predictive density** ...

- But rather an asset-price-based **risk-neutral probability density** (RNPD).

Intuition

- From an ex ante perspective, resources may be more valuable in one state than in another state.

- Optimal decisions should reflect these relative resource valuations.

- RNPDs are derived from financial market *prices*.

- Hence, an outlook based on an RNPD *does* reflect the relative values of resources in different states.

- But an outlook based on a statistical forecast *does not*.

Outline

1. General Policy Problem

2. Risk-Neutral Probabilities

3. Example: Inflation-Targeting

4. Conclusions

GENERAL POLICY PROBLEM

Choice Problem

- Policymaker (**P**) chooses an action a.

- The result of the action next period depends on the realization of x.

  - The random variable x has realizations $\{x_n\}_{n=1}^{N}$.

- The outcome (a, x) results in a welfare loss of $L(a, x)$ dollars.

  - The loss $L(a, x)$ may be positive or negative.

Possible Losses

- When **P** chooses an action a, there is a vector of possible social losses:

$$(L(a, x_n))_{n=1}^{N}$$

- Dollars in different states are really different goods.

- Hence, each choice of a results in a distinct *bundle* of different goods.

- How should **P** compare these bundles?

Simple Fruit Analogy

- I face a choice between giving up two baskets of fruit:
  - A apples and B bananas
  - OR A' apples and B' bananas

- I need a way to combine apples and bananas together.
  - Should I just add the number of apples and bananas?
  - Should I estimate CES preferences over apples/bananas?

Using Prices

- Right approach: How much will it cost me to replace the lost fruit?

- Hence, I need to compare:

$$p_A A + p_B B$$

$$\text{vs. } p_A A' + p_B B'$$

- This comparison requires the use of appropriate market prices.

Replacement Cost Approach

- If **P** chooses a, then society suffers a random loss $L(a, x)$.
- By buying a portfolio with random payoff $L(a, x)$, **P** can replace the losses incurred by the action a.
- Hence, the value of that portfolio is the *current* (replacement) cost of taking action a.
- **P** should choose a so as to minimize this cost.
- This comparison requires the use of appropriate market prices.

RISK-NEUTRAL PROBABILITIES

State Prices

- If **P** chooses a, then society loses $L(a, x_n)$ if $x = x_n$.

- How much would it cost *today* to reimburse society for the loss in that state?

- To answer this question, we need to know q_n - the current price of a dollar received in the event that $x = x_n$.
  - The vector $(q_n)_{n=1}^N$ is the vector of *state prices*.

- Given q, it would cost:

$$\sum_{n=1}^{N} q_n L(a, x_n)$$

to reimburse society for the losses incurred with action a.

- **P** should choose a so as to minimize $\sum_{n=1}^{N} q_n L(a, x_n)$.

Risk-Neutral Probabilities

- We don't affect decisions if we divide q_n by a constant.

- Define:

$$q_n^* = \frac{q_n}{\sum_{m=1}^{N} q_m}$$

- q^* is called the *risk-neutral probability density* (RNPD) of x.
  - Probability means: q^* sums to one and q_n^* is nonnegative for all n.

Risk-Neutral and "True" Probabilities

- The RNPD q^* of x is not the same as the "true" probability density of x.
  - And what exactly is the "true" probability density of x?

- q^* reflects asset traders' aversion to risk.

- And q^* reflects asset traders' assessments of the likelihood of x.

$\mathbf{E}^*$

- For any function ϕ of x, define:

$$E^*(\phi(x)) = \sum_{n=1}^{N} q_n^* \phi(x_n)$$

- **P** can optimally choose a by minimizing:

$$E^*(L(a, x))$$

- If L is differentiable with respect to a:

$$E^*\{\frac{\partial L}{\partial a}(a^* \ , x)\} = 0$$

Verbal Summary

- Standard: Policymaker's optimal choice sets the *outlook* for L_a equal to zero.

- **Novel: The appropriate notion of the outlook is given by E^*.**

- Intuitively, policymaker makes choices so as to balance losses across states of the world.

- The relevant trade-offs are governed by state prices, not statistical forecasts.

Aside: Endogeneity of State Prices

- Above: I've treated q^* as exogenous to **P**.

- More realistic: Risk-neutral probability density q^* depends on a.

- Then, **P**'s problem is to choose a to minimize:

$$\sum_{n=1}^{N} q_n^*(a) L(a, x_n)$$

- Suppose **P** ignores endogeneity and chooses a^* so that:

$$E^*[\frac{\partial L}{\partial a}(a^*, x_n)] = 0$$

- Result: This choice is nearly optimal as long as this second moment:

$$Cov^*(L(a^*, x), \frac{\partial \ln q^*(a^*)}{\partial a})$$

  is sufficiently small.

- Note: This second moment is calculated using the RNPD $q^*(a^*)$.

EXAMPLE:

INFLATION-TARGETING

Model of Inflation-Targeting

- Consider a hypothetical central bank (CB) with a single mandate: inflation target $\overline{\pi}$.

- CB chooses accommodation a that, next period, results in:
  - inflation rate $\pi = (a + x)$
  - where x is random

- Sticky prices imply that there is an efficiency loss if π differs from the target $\overline{\pi}$.

- The gap $|\pi - \pi^*|$ generates an approximate *dollar* loss:

$$\kappa(\pi - \overline{\pi})^2$$

- That is, the CB's loss function is well approximated by:

$$L(a, x) = \kappa(a + x - \overline{\pi})^2$$

First Order Condition

- The CB chooses a to minimize:

$$E^*(a + x - \overline{\pi})^2$$

- This results in the first-order condition:

$$E^*(\pi) = \overline{\pi}$$

- The inflation-targeting CB ensures that the outlook for π is kept near $\overline{\pi}$.

- Standard result - except the relevant outlook is given by E^*, not E.

Intuition

- $E^*(\pi)$ can be measured with inflation *break-evens*.
  - on TIPS bonds or on zero coupon inflation swaps

- These break-evens imply that $E^*(\pi)$ is generally larger than (usual measures of) $E(\pi)$.

- Keeping $E^*(\pi)$ equal to $\overline{\pi}$ will result in $E(\pi)$ being less than $\overline{\pi}$.

- Why is this desirable?

- $E^*(\pi) > E(\pi) \Rightarrow$ state prices tend to be high when inflation is high.

- This means that $\pi > \overline{\pi}$ is more costly to society than $\pi < \overline{\pi}$.

- Hence, optimal monetary policy should lead to $E(\pi)$ being *lower* than $\overline{\pi}$.

CONCLUSIONS

RNPDs and Predictions

- FAQ: Do RNPDs forecast the future better than statistical models?

- Similar: Did RNPDs in 2006 reveal the coming asset price corrections?

- **My point today is that these are the wrong questions for policymakers to ask**.

Financial Market Data and Decisions

- Policymakers form future outlooks so as to make current decisions with future outcomes.

- Optimal decisions trade off benefits/costs in future states of the world.

- The trade-off should *not* be based on ex ante (or ex post!) assessments of the states' probabilities.

- Instead, the trade-off should be based on the ex ante relative *values* of resources in those states.

Hence, the relevant outlook for a policymaker is an RNPD.

Implementation Challenges

- Decision-making using RNPDs is not necessarily easy.
  - Need to determine appropriate financial proxy.
  - Even then: Available options may not cover longer horizons or extreme tail events.

- Nothing new: Good decisions are always based on a mix of good judgment, good data, and good modeling choices.

BUT:

The right goal is to model/estimate RNPDs, not statistical forecasts.

Ninth District Activities

- Minneapolis Fed's Banking Group uses options data to compute RNPDs.
- They report the results on the public website for a wide range of assets.
  - Gold, silver, wheat, S&P 500, exchange rates, etc.
- They report and archive the results on a biweekly basis.
- See http://www.minneapolisfed.org/banking/assetvalues/index.cfm.