Optimal Outlooks

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Disclaimer and Acknowledgements

Disclaimer: I am not speaking for others in the Federal Reserve System.

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Need for Outlooks

- A policymaker needs to make a decision today.

- The *current* decision results in random *future* net losses to society.

- Hence, the policymaker's decision depends on his or her outlook about those net losses.

Question

What's the appropriate notion of an outlook for this policymaker?

Answer

- The needed outlook is not a statistically motivated **predictive density** ...

- But rather an asset-price-based **risk-neutral probability density** (RNPD).

Intuition

- From an ex ante perspective, resources may be more valuable in one state than in another state.
- Optimal decisions should reflect these relative resource valuations.
- RNPDs are derived from financial market *prices*.
- Hence, an outlook based on an RNPD *does* reflect the relative values of resources in different states.
- But an outlook based on a statistical forecast *does not*.

Outline

1. General Policy Problem

2. Risk-Neutral Probabilities

3. Example: Macro-Prudential Supervision

4. Conclusions

GENERAL POLICY PROBLEM

Choice Problem

- Policymaker (**P**) choose an action a.

- The result of the action next period depends on the realization of x.
  - The random variable x has realizations $\{x_n\}_{n=1}^{N}$.

- The outcome (a, x) results in a welfare loss of $L(a, x)$ dollars.
  - The loss $L(a, x)$ may be positive or negative.

Possible Losses

- When **P** chooses an action a, there is a vector of possible social losses:

$$(L(a, x_n))_{n=1}^{N}$$

- Dollars in different states are really different goods.

- Hence, each choice of a results in a distinct *bundle* of different goods.

- How should **P** compare these bundles?

Simple Fruit Analogy

- I face a choice between giving up two baskets of fruit:
  - A apples and B bananas
  - OR A' apples and B' bananas
- I need a way to combine apples and bananas together.
  - Should I just add the number of apples and bananas?
  - Should I estimate CES preferences over apples/bananas?

Using Prices

- Right approach: How much will it cost me to replace the lost fruit?

- Hence, I need to compare:

$$p_A A + p_B B$$

$$\text{vs. } p_A A' + p_B B'$$

- This comparison requires the use of appropriate market prices.

Replacement Cost Approach

- If **P** chooses a, then society suffers a random loss $L(a,x)$.

- By buying a portfolio with random payoff $L(a,x)$, **P** can replace the losses incurred by the action a.

- Hence, the value of that portfolio is the *current* (replacement) cost of taking action a.

- **P** should choose a so as to minimize this cost.

- This comparison requires the use of appropriate market prices.

RISK-NEUTRAL PROBABILITIES

State Prices

- If **P** chooses a, then society loses $L(a, x_n)$ if $x = x_n$.

- How much would it cost *today* to reimburse society for the loss in that state?

- To answer this question, we need to know q_n - the current price of a dollar received in the event that $x = x_n$.
  - The vector $(q_n)_{n=1}^{N}$ is the vector of *state prices*.

- Given q, it would cost:

$$\sum_{n=1}^{N} q_n L(a, x_n)$$

to reimburse society for the losses incurred with action a.

- **P** should choose a so as to minimize $\sum_{n=1}^{N} q_n L(a, x_n)$.

Risk-Neutral Probabilities

- We don't affect decisions if we divide q_n by a constant.

- Define:

$$q_n^* = \frac{q_n}{\sum_{m=1}^{N} q_m}$$

- q^* is called the *risk-neutral probability density* (RNPD) of x.
  - Probability means: q^* sums to one and q_n^* is nonnegative for all n.

Risk-Neutral and "True" Probabilities

- The RNPD q^* of x is not the same as the "true" probability density of x.
  - And what exactly is the "true" probability density of x?

- q^* reflects asset traders' aversion to risk.

- And q^* reflects asset traders' assessments of the likelihood of x.

$\mathbf{E}^*$

- For any function ϕ of x, define:

$$E^*(\phi(x)) = \sum_{n=1}^{N} q_n^* \phi(x_n)$$

- **P** can optimally choose a by minimizing:

$$E^*(L(a, x))$$

- If L is differentiable with respect to a:

$$E^*\{\frac{\partial L}{\partial a}(a^*\ , x)\} = 0$$

Verbal Summary

- Standard: Policymaker's optimal choice sets the *outlook* for L_a equal to zero.

- **Novel: The appropriate notion of the outlook is given by E^*.**

- Intuitively, policymaker makes choices so as to balance losses across states of the world.

- The relevant trade-offs are governed by state prices, not statistical forecasts.

Aside: Endogeneity of State Prices

- Above: I've treated q^* as exogenous to **P**.

- More realistic: Risk-neutral probability density q^* depends on a.

- Then, **P**'s problem is to choose a to minimize:

$$\sum_{n=1}^{N} q_n^*(a) L(a, x_n)$$

- Suppose **P** ignores endogeneity and chooses a^* so that:

$$E^*[\frac{\partial L}{\partial a}(a^*, x_n)] = 0$$

- Result: This choice is nearly optimal as long as this second moment:

$$Cov^*(L(a^*, x), \frac{\partial \ln q^*(a^*)}{\partial a})$$

  is sufficiently small.

- Note: This second moment is calculated using the RNPD $q^*(a^*)$.

EXAMPLE:

MACRO-PRUDENTIAL SUPERVISION

Dividend Payouts

- Regulatory question: Large banks want to pay dividends.
- How large a dividend payment should they be allowed to make?
- A low dividend payment today allows banks to have more capital in the future ...
- Which will prove valuable if financial markets are strained in the future.

Model

- Let S be the level of financial market stress next period.

- Let $L(a, S)$ be the *net* social loss (next period) of a current dividend payment a.

- We know that the optimal a^* satisfies:

$$E^*\{\frac{\partial L(a^*, S)}{\partial a}\} = 0$$

A Comparative Statics Result

- Intuitively: The approved level of current bank dividends should depend on the outlook for future financial market strains.

- To see how: Consider two different RNPDs for S denoted by q^* and q^{**}.

- Assume q^* puts more weight on high realizations of S than q^{**}.
  - Formally: q^* dominates q^{**} in a first-order sense.

- Suppose L is supermodular in (a, S).
  - Increasing dividends raises social loss by more when financial markets are strained.

- Then:

$$a^*(q^*) < a^*(q^{**})$$

- **Summary: A regulator should approve lower levels of bank dividends when the RNPD of S' puts more weight on high realizations.**

Implementation Challenges

- We need an appropriate proxy S' for S.
  - S' must be highly correlated with S.
  - There are enough options on S' so that we can construct q^*.
- One possibility: treat (the negative of the) logged S&P 500 index as S'.
- With options on the S&P 500, we can estimate an RNPD for S'.
- Then, if the S&P 500 RNPD has a longer left tail, bank dividends should be lower.

CONCLUSIONS

RNPDs and Predictions

- RNPDs are an ex ante measure of the relative *values* of resources in future states of the world.

- Resources are, all else equal, more valuable in states that are more likely to occur.

- But all else is never equal: RNPDs are shaped by factors other than relative likelihoods.

- So, an RNPD is not the same as a predictive density.

Financial Market Data and Decisions

- BUT, this distinction between RNPDs and predictive densities is exactly what makes RNPDs more useful for policymakers.

- Policymakers form future outlooks so as to make current decisions with future outcomes.

- Optimal decisions trade off future benefits/costs in future states of the world.

- That trade-off should be based on the relative *values* of resources in those states, not their relative likelihods.

For a decision-maker, the relevant outlook is given by an RNPD.

Implementation Challenges

- Decision-making using RNPDs is not necessarily easy.
  - Need to determine appropriate financial proxy.
  - Even then: Available options may not cover longer horizons or extreme tail events.

- Nothing new: Good decisions are always based on a mix of good judgment, good data, and good modeling choices.

BUT:

The right goal is to model/estimate RNPDs, not statistical forecasts.

Ninth District Activities

- Minneapolis Fed's Banking Group uses options data to compute RNPDs.
- They report the results on the public website for a wide range of assets.
  - Gold, silver, wheat, S&P 500, exchange rates, etc.
- They report and archive the results on a biweekly basis.
- See http://www.minneapolisfed.org/banking/assetvalues/index.cfm.