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Time-Varying Risk and International Portfolio Diversification with Contagious Bear Markets

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ABSTRACT

In this paper we estimate and test a conditional version of the international CAPM. By using a parsimonious parameterization recently proposed by Ding and Engle (1994), we allow risk premia, betas, and correlations to very through time and test the cross-section restrictions of the model using a relatively large number of assets. One advantage of our test is is that it does not require the market weights to be observed in each period. In support of the international CAPM, we find that world-wide risk is priced whereas country-specific risk is not. Further, we find that the price of world risk is time-varying and has a strong January seasonal. When the price of risk is allowed to vary, a January dummy and the world dividend yield are driven out as independently priced factors. However, contrary to the prediction of the model, differences in risk premia across countries are explained not only by world-wide risk, but also by a constant country-specific factor. The estimated correlations reveal three main facts, cross-country correlations vary through time; they have been affected only to a limited extent by the process of liberalization of the last decade; they tend to increase during severe bear markets in the U.S. However, international correlations are smaller than correlations among U.S. assets. Therefore, investors gain from global diversification, even with contagious bear markets.

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A large body of evidence shows that risk premia vary through time both in the U.S. and in foreign financial markets. As a consequence, researchers have increasingly focused their attention on empirical methods that can be used to estimate and test conditional versions of asset pricing models. Two approaches, in particular, have been widely used in recent work: generalized autoregressive conditionally heteroskedastic (GARCH) models, and non-parametric techniques based on the generalized method of moments (GMM). The most appealing feature of GARCH models is their ability to accommodate volatility clustering, which is a common phenomenon in financial time series. Applications of GARCH models to international financial markets include, among others, Engel and Rodrigues (1989); Giovannini and Jorion (1989); Chan, Karolyi and Stulz (1991); De Santis (1993). Unfortunately, multivariate versions of these models are hard to estimate due to the large proliferation of parameters. For this reason most GARCH applications are limited to a small number of assets and often introduce strong restrictions on the model (e.g. constant betas or constant correlations).

Non-parametric methods also have several appealing features: they do not require any assumption on the distribution of the data generating process, they can easily handle corrections for heteroskedasticity and autocorrelation in the data, and they are easily generalized to incorporate conditioning information. Harvey (1991) uses a GMM approach to test an international version of the conditional CAPM. However, these techniques have also some limitations. First, if the underlying model is correctly specified, they have less power than parametric methods. Second, their reliability decreases as the number of orthogonality conditions being tested increases.

Because they can only handle a small number of assets, both approaches have a limited ability to address some interesting questions . For example, cross-sectional restrictions on asset returns are a main component of any test of asset pricing models. Also, when addressing problems of optimal portfolio choice, investors face a wide variety of securities, and therefore, applications that have to impose severe restrictions on the investment opportunity set are of limited interest in practice.

In this paper we use a new parameterization of a multivariate GARCH process, recently proposed by Ding and Engle (1994), to derive a test of an international version of the conditional CAPM. Our study contains several contributions. First, it presents a simultaneous analysis of several international equity markets where both the price of covariance risk and conditional betas are allowed to be time-varying. Second, it proposes a test of the conditional CAPM that can be used as an alternative to the test developed by Bollerslev, Engle and Wooldridge (1988) when a market-wide index is available, but the weights used to compute the index at each point in time are not. Lastly, it allows us to evaluate whether the benefits of international diversification vary across time and across bear and bull markets.

The empirical evidence supports the hypothesis of international integration. In particular, we find that the price of covariance risk is equal across countries and changes over time, whereas the price of country-specific risk is not different from zero. We also find that allowing for a time-varying price of risk drives out a January dummy and the dividend yield on the world index as independently priced factors.

Our results have a series of interesting implications for investors who want to diversify their portfolios internationally. First, both the betas and the cross country correlations vary over time. In particular, we find some evidence that correlations tend to increase when the U.S. market experiences large negative shocks. However, we also find that the estimated correlations are smaller than the corresponding values observed within the U.S. market. This implies that investors can still improve the performance of their portfolio by going international. In particular, we document positive gains from international diversification, even during bear markets. Finally, based on our results, the benefits of diversification have been reduced, but not eliminated, by the market liberalization of the last decade.

The rest of the paper is organized as follows. Section 1 presents the asset pricing model and the methodology used to test it. Section 2 describes the data. The empirical results are discussed in section 3. Section 4 concludes the paper.

1. Model and Empirical Methods

1.1. A Model of International Asset Pricing

One of the most widely used models in finance is the capital asset pricing model (CAPM) originally derived by Sharpe (1964) and Lintner (1965). In a two-period framework, the model predicts that the expected return on any traded asset, in excess of a risk-free return, is proportional to the nondiversifiable risk of the asset, as measured by its covariance with a market-wide portfolio return.

The extension of this result to a dynamic framework requires additional assumptions. A well known example is that of the log-utility agents in Merton (1973).

Recently, Campbell (1993) has shown that the linear relation between conditional excess returns and volatility holds if a log-linear approximation is used for the intertemporal budget constraint. In a multiperiod framework, the conditional version of the CAPM can be formalized as follows

$$E(R_{it}|\Im_{t-1}) - R_{ft} = \delta_t cov(R_{it}, R_{mt}|\Im_{t-1}) \qquad \forall i \tag{1}$$

where R_{it} is the return on asset i between time $t-1$ and t; R_{ft} is the return on a (conditionally) risk-free asset; R_{mt} is the return on the market portfolio and $\Im_{t-1}$ is the set of market-wide information available at the end of time $t-1$. As equation (1) obviously implies, the δ_t coefficient can be interpreted as the price of market risk. Alternatively, δ_t is often used as a measure of the aggregate relative risk-aversion in the economy.

The same model is often used in an international framework (see, among others, Engel and Rodrigues (1989), Giovannini and Jorion (1989), Harvey (1991) and Chan, Karolyi and Stulz (1991)). Simply applying equation (1) to international data implies assuming that investors do not cover their exposure to exchange rate risk or, equivalently, that the price of exchange rate risk is equal to zero.[1] A more general framework, which assumes that agents use optimal hedging strategies, is discussed in Solnik (1974a), Sercu (1980), Stulz (1981), and Adler and Dumas (1983). In this study, we limit our analysis to the first model; however, the methodology proposed here is easily applied to the more general framework (see Buraschi, De Santis and Gerard (1994)).

1.2. Empirical Methods

Although equation (1) appears to be the obvious relation to use in empirical tests of the CAPM, a large body of research uses the unconditional version of the model. One of the main reasons for this choice is that the conditional specification of the CAPM is, in general, harder to estimate. However, since the seminal paper on autoregressive conditionally heteroskedastic (ARCH) processes by Engle (1982), an increasing number of studies have focused their attention on conditional moments. In particular, a multivariate version of a Generalized ARCH (GARCH) process can be used to test the pricing implications of the CAPM.

Start from equation (1) and note that the model has to hold for every asset, including the market portfolio. Therefore, in an economy with N risky assets, the

[1]See Stulz (1994) for a detailed discussion of the conditions under which the Sharpe-Lintner version of the CAPM holds internationally.

following system of asset pricing restrictions has to be satisfied at each point in time

$$\begin{array}{rcl} E(R_{1t}|\Im_{t-1}) - R_{ft} & = & \delta_t cov(R_{1t}, R_{mt}|\Im_{t-1}) \\ \vdots & & \vdots \\ E(R_{N-1t}|\Im_{t-1}) - R_{ft} & = & \delta_t cov(R_{N-1t}, R_{mt}|\Im_{t-1}) \\ E(R_{mt}|\Im_{t-1}) - R_{ft} & = & \delta_t var(R_{mt}|\Im_{t-1}) \end{array} \tag{2}$$

The above system includes only $(N-1)$ risky securities plus the market portfolio to avoid redundancies. In fact, if all the risky securities were included into the system, the last equation would just be a linear combination of the first N equations, in each period. In empirical work, any subset of the risky securities can be used if N is too large. Of course, there are some costs in eliminating too many securities: information on cross-correlations is lost and tests of the asset pricing restrictions imposed by the model have less power.

Formally, let R_t denote the $(N \times 1)$ vector which includes $(N-1)$ risky assets and the market portfolio. Then, the following system of equations can be used as a benchmark model to test the conditional CAPM

$$R_t - R_{ft}\iota = \alpha + \delta_t h_{Nt} + \epsilon_t \qquad \epsilon_t|\Im_{t-1} \sim N(0, H_t) \tag{3}$$

where ι is an N-dimensional vector of ones, α is an $(N \times 1)$ vector of constants, H_t is the $(N \times N)$ conditional covariance matrix of asset returns and h_{Nt} is the N^{th} column of H_t.

Equation (3) follows directly from the conditional version of the CAPM. However, the asset pricing model does not impose any restrictions on the dynamics of the conditional second moments. GARCH models can be used to fill this gap and obtain a testable version of the model.

A typical GARCH parameterization for H_t is

$$H_t = C'C + \sum_{i=1}^{p} A_i'\epsilon_{t-i}\epsilon_{t-i}'A_i + \sum_{j=1}^{q} B_j'H_{t-j}B_j \tag{4}$$

where C is an $(N \times N)$ symmetric matrix and A_i and B_j are $(N \times N)$ matrices of constant coefficients.[2]

This specification of the model is very appealing given the success of univariate GARCH processes in fitting financial time series. Unfortunately, the multivariate version of the model described above is very costly to estimate, due the large number of unknown parameters in (4).

[2]See Baba, Engle, Kraft and Kroner (1991) for a detailed discussion of this parameterization.

To get around this problem, most studies that use a multivariate GARCH process have limited the analysis to a small number of assets and/or imposed several restrictions on the process generating H_t. Typically, a GARCH(1,1) specification is assumed (i.e. p and q in equation (4) are set equal to 1) and both A and B are restricted to be diagonal matrices. This implies that the variances in H_t depend only on past square residuals and an autoregressive component, while the covariances depend upon past cross-products of residuals and an autoregressive component (see, for example, Bollerslev, Engle and Wooldridge (1988)). In this case, equation (4) can be written in a simpler form

$$H_t = C'C + \mathbf{aa}' * \epsilon_{t-1}\epsilon_{t-1}' + \mathbf{bb}' * H_{t-1} \tag{5}$$

where $\mathbf{a}$ and $\mathbf{b}$ are $(N \times 1)$ vectors which include the diagonal elements of A and B respectively, and $*$ denotes the Hardmard matrix product (element by element).

Alternatively, all variances and covariances are allowed to change over time, but correlations are assumed to be constant (Bollerslev (1990) and Ng (1991)).

Unfortunately, if the researcher is interested in analyzing a relatively large number of assets, the model is still very costly to estimate, unless more restrictions are imposed on the process driving H_t. For example, suppose that both q and p are set equal to 1 (GARCH(1,1) specification), δ in equation (3) is constant and N is equal to 10. The number of parameters to estimate is equal to 86. If the constant correlation model is used instead of the diagonalization of A and B, the number of unknown parameters is also equal to 86.

In this study, we choose the diagonal representation because the constant correlation model appears to be too restrictive. In particular, some authors (see, for example, Longin and Solnik (1991)) have suggested that correlations among asset returns change with market conditions and this feature cannot be accommodated by a constant correlation model. To further reduce the number of parameters to estimate, we introduce additional restrictions, recently proposed by Ding and Engle (1994), on the conditional covariance process. Under the assumption that the system is covariance stationary, the unconditional variance-covariance matrix of asset returns is equal to

$$H_0 = C'C * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}')^{-1}.$$

Therefore, they propose to replace $C'C$ in equation (5) with the matrix $H_0 * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}')$, where H_0 can be approximated by the sample covariance matrix. In a diagonal system with N assets, this implies that the number of unknown parameters in the conditional variance equation is equal to $2N$. In particular, with

10 assets, the total number of parameters to estimate for the entire benchmark model is now reduced to 31. Formally, equation (5) is replaced by

$$H_t = H_0 * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' * \epsilon_{t-1}\epsilon_{t-1}' + \mathbf{bb}' * H_{t-1} \tag{6}$$

Although it may appear restrictive, the assumption of covariance stationarity is implicit, if the CAPM is the model to be tested. In fact, it would not make sense to use mean-variance analysis if the second moments were not defined. As a consequence, the additional restrictions do not imply any loss of generality with respect to the diagonal models previously used in related studies. Moreover, the considerable reduction in the number of parameters to estimate makes the model applicable to relatively large systems, without having to assume constant correlations.

We use equations (3) and (6) as our benchmark model. Under the assumption of conditional normality, the log-likelihood function for the model can be written as follows

$$\ln L(\theta) = -\frac{TN}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}\ln|H_t(\theta)| - \frac{1}{2}\sum_{t=1}^{T}\epsilon_t(\theta)'H_t(\theta)^{-1}\epsilon_t(\theta) \tag{7}$$

where θ is the vector of unknown parameters in the model. Maximum likelihood estimation is obtained using the BFGS (Broyden, Fletcher, Goldfarb and Shanno) algorithm and the gradient, at each point in time, is approximated using numerical derivatives. If standard regularity conditions are satisfied, the maximum likelihood estimator of θ is consistent and asymptotically normal and, therefore, statistical inference can be carried out using traditional methods.

A battery of specification tests and of asset pricing restrictions is discussed in detail in the empirical results section.

1.3. Related Work

In this section we briefly review some of the existing studies on the conditional version of CAPM and, in each case, we discuss the contribution of our analysis.

Multivariate GARCH models have been widely used to test the pricing restrictions of the conditional CAPM. A necessarily incomplete list includes Bollerslev, Engle and Wooldridge (1988); Giovannini and Jorion (1989); Engel and Rodrigues (1989); Ng (1991) and Chan, Karolyi and Stulz (1992). The parameterization used by these studies can be formalized as follows[3]

$$R_t - R_{ft}\iota = \alpha + \delta\, H_t \omega_{t-1} + \epsilon_t \qquad \epsilon_t|\Im_{t-1} \sim N(0, H_t) \tag{8}$$

where ι, α and H_t are defined in the previous section, and ω_{t-1} is an $(N \times 1)$ vector of market weights for the risky assets, measured at the end of time $t-1$.

Our specification of the model differs from previous studies that use GARCH processes in two ways.

First, consider the system of pricing restrictions in (8). Once the dynamic process driving H_t is specified, the model can be estimated and tested, as long the market weights are observed by the econometrician, at each time t. Unfortunately, data on ω_t are not always readily available. Most studies use different sources to estimate the amount of wealth invested in each one of the assets being tested. As a consequence, they often face problems of measurement errors. The parameterization that we propose in equation (3) assumes only that a market index is observed at each t. The model can be estimated and tested even though the weights used to compute the index are not available to the econometrician.

Second, previous studies are mostly limited to a small number of securities because they only impose the diagonalization of the parameter matrices in the GARCH equations. In some studies (for example Ng (1991)), a relatively large number of assets is analyzed but correlations across returns are assumed to be constant. Using the new parameterization proposed by Ding and Engle (1994) we are able to test the pricing restrictions of the CAPM simultaneously on a large number of assets without imposing the assumption of constant correlations. As discussed in the previous section, this improvement is particularly important in light of recent studies that argue that correlations change during different cycles of the U.S. market.

A different approach to testing the conditional version of the international CAPM is proposed by Harvey (1991). In his paper, Harvey uses a GMM-based test of the pricing restrictions of the model. One of the appealing features of the test is that it can be computed without pre-specifying the dynamic behavior of the conditional second moments.

Unfortunately, this feature also imposes some limitations on the questions that the method can address. In general, any variable of interest which is a function of conditional second moments, cannot be recovered. For example, investors are

[3]The notation does not necessarily reflect all the features of each of the studies cited above. However, it provides a good framework to characterize the common aspects among them.

interested in knowing how the market risk of an asset, measured by its conditional beta, changes over time. Also, investors need to know the conditional covariances among assets, as well as conditional expected returns, to make their portfolio allocation decisions at the beginning of each period. Finally, the time series of the cross-country conditional covariances is necessary to assess the variability of the benefits of international diversification during different cycles in the U.S. stock market.

Of course, the method proposed by Harvey can be generalized to incorporate a parameterization of the conditional second moments. However, this would imply a large proliferation of orthogonality conditions and, therefore, reduce the reliability of the results, especially if the analysis has to be conducted simultaneously on several assets.

Our approach allows us to address all the questions mentioned above. The cost we have to pay is some loss of generality compared to Harvey's method. First, we impose a parameterization on the dynamic behavior of the conditional second moments. Second, we need to assume a functional form for the conditional density of the asset returns. Both assumptions might generate incorrect inferences in case of model misspecification. However, using quasi-maximum likelihood estimates for the standard errors yields inferences which are robust to violations of the normality assumption.[4]

2. Data

In this study we use monthly returns on stock indices for eight countries plus a value-weighted world index during the period January 1970 through December 1992. The indices are obtained from Morgan Stanley Capital International (MSCI). The MSCI data set contains indices for more than twenty developed financial markets plus several regional indices. For our analysis we select the G7 countries (Canada, France, Germany, Italy, Japan, United Kingdom and U.S.A.) and the largest European market not included in the G7: Switzerland.

The MSCI indices have several features that make them particularly attractive for empirical analysis. First, they provide a good representation of national markets. The combined market value of the companies included in the indices equals approximately 60% of the total market capitalization for all the countries contained in the MSCI data set. The companies are selected on the basis of national and industry representation. Investment companies and foreign domiciled

[4]See White (1982) and Hamilton (1994).

companies are excluded to avoid double-counting. Second, most of the stocks included in the MSCI indices are actually available to non-national investors (see Cumby and Glen (1990)). Third, the indices have statistical properties that are very similar to those of other national indices commonly used in empirical studies. For example, Harvey (1991) finds that the MSCI indices for the U.S. and Japan are highly correlated with the value-weighted NYSE index (the correlation is 0.991) and the Nikkei 255 (the correlation is 0.938) respectively. Finally, all indices are computed with and without dividends reinvested so that returns can be evaluated that include both capital gains and dividend yields. The monthly dividend yield is equal to 1/12 of the previous year dividend divided by the index at the end of each month. All returns are computed in U.S. dollars based on the closing European interbank currency rates from MSCI. In this sense, this study takes the point of view of a U.S. investor who is not hedged against exchange rate risk at any point in time.

Table 1 contains summary statistics for the eight national stock returns and for the value-weighted world index. Panel a in the table reports means, standard deviations, the largest and the smallest returns over the entire sample. Japan is the country with the largest average return (1.50%) even though it is not the most volatile market (the standard deviation is equal to 6.63%). On the other hand, Italy has the lowest average return (0.67%) and the largest standard deviation (7.69%) among the countries analyzed. Some of these differences may be due to discrepancies in exchange rate movements across theses countries (see, for example, Odier and Solnik (1993) and Jorion (1989)).

Panel b in the table contains the unconditional correlation matrix computed over the whole sampling period. The numbers in the table clearly show that cross-country correlations for asset returns tend to be much lower than domestic correlations for the U.S.. Most values are below 0.5 and the average (excluding the world index) is equal to 0.435.

The model analyzed in this study is derived assuming the existence of a conditionally risk-free asset. For this purpose we use the monthly return on the U.S. Treasury Bill closest to 30 days to maturity. The data on the T-Bill are obtained from the CRSP files. In each period we compute the monthly excess returns by subtracting the T-Bill rate from each one of the MSCI country returns.

3. Empirical Evidence

In the empirical analysis we consider several specifications both for the mean equation and for the GARCH process that drives the conditional variance-covariance matrix. In this section we discuss a battery of statistical tests that we use to compare the different specifications. As mentioned before, the CAPM imposes only cross-section restrictions on the mean equation, at any point in time. Therefore, all the tests on the conditional covariance process are used to fine tune the GARCH parameterization but have no direct economic interpretation.

3.1. Model Specification

First, we specify the benchmark mean equation used to test the cross-section restrictions of the conditional CAPM.

Equation A:

$$R_{it} - R_{ft} = \alpha_i + \delta\, cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \qquad \forall i$$

This parameterization assumes that: the price of world covariance risk δ is equal across countries and constant over time; the conditional covariance of each asset return with the world portfolio is time-varying and the α's are country-specific constants, which may reflect factors other than covariance risk affecting expected returns.

Equation B:

$$R_{it} - R_{ft} = \alpha_i + \delta_i\, cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \qquad \forall i$$

In this equation, the price of world covariance risk as well as the constant factors are allowed to vary across countries.

Equation C:

$$R_{it} - R_{ft} = \alpha_i + \delta\, cov(R_{it}, R_{mt}|\Im_{t-1}) + \gamma_i var(R_{it}|\Im_{t-1}) + \epsilon_{it} \qquad \forall i$$

In this specification, a country's premium over the risk-free rate is determined by its specific risk as well as its world covariance risk. The price of world risk is assumed to be equal across all countries whereas the price of country risk is allowed to differ.

Equations B and C can be compared to equation A to test two of the most important pricing restrictions of the model. If markets are integrated, in the sense that the same model consistently prices assets from different countries, the price of covariance risk should be equal across countries and country-specific risk should not be priced.

Equation D:

$$R_{it} - R_{ft} = \delta\, cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \qquad \forall i$$

In this equation we impose the restriction that the α_i parameters are equal to zero for all assets. This restriction follows directly from the Sharpe-Lintner version of the CAPM and is used in most tests of the model. However, the CAPM might still be a good approximation of the true asset pricing model, even if the restriction imposed in equation D is rejected. For example, in an international framework, the α's might reflect country-specific factors (e.g. differential tax-treatment) which are not captured by market risk.

Equation E:

$$R_{it} - R_{ft} = \alpha + \delta\, cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \qquad \forall i$$

In this specification the constant factor α is assumed to be equal across all assets. A significant α parameter across countries would suggest the existence of some common world-wide priced factor unaccounted for by the country's covariance risk.

All the parameterizations described so far assume that the price of market-risk δ is constant over time. This is a common feature in most empirical studies that use GARCH-M models. However, it is not an assumption of the theoretical model. For this reason, we also consider a different specification of equation A in which the parameter δ is allowed to vary as a function of a set of information variables. The proposed parameterization is

Equation F:

$$R_{it} - R_{ft} = \alpha_i + (\delta + \kappa' Z_{t-1})\, cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \qquad \forall i$$

where Z_{t-1} is a $(K \times 1)$ vector of variables included in $\Im_{t-1}$. The assumption that δ_t is a linear function of the instruments in Z_{t-1} is discussed later in this section.

Panel a in Table 2 summarizes the specifications used for the asset pricing equation. Panel b in the same table reports the three parameterizations that we consider for the process driving the conditional covariance matrix. The benchmark GARCH process corresponds to equation (6); the first alternative assumes that all the elements of the **b** vector are equal and the second alternative assumes that all the elements of the **a** vector are equal. The main purpose of the two alternative specifications is to reduce further the number of parameters to be estimated.

3.2. Estimation and Testing Results

Maximum likelihood estimates of the parameters of our benchmark model (equation A for the asset pricing restrictions and equation (6) for the conditional covariance process) are reported in Table 3.

First, consider the parameters of the GARCH process. All the parameters in **a** and **b** are statistically significant at any reasonable level (the only exception is the estimate of b for Germany, which is less than two times its standard error). Also, all the estimates satisfy the stationarity conditions for all the variance and covariance processes.[5] Finally, as it is typical in most studies that use GARCH models, all processes display high persistence[6] and the estimates of the b_i coefficients (which link the current second moment to its lagged value) are considerably larger than the corresponding estimates of the a_i's (which link the current second moment to past innovations).

Second, consider the parameters of the asset pricing equations. The estimated measure of aggregate risk aversion (or, alternatively, the constant price of market-risk) is equal to 7.095 and is more than three times its asymptotic standard error.[7] This measure is within the range of estimates obtained in related work (see, for example, Harvey (1991) and Ng (1991)). On the other hand, all the estimates of the alpha parameters are negative and, in most cases, statistically significant, at least on an individual basis.

Table 3 also includes some specification tests for the model. In particular, we construct the (estimated) standardized residuals ($\epsilon_t h_t^{-1/2}$) and the (estimated) standardized residuals squared ($\epsilon_t^2 h_t^{-1}$) and then compute the Ljung-Box port-

[5]Theorem 1 in Bollerslev (1986) implies that for each process in H_t to be covariance stationary, the condition $a_i a_j + b_i b_j < 1 \quad \forall i, j$ has to be satisfied.

[6]This is because the value of $a_i a_j + b_i b_j$ is close to one in all instances. See Bollerslev and Engle (1986).

[7]The value reported in Table 3 is equal to 0.0795 because percentage returns are used during the estimation.

manteau statistic for each series to test the hypothesis of no autocorrelation up to order 12. The results in the table show that our specification is satisfactory in most cases. The only countries for which some residual correlation is left in the mean equation are Japan and Italy. However, for both countries, the estimated autocorrelation coefficients are all very small.

Table 4 contains the results of a battery of tests of the cross-sectional restrictions imposed by the international version of the CAPM used in this study.

The first test reported in the table evaluates the null hypothesis that the price of market risk δ is the same across countries. The alternative hypothesis relaxes this assumption by allowing the coefficient to be different for each one of the assets included in the system. Formally, this amounts to comparing equations A and B. The likelihood ratio test statistic has a p-value of 0.6634 which implies that this CAPM restriction cannot be rejected at any statistically reasonable level.

Second, we compute a test of the hypothesis that country specific risk (measured by the conditional variance of each country index) is also priced in international financial markets. In this case, a likelihood ratio test is computed to compare equations A and C. The p-value for this test is equal to 0.7368. Again, the evidence supports the pricing restriction of the CAPM and suggests that the financial markets of the countries investigated are integrated.

The third test evaluates the null hypothesis that the entire vector of the alpha parameters is equal to zero. In particular, a likelihood ratio test is computed to compare equations A and D. The p-value of the test indicates that the null hypothesis is rejected at any level larger than 1.2%. This result implies that this pricing implication of the international CAPM is violated, at least under the assumption that the price of covariance risk is constant through time. However, before discarding the model, some additional considerations are in order. The theoretical model is based on a set of rather strong assumptions which are likely to be violated. In fact, other studies of the conditional CAPM that use an approach similar to ours, obtain negative estimates for most of the α's (see, for example, Engle, Lilien and Robins (1987); Bollerslev, Engle and Wooldridge (1988) and Ng (1991)). Different explanations have been proposed for this finding. For example, the model may fail to capture differences in tax treatment or other institutional arrangements across countries. Alternatively, restricting the relationship between expected excess returns and covariance risk to be linear might be incorrect. Finally, as mentioned in section 1, the estimated model assumes that investors do not hedge against currency risk. If this assumption is violated, the α's could be functions of the price of exchange rate risk for each one of the countries included in our study.

Firnally, we restrict all the α parameters to be equal across countries (equation E). The purpose of this test is to determine whether there is a world-wide constant factor common to all assets, that helps explain risk premia. The p-value of the likelihood ratio test is equal to 0.007. Therefore, the evidence supports the hypothesis that variations in country risk premia, not explained by world covariance risk, are due to country-specific constant factors and not to a world-wide constant factor.

To summarize, our tests suggest that investors in international financial markets are rewarded for their exposure to world risk but not to country-specific risk. Consistently with the conditional version of the CAPM, world risk is measured by the conditional covariance between the return on each asset and the return on a worldwide portfolio. Moreover, the price of world risk is equal across all the countries included in the sample. The only pricing implication of the CAPM that is rejected by our tests is that nondiversifiable risk is the only source of cross-sectional differences in risk premia. In particular, we detect the presence of a constant country-specific factor.

We also estimate the model and perform the tests described above using two alternative parameterizations for the conditional covariance process. In the first specification we restrict all the parameters in the **a** vector to be equal, whereas in the second specification we impose the equality restriction on the components of **b**. Based on the likelihood ratio test statistic, we cannot reject the first set of restrictions. This result implies that, at least for the set of countries included in this study, a more parsimonious parameterization of the GARCH process could be used. However, the results of the cross-sectional tests of the international CAPM are unchanged when using this parameterization.

3.3. Is the Price of Market Risk Constant?

In all the specifications analyzed so far, we assume that the price of market risk δ is constant through time. This is not an implication of the conditional asset pricing model. In fact, under the assumption that the α parameters are equal to zero, δ measures the slope of the tangent to the conditional mean-variance frontier at any point in time (for a given value of the conditionally risk-free rate). Since both the first and the second order conditional moments are allowed to change over time, there is no reason to believe that the slope has to stay constant. In order to accommodate a time-varying slope, the price of covariance risk is parameterized as a linear function of a set of instrumental variables (equation F). This specification makes the hypothesis of time-varying price of market risk easily testable, under the implicit assumption that the unknown function (possibly nonlinear) of the state variables, which determines δ_t, is well approximated by a linear function of the instruments.

The set of instruments includes a constant, a dummy variable for the month of January and the lagged dividend-price ratio for the world index computed by MSCI.[8] Table 5 reports maximum likelihood estimates of the parameters for the model with a time varying δ. A likelihood ratio test reveals that the hypothesis that δ is constant is rejected at any level higher than 1.7%. As documented in the table, the structure of the conditional second moments does not change when the price of risk is allowed to vary. The α's in the mean equation are larger in absolute value than those reported in Table 4. However, their standard errors are also larger, so that, statistically, the change is of little relevance.

Figure 1 contains a plot of the time-varying price of covariance risk, obtained using the results in Table 5. The point estimates follow a *wave* which reaches its high values between the end of the Seventies and the beginning of the Eighties. A striking feature of the plot is the clear detection of a January effect in international financial markets. Seasonal spikes are observed, for the month of January, over the whole sample. In other words, our model can explain the observed excess returns in January only by allowing the price of covariance risk in that month to be much higher compared to the remaining months of the year. Although based on a different methodology, our results extend the findings of Gultekin and Gultekin (1983) to the Eighties and the beginning of the Nineties.

Given our findings, a natural question to ask is whether some of the anomalies often detected in empirical tests of the CAPM can be explained by allowing the price of risk to vary over time. Typically, the instrumental variables used in our analysis are treated as independently priced factors that help explain the variation of expected excess returns, once market risk has been taken into account. This evidence is often interpreted as a rejection of the CAPM. However, most studies are conducted under the assumption that the price of market risk is constant. Our approach can be used to directly test the hypothesis that the January dummy and the dividend yield are not priced by the market when the price of risk is allowed to vary.

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[8]Other studies have shown the predictive ability of these instruments (see, for example, Harvey (1991) and Ferson and Harvey (1993)).

Formally, we estimate and compare three different specification of the asset pricing equations. The first specification assumes a three-factor model with a constant δ. The three factors are market risk, a January dummy and the world dividend yield. The second model (which corresponds to equation F) assumes that the price of risk changes as a linear function of the January dummy and of the dividend yield, but covariance risk is the only factor priced by the market. Both models are nested into the third specification. In this case, risk premia are assumed to be a function of three factors and δ is allowed to vary over time. Likelihood ratio tests can be used to evaluate the competing models. First, we compare the first specification to the third. The p-value for this test is equal to 0.024 suggesting that allowing for a time-varying price of risk improves the performance of the model, even when the January dummy and the dividend yield are used as independent factors. On the other hand, when the benchmark model assumes a variable δ, the null hypothesis that the two instrumental variables are not independently priced cannot be rejected at any level lower than 8.7%. In other words, the evidence suggests that a time-varying price of risk drives out the January dummy and the dividend yield as independent factors.

These results are supportive of the conditional CAPM and suggest that rejections of the model, in favor of multifactor alternatives, might be due to incorrect assumptions about the price of covariance risk.

3.4. Should U.S. Investors Diversify Internationally?

The evidence presented above suggests that the international assets included in our study satisfy, to a large extent, the pricing restrictions imposed by the conditional CAPM. In this section we use the results of our empirical analysis to discuss some issues concerning global diversification that have been recently brought up by both researchers and professionals.

In a mean-variance framework, investors can improve the reward-to-risk ratio of their portfolios by diversifying across a large number of assets. This result is mostly driven by the correlations that link the returns on different securities. The lower the correlations, the higher the benefits of diversification. Therefore, international diversification appears to be the best way to improve portfolio performance. In fact, as long as financial markets are affected by country specific factors, correlations between asset returns from different countries are likely to be lower than correlations within the same country. Based on this reasoning, researchers (starting with Solnik (1974b)) and investment advisors alike have largely promoted the idea of global diversification. However, more recently, the enthusiasm for international diversification has subsided. In particular, two main arguments have contributed to a more cautious attitude. First, international financial markets have become increasingly more integrated, due to the lifting of many restrictions to international investment during the Eighties. If the legal barriers were binding, cross-country correlations may have increased in recent years, thus reducing the benefits of diversification. Second, recent studies suggest that global diversification may not be as good a safety net against bear markets in the U.S. as most people think. More precisely, Solnik and Odier (1993) and Lin, Engle and Ito (1994) suggest that bear markets are contagious at the international level; when the U.S. market tumbles, volatility increases and, most important, cross-country correlations increase. In this sense, the benefits of portfolio globalization may be reduced even further when investors need them the most.

In our model, the benefits of international diversification are affected by both the time-varying price of risk and time-varying correlations. For example, at the level of volatility of the U.S. portfolio, the expected gains from international diversification are measured by the following expression

$$E(R_{d\,t} - R_{US\,t}|\Im_{t-1}) = \delta_t[var(R_{US\,t}|\Im_{t-1}) - cov(R_{US\,t}, R_{m\,t}|\Im_{t-1})] \qquad (9)$$

where $R_{d\,t}$ is the return on a portfolio which includes R_m and R_f and has the same volatility as the U.S. portfolio. For this reason, we discuss the two questions at hand considering the dynamics of both correlations and price of risk.

We approach each question from a different perspective. Since the process of liberalization has taken place over several years, its effects on asset prices should be mainly identified with the trend component in the data. On the other hand, the behavior of international returns during bear and bull cycles is mostly a short term phenomenon and, therefore, is reflected by the high frequency component in the data. Based on this argument, we use an Hodrick-Prescott (HP) filter to separate the trend and cycle components in the series of interest.

First, consider the effect of liberalization on return correlations from the perspective of a U.S. investor. Figure 2 contains plots of the time-varying correlations between the U.S. market and an equally weighted portfolio of the foreign assets included in this study.[9] The lowest correlations are observed mostly in the first part of the sample; however, it is hard to detect any radical change over the whole period by looking at the estimated correlations. The trend component of the series

[9] Since the correlation between the U.S. and Canada is condiderably higher than the other correlations, we also construct a weighted portfolio which does not include Canada. The results discussed below are qualitatively unaffected.

is more revealing. In the first part of the sample, the low frequency component of the correlation moves around the sample mean, crossing its value four times. On the other hand, for most of the last nine years the filtered correlations are larger than the overall mean suggesting that an upward trend in correlations might be associated with international liberalization. However, it should be pointed out that the increase in correlations is relatively small thus leaving even recent values below the corresponding estimates for U.S. assets. In panel b of Table 6 we report the average conditional correlations for three different periods included in our sample. The figures in the table confirm a slow increase in average conditional correlations over time.

Figure 3 contains a plot of the expected gains from diversification, measured according to equation (9). As discussed above, the estimated gains are affected by the dynamics of both the price of risk and cross country correlations. A comparison of the HP filtered gains and the average gains for the whole sample reveals that, with the exception of the period prior to the collapse of the Bretton Woods agreement, the trend in expected gains from diversification is downward sloping. More precisely, the filtered gains are above the overall mean from February 1974 to August 1983 and almost always below the mean thereafter. On an annual basis, the average gains in the '74-'83 period are equal to 6.58%, while they drop to 3.62% in the second subperiod.

Based on these results, we conclude that market liberalization has reduced, but not yet wiped away, the potential benefits of international diversification.

Second, consider the issue of whether bear markets are contagious across countries. In Figure 2 we identify seven bear cycles and seven bull cycles for the U.S. stock market, based on a definition used by Forbes magazine to evaluate mutual fund performance. The cycles are identified using the S&P 500 index as a benchmark. Table 7 contains a detailed description of the starting and ending date as well as a measure of market performance for each cycle. The two most severe bear cycles in our samples are the periods from January 1973 to September 1974 (the U.S. market dropped 45.06%) and from September 1987 to November 1987 (the U.S. market dropped 29.42%). The plots clearly show that, in both periods, the estimated correlations tend to increase during the bear cycle and reach a peak just before the U.S. market starts to recover. This evidence is not as strong for the less severe bear markets of the Seventies and early Eighties. However, in most cases when a large drop[10] is observed in the U.S. index (even for just one period),

[10]A month in which the US market experiences a large drop is defined as a month during

our model predicts positive jumps in correlations.

These results appear to support the claim that international diversification is not a very good safety net when the U.S. market tumbles. However, the empirical evidence also suggests that portfolio globalization should not be dismissed too quickly. First, in most cases, when correlations increase they are still lower than 0.7 even at their peaks. Second, after reaching their peaks, the estimated correlations appear to reverse to lower values rather quickly. Lastly, as documented in panel c of Table 6, the difference between average conditional correlations over bear and bull cycles appears to be negligible.

De Santis and Gerard (1994) use the methodology described in this paper to estimate time-varying correlations among U.S. decile portfolios. They find that, over the same sampling period, the estimated correlations for U.S. portfolios are higher than the cross-country correlations documented in this study. In particular, the overall average correlation is equal to 0.84 and the average correlations for the smallest and largest decile portfolios are equal to 0.62 and 0.99 respectively. Therefore, although investors may not be as protected against U.S. market crashes as previously believed, it seems unlikely that the benefits of international diversification can be replicated by domestic diversification alone.

This intuition is confirmed by the plot of the expected gains from diversification in Figure 3. During most of the bear cycles, the gains are below the overall mean. As expected, this evidence is even stronger during the most severe bear markets. However, immediately after the end of the '73-'74 and the '87 bear cycles, the joint effect of lower correlations and higher price of covariance risk induces large positive jumps in the expected gains.

Therefore, also in this case, our findings are consistent with the suggestion that bear markets are contagious across countries. However, investors can still gain from being internationally diversified, especially immediately after severe bear cycles.

4. Conclusions

In this paper we test an international version of the conditional CAPM using a multivariate GARCH process. A parsimonious parameterization, recently proposed by Ding and Engle (1994), allows us to test the asset pricing implications of the model using a relatively large number of assets. Our analysis is more general

which the US equity index excess return is more than two standard deviations below its mean. Panel b in Table 7 identifies the nine instances of large drop observed in our sample.

than previous related work that uses GARCH processes; in particular, we allow risk premia, betas and correlations to vary through time. In addition, we propose a test of the conditional CAPM that can be used as an alternative to the one developed by Bollerslev, Engle and Wooldridge (1988). To implement our test, the econometrician needs to observe a market-wide index, but he does not need to observe the weights of the individual assets in the market portfolio.

Compared to tests of the conditional CAPM based on GMM, our approach can be used to the asset pricing restrictions of the model simultaneously. At the same time, the analysis provides estimates of the conditional second moments which are of primary interest to portfolio investors. The entire set of hypotheses tested in our study could be theoretically specified in a GMM framework. However, this would generate a proliferation of moment restrictions that would make the problem hard to implement in practice.

Our findings can be summarized as follows. Market-wide risk, measured by the conditional covariance between the return on each asset and a world-index return, is equally priced across all countries. On the other hand, the price of country-specific risk, measured by the conditional variance of each country index, is equal to zero. Although these two results are consistent with the prediction of the model, we also find some evidence against it. Namely, differences in risk premia across countries are explained by two factors: time-varying covariance risk and a constant country-specific factor. We also find evidence that the price of covariance risk varies over time and has a strong January seasonal. Interestingly, allowing for a time-varying price of risk drives out a January dummy and the world dividend yield as independently priced factors.

The estimated correlations have interesting implications for portfolio management. First, cross-country correlations fluctuate considerably throughout the sampling period but, in general, are much lower than correlations among U.S. assets. Second, the benefits of diversification have declined but not disappeared after the liberalization of most financial markets. Third, our estimates indicate that cross-country correlations tend to increase during severe bear cycles of the U.S. markets. However, even at their peak, international correlations are rather low. Therefore, U.S. investors can still improve the reward-to-risk ratio of their portfolios through global diversification.

References

[1] Adler, Michael and Bernard Dumas, 1983, International Portfolio Selection and Corporation Finance: A Synthesis, *Journal of Finance* 46, 925-984.

[2] Baba, Yoshi, Robert F. Engle, Dennis F. Kraft and Kenneth F. Kroner, 1991, Multivariate Simultaneous Generalized ARCH, manuscript, University of California at San Diego.

[3] Bollerslev, Tim, 1986, Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics* 31, 307–327.

[4] Bollerslev, Tim, 1990, Modeling the Coherence in Short Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model, *Review of Economics and Statistics* 69, 542-546.

[5] Bollerslev, Tim and Robert F. Engle, 1986, Modeling the Persistence of Conditional Variance, Econometric Review 5.

[6] Bollerslev, Tim, Robert F. Engle and Jeffrey M. Wooldridge, 1988, A Capital Asset Pricing Model with Time-Varying Covariances, *Journal of Political Economy* 96, 116-131.

[7] Buraschi, Andrea, Giorgio De Santis and Bruno Gerard, 1994, Optimal Dynamic Hedging in Currency Markets, work in progress, Department of Finance and Business Economics - University of Southern California.

[8] Campbell, John Y., 1993, Intertemporal Asset Pricing without Consumption Data, *The American Economic Review* 83, 487-512.

[9] Chan, K. C., G. A. Karolyi and R. M. Stulz, 1991, Global Financial Markets and the Risk Premium on U.S. Equity, *Journal of Financial Economics* 32, 137-168.

[10] Cumby, Robert E. and Jack D. Glen, 1990, Evaluating the Performance of International Mutual Funds, *Journal of Finance* 45, 497-521.

[11] De Santis, Giorgio, 1993, Globalization of Equity Markets: A Test Using Time-Varying Conditional Moments, working paper, Department of Finance and Business Economics - University of Southern California.

[12] De Santis, Giorgio and Bruno Gerard, 1994, Another Look at the Conditional CAPM, work in progress, Department of Finance and Business Economics - University of Southern California.

[13] Ding, Zhuanxin and Robert F. Engle, 1994, Large Scale Conditional Covariance Matrix Modeling, Estimation and Testing, manuscript, University of California at San Diego.

[14] Engel, Charles and Anthony P. Rodrigues, 1989, Tests of International CAPM with Time-Varying Covariances, *Journal of Applied Econometrics* 4, 119-138.

[15] Engle, Robert F., 1982, Autoregressive Conditional heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica* 50.

[16] Engle, Robert F., David M. Lilien, and Russell P. Robins, 1987, Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model, *Econometrica* 55, 391–407.

[17] Ferson, Wayne E. and Campbell R. Harvey, 1993, The Risk and Predictability of International Equity Returns

[18] Giovannini, Alberto and Philippe Jorion, 1989, The time variation of risk and return in the foreign exchange and stock markets, *Journal of Finance* 44, 307-325.

[19] Gultekin, Mustafa and N. Bulent Gultekin, 1983, Stock Market Seasonality: International Evidence, *Jouranl of Finance* 12, 469-481.

[20] Hamilton, James D., 1994, *Time Series Analysis*, Priceton University Press.

[21] Harvey, Campbell R., 1991, The World Price of Covariance Risk, *Journal of Finance* 46, 111-157.

[22] Jorion, Philippe, 1989, The Linkages Between National Stock Markets, in Robert Aliber, ed.: *The Handbook of International Financial Management* (Dow Jones - Irwin).

[23] Lin, Weng-Ling, Robert F. Engle and Takatoshi Ito, 1994, Do Bulls and Bears Move across Borders? International Transmission of Stock Returns and Volatility, *Review of Financial Studies* 7, 507-538.

[24] Lintner, John, 1965, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics* 47, 13-37.

[25] Longin and Bruno Solnik, 1991, Conditional Correlation in International Equity Returns, working paper, HEC, Jouy en Josas, France.

[26] Odier, Patrick and Bruno Solnik, 1993, Lessons for International Asset Allocation, *Financial Analyst Journal* 49, 63-77.

[27] Ng, Lilian, Tests of the CAPM with Time-Varying Covariances: A Multivariate GARCH Approach, *Journal of Finance* 46, 1507-1521.

[28] Sercu, Piet, 1980, A Generalization of the International Asset Pricing Model, *Revue de l'Association Francaise de Finance* 1, 91-135.

[29] Sharpe, William, 1964, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk, *Journal of Finance* 19, 425-442.

[30] Solnik, Bruno, 1974a, The International Pricing of Risk: An Empirical Investigation of the World Capital Market Structure, *Journal Finance* 29, 48-54.

[31] Solnik, Bruno, 1974b, Why Not Diversify Internationally Rather than Domestically?, *Financial Analysts Journal* 30, 48-54.

[32] Stulz, René M., 1981, A Model of International Asset Pricing, *Journal of Financial Economics* 9, 383-406.

[33] Stulz, René M., 1994, International Portfolio Choice and Asset Pricing: An Integrative Survey, NBER working paper n.4645.

[34] White, Halbert, 1982, Maximum Likelihood Estimation of Misspecified Models, *Econometrica* 50, 1-25.

Table 1: Summary Statistics and Unconditional Correlations of Index Returns for Eight Coutries and the World.

The statistics are based on the monthly U.S. $ denominated raw returns on the equity indices of eight countries, Canada, Japan, France, Germany, Italy, Switzerland, United Kingdom and the United States, and to the value weighted world index. Data are obtained from the MSCI database for the period January 1970 to December 1992 (276 observations).

a. Summary Statistics

Country	Mean	Standard Deviation	Minimum	Maximum	Weights*
Canada	0.89	5.59	-22.04	17.98	.028
Japan	1.50	6.63	-19.38	24.26	.312
France	1.22	7.11	-23.18	26.83	.034
Germany	1.07	6.17	-17.64	20.23	.042
Italy	0.67	7.69	-21.42	30.99	.015
Switzerland	1.11	5.69	-17.64	24.58	.019
U.K.	1.29	7.66	-21.53	56.41	.108
U.S.A.	0.96	4.59	-21.22	17.79	.350
World	0.98	4.27	-16.96	14.71	1.000

* As of December 31, 1990.

b. Unconditional Correlations

	Canada	Japan	France	Germany	Italy	Switz.	U.K.	U.S.
Canada	1							
Japan	0.264	1						
France	0.418	0.403	1					
Germany	0.290	0.405	0.595	1				
Italy	0.292	0.377	0.445	0.382	1			
Switz.	0.457	0.439	0.620	0.715	0.385	1		
U.K.	0.510	0.364	0.533	0.418	0.349	0.567	1	
U.S.	0.701	0.268	0.428	0.341	0.227	0.499	0.500	1
World	0.701	0.679	0.616	0.556	0.431	0.683	0.681	0.833

Table 2: Specifications of the Returns Equation and of the Time–Varying Conditional Covariance Process.

This table reports the functional form and the number of parameters for each specification of the returns equation and each specification of the time varying covariance process. N is the number of portfolios for which the model is estimated. $R^e_{i,t}$ is the excess return on portfolio (country index) i in period t, $\epsilon_{i,t}$ is the period t innovation in portfolio i excess returns, α, δ, γ are scalars, κ is a $(K \times 1)$ vector of scalars and Z_{t-1} a $(K \times 1)$ vector of information variables. Conditional on $\Im_{t-1}$, the information available at the end of period $t-1$, ϵ_t is distributed $N(0, H_t)$. The conditional covariance process H_t is specified in panel (b). $*$ denotes the Hardmard matrix product (element by element), $\mathbf{a}, \mathbf{b}$ are $(N \times 1)$ vectors of constants, ι is a $(N \times 1)$ unit vector, a, b are scalars, and Σ_{t-1} is the matrix of cross products of error terms, $\epsilon_{t-1}\epsilon'_{t-1}$.

	(a) Returns Equation Functional Form	# of par.	if N = 9
A	$R^e_{it} = \alpha_i + \delta cov(R_{it}, R_{mt} \mid \Im_{t-1}) + \epsilon_{it}$	N+1	10
B	$R^e_{it} = \alpha_i + \delta_i cov(R_{it}, R_{mt} \mid \Im_{t-1}) + \epsilon_{it}$	2N	18
C	$R^e_{it} = \alpha_i + \delta cov(R_{it}, R_{mt} \mid \Im_{t-1}) + \gamma_i var(R_{it} \mid \Im_{t-1}) + \epsilon_{it}$	2N	18
D	$R^e_{it} = \delta cov(R_{it}, R_{mt} \mid \Im_{t-1}) + \epsilon_{it}$	1	1
E	$R^e_{it} = \alpha + \delta cov(R_{it}, R_{mt} \mid \Im_{t-1}) + \epsilon_{it}$	2	2
F	$R^e_{it} = \alpha_i + (\delta + \kappa' Z_{t-1}) cov(R_{it}, R_{mt} \mid \Im_{t-1}) + \epsilon_{it}$	N+K+1	10+K

	(b) Conditional Covariance Process Functional Form	# of par.	if N =9
1	$H_t = H_0 * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' * \Sigma_{t-1} + \mathbf{bb}' * H_{t-1}$	2N	18
2	$H_t = H_0 * (\iota\iota'(1-b^2) - \mathbf{aa}') + \mathbf{aa}' * \Sigma_{t-1} + b^2 H_{t-1}$	N+1	10
3	$H_t = H_0 * (\iota\iota'(1-a^2) - \mathbf{bb}') + a^2 \Sigma_{t-1} + \mathbf{bb}' * H_{t-1}$	N+1	10

Table 3: Maximum Likelihood Parameter Estimates of the International CAPM and of the Multivariate GARCH model for Eight Countries plus the World.
This table reports the maximum likelihood estimates of the parameters of the CAPM returns equations and of the time–varying conditional covariance process of returns. The estimation uses monthly US $ denominated excess returns on the equity indices of eight countries, Canada, Japan, France, Germany, Italy, Switzerland, United Kingdom and the United States, and on the world portfolio. Data are obtained from the MSCI database for the period January 1970 to December 1992. The estimates are reported for the CAPM parameterization of the returns equation which relates linearly country excess returns $R^e_{i,t}$ to their time–varying world covariance risk $cov(R_{it}, R_{mt}|\Im_{t-1})$, where α_i and δ are constants,

$$R^e_{i,t} = \alpha_i + \delta cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \qquad \text{where} \qquad \epsilon_t|\Im_{t-1} \sim N(0, H_t)$$

The time–varying conditional covariance H_t is modelled as

$$H_t = H_0 * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' * \Sigma_{t-1} + \mathbf{bb}' * H_{t-1},$$

where $*$ denotes the Hardmard matrix product, $\mathbf{a}, \mathbf{b}$ are $(N \times 1)$ vectors of constants, ι is a $(N \times 1)$ unit vector, and Σ_{t-1} is the matrix of cross products of error terms, $\epsilon_{t-1}\epsilon'_{t-1}$. Quasi maximum likelihood standard errors are reported in parentheses. P-values for the Liung-Box statistics of order 12 are reported for the standardized residuals $z = \epsilon/h^{1/2}$, and the standardized residuals squared $z^2 = (\epsilon/h^{1/2})^2$. The table also reports the summary statistics for the estimated risk–premia.

	Canada	Japan	France	Germ.	Italy	Switz.	U.K.	U.S.	World
α_i	-0.8986	-0.6511	-0.9980	-0.8471	-1.4742	-0.7370	-0.9449	-0.6252	-0.8472
	(0.3320)	(0.3985)	(0.3809)	(0.2830)	(0.4440)	(0.3247)	(0.3981)	(0.3125)	(0.2458)
δ					.0709				
					(.0228)				
a_i	0.1860	0.2013	0.2294	0.2786	0.2141	0.1749	0.2496	0.2294	0.2133
	(0.0468)	(0.0349)	(0.0704)	(0.0780)	(0.0729)	(0.0790)	(0.0461)	(0.0445)	(0.0551)
b_i	0.9617	0.9580	0.8797	0.7526	0.8654	0.9207	0.9421	0.9597	0.9616
	(0.0169)	(0.0150)	(0.0414)	(0.4184)	(0.0650)	(0.0257)	(0.0208)	(0.0165)	(0.0205)
$LB_{12}(z)$	.397	.021	.424	.104	.049	.923	.733	.679	.182
$LB_{12}(z^2)$	.449	.863	.602	.459	.648	.406	.882	.552	.834

Table 4: Cross–Sectional Tests of International CAPM Restrictions.

This table reports likelihood ratio tests of the cross–sectional restrictions of international asset pricing models for two specifications of the time–varying conditional covariance process of asset excess returns. The tests use monthly US $ denominated excess returns on the equity indices of eight countries, Canada, Japan, France, Germany, Italy, Switzerland, United Kingdom and the United States, and on the world portfolio. Data are obtained from the MSCI database for the period January 1970 to December 1992. The test statistics are asymptotically chi-square distributed and their p-values and degrees of freedom are reported accordingly. The returns equation relates linearly country excess returns $R^e_{i,t}$ to their time–varying world covariance risk $cov(R_{it}, R_{mt}|\Im_{t-1})$ and own country risk $var(R_{it}|\Im_{t-1})$, where α_i, δ_i, and γ_i are constants.

$$R^e_{i,t} = \alpha_i + \delta_i cov(R_{it}, R_{mt}|\Im_{t-1}) + \gamma_i var(R_{it}|\Im_{t-1}) + \epsilon_{it} \quad \text{where} \quad \epsilon_t|\Im_{t-1} \sim N(0, H_t)$$

The tests are reported for the following parametrization of the conditional covariance process H_t,

$$H_t = H_0 * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' * \Sigma_{t-1} + \mathbf{bb}' * H_{t-1}$$

where $*$ denotes the Hardmard matrix product, $\mathbf{a}, \mathbf{b}$ are $(N \times 1)$ vectors of constants, ι is a $(N \times 1)$ unit vector, and Σ_{t-1} is the matrix of cross products of error terms, $\epsilon_{t-1}\epsilon'_{t-1}$.

Null hypothesis Eq.	Restriction		Alt. hypothesis Restriction	Eq.	χ^2	df	p- level
A	$\alpha_i \neq 0$, $\delta_i = \delta$, $\gamma_i = 0$	vs.	$\alpha_i \neq 0$, $\delta_i \neq 0$, $\gamma_i = 0$	B	5.855	8	.6634
A	$\alpha_i \neq 0$, $\delta_i = \delta$, $\gamma_i = 0$	vs.	$\alpha_i \neq 0$, $\delta_i = \delta$, $\gamma_i \neq 0$	C	5.193	8	.7368
D	$\alpha_i = 0$, $\delta_i = \delta$, $\gamma_i = 0$	vs.	$\alpha_i \neq 0$, $\delta_i = \delta$, $\gamma_i = 0$	A	21.191	9	.0118
E	$\alpha_i = \alpha$, $\delta_i = \delta$, $\gamma_i = 0$	vs.	$\alpha_i \neq 0$, $\delta_i = \delta$, $\gamma_i = 0$	A	20.992	8	.0072

Table 5: Maximum Likelihood Parameter Estimates of the Conditional International CAPM with Time–Varying Price of World Covariance Risk.

This table reports the maximum likelihood estimates of the parameters of the returns equation and of the time–varying conditional covariance process of returns. The estimation uses monthly US $ denominated excess returns on the equity indices of eight countries, Canada, Japan, France, Germany, Italy, Switzerland, U.K. and United States, and on the world portfolio. Data are obtained from the MSCI database for the period January 1970 to December 1992. The estimates are reported for a parametrization of the returns equation which relates country indices excess returns $R^e_{i,t}$ to their time–varying world covariance risk $cov(R_{it}, R_{mt}|\Im_{t-1})$, where the price of covariance risk is equal to a constant plus a time–varying component conditional on investor's information Z_{t-1}. The set of predetermined information variables are: the world index dividend price ratio (DPRat), a dummy variable for the month of January (JanD). α_i, δ and κ_k are constants,

$$R^e_{i,t} = \alpha_i + (\delta + \kappa' Z_{t-1})cov(R_{it}, R_{mt}|\Im_{t-1}) + \epsilon_{it} \quad \text{where} \quad \epsilon_t|\Im_{t-1} \sim N(0, H_t)$$

The time–varying conditional covariance H_t is modelled as

$$H_t = H_0 * (\iota\iota' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' * \Sigma_{t-1} + \mathbf{bb}' * H_{t-1},$$

where $*$ denotes the Hardmard matrix product, $\mathbf{a}, \mathbf{b}$ are $(N \times 1)$ vectors of constants, ι is a $(N \times 1)$ unit vector, and Σ_{t-1} is the matrix of cross products of error terms, $\epsilon_{t-1}\epsilon'_{t-1}$. Quasi maximum likelihood standard errors are reported in parentheses.

	Canada	Japan	France	Germ.	Italy	Switz.	U.K.	U.S.	World
α_i	-1.1160	-0.9555	-1.2650	-1.0640	-1.6860	-0.9860	-1.2260	-0.8388	-1.0922
	(0.728)	(0.759)	(0.576)	(0.509)	(0.602)	(0.562)	(0.635)	(0.704)	(0.682)
δ (const)				-0.0227	(0.042)				
κ_1 (DPRat)				0.3364	(0.103)				
κ_2 (JanD)				0.0855	(0.051)				
a_i	0.1864	0.2018	0.2434	0.2737	0.2138	0.1864	0.2534	0.2371	0.2165
	(0.046)	(0.030)	(0.056)	(0.077)	(0.070)	(0.062)	(0.047)	(0.038)	(0.046)
b_i	0.9603	0.9583	0.8767	0.7686	0.8618	0.9163	0.9404	0.9569	0.9604
	(0.018)	(0.013)	(0.041)	(0.300)	(0.069)	(0.025)	(0.020)	(0.014)	(0.017)

Likelihood Function:	-5149.645
Likelihood Ratio Test of $\kappa_k = 0$, $\forall k$:	$\chi^2_2 = 8.100$, (p=.017)

Table 6: Comparison of Unconditional and Time-varying Conditional Correlations of Returns Between Seven Countries and the US

This table reports the unconditional correlations and summary statistics of the time-varying conditional correlations between the excess returns of 7 country indices and the US index over several subsamples, using the maximum likelihood estimates of the conditional covariance process reported in Table 5. Ptf denotes the equally weighted portfolio of the 7 foreign country indices.

a. Overall Sample Correlations

	Conditional Correlations				Uncond.	
Country	Mean	Std. Dev.	Minimum	Maximum	Corr.	Difference
Can	0.710	0.046	0.556	0.832	0.704	-0.006
Jap	0.280	0.090	0.066	0.554	0.277	-0.003
Fr	0.431	0.086	0.128	0.671	0.434	0.003
Ger	0.347	0.090	0.049	0.660	0.347	-0.000
It	0.232	0.078	-0.019	0.506	0.229	-0.003
Swit	0.504	0.079	0.194	0.666	0.506	0.002
U.K.	0.492	0.094	0.225	0.723	0.505	0.013
Ptf	0.574	0.069	0.335	0.765	0.580	0.005

b. Subsamples Correlations

	Can.	Japan	France	Germ.	Italy	Switz.	U.K.	Ptf
January 1970 to December 1973 Subsample (N = 48)								
Uncond.	0.758	0.464	0.193	0.372	0.035	0.439	0.429	0.511
Avg. Cond.	0.709	0.332	0.354	0.349	0.197	0.485	0.452	0.548
January 1974 to June 1983 Subsample (N = 114)								
Uncond.	0.682	0.247	0.439	0.322	0.211	0.515	0.464	0.582
Avg. Cond.	0.697	0.263	0.437	0.336	0.222	0.498	0.454	0.568
July 1983 to December 1992 Subsample (N = 114)								
Uncond.	0.749	0.257	0.497	0.363	0.299	0.518	0.611	0.596
Avg. Cond.	0.722	0.275	0.457	0.358	0.256	0.519	0.546	0.592
Difference between subsample 2 and subsample 3 estimated correlations								
Uncond.	0.067	0.010	0.059	0.041	0.088	0.003	0.147	0.013
Avg. Cond.	0.025	0.011	0.020	0.022	0.034	0.021	0.092	0.025

c. U.S. Bear vs Bull Markets Correlations

	Can.	Japan	France	Germ.	Italy	Switz.	U.K.	Ptf
Months During which the U.S. experience a bear market (N = 82)								
Uncond.	0.649	0.406	0.407	0.477	0.150	0.542	0.513	0.594
Avg. Cond.	0.708	0.312	0.415	0.376	0.204	0.514	0.478	0.579
Months During which the U.S. is not in a bear market (N = 194)								
Uncond.	0.666	0.150	0.362	0.214	0.198	0.426	0.453	0.492
Avg. Cond.	0.711	0.266	0.438	0.335	0.243	0.501	0.497	0.573
Difference between bear market months and other months								
Uncond.	-0.017	0.256	0.045	0.262	-0.048	0.116	0.060	0.102
Avg. Cond.	-0.003	0.046	-0.023	0.041	-0.040	0.013	-0.019	0.006

Table 7: Up and Down Markets for US Equities

This table reports the periods during which the US equity index experienced up and down markets and the total index return during those periods. The Up and Down Markets are based on the definitions of *Forbes* magazine. Up markets start at the beginning of the month following the month in which the maket index reaches the bottom of a through and end in the month the market index reaches a peak. Down markets start at the beginning of the month following at a market peak and end at the end of the month in which the markets reach the bottom of the through.

a. Up and Down Markets

Down markets			Up markets		
Start Date	End Date	Total Return	Start date	End Date	Total return
Jan. 1, 70	June 30, 70	-18.33%	July 1, 70	Dec. 31, 72	69.82%
Jan. 1, 73	Sep. 30, 74	-45.06	Oct. 1, 74	Dec. 31, 76	84.58
Jan. 1, 77	Feb. 28, 78	-15.17	Mar. 1, 78	Nov. 30, 80	77.17
Dec. 1, 80	Jul. 31, 82	-15.52	Aug. 1, 82	June 30, 83	62.32
Jul. 1, 83	Jul. 31, 84	-6.01	Aug. 1, 84	Aug. 31, 87	143.34
Sep. 1, 87	Nov. 30, 87	-29.42	Dec. 1, 87	May 31, 90	69.49
Jun. 1, 90	Oct. 31, 90	-14.05	Nov. 1, 90	Dec 31, 92	54.46

b. Largest Negative Excess Returns Months for the US Index.

Rank	Month	Return	Rank	Month	Return	Rank	Month	Return
1	Oct. 87	-21.58%	4	Mar. 80	-10.08	7	Apr. 70	-9.17
2	Sep. 74	-12.00	5	Aug. 90	-9.75	8	Oct. 78	-9.15
3	Nov. 73	-11.13	6	Aug. 74	-9.70	9	Sept. 86	-8.92

Fig.1 Price of Covariance Risk (with stderr bounds)

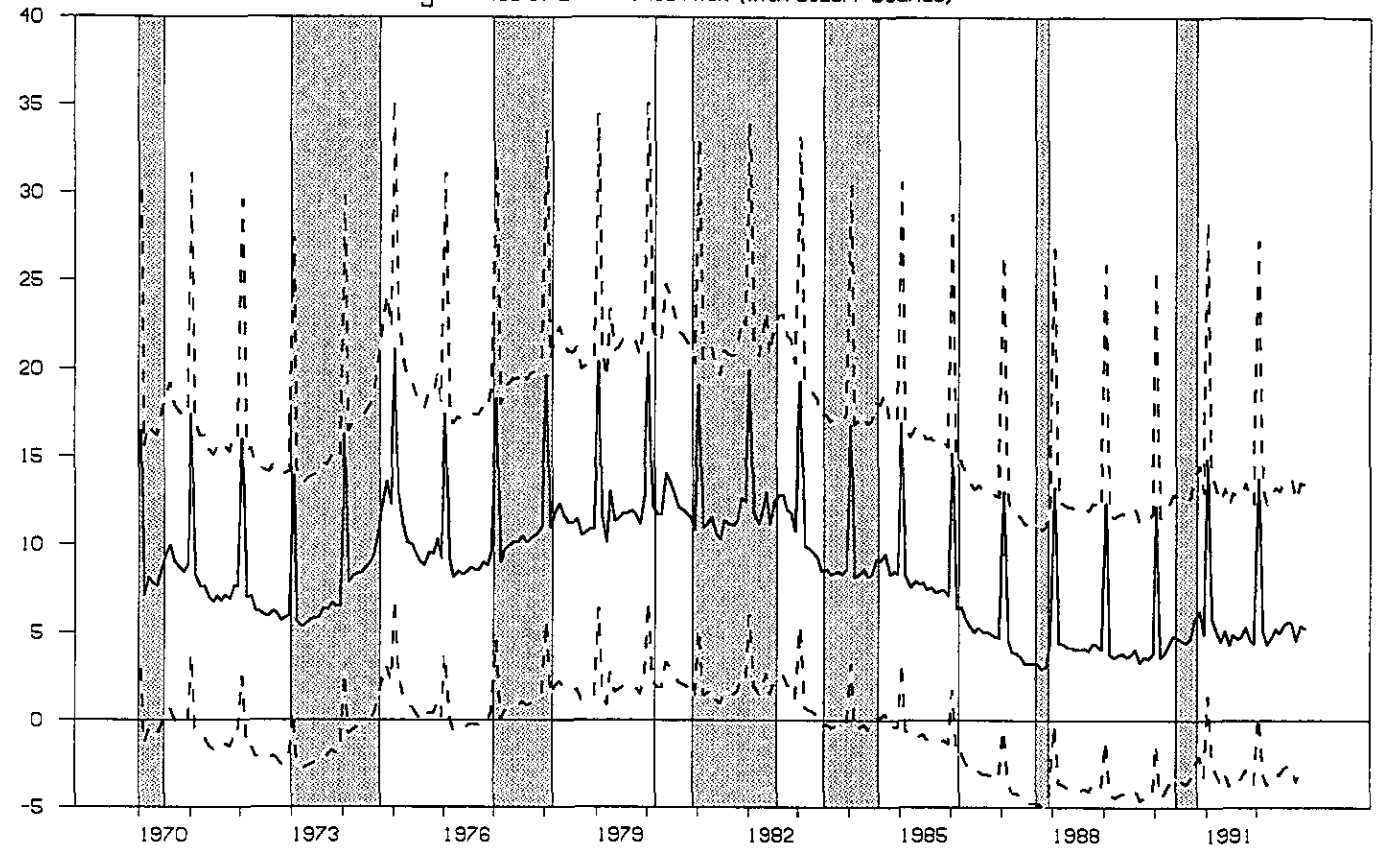

Fig.2 Average Correlation with U.S.

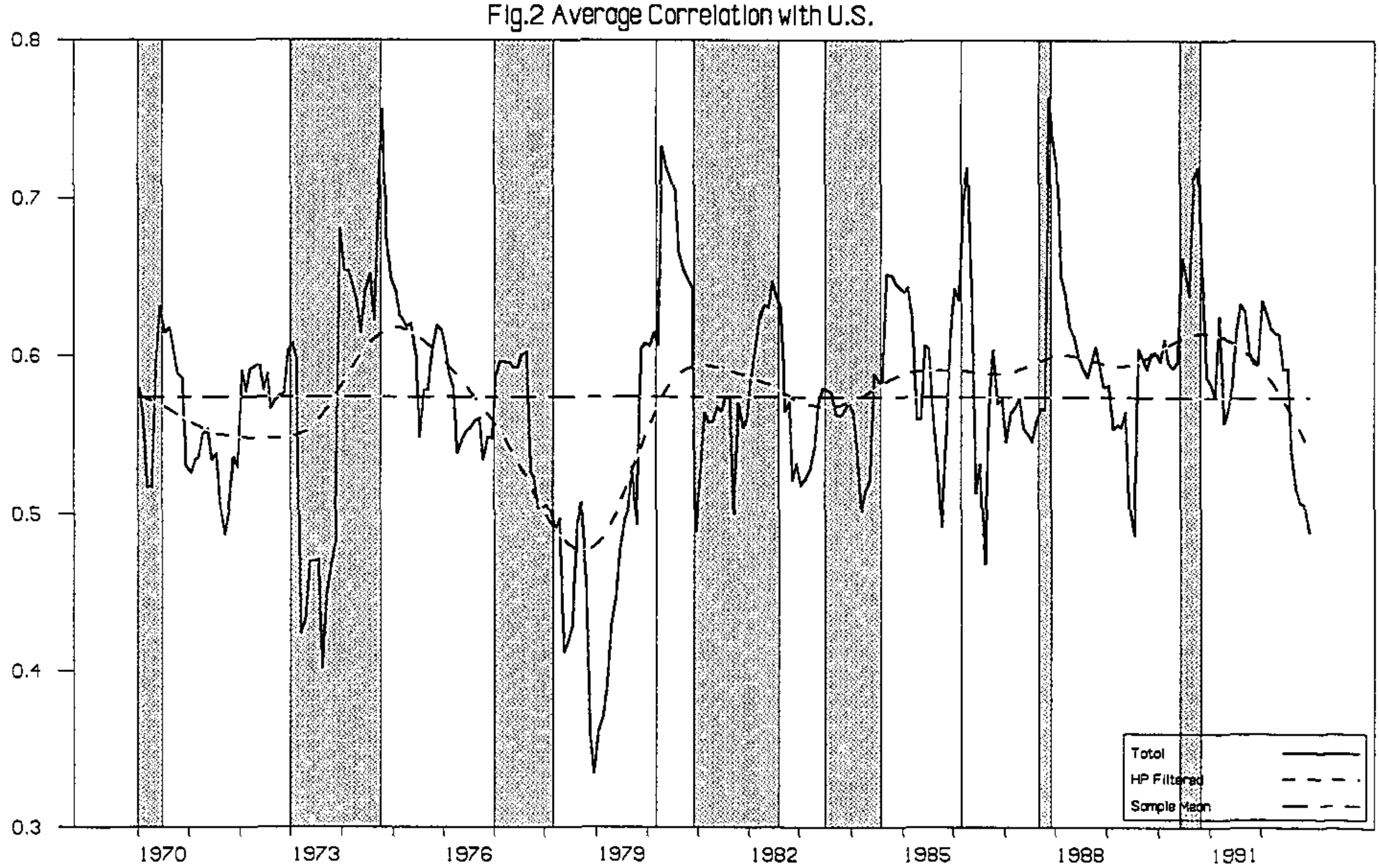

Fig.3 Expected Gains from International Diversification