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THE EFFECTS OF STOCHASTIC INFLATION ON ASSET PRICES

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ABSTRACT

Stochastic inflation affects the risk characteristics, measured by the equity premium and the correlation of the equity's return with consumption, in a fundamental way. The riskiness of a dollar–denominated asset depends on two conditional covariances: the covariance of the marginal rate of substitution (MRS) with the equity price and the covariance of the MRS with the rate of appreciation in the purchasing power of money. The second covariance may take either sign which becomes significant when the risk characteristics of the dollar–denominated asset are compared with the risk characteristics of an indexed asset constructed in a real version of the model.

The effects of stochastic inflation on the assets' risk characteristics are studied in a parameterized version of a cash–in–advance asset–pricing model. The growth rates of the endowment and monetary transfer evolve according to a VAR. The equity price is a geometric distributed lead of log–normally distributed random variables; an algorithm to express the price as an explicit function of the state variables is described.

How are asset prices affected by stochastic inflation? To answer this question, the risk characteristics of several assets are studied in real and monetary versions of a pure exchange, representative agent, asset–pricing model. The risk characteristics of an asset are summarized by the equity premium, which measures the difference between the equity return and the return to a risk–free asset, and by the β–coefficient, which measures the correlation of the equity return with the marginal rate of substitution in consumption.

Money is incorporated by way of a cash–in–advance constraint. Stochastic monetary transfers affect the real value of dollar–denominated assets by way of an inflation tax. Risk averse agents assess the inflation tax in their decision making which affects the determination of the equilibrium asset prices. If the cash–in–advance constraint binds resulting in unitary velocity, the random inflation tax is the only mechanism by which random money growth can affect real equilibrium asset prices in the model described in Section 2. The effects of the inflation tax on asset prices are determined by studying the assets' risk characteristics.

The effect of the inflation tax on asset prices is illustrated by comparing and simulating real and monetary versions of the model. The simulations show that, for identical endowment streams, the expected equity return in the monetary model is uniformly less than the expected equity return in the real model. Moreover, the monetary model's equity premium when compared to the real model is larger and displays much greater volatility. These results hold when the conditional covariance of the marginal rate of substitution (MRS) with the rate of appreciation in the purchasing power of money is positive.

A key variable that determines the behavior of the risk premium and the β–coefficient is the conditional covariance of the intertemporal marginal rate of substitution with the rate of appreciation in the purchasing power of money. This covariance can take either sign depending on the sign of the conditional covariance of the contemporaneous endowment and monetary shocks. Because the conditional covariance of the MRS with the rate of appreciation in the purchasing power of money can take either sign, the β–coefficient of the dollar–denominated asset may be smaller (in absolute value) than the β–coefficient of the inflation–indexed asset (in absolute value), other things being constant.

The risk characteristics of a nominal bond, a bond that pays with certainty one unit of the currency one period hence, are also examined. The conditional covariance of the MRS with the rate of appreciation in the purchasing power of money determines the risk premium of the nominal bond. The time–varying risk premium affects the Fisher equation in important ways. The inflation premium can be quite large; for example the simulation summarized in Table 5 results in average inflation premiums of 1.37 percent.

The effect of stochastic inflation on the risk characteristics of several assets is described in Section 1. Determining the magnitude of these effects is difficult without formulating an explicit model. A model is described in Section 2; it is a parameterized version of the models devised by Lucas (1978,1980,1982). There is a representative agent with preferences described by a time–additive, isoelastic utility function. In the monetary version, the equity holder receives at the end of each period a dollar–denominated dividend whose return is subject to two sources of uncertainty: endowment uncertainty and a random inflation tax. The growth rate of the endowment good and the monetary transfer evolve over time according to a covariance stationary bivariate autoregressive process. A recursive scheme is devised in order to express the equity price and its conditional expected value as explicit functions of the state variables. The solution method can be used to evaluate geometric distributed leads of log–normally distributed random variables. The equity price and its return are not log–normally distributed and the conditional variance of the equity price is heteroscedastic and displays a motion over time that cannot be described by a smooth autoregressive process. The model and the solution method are described in Section 2. Comparative dynamics and simulations are reported in Section 3.

The effects of stochastic inflation on stock prices have been studied by Fama (1983), Fama and Gibbons (1983), LeRoy (1984a,1984b), and Stulz (1986). The effect of time–varying inflation on the Fisher equation has been studied by LeRoy (1984a,1984b) and Svensson (1985). The point that inflation affects the underlying risk characteristics of a stock has been made by Stulz (1986).

I. Asset Returns and Inflation

The main question to be addressed is: How does stochastic inflation affect the risk characteristics of an asset? The risk characteristics are defined as the risk premium, which is the difference between the asset's return and the return to a certain risk–free asset, and the correlation of the asset's return with consumption. The risk characteristics are determined by studying the empirical implications of the intertemporal, discrete–time, representative agent model devised by Hansen, Richard, and Singleton (1981). The price and return of two special assets, an asset whose return is perfectly correlated with the marginal rate of substitution and an asset whose return displays zero correlation, are used to characterize the risk premiums of other assets and to evaluate the effect of inflation on stock returns.

When the assets are in zero net supply, a discrete–time, asset–pricing model with an infinite–lived, representative agent and an exogenous endowment will result in an equilibrium asset–pricing equation of the form

$$U'(y_t)\bar{q}_{it} = \beta E_t U'(y_{t+1})(\bar{q}_{it+1}+\bar{d}_{it+1}),\ 0 < \beta < 1 \tag{1.1}$$

where y_t is the exogenous endowment at time t, $U'(y_t)$ is the diminishing marginal utility from consuming the endowment, $\bar{q}_{it}$ is the equilibrium real price of equity i at time t, E_t is the expectations operator conditioned on information at time t, and $\bar{d}_{it}$ is the real dividend paid to the holder of asset i that is available for consumption in period t. An example of this type of model is provided in the next section. The ex post real return to equity i is

(1.2) $$1 + \overline{R}^i_{t+1} = (\overline{q}_{it+1}+\overline{d}_{it+1})(\overline{q}_{it})^{-1}.$$

If the time t dividends are paid in dollars at the end of the period and are unavailable for consumption expenditures until the next period, the real value of the dividend will depend on the inflation shock between t and t + 1. Let π_{t+1} denote the gross inflation rate at t + 1; the real value of the dividend at t + 1 is $\pi^{-1}_{t+1}d_{jt}$ and the equilibrium condition is

(1.3) $$U'(y_t)q_{jt} = \beta E_t\left[U'(y_{t+1})(q_{jt+1}+\pi^{-1}_{t+1}d_{jt})\right]$$

where q_{jt} denotes the equilibrium price of equity j when the dividends are paid in dollars. The return is

(1.4) $$1 + R^j_{t+1} = (q_{jt+1}+\pi^{-1}_{t+1}d_{jt})(q_{jt})^{-1}.$$

Two special assets, a perfectly–correlated asset and a zero–correlated asset are introduced next. Let S_{t+1} denote the marginal rate of substitution

(1.5) $$S_{t+1} = \beta\frac{U'(y_{t+1})}{U'(y_t)}.$$

An asset that is perfectly correlated with the marginal rate of substitution has a price q^s_t that satisfies

(1.6) $$q^s_t = E_tS^2_{t+1}$$

in equilibrium. Let $(1+R^s_{t+1})$ be the return to this asset so that

(1.7) $$1 + R^s_{t+1} = S_{t+1}(E_tS^2_{t+1})^{-1}.$$

An asset that displays zero correlation with S pays one unit of the consumption good with certainty at time t + 1 and has a price q_t^0, where

$$q_t^0 = E_t S_{t+1}, \tag{1.8}$$

in equilibrium. Its return $(1+R_{t+1}^0)$ satisfies

$$1 + E_t R_{t+1}^0 = (E_t S_{t+1})^{-1}.$$

Hansen, Richard, and Singleton (1981) show that the intertemporal CAPM implies:

$$E_t[R_{t+1}^S - R_{t+1}^0] = -\frac{\text{var}_t\,(R_{t+1}^s)}{E_t(1+R_{t+1}^s)} \tag{P1}$$

$$E_t[R_{t+1}^i - R_{t+1}^0] = -\frac{\text{cov}_t(R_{t+1}^i, R_{t+1}^s)}{E_t[1+R_{t+1}^s]} \tag{P2}$$

$$= \beta_t^i E_t[R_{t+1}^s - R_{t+1}^0]$$

where

$$\beta_t^i = \frac{\text{cov}_t(R_{t+1}^i, R_{t+1}^s)}{\text{var}_t\,(R_{t+1}^s)}, \tag{1.9}$$

and cov_t and var_t denote the conditional covariance and conditional variance, respectively.

The coefficient β_t^i measures the riskiness of an asset because it measures the correlation of the asset's return with consumption. An asset is risky if the negative covariance of the asset return with the marginal rate of substitution is large in absolute value. The perfectly correlated asset has a β–coefficient equal to unity whereas the zero–correlated asset has a zero β–coefficient since, by definition, the covariance of S with R^0 is zero. Notice from (P2) that the risk premium for the perfectly correlated asset is negative since $E_t(1+R^S_{t+1})$ is positive.[1] If asset i has a negative β–coefficient, $cov_t(R^i_{t+1},R^s_{t+1})$ is negative and the asset is risky in the sense that the conditional risk premium $E_t[R^i_{t+1}-R^0_{t+1}]$ is positive. If asset i has a positive β–coefficient, the $cov_t(R^i_{t+1},R^s_{t+1})$ is positive and the asset's conditional premium $E_t[R^i_{t+1}-R^0_{t+1}]$ is negative. This means that the asset is a good hedge against future uncertainty.

The equity premiums and the risk characteristics of the monetary and real models are now compared.

An equity that is a claim to the nominal endowment stream $\{p_{t+j}y_{t+j};j=0,...\}$ paid at the end of the period, where p_{t+j} is the nominal price per unit of the endowment and y_{t+j} is the endowment, has a real price q_t that satisfies

$$U'(y_t)q_t = \beta E_t U'(y_{t+1})(q_{t+1}+\pi^{-1}_{t+1}y_t) \tag{1.10}$$

and a return

$$1 + R^q_{t+1} = (q_{t+1}+\pi^{-1}_{t+1}y_t)(q_t)^{-1}. \tag{1.11}$$

Substituting (1.11) into the equilibrium first order condition (1.10) results in

$$1 = E_t\left[S_{t+1}(1+R^q_{t+1})\right]. \tag{1.12}$$

This expression can be rewritten using the covariance as

$$E_t(1+R^q_{t+1}) = E_t(1+R^0_{t+1})\left[1-cov_t(S_{t+1},R^q_{t+1})\right]$$

so that the conditional equity premium is

$$\text{(1.13)} \qquad E_t[R^q_{t+1} - R^0_{t+1}] = -(1+E_tR^0_{t+1})\mathrm{cov}_t(S_{t+1}, R^q_{t+1}).$$

From (P2), the conditional equity premium is also

$$\text{(1.14)} \qquad E_t[R^q_{t+1} - R^0_{t+1}] = -\frac{\mathrm{cov}_t[R^q_{t+1}, R^s_{t+1}]}{E_t[1+R^s_{t+1}]}.$$

After some simplification, the conditional covariance of the equity return and the return to the perfectly correlated asset is

$$\text{(1.15)} \quad \mathrm{cov}_t(R^q_{t+1}, R^s_{t+1}) = [q_t E_t S^2_{t+1}]^{-1}\left[y_t \mathrm{cov}_t(S_{t+1}, \pi^{-1}_{t+1}) + \mathrm{cov}_t(q_{t+1}, S_{t+1})\right].$$

Since

$$\text{(1.16)} \qquad E_t(1+R^s_{t+1}) = E_t S_{t+1}(E_t S^2_{t+1})^{-1},$$

substituting (1.15) and (1.16) into (1.14) results in

$$\text{(1.17)} \qquad E_t[R^q_{t+1} - R^0_{t+1}] = -\frac{[q_t E_t S^2_{t+1}]^{-1}[y_t \mathrm{cov}_t(S_{t+1}, \pi^{-1}_{t+1}) + \mathrm{cov}_t(q_{t+1}, S_{t+1})]}{E_t S_{t+1}(E_t S^2_{t+1})^{-1}}$$

$$= -(q_t)^{-1}[1+E_tR^0_{t+1}]\left[y_t \mathrm{cov}_t(S_{t+1}, \pi^{-1}_{t+1}) + \mathrm{cov}_t(q_{t+1}, S_{t+1})\right].$$

By substituting (1.17) into (1.13), the conditional covariance of the return with the marginal rate of substitution is expressed as

$$\text{(1.18)} \qquad \mathrm{cov}_t(S_{t+1}, R^q_{t+1}) = (q_t)^{-1}\left[y_t \mathrm{cov}_t(S_{t+1}, \pi^{-1}_{t+1}) + \mathrm{cov}_t(q_{t+1}, S_{t+1})\right].$$

The expression $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1})$ is the conditional covariance of the marginal rate of substitution with the rate of appreciation in the purchasing power of money. The conditional equity premium is a decreasing function of the conditional covariances on the right–hand side of (1.18). The sign of $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1})$ can be positive or negative. To see this, notice that, by assumption, the marginal utility $U'(y_{t+1})$ is decreasing in y_{t+1} so that S_{t+1} is decreasing in y_{t+1}. The rate of appreciation in the purchasing power of money π_{t+1}^{-1} generally has a positive partial correlation with output and a negative partial correlation with the monetary transfer. Since the monetary transfer and the endowment may be correlated, as in the case of a negatively–sloped Phillips curve, the simple correlation of the endowment with the rate of appreciation in the purchasing power of money may be positive or negative. Hence, the sign of $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1})$ can be positive or negative. For the model described in the next section, the sign of $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1})$ is shown to depend on the sign of the covariance of the contemporaneous endowment and monetary shocks and on the size of this covariance relative to the variance of the endowment shock. If the contemporaneous covariance is positive, so that a large endowment shock tends to coincide with a large monetary transfer, as in the case of a negatively–sloped Phillips curve, and if it is large in absolute value relative to the variance of the endowment shock, the covariance $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1})$ is positive. If the covariance is positive, a dollar–denominated dividend that comes in periods of low marginal utility also tends to come in a period when the inflation tax is high, other things being constant. The converse of this statement is also true. An increase in the covariance $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1})$ will decrease the conditional equity premium (1.17), other things being constant.

The other important covariance in (1.18), $\mathrm{cov}_t(S_{t+1},q_{t+1})$, is a complicated function of the model's state variables and parameters. For a markov economy, the effect of changes in the endowment on the asset price is generally indeterminate; this indeterminacy occurs because of the information about the future that is signaled by the current endowment realization.[2]

The effect of stochastic inflation on asset returns is illustrated by comparing the equity that is a claim to the nominal endowment stream to the equity that is a claim to the real endowment stream. An equity that is a claim to the real endowment stream $\{y_{t+j}, j=1,2,...\}$ has an equilibrium price $\overline{q}_t$ that satisfies

$$U'(y_t)\overline{q}_t = \beta E_t U'(y_{t+1})(\overline{q}_{t+1}+y_{t+1}) \tag{1.19}$$

and a return

$$1 + \overline{R}^q_{t+1} = (\overline{q}_{t+1}+y_{t+1})(\overline{q}_t)^{-1}. \tag{1.20}$$

The conditional equity premium for the real endowment model is

$$E_t[\overline{R}^q_{t+1}-R^0_{t+1}] = -(1+R^0_{t+1})\mathrm{cov}_t(1+\overline{R}^q_{t+1}, S_{t+1}) \tag{1.21}$$

$$= -\frac{\mathrm{cov}_t(1+\overline{R}^q_{t+1}, R^s_{t+1})}{E_t(1+R^s_{t+1})}$$

where the second equality uses (P2). The conditional covariance is

$$\mathrm{cov}_t(\overline{R}^q_{t+1}, R^s_{t+1}) = [\overline{q}_t E_t S^2_{t+1}]^{-1}\left[\mathrm{cov}_t(S_{t+1}, \overline{q}_{t+1}) + \mathrm{cov}_t(S_{t+1}, y_{t+1})\right]. \tag{1.22}$$

Substituting $(1+\overline{R}^q_{t+1})$ and (1.21) into (1.22) results in

$$E_t[\overline{R}^q_{t+1}-R^0_{t+1}] = -(\overline{q}_t)^{-1}(1+R^0_{t+1})\left[\mathrm{cov}_t(S_{t+1}, \overline{q}_{t+1}) + \mathrm{cov}_t(S_{t+1}, y_{t+1})\right]. \tag{1.23}$$

The most striking difference between the conditional risk premiums of the two models (1.23) and (1.17) is reflected in the conditional covariances of the marginal rate of substitution with the time (t+1) real value of the dividend. The conditional covariance

$cov_t(S_{t+1},y_{t+1})$ is always negative, whereas the conditional covariance $cov_t(S_{t+1},\pi_{t+1}^{-1})$ can take either sign, as discussed earlier.

The first important result is that the β–coefficients for the two types of equities, one of which is a claim to the nominal endowment stream and the other of which is a claim to the real endowment stream, are different. In particular, the dollar–denominated equity may act as a hedge against future uncertainty in the sense that its β–coefficient may be smaller in absolute value than the corresponding β–coefficient for the real endowment model.

In an economy that has many dollar–denominated assets, there is a distribution of asset returns at each point in time with each asset differing in riskiness as measured by its β–coefficient. The β–coefficient can take either sign as can the correlation of the asset's return with inflation. To see this, consider a monetary model with ℓ–types of assets that are in zero net supply with dollar–denominated dividend payments. The equilibrium first order condition for asset i is (1.3) and the return is (1.4). From (1.9),

$$(1.24) \qquad E_t[R_{t+1}^i - R_{t+1}^0] = \beta_t^i E_t[R_{t+1}^s - R_{t+1}^0]$$

$$= -(q_{it})^{-1}[1+E_t R_{t+1}^0]\Big[d_{it} cov_t(S_{t+1},\pi_{t+1}^{-1}) + cov_t(q_{it+1},S_{t+1})\Big].$$

As described earlier, the two conditional covariances in (1.25) can take either sign.

As the $cov_t(S_{t+1},\pi_{t+1}^{-1})$ increases, all else constant, the β^i–coefficient will increase. Consider the following four examples of the relationship between inflation and the equity's return.

<u>Case I</u>: $cov_t(S_{t+1},\pi_{t+1}^{-1}) > 0$.

In this case, periods when marginal utility is below its conditional mean (higher than expected endowment) are likely to occur when inflation is above its conditional mean.

Case (Ia) $\text{cov}_t(R^i_{t+1}, R^S_{t+1}) > 0$.

The equity return tends to be below its conditional mean when the endowment is above its conditional mean. Since $\text{cov}_t(S_{t+1}, \pi^{-1}_{t+1})$ is positive, the equity return and inflation will tend to be negatively correlated. The β–coefficient for the asset is positive which corresponds to a negative equity premium. The asset is a hedge against future uncertainty despite the negative correlation between inflation and the equity return.

Case (I.b) $\text{cov}_t(R^i_{t+1}, R^S_{t+1}) < 0$.

The return to asset i tends to be above its conditional mean when the endowment is above its conditional mean (S_{t+1} and hence R^s_{t+1} are below their conditional means). Since the endowment and inflation are positively correlated, the equity's return and inflation tend to be positively correlated. The β–coefficient for the asset is negative and its risk premium is positive so that the asset is risky despite the positive correlation between inflation and the equity return.

<u>Case II</u>: $\text{cov}_t(S_{t+1}, \pi^{-1}_{t+1}) > 0$.

Periods when marginal utility is above its conditional mean (lower than expected endowment) tend to occur when inflation is above its conditional mean.

Case (II.a) $\text{cov}_t(R^i_{t+1}, R^s_{t+1}) > 0$.

Since inflation and the endowment tend to be negatively conditionally correlated, the positive conditional covariance of the returns will result in a positive correlation between the equity return and inflation. The β–coefficient is positive and the risk premium is negative so that the asset is a hedge against future uncertainty.

Case (II.b) $\text{cov}_t(R^i_{t+1}, R^s_{t+1}) < 0$.

Since the equity return tends to be above its conditional mean when the endowment is above its conditional mean, the β-coefficient is negative and risk premium is positive. The equity's return is negatively correlated with inflation.

The underlying assumption in the four examples is this: since the sign of $\text{cov}_t(R^s_{t+1}, \pi_{t+1})$ is the same as the sign of $\text{cov}_t(S_{t+1}, \pi_{t+1})$, and since the sign of $\text{cov}_t(R^i_{t+1}, R^s_{t+1})$ is the same as the sign of $\text{cov}_t(R^i_{t+1}, S_{t+1})$, the sign of $\text{cov}_t(R^i_{t+1}, \pi_{t+1})$ can be determined indirectly. In fact, there are other possibilities and so this list is not exhaustive.

The covariance of the equity return $(1+R^q)$ with inflation in the single–asset real and monetary versions of the model described in Section 2 are computed explicitly and reported in Section 3.

Nominal Bonds

A nominal asset of particular interest is a one period bond that pays with certainty one unit of the currency one period hence. The nominal price B_t of the bond satisfies

$$U'(y_t)B_t = \beta E_t\left[U'(y_{t+1})\pi^{-1}_{t+1}\right]$$

and the nominal interest rate is

$$1 + i_t = (B_t)^{-1}.$$

The expected real return is

$$\begin{aligned} 1 + E_t r_{t+1} &= (1+i_t)E_t \pi^{-1}_{t+1} \\ &= E_t \pi^{-1}_{t+1}[E_t S_{t+1} \pi^{-1}_{t+1}]^{-1}. \end{aligned}$$

The risk premium of the nominal bond is defined by

$$E_t[r_{t+1}-R^0_{t+1}] = \left[\frac{E_t\,\pi^{-1}_{t+1}}{E_t S_{t+1}\,\pi^{-1}_{t+1}} - \frac{1}{E_t S_{t+1}}\right]$$

$$= \beta^r_t E_t[R^s_{t+1}-R^0_{t+1}].$$

The β-coefficient for the nominal bond equals zero only if the marginal rate of substitution and the rate of appreciation in the purchasing power of money are uncorrelated.

The sign of β^r_t depends on the sign of $cov_t(r_{t+1},R^s_{t+1})$. Upon examining this covariance more carefully, one finds that

$$cov_t[r_{t+1},R^s_{t+1}] = cov_t(S_{t+1},\pi^{-1}_{t+1})[E_t S^2_{t+1} E_t S_{t+1}\pi^{-1}_{t+1}]^{-1}$$

and so it becomes apparent that the β-coefficient and hence the risk premium have the same sign as $cov_t(S_{t+1}\pi^{-1}_{t+1})$. In general, the risk premium on the nominal bond is time–varying; this has important implications for extracting real interest rates from nominal rates by use of a Fisher equation.

The effect on the Fisher equation of the presence of a time–varying risk premium is summarized in three cases:

<u>Case 1</u>. $cov_t(S_{t+1},\pi^{-1}_{t+1}) > 0$.

The covariance $cov_t(r_t,R^s_{t+1})$ is positive, the risk premium is negative and the β-coefficient is positive so that the nominal bond is a hedge against future uncertainty relative to the zero–correlated asset. Approximating the zero–correlated asset return by subtracting ex post inflation from the nominal interest will <u>underpredict</u> the real return R^0_{t+1}.

Case 2. $\text{cov}_t(S_{t+1},\pi_{t+1}^{-1}) = 0$.

In this case, the β-coefficient and risk premium equal zero. Computing R_{t+1}^0 by use of the Fisher equation will result in a good approximation.

Case 3. $\text{cov}_t(S_{t+1},\pi_{t+1}^{-1}) < 0$.

The covariance $\text{cov}_t(r_t,R_{t+1}^s)$ is negative and hence the β-coefficient is negative and risk premium is positive. The nominal bond is riskier than the zero-correlation asset. Approximating the zero-correlated asset return by use of the Fisher equation will overpredict the real return R_{t+1}^0.

The time series properties of the risk premium for the nominal bond are described in Section 3 for a particular time series model of the endowment and monetary shocks and parameterization of preferences.

2. The Model and the Solution Method

Two versions of a pure endowment representative agent model are described in this section. In the first version, currency is incorporated by way of a cash-in-advance constraint. The dividend is paid in units of currency. The second version is a real model in which the dividend is paid in units of the endowment good. In both instances, the equilibrium equity price is a geometric distributed lead of log-normally distributed random variables. An iterative scheme to express the equity price as an explicit function of the model's state variables is described.

The growth rate of the nonstorable endowment and the growth rate of the monetary transfer evolve over time according to a covariance stationary, bivariate autoregressive process. The innovations are normally distributed; as a result the levels of the endowment shock and the monetary transfer are log-normally distributed. The process is specified fully below.

Each member of the identical and fixed population maximizes an isoelastic time–additive utility function over an infinite planning horizon. The representative agent carries wealth from the previous period in the form of equities, which are fractional claims to the current and future dollar–denominated dividend stream, and in the form of currency M_{t-1}. All variables are expressed as per capita.

At the beginning of each period and prior to any trading, the realizations of the monetary and endowment shocks, $(1+w)$ and λ respectively, are observed by all; the lump–sum monetary transfer $w_t M_{t+1}$ is made. The agent's post–transfer, pre–trade currency holdings are

$$M_t = (1+w_t)M_{t-1} \tag{2.1}$$

where $(1+w_t)$ is the current realization of the stochastic money growth rate. Let $\phi_t = (1+w_t)^{-1}$ for notational convenience.

The endowment at time t, denoted as y_t, evolves over time according to

$$y_t = \lambda_t y_{t-1} \tag{2.2}$$

where λ_t is the endowment shock at t.

The exchange of currency, equities, and goods takes place in two phases. In the first trading phase, currency holdings and asset holdings are adjusted; the agent divides his post–transfer wealth between currency M_t^D, where each currency unit has value $(p_t)^{-1}$ in units of the time t endowment, and equity shares z_t where the real equity price at time t is q_t.

Goods trading and nominal dividend collection takes place during the second trading phase. Consumption purchases must be financed with currency acquired in the first trading phase; the constraint is

(2.3) $$p_t c_t \leq M_t^D$$

where c_t is real consumption purchases. The dollar–denominated dividends collected are $p_t z_t y_t$ and, by construction, these funds are unavailable for spending in the current period. If the constraint (2.3) is binding, the dollar denominated, pre–transfer, pre–trade wealth at $t + 1$ is

(2.4) $$z_t[p_t y_t + q_{t+1} p_{t+1}]$$

and the post–transfer, pre–trade nominal wealth is

(2.5) $$z_t[p_t y_t + q_{t+1} p_{t+1}] + w_{t+1} M_t.$$

The maximization problem solved by the representative agent and the equilibrium conditions can now be stated fully and precisely. The representative agent solves

$$\max_{\{z_t\,,\, c_t, M_t^D\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\gamma}\right) c_t^{1-\gamma} \right\}, \; 0 < \beta < 1, \; \gamma > 0$$

subject to the constraint (2.3) and

(2.6) $$z_{t-1}\left[\frac{y_{t-1}p_{t-1}}{p_t} + q_t\right] + \left[\frac{M_{t-1}^D - p_{t-1}c_{t-1}}{p_t}\right] + \frac{w_t M_{t-1}}{p_t} \geq \frac{M_t^D}{p_t} + z_t q_t$$

where E_t denotes the expectations operator conditioned on information at time t. The first order conditions are summarized by

$$c_t^{-\gamma} q_t = \beta E_t\left[c_{t+1}^{-\gamma}\right]\left[z_t(\pi_{t+1}^{-1} y_t + q_{t+1})\right].$$

There are three market–clearing conditions. The first is that all output is consumed, or $c_t = y_t$. The second is that all claims are held, or $z_t = 1$. The third is that all money is held at the end of the period. If the constraint (2.3) is binding, M_t^D equals M_t in equilibrium.

Let q(.) and p(.) denote the equilibrium equity price function and the equilibrium consumption goods price function and let s_t denote the state vector at time t. If markets are cleared and if the constraint (2.3) is binding, the equilibrium consumption goods price function is

(2.7) $$p_t = p(y_t, M_t) = \frac{M_t}{y_t}.$$

A binding constraint will result if nominal interest rates are nonnegative or equivalently the price of the nominal bond

$$B_t = \beta E_t[\lambda_{t+1}^{1-\gamma}\phi_{t+1}]$$

is less than one. This imposes some restrictions on the parameter space which are difficult to determine analytically; some conditions are described in the Appendix. The equilibrium first order condition is

(2.8) $$y_t^{-\gamma} q(s_t) = \beta E_t y_{t+1}^{-\gamma}\left[\pi_{t+1}^{-1} y_t + q(s_{t+1})\right]$$

$$= \beta E_t y_{t+1}^{-\gamma}\left[y_{t+1}\phi_{t+1} + q(s_{t+1})\right]$$

where the second equality follows from (2.1), (2.2), and (2.7).

The equilibrium equity price function is assumed to be a fixed function of the current state vector. Solving for an explicit equity price function requires the evaluation of the conditional expectation in (2.8). To this end, the following assumption for the exogenous state variables is made.

(A1) Assumption. The motion of $(\ell n\ \lambda, \ell n\ \phi)$ is described by the bivariate system

$$\begin{bmatrix} \ell n\ \lambda_{t+1} \\ \ell n\ \phi_{t+1} \end{bmatrix} = \begin{bmatrix} \delta_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} \delta_1 & \eta_1 \\ \psi_1 & \theta_1 \end{bmatrix} \begin{bmatrix} \ell n\ \lambda_t \\ \ell n\ \phi_t \end{bmatrix} + \begin{bmatrix} \delta_2 & \eta_2 \\ \psi_2 & \theta_2 \end{bmatrix} \begin{bmatrix} \ell n\ \lambda_{t-1} \\ \ell n\ \phi_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \delta_3 & \eta_3 \\ \psi_3 & \theta_3 \end{bmatrix} \begin{bmatrix} \ell n\ \lambda_{t-2} \\ \ell n\ \phi_{t-2} \end{bmatrix} + \begin{bmatrix} v_t \\ u_t \end{bmatrix}.$$

Let $\ell n\ \overline{\lambda}$ and $\ell n\ \overline{\phi}$ denote the deviation from the conditional mean of $\ell n\ \lambda$ and $\ell n\ \phi$, respectively. The bivariate process $(\ell n\ \overline{\lambda}, \ell n\ \overline{\phi})$ is assumed to have a Wold moving average representation

(2.10)
$$\begin{bmatrix} \ell n\ \overline{\lambda}_t \\ \ell n\ \overline{\phi}_t \end{bmatrix} = C(L) \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix}$$

where z_{1t}, z_{2t} are jointly fundamental, L denotes the lag operator and C is a matrix polynomial in L. Furthermore, $z_t' = [z_{1t}, z_{2t}]$ is normally distributed with variance–covariance matrix

(2.11)
$$Ez_t'z_t = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}.$$

The linear least squares projection errors are

(2.12)
$$\ell n\ \lambda_t - P\left[\ell n\ \lambda_t \mid \ell n\ \lambda_{t-1}, \ldots, \ell n\ \phi_{t-1}, \ldots\right] = c_{11}z_{1t} + c_{12}z_{2t} = v_t$$

$$\ell n\ \phi_t - P\left[\ell n\ \phi_t \mid \ell n\ \lambda_{t-1}, \ldots, \ell n\ \phi_{t-1}, \ldots\right] = c_{21}z_{1t} + c_{22}z_{2t} = u_t$$

so that the variance–covariance matrix of the projection errors is

$$(2.13)\qquad \begin{bmatrix} \sigma_v^2 & \sigma_{vu} \\ \sigma_{vu} & \sigma_u^2 \end{bmatrix} = \begin{bmatrix} c_{11}^2\sigma_1^2 + c_{12}^2\sigma_2^2, & c_{11}c_{21}\sigma_1^2 + c_{12}c_{22}\sigma_2^2 \\ c_{11}c_{21}\sigma_1^2 + c_{12}c_{22}\sigma_2^2, & c_{21}^2\sigma_1^2 + c_{22}^2\sigma_2^2 \end{bmatrix}.$$

The state vector at time t is

$$s_t = (M_t, y_t, \lambda_t, \lambda_{t-1}, \lambda_{t-2}, \phi_t, \phi_{t-1}, \phi_{t-2}).$$

Computing an Explicit Solution

A method to express the equilibrium equity price as an explicit function of the state vector is now described. It is an iterative procedure by which a geometric distributed lead of lognormally distributed random variables can be evaluated.

The equilibrium first order condition is

$$(2.14)\qquad y_t^{-\gamma} q(s_t) = \beta E_t\left[y_{t+1}^{-\gamma}(y_{t+1}\phi_{t+1} + q(s_{t+1}))\right].$$

Define the function h by

$$(2.15)\qquad h(s_t) = y_t^{-\gamma} q(s_t);$$

then (2.14) can be transformed into a linear equation in the function h, or

$$(2.16)\qquad h(s_t) = \beta E_t\left[h(s_{t+1}) + y_{t+1}^{\rho}\phi_{t+1}\right]$$

where $\rho = 1 - \gamma$ for notational convenience. Equation (2.16) may be solved forward as

$$(2.17)\qquad h(s_t) = E_t\left\{\sum_{j=1}^{\infty} \beta^j y_{t+j}^{\rho}\phi_{t+j}\right\}.$$

An assumption to ensure that (2.17) is well–defined is more conveniently stated below.

The linearity of a conditional expectations operator permits (2.17) to be evaluated term by term. In doing so, the following property of a log–normally distributed random variable is used repeatedly; the property is

$$\ell n\, E_t x_{t+1}^{a} = E_t(a\, \ell n\, x_{t+1}) + \frac{a^2}{2} \operatorname{var}_t(\ell n\, x_{t+1}) \tag{2.18}$$

where var_t is the conditional variance and a is a positive or negative scaler. Applying (2.18) to each term in (2.17) allows h to be evaluated iteratively; this process is summarized by the following theorem.

Theorem 1. Assume that $\lim_{j\to\infty} \beta A_j < 1$ where A_j is defined below. Under the distribution assumption (A1), the equilibrium equity price is

$$q(s_t) = y_t \sum_{j=1}^{\infty} A_j \lambda_t^{a_j} \lambda_{t-1}^{b_j} \lambda_{t-2}^{c_j} \phi_t^{d_j} \phi_{t-1}^{e_j} \phi_{t-2}^{f_j} \tag{2.19}$$

where

$$A_{j+1} = A_j \beta \exp\left[(a_j+\rho)\left[\delta_0 + .5(a_j+\rho)\sigma_v^2\right] + d_j(\theta_0 + .5 d_j \sigma_u^2) + (a_j+\rho) d_j \sigma_{vu}\right]$$

$$a_{j+1} = \delta_1(a_j+\rho) + b_j + d_j \psi_1$$

$$b_{j+1} = \delta_2(a_j+\rho) + c_j + d_j \psi_2 \tag{2.20}$$

$$c_{j+1} = \delta_3(a_j+\rho) + d_j \psi_3$$

$$d_{j+1} = \theta_1 d_j + e_j + \eta_1(a_j+\rho)$$

$$e_{j+1} = \theta_2 d_j + f_j + \eta_2(a_j+\rho)$$

$$f_{j+1} = \theta_3 d_j + \eta_3(a_j+\rho)$$

and

(2.21)
$$A_1 = \beta \exp\left[\rho(\delta_0+.5\sigma_v^2)+\theta_0+.5\sigma_u^2+\rho\sigma_{uv}\right]$$

$$a_1 = \rho\delta_1 + \psi_1$$

$$b_1 = \rho\delta_2 + \psi_2$$

$$c_1 = \rho\delta_3 + \psi_3$$

$$d_1 = \rho\eta_1 + \theta_1$$

$$e_1 = \rho\eta_2 + \theta_2$$

$$f_1 = \rho\eta_3 + \theta_3.$$

Proof. The proof is in the Appendix.◇

The rational expectations price (2.19) can be approximated arbitrarily well with the following algorithm for fixed parameter values, a fixed sample realization $\{\lambda_t,\phi_t\}$ and initial values (y_0,M_0).

Algorithm

1. Define $(A_1,a_1\text{–}f_1)$ as in (2.21). Let

$$h_t^1 = A_1\lambda_t^{a_1}\lambda_{t-1}^{b_1}\lambda_{t-2}^{c_1}\phi_t^{d_1}\phi_{t-1}^{e_1}\phi_{t-2}^{f_1}$$

and set $H_t^1 = h_t^1$.

2. Use the system of deterministic linear difference equations (2.20) to define $(A_2, a_2 - f_2)$ and set

$$h_t^2 = A_2 \lambda_t^{a_2} \lambda_{t-1}^{b_2} \lambda_{t-2}^{c_2} \phi_t^{d_2} \phi_{t-1}^{e_2} \phi_{t-2}^{f_2}.$$

Use this to compute

$$H_t^2 = H_t^1 + h_t^2.$$

3. Repeat step 2 for each iteration so that, at the j^{th} iteration,

$$H_t^j = H_t^{j-1} + h_t^j = \sum_{i=1}^{j-1} A_i \lambda_t^{a_i} \lambda_{t-1}^{b_i} \lambda_t^{c_i} \phi_t^{d_i} \phi_{t-1}^{e_i} \phi_{t-2}^{f_i} + h_t^j.$$

4. Repeat step 3 using (2.20) for increasing values of j until

(2.22) $$\max_t |H_t^n - H_t^{n-1}| = \max_t |h_t^n| < \epsilon$$

where ϵ is a predetermined small and positive constant. Let N denote the iteration number at which (1.22) is satisfied; then $(H_t^N y_t^\gamma)$ is an arbitrarily good approximation to the rational expectations price (2.19). The approximation error is an increasing function of ϵ.

The conditional expected value of the equity price can also be computed iteratively. From (2.19), the conditional expected equity price is

(2.23) $$E_t q_{t+1} = E_t \left\{ y_{t+1} \sum_{j=1}^{\infty} A_j \lambda_{t+1}^{a_j} \lambda_t^{b_j} \lambda_{t-1}^{c_j} \phi_{t+1}^{d_j} \phi_t^{e_j} \phi_{t-1}^{f_j} \right\}$$

$$= y_t E_t \left\{ \sum_{j=1}^{\infty} A_j \lambda_{t+1}^{a_j+1} \lambda_t^{b_j} \lambda_{t-1}^{c_j} \phi_{t+1}^{d_j} \phi_t^{e_j} \phi_{t-1}^{f_j} \right\}$$

since $y_{t+1} = \lambda_{t+1} y_t$. The linearity of the expectations operator permits the term–by–term evaluation of (2.23), which results in a recursive exponential system that is described in the next theorem.

Theorem 2. Assume that $\lim_{j\to\infty} \beta A_j \leq 1$ where A_j is defined in (2.20). If (λ,ϕ) are distributed according to (A1), the conditional expected equilibrium price is

$$E_t q_{t+1} = y_t \sum_{j=1}^{\infty} B_j \lambda_t^{\bar{a}_j} \lambda_{t-1}^{\bar{b}_j} \lambda_{t-2}^{\bar{c}_j} \phi_t^{\bar{d}_j} \phi_{t-1}^{\bar{e}_j} \phi_{t-2}^{\bar{f}_j}$$

where

$$B_j = A_j \exp\left[(a_j+1)\delta_0 + .5(a_j+1)^2\sigma_v^2 + .5d_j^2\sigma_u^2 + (a_j+1)d_j\sigma_{vu}\right]$$

$$\bar{a}_j = \delta_1(a_j+1) + b_j + \psi_1 d_j$$

$$\bar{b}_j = \delta_2(a_j+1) + c_j + \psi_2 d_j$$

$$\bar{c}_j = \delta_3(a_j+1) + \psi_3 d_j$$

$$\bar{d}_j = \theta_1 d_j + \eta_1(a_j+1) + e_j$$

$$\bar{e}_j = \theta_2 d_j + \eta_2(a_j+1) + f_j$$

$$\bar{f}_j = \theta_3 d_j + \eta_3(a_j+1).$$

Proof. The proof is in the Appendix. ◇

For fixed parameters and a fixed sample of the state variables, the conditional expected equity price can be evaluated using the algorithm described earlier by computing the $(B_j, \bar{a}_j - \bar{f}_j)$ at each iteration.

The equilibrium equity price and its conditional mean, while geometric leads of log–normally distributed random variables, are not log–normally distributed. The conditional variances of the equity price and return are heteroscedastic and do not vary according to a simple linear autoregressive process.[3]

A Real Model

In the real version of the model, a different type of asset is exchanged. A purchase of an equity share at time t is a claim to the future dividend stream; it differs from the monetary model since the equity of that model is a claim to the current and future dollar–denominated dividend stream. The first order condition for the real model is

$$y_t^{-\gamma}\,\overline{q}_t = \beta E\left[y_{t+1}^{-\gamma}(\overline{q}_{t+1}+y_{t+1})\right] \tag{2.24}$$

where $\overline{q}_t$ denotes the value of the equilibrium price function $\overline{q}(.)$ evaluated at the current state. There are two market–clearing conditions: all goods are consumed ($c_t=y_t$), and all claims are held ($z_t=1$).

(A2) Assumption: The endowment process evolves according to

$$y_{t+1} = \lambda_{t+1}y_t$$

where

$$\ell n\,\lambda_{t+1} = \alpha_0 + \alpha_1\,\ell n\,\lambda_t + \alpha_2\,\ell n\,\lambda_{t-1} + \alpha_3\,\ell n\,\lambda_{t-2} + \alpha_4\,\ell n\,\lambda_{t-3} + e_{t+1} \tag{2.25}$$

and e is an independently and identically distributed normal random variable with zero mean and finite variance σ_e^2.

The representative agent's wealth, measured in units of time t + 1 goods, is

$$z_{t+1}[y_{t+1}+\overline{q}_{t+1}].$$

Recall that the post–transfer pre–trade wealth of the agent is

$$z_t\left[\frac{p_t y_t}{p_{t+1}} + q_{t+1}\right] + \frac{w_{t+1} M_t}{p_{t+1}} \tag{2.26}$$

in the monetary model. Using (2.7), the second term in (2.26) is

$$w_{t+1}M_t(p_{t+1})^{-1} = w_{t+1}y_{t+1}(1+w_{t+1})^{-1},$$

so that (2.26) is expressed as

$$z_t[\pi_{t+1}^{-1}y_t+q_{t+1}] + \frac{w_{t+1}M_t}{p_{t+1}} = q_{t+1} + y_{t+1}$$

in equilibrium. The models are meaningfully compared because the post–transfer, pretrade real dividend income available for spending is equal in the two models for the identical endowment stream. As explained in Section 2, the assets' prices and returns are different and display different risk characteristics.

The equilibrium equity price and its conditional expected value can be computed iteratively using the algorithm described earlier.

Comparative Dynamics and Simulations

The analysis provided in Section 1 of the risk characteristics of the real and dollar–denominated assets demonstrates that there are important differences between the two types of assets as measured by the risk premium and the β–coefficients. An important source of the difference is in the sign and magnitude of the conditional covariance of the marginal rate of substitution with the dividend currently available for spending. It is difficult to determine analytically whether the covariance is important empirically. A meaningful comparison is possible if the conditional expected equity premium and the β-coefficient are computed for each model using the same realization of the endowment process and fixed parameter values (γ,β). Of course, two different probability models are assumed to generate the realized endowment process; for the monetary model, (A1) is assumed to describe the motion of λ and (A2) is assumed to describe the motion of λ for the real model.

The procedure for simulating the endowment shock and monetary transfer is now described. Two realizations of standard normal random variables (z_{1t},z_{2t}) of length 203 are drawn. The variance–covariance matrix (2.13) selected is[4]

$$(3.1)\qquad \begin{bmatrix} \sigma_v^2 & \sigma_{vu} \\ \sigma_{vu} & \sigma_u^2 \end{bmatrix} = \begin{bmatrix} .003159 & -.003247 \\ -.003247 & .00532 \end{bmatrix}$$

and a Cholesky decomposition is used to compute $(c_{11},c_{12},c_{21},c_{22})$ in (2.13). This results in two sequences of normally distributed shocks $\{u_t,v_t\}$ that are contemporaneously correlated but serially uncorrelated. Initial values for the state vector s_t are selected. Parameter values for the system (2.9) are chosen; these values are reported in the tables. The time series process is

$$(3.2)\qquad \begin{bmatrix} \ln \lambda_t \\ \ln \phi_t \end{bmatrix} = \begin{bmatrix} \delta_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} \delta_1 & 0 \\ \varphi_1 & \theta_1 \end{bmatrix} \begin{bmatrix} \ln \lambda_{t-1} \\ \ln \phi_{t-1} \end{bmatrix} + \begin{bmatrix} v_t \\ u_t \end{bmatrix}.$$

The sample $\{u_t,v_t\}$, initial state s_0, and fixed parameters are used to generate a realization $\{\overline{\phi}_t,\overline{\lambda}_t\}$ which is then used to compute the prices and returns to a variety of assets. The formulas for the assets in the monetary model are contained in Table 1. The simulations are contained in Tables 3 and 5.

By setting η_1 equal to zero in (3.2), the time series model describing the evolution of λ_t is

$$\ell n\, \lambda_t = \delta_0 + \delta_1\, \ell n\, \lambda_{t-1} + v_t \qquad (3.3)$$

where v_t is a serially uncorrelated, log-normally distributed random variable. To simulate the real model, the same sample realization $\{\overline{\lambda}_t, v_t\}$ and initial state s_0 are used as data. The time series model (3.3) describes the motion of the endowment shock. The asset prices and returns for the real model are computed with the pricing formulas summarized in Table 2; the simulations are reported in Table 4.

Monetary Model: Comparative Dynamics

The variables of interest in the monetary model are the conditional covariance of the marginal rate of substitution with the purchasing power of money, the conditional covariance of the marginal rate of substitution with the equity price, and the conditional expected returns of the equity, the perfectly correlated asset and the zero-correlated asset. The distribution assumption (A1) for ($\ell n\, \lambda$, $\ell n\, \phi$) is made; the explicit formula for each variable is contained in Table 1. Some comparative dynamics results are now summarized.

1. The return to the perfectly correlated asset that is evaluated in Table 1 under distribution assumption (A1) is: decreasing in β; increasing in δ_0; decreasing σ_v^2; indeterminate in γ.
2. The return to the zero correlated asset, also evaluated in Table 1, is: decreasing in β; increasing in δ_0; decreasing in σ_v^2; indeterminate in γ.
3. The conditional mean $E_t[R^s_{t+1} - R^0_{t+1}]$ is increasing in δ_0, decreasing in β, and decreasing in σ_v^2.
4. The conditional variance of the perfectly correlated asset is state independent, increasing σ_v^2, and increasing in γ. The sign of $\text{cov}_t(S_{t+1}, \pi^{-1}_{t+1})$ depends directly on the sign of

$$(3.3) \qquad \exp\left[-\gamma(\sigma_v+\sigma_{vu})\right]-1.$$

If $\sigma_{vu} < 0$ and if $|\sigma_{vu}| > \sigma_v^2$, then (3.1) is positive.

5. The conditional variances of R^0 and R^q are state dependent as is the β–coefficient. The equity return is not lognormally distributed even though the endowment is, and its conditional variance is state–dependent.

6. The risk premium of the nominal bond, equation 9 in Table 1, is state dependent and time–varying. It is decreasing in β and its sign depends on the sign of $cov_t(S_{t+1},\pi_{t+1}^{-1})$. The risk premium is log–normally distributed with a conditionally heteroscedastic variance.

Real Model: Comparative Dynamics

The variables of interest for the real model are the conditional covariances of the marginal rate of substitution with the dividend and with the equity price and the conditional expected returns of the equity, the perfectly correlated asset and the zero correlated asset. The formula for each variable is contained in Table 2. Some comparative dynamic results are now summarized.

1. The conditional covariance $cov_t(S_{t+1},\lambda_{t+1})$ is negative, increasing in β and α_0 and decreasing in σ_e^2.
2. The conditional expected returns $E_t(1+R_{t+1}^s)$ and $E_t(1+R_{t+1}^0)$ are decreasing in β, increasing in α_0 and decreasing in σ_e^2.
3. The conditional expected premium $E_t[R_{t+1}^s-R_{t+1}^0]$ is decreasing in β, increasing in α_0 and decreasing in σ_e^2.

The conditional covariance of the equity return with the return to the perfectly correlated asset is a function of the conditional covariances of the marginal rate of substitution with the current dividend and the marginal rate of substitution with the equity price. The covariance of the MRS with the equity price for the monetary model can be shown to equal

$$\text{(3.4)} \qquad \text{cov}_t(q_{t+1},S_{t+1}) = \beta y_t \sum_{j=1}^{\infty} \lambda_t^{\hat{a}_j}\lambda_{t-1}^{\hat{b}_j}\lambda_{t-2}^{\hat{c}_j}\phi_t^{\hat{d}_j}\phi_{t-1}^{\hat{e}_j}\phi_{t-2}^{\hat{d}_j}\left[C_j - B_j \exp(-\gamma\delta_0 + .5\gamma\sigma\right]$$

where

$$\hat{a}_j = \delta_1(a_j+\rho) + b_j + \psi_1 d_j$$

$$\hat{b}_j = \delta_2(a_j+\rho) + c_j + \psi_2 d_j$$

$$\hat{c}_j = \delta_3(a_j+\rho) + \psi_3 d_j$$

$$\hat{d}_j = \theta_1 d_j + \eta_1(a_j+\rho) + e_j$$

$$\hat{e}_j = \theta_2 d_j + \eta_2(a_j+\rho) + f_j$$

$$\hat{f}_j = \theta_3 d_j + \eta_3(a_j+\rho),$$

$$C_j = A_j \exp\left[(a_j+\rho)\delta_0 + .5(a_j+\rho)^2\sigma_v^2 + .5 d_j^2\sigma_u^2 + (a_j+\rho)d_j\sigma_{vu}\right],$$

and A_j, B_j, $(a_j–f_j)$ are defined in (2.20) and (2.21). Obviously, this covariance is a complicated function of the model's parameters and state variables. This covariance can be computed iteratively using the algorithm described earlier. The conditional covariance $\text{cov}_t(R^s_{t+1},R^q_{t+1})$ is computed as

$$\text{cov}_t(R^s_{t+1},R^q_{t+1}) = -E_t(R^q_{t+1}-R^0_{t+1})E_t(1+R^0_{t+1})$$

which follows from (1.16).

The conditional covariance of the equity return and inflation can be computed iteratively by observing that

$$\text{cov}_t\left[(q_t)^{-1}(q_{t+1}+\pi^{-1}_{t+1}y_t),\pi_{t+1}\right]$$

$$= (q_t)^{-1}\left\{E_t(q_{t+1}\pi_{t+1}+y_t)-E_t\left[(q_{t+1}+\pi^{-1}_{t+1}y_t)\right]E_t\pi_{t+1}\right\}$$

and that

$$E_tq_{t+1}\pi_{t+1} = E_t\left[y_t\sum_{j=1}^{\infty}A_j\lambda^{a_j}_{t+1}\lambda^{b_j}_t\lambda^{c_j}_{t-1}\phi^{d_j}_{t+1}\phi^{e_j}_t\phi^{f_j}_{t-1}\right].$$

This conditional expectation can be evaluated using the approach described in the proof of Theorem 2.[5]

Simulations

The simulation results for the monetary model are reported in Table 3. The simulation results for the real model are reported in Table 4. The information in the two tables is comparable because the identical endowment realization and initial state is used for both tables. The preference parameters are fixed at $\gamma = 2.00$ and $\beta = 0.95$.

The conditional expected equity returns for the monetary model are uniformly less than the conditional expected returns for the real model. The conditional risk premiums of the monetary model are uniformly greater than the conditional risk premiums of the real model. The β-coefficients are negative and the equity of the monetary model is less risky than the equity of the real model since $|\beta^q| < |\bar{\beta^q}|$. The conditional covariance $\text{cov}_t(S,\pi^{-1})$ in Table 3 is positive and the term $\text{cov}_t(S,\lambda)$ is negative in Table 4 as predicted.

In the monetary model, the risk premium of the nominal bond is negative and time–varying. This means that the nominal bond is a hedge against future uncertainty and that the Fisher equation will yield a poor approximation to the zero–correlated return R^0.

The simulation results for the monetary model reported in Table 3 assume that $\sigma_{uv} = -0.003247$. As discussed, this covariance is an important determinant of the risk characteristics of a dollar–denominated asset. To determine how the results change if this covariance is positive, the parameter values reported in Table 3 with the exception that $\sigma_{uv} = 0.003247$ are used to generate another realization $\{u_t,v_t\}$. The same initial state s_0 is used to generate a sample $\{\lambda_t,\phi_t\}$. The formulas summarized in Table 1 are used to evaluate the asset prices and returns. The results are reported in Table 5. As predicted, the conditional covariance of MRS with the inverse of inflation is uniformly negative. The conditional equity premium is uniformly greater when $\sigma_{uv} > 0$ than when $\sigma_{uv} < 0$. The risk premium on the nominal bond is positive and in the range of 1.37 percent. This again implies that the approximation of the zero–correlated asset return by use of the Fisher equation is biased.

4. Conclusion

The effect of stochastic inflation on stock prices is explored using the empirical implications of the intertemporal CAPM described by Hansen, Richard, and Singleton (1981). The result is that stochastic inflation affects the risk characteristics, measured by the equity premium and the correlation of the asset's return with consumption, in a fundamental way. The riskiness of a dollar–denominated asset depends on two conditional covariances: the covariance of the MRS with the equity price and the covariance of the MRS with the rate of appreciation in the purchasing power of money. The second covariance may take either sign depending on the covariance of the endowment process and the monetary transfer. This becomes significant when the riskiness of the dollar–denominated asset is compared with the risk characteristics of the indexed asset constructed in a real version of the model.

Currency is incorporated into a pure endowment representative agent model by way of a cash–in–advance constraint. Timing of trade and information acquisition is such that the constraint is binding under the hypothesis that nominal interests are nonnegative in all states. Alternative model specifications that result in a potentially nonbinding constraint are studied by Svensson (1985), for example. The growth rates of the monetary transfer and the endowment are assumed to evolve according to a bivariate autoregressive process with normally distributed error terms. As a result of this assumption and the parameterization of the utility function, the equity price is a geometric distributed lead of log–normally distributed random variables. An algorithm to express the equilibrium equity price as an explicit function of the state variables is described.

Comparative dynamic results and simulations of the real and monetary versions of the model are presented and discussed in Section 3. The simulations reinforce the conclusion that stochastic inflation has an important effect on the risk characteristics of the assets. The particular model used minimizes the role of money in the economy; if stochastic inflation affects the risk characteristics in this model, the effect should be magnified as velocity is allowed to vary and production decisions are introduced.

Footnotes

[1]The covariance is

$$\mathrm{cov}_t(R^s_{t+1}, S_{t+1}) = (E_t S^2_{t+1})^{-1}\left[E_t S^2_{t+1} - (E_t S^2_{t+1})^2\right]$$

$$= (E_t S^2_{t+1})^{-1}\,\mathrm{var}(R^s_{t+1}) > 0.$$

[2]This point is illustrated by Lucas (1978) for a pure exchange markov economy that is stationary in the endowment level.

[3]The conditional variance of the price,

$$\mathrm{var}_t(q_{t+1}) = E_t\left[(q_{t+1} - E_t q_{t+1})^2\right],$$

can be shown to equal

$$\mathrm{var}_t(q_{t+1}) = y_t^2\left[\sum_{j=1}^{\infty}\sum_{i=1}^{\infty} A_i A_j \lambda_t^{\bar{a}_i+\bar{a}_j}\lambda_{t-1}^{\bar{b}_i+\bar{b}_j}\lambda_{t-2}^{\bar{c}_i+\bar{c}_j}\phi_t^{\bar{d}_i+\bar{d}_j}\phi_{t-1}^{\bar{e}_i+\bar{e}_j}\phi_{t-2}^{\bar{f}_i+\bar{f}_j}\right.$$

$$\exp\left[\left[(a_i+1+a_j+1)\right]\delta_0+(d_i+d_j)\theta_0\right]$$

$$\exp\left[\left[(a_j+1)+(a_i+1)\right]^2(.5)\sigma_v^2+.5(c_i+c_j)^2\sigma_u^2\right]$$

$$\left.-\exp\left[\left[(a_j+1)^2+(a_i+1)^2\left[\right]\right](.5)\sigma_v^2+(c_j^2+c_i^2).5\sigma_u^2\right]\right].$$

This variance is state–dependent and does not display any smooth autoregressive structure.

[4]These values are estimated from the Mehra–Prescott data (Mehra and Prescott 1985) and are reported in Labadie (1988).

[5]The solution is

$$E_t q_{t+1}\pi_{t+1} = y_t \sum_{j=1}^{\infty} \tilde{B}_j \lambda_t^{\tilde{a}_j} \lambda_{t-1}^{\tilde{b}_j} \lambda_{t-2}^{\tilde{c}_j} \phi_t^{\tilde{d}_j} \phi_{t-1}^{\tilde{e}_j} \phi_{t-2}^{\tilde{f}_j}$$

where

$$\tilde{B}_j = A_j \exp\left[a_j\delta_0 + .5a_j^2\sigma_v^2 + (d_j-1)\theta_0 + .5(d_j-1)^2\sigma_u^2 + a_j(d_j-1)\sigma_{uv}\right]$$

$$\tilde{a}_j = \delta_1 a_j + b_j + \psi_1(d_j-1)$$

$$\tilde{b}_j = \delta_2 a_j + c_j + \psi_2(d_j-1)$$

$$\tilde{c}_j = \delta_3 a_j + \psi_3(d_j-1)$$

$$\tilde{d}_j = \theta_1(d_j-1) + \eta_1 a_j + e_j$$

$$\tilde{e}_j = \theta_2(d_j-1) + \eta_2 a_j + f_j$$

$$\tilde{f}_j = \theta_3(d_j-1) + \eta_3 a_j$$

so that

$$\text{cov}_t(R_{t+1}^q, \pi_{t+1}) = (q_t)^{-1}\left[y_t + E_t q_{t+1}\pi_{t+1} - E_t(1+R_{t+1}^q)E_t\pi_{t+1}\right]$$

where

$$E_t\pi_{t+1} = E_t(\lambda_{t+1}\phi_{t+1})^{-1}$$

$$= \exp\left[-(\theta_0+\delta_0) + .5\sigma_u^2 + .5\sigma_v^2 + \sigma_{vu}\right]$$

$$\left[\lambda_t^{\delta_1+\psi_1}\lambda_{t-1}^{\delta_2+\psi_2}\lambda_{t-2}^{\delta_3+\psi_3}\phi_t^{\theta_1+\eta_1}\phi_{t-1}^{\theta_2+\eta_2}\phi_{t-2}^{\theta_3+\eta_3}\right]^{-1}$$

and $E_t(1+R_{t+1}^q)$ is defined in Table 2.

Appendix

Proof of Theorem 1

The first term in (2.14) is

$$\beta E_t y_{t+1}^{\rho} \phi_{t+1} = \beta y_t^{\rho} E_t \lambda_{t+1}^{\rho} \phi_{t+1}$$

where

$$\beta y_t^{\rho} E_t y_{t+1}^{\rho} \phi_{t+1} = \beta y_t^{\rho} \exp\left[\rho(\delta_0 + .5\rho\sigma_v^2) + \theta_0 + .5\sigma_u^2 + \rho\sigma_{uv}\right]$$

$$\lambda_t^{\rho\delta_1+\psi_1} \lambda_{t-1}^{\rho\delta_2+\psi_2} \lambda_{t-2}^{\rho\delta_3+\psi_3} \phi_t^{\rho\eta_1+\theta_1} \phi_{t-1}^{\rho\eta_2+\theta_2} \phi_{t-2}^{\rho\eta_3+\theta_3}.$$

The coefficients are used to define $(A_1;a_1-f_1)$ in 2.18.

The second term in (2.14) is

$$\beta^2 E_t y_{t+2}^{\rho} \phi_{t+2} = \beta^2 y_t^{\rho} E_t[\lambda_{t+1}^{\rho} \lambda_{t+2}^{\rho} \phi_{t+2}]$$

$$= \beta^2 y_t^{\rho} \exp(\rho\delta_0 + .5\rho^2\sigma_v^2 + \theta_0 + .5\sigma_u^2 + \rho\sigma_{uv})$$

$$E_t(\lambda_{t+1}^{\rho+a_1} \lambda_t^{b_1} \lambda_{t-1}^{c_1} \phi_{t+1}^{d_1} \phi_t^{e_1} \phi_{t-1}^{f_1})$$

$$= \beta y_t^{\rho} A_1 \exp\left[(a_1+\rho)\delta_0 + (a_1+\rho)^2.5\sigma_v^2 + d_1(\theta_0 + d_1.5\sigma_u^2) + (a_1+\rho)d_1\sigma_{uv}\right]$$

$$\lambda_t^{a_2} \lambda_{t-1}^{b_2} \lambda_{t-2}^{c_2} \phi_t^{d_2} \phi_{t-1}^{e_2} \phi_{t-2}^{f_2}$$

where

$$a_2 = \delta_2\rho + \psi_2 + \delta_1\left[\rho(1+\delta_1) + \psi_1\right] + (\theta_1 + \rho\eta_1)\psi_1$$

$$= b_1 + \delta_1(a_1+\rho) + d_1\psi_1$$

$$b_2 = \rho\delta_3 + \psi_3 + \delta_2\left[\rho(1+\delta_1)+\psi_1\right] + (\theta_1+\rho\eta_1)\psi_2$$

$$= c_1 + \delta_2(a_1+\rho) + d_1\psi_1$$

$$c_2 = \delta_3\left[(1+\delta_1)+\psi_1\right] + (\theta_1+\rho\eta_1)\psi_3$$

$$= \delta_3(a_1+\rho) + d_1\psi_3$$

$$d_2 = \theta_2 + \rho\eta_2 + \theta_1(\theta_1+\rho\eta_1) + \left[\rho(1+\delta_1)+\psi_1\right]\eta_1$$

$$= e_1 + \theta_1 d_1 + \eta_1(a_1+\rho)$$

$$e_2 = \theta_3 + \rho\eta_3 + \theta_2(\theta_1+\rho\eta_1) + \left[\rho(1+\delta_1)+\psi_1\right]\eta_2$$

$$= f_1 + \theta_2 d_1 + \eta_2(a_1+\rho)$$

$$f_2 = \theta_3(\theta_1+\rho\eta_1) + \eta_3\left[\rho(1+\delta_1)+\psi_1\right]$$

$$= \theta_3 d_1 + \eta_3(a_1+\rho).$$

Evaluation of the third term results in

$$\beta^3 y_t^\rho E_t \lambda_{t+3}^\rho \phi_{t+3} = \beta^3 y_t^\rho E_t \lambda_{t+3}^\rho \lambda_{t+2}^\rho \lambda_{t+1}^\rho \phi_{t+3}$$

$$= \beta^3 y_t^\rho A_1 E_t \lambda_{t+2}^{\rho(1+\delta_1)+\psi_1} \lambda_{t+1}^{\rho(1+\delta_2)+\psi_2} \lambda_t^{\rho(1+\delta_3)+\psi_3}$$

$$\phi_{t+2}^{\theta_1+\rho\eta_1} \phi_{t+1}^{\theta_2+\rho\eta_2} \phi_t^{\theta_3+\rho\eta_3}$$

$$= \beta^2 y_t^\rho A_1 E_t \lambda_{t+2}^{\rho+a_1} \lambda_{t+1}^{\rho+b_1} \lambda_t^{c_1} \phi_{t+2}^{d_1} \phi_{t+1}^{e_1} \phi_t^{f_1}$$

$$= \beta y_t^\rho A_2 E_t \lambda_{t+1}^{a_2} \lambda_t^{b_2} \lambda_{t-1}^{c_2} \phi_{t+1}^{d_2} \phi_t^{e_2} \phi_{t-1}^{f_2}$$

$$= y_t^\rho A_3 \lambda_t^{a_3} \lambda_{t-1}^{b_3} \lambda_{t-2}^{c_3} \phi_t^{d_3} \phi_{t-1}^{e_3} \phi_{t-2}^{f_3}.$$

Since there is nothing special about the second and third iterations, the recursive scheme defined by (2.17) and (2.18) follows.

To determine if

$$\lim_{j \to \beta} \beta A_j < 1,$$

the steady state of the system must be examined. Define

$$J = \delta_1 + \delta_2 + \delta_3 + \frac{(\psi_3+\psi_2+\psi_1)(\eta_1+\eta_2+\eta_3)}{1 - \theta_1 - \theta_2 - \theta_3}.$$

Then the steady state of the system (2.17) is:

$$a = \frac{\rho J}{1 - J}$$

$$d = \left[\frac{\eta_3+\eta_2+\eta_1}{1-\theta_1-\theta_2-\theta_3}\right]\left[\frac{\rho}{1-J}\right]$$

$$b = (\delta_2+\delta_3)\left[\frac{\rho}{1-J}\right] + (\psi_2+\psi_3)d$$

$$c = \frac{\delta_3\rho}{1 - J} + \psi_3 d$$

$$e = (\theta_2+\theta_3)d + (\eta_2+\eta_3)\frac{\rho}{1-J}$$

$$f = \theta_3 d + \frac{\eta_3\rho}{1-J}.$$

The assumptions made are

i. $1-\theta_1-\theta_2-\theta_3 < 1$

ii. $J \neq 1$

iii. $\left|\frac{\rho}{1-J}\right| < \infty$

in addition to the assumptions in A1. The term

$$\exp\left[(a_j+\rho)\left[\delta_0+.5(a_j+\rho)\sigma_v^2\right]+d_j(\theta_0+.5d_j\sigma_u^2)\right]$$

will tend to a constant K as $j \to \infty$. If

$$\beta K < 1$$

then the sum in Theorem 1 converges.

Proof of Theorem 2

Equation (2.20) is

$$E_t q_{t+1} = y_t\left\{\sum_{j=1}^{\infty} A_j \lambda_{t+1}^{a_j+1} \lambda_t^{b_j} \lambda_{t-1}^{c_j} \phi_{t+1}^{d_j} \phi_t^{e_j} \phi_{t-1}^{f_j}\right\}.$$

Evaluation of the j^{th} term results in

$$E_t A_j \lambda_{t+1}^{a_j+1} \lambda_t^{b_j} \lambda_{t-1}^{c_j} \phi_{t+1}^{d_j} \phi_t^{e_j} \phi_{t-1}^{f_j}$$

$$= A_j \exp\left[(a_j+1)\delta_0 + .5(a_j+1)^2\sigma_v^2 + d_j\theta_0 + .5d_j^2\sigma_u^2 + (a_j+1)d_j\sigma_{vu}\right]$$

$$\lambda_t^{b_j+\delta_1(a_j+1)+d_j\psi_1} \lambda_{t-1}^{c_j+\delta_2(a_j+1)+d_j\psi_2} \lambda_{t+2}^{\delta_3(a_j+1)+d_j\psi_3}$$

$$\phi_t^{e_j+d_j\theta_1+(a_j+1)\eta_1} \phi_{t-1}^{f_j+d_j\phi_2+(a_j+1)\eta_2} \phi_{t-2}^{d_j\theta_3+(a_j+1)\eta_3}$$

which determines the linear deterministic system of exponents defined in Theorem 2.

Table 1

Formulas for Variables in the Monetary Model Expressed as Explicit Functions of the Current State Variables and Parameters

1. $\mathrm{cov}_t(S_{t+1},\pi_{t+1}^{-1}) = \mathrm{cov}_t(\beta\lambda_{t+1}^{-\gamma},\lambda_{t+1}\phi_{t+1})$

$$= \beta\lambda_t^{\delta_1\rho+\psi_1}\lambda_{t-1}^{\delta_2\rho+\psi_2}\lambda_{t-2}^{\delta_3\rho+\psi_3}\phi_t^{\eta_1\rho+\theta_1}\phi_{t-1}^{\eta_2\rho+\theta_2}$$

$$\phi_{t-2}^{\eta_3\rho+\theta_3}\exp(\rho\delta_0+\theta_0+.5\sigma_u^2)\left[\exp(\rho^2.5\sigma_v^2+\rho\sigma_{vu})-\exp\left[\sigma_{vu}+.5(1+\gamma^2)\sigma^2\right]\right].$$

2. $E_t(1+R_{t+1}^0) = (E_tS_{t+1})^{-1} = (E_t\beta\lambda_{t+1}^{-\gamma})^{-1}$

$$= \left\{\left[\beta\exp(-\gamma\delta_0+.5\gamma^2\sigma_v^2)\right]^{-1}(\lambda_t^{\delta_1}\lambda_{t-1}^{\delta_2}\lambda_{t-2}^{\delta_2}\phi_t^{\eta_1}\phi_{t-1}^{\eta_2}\phi_{t-2}^{\eta_3})^{-\gamma}\right\}.$$

3. $E_t(1+R_{t+1}^s) = E_tS_{t+1}(E_tS_{t+1}^2)^{-1}$

$$= (\lambda_t^{\delta_1}\lambda_{t-1}^{\delta_2}\lambda_{t-2}^{\delta_3}\phi_t^{\eta_1}\phi_{t-1}^{\eta_2}\phi_{t-2}^{\eta_3})^{-\gamma}\exp\left[\gamma\delta_0-(1.5)\gamma^2\sigma_v^2\right]$$

4. $E_t(1+R_{t+1}^q) = E_t[q_{t+1}+\lambda_{t+1}\phi_{t+1}](q_t)^{-1}$

$$= q_t^{-1}\Big[E_tq_{t+1}+y_t\exp(\delta_0+\theta_0+.5\sigma_u^2+\sigma_{vu}+.5\sigma_v^2)$$

$$\lambda_t^{\delta_1+\psi_1}\lambda_{t-1}^{\delta_2+\psi_2}\lambda_{t-2}^{\delta_3+\psi_3}\phi_t^{\theta_1+\eta_1}\phi_{t-1}^{\theta_2+\eta_2}\phi_{t-2}^{\theta_3+\eta_3}\Big]$$

where E_tq_{t+1} is defined in (2.20).

5. $\mathrm{var}_t(R_{t+1}^s) = (E_tS_{t+1}^2)^{-2}\,E_t\left[(\beta\lambda_{t+1}^{-\gamma}-E_t\beta\lambda_{t+1}^{-\gamma})^2\right] = 1-\exp(-\gamma^2\sigma_v^2).$

6. $E_t[R_{t+1}^s-R_{t+1}^0] = \frac{1}{\beta}(\lambda_t^{\delta_1}\lambda_{t-1}^{\delta_2}\lambda_{t-2}^{\delta_3}\phi_t^{\psi_1}\phi_{t-1}^{\psi_2}\phi_{t-2}^{\psi_3})^{\gamma}$

$$\exp(\gamma\delta_0)\left[\exp\left[-(1.5)\gamma^2\sigma_v^2\right]-\exp(-.5\gamma^2\sigma_v^2)\right].$$

Table 1 continued

7. $1 + i_t = [E_t S_{t+1} \pi_{t+1}^{-1}]^{-1} = [\beta E_t \lambda_{t+1}^{\rho} \phi_{t+1}]^{-1}$

$$= \beta^{-1} \left[\exp(\rho\delta_0 + .5\rho^2\sigma_v^2 + \theta_0 + .5\sigma_u^2 + \rho\sigma_{uv})\right]^{-1} \cdot$$

$$[\lambda_t^{\rho\delta_1+\psi_1} \lambda_{t-1}^{\rho\delta_2+\psi_2} \lambda_{t-2}^{\rho\delta_3+\psi_3} \phi_t^{\theta_1+\rho\eta_1} \phi_{t-1}^{\theta_2+\rho\eta_2} \phi_{t-2}^{\theta_3+\rho\eta_3}]^{-1}.$$

8. $1 + E_t r_{t+1} = [E_t S_{t+1} \pi_{t+1}^{-1}]^{-1} E_t \pi_{t+1}^{-1} = [\beta E_t \lambda_{t+1}^{\rho} \phi_{t+1}]^{-1} E_t \lambda_{t+1} \phi_{t+1}$

$$= \left[\beta \exp\left[-\gamma\delta_0 + .5(\gamma^2 - 2\gamma)\sigma_v^2 - \gamma\sigma_{uv}\right]\right]^{-1} [\lambda_t^{\delta_1} \lambda_{t-1}^{\delta_2} \lambda_{t-2}^{\delta_3} \phi_t^{\eta_1} \phi_{t-1}^{\eta_2} \phi_{t-2}^{\eta_3}]^{-\gamma}.$$

9. $E_t[r_{t+1} - R_{t+1}^0] = E_t\left[(E_t S_{t+1} \pi_{t+1}^{-1})^{-1} \pi_{t+1}^{-1} - (E_t S_{t+1})^{-1}\right]$

$$= \left[\beta \exp(-\gamma\delta_0 + .5\gamma^2\sigma_v^2)\right]^{-1} [\lambda_t^{\delta_1} \lambda_{t-1}^{\delta_2} \lambda_{t-2}^{\delta_3} \phi_t^{\eta_1} \phi_{t-1}^{\eta_2} \phi_{t-2}^{\eta_3}]^{\gamma}$$

$$\left[\exp\left[\gamma(\sigma_v^2 + \sigma_{uv})\right] - 1\right]$$

Note: (1) is the covariance of the MRS with the inverse of gross inflation. (2) is the expected return to the zero-correlated asset. (3) is the expected return to the perfectly correlated asset. (4) is the return to the equity in the monetary model. (5) is the variance of the perfectly correlated return. (6) is the expected risk premium of the perfectly correlated return. (7) is the nominal interest rate. (8) is the expected real return on the nominal bond. (8) is the expected premium on the real return to the nominal bond. All covariances and expectations are conditioned on information available at time t.

Table 2

Formulas for Variables in the Real Model Expressed as Explicit Functions of the Current State Variables and Parameters

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1. $\mathrm{cov}(S_{t+1},\lambda_{t+1}) = \beta(\lambda_t^{\alpha_1}\lambda_{t-1}^{\alpha_2}\lambda_{t-2}^{\alpha_3}\lambda_{t-3}^{\alpha_4})^{\rho}\exp(\rho\alpha_0)\left[\exp(.5\rho^2\sigma_e^2)-\exp\left[.5(1+\gamma^2)\sigma_e^2\right]\right].$

2. $E_t(1+R^0_{t+1}) = \frac{1}{\beta}\exp(\gamma\alpha_0-.5\gamma^2\sigma_e^2)(\lambda_t^{\alpha_1}\lambda_{t-1}^{\alpha_2}\lambda_{t-2}^{\alpha_3}\lambda_{t-3}^{\alpha_4})^{\gamma}.$

3. $E_t(1+R^s_{t+1}) = \frac{1}{\beta}(\lambda_t^{\alpha_1}\lambda_{t-1}^{\alpha_2}\lambda_{t-2}^{\alpha_3}\lambda_{t-3}^{\alpha_4})^{\gamma}\exp(\gamma\alpha_0-1.5\gamma^2\sigma_v^2).$

4. $E_t(1+R^{\bar{q}}_{t+1}) = (\bar{q}_t)^{-1}\left[E_t\bar{q}_{t+1}+y_t\exp(\alpha_0+.5\sigma_e^2)\lambda_t^{\alpha_1}\lambda_{t-1}^{\alpha_2}\lambda_{t-2}^{\alpha_3}\lambda_{t-3}^{\alpha_4}\right]$

   where $E_t\bar{q}_{t+1}$ is defined in Theorem 2.

5. $\mathrm{var}(R^s_{t+1}) = 1-\exp(-\gamma^2\sigma_e^2).$

6. $E_t[R^s_{t+1}-R^0_{t+1}] = \frac{1}{\beta}(\lambda_t^{\alpha_1}\lambda_{t-1}^{\alpha_2}\lambda_{t-2}^{\alpha_3}\lambda_{t-3}^{\alpha_4})^{\gamma}\exp(\gamma\alpha_0)\left[\exp(-.5\gamma^2\sigma_u^2)-\exp(-.5\gamma^2\sigma_v^2)\right].$

Note: (1) is the covariance of MRS and the endowment shock. (2) is the expected return to the zero–correlated asset, (3) is the expected return to the perfectly correlated asset. (4) is the expected return to the equity in the real model, (5) is the variance of the return to the perfectly correlated asset. (6) is the risk premium on the perfectly correlated asset. All covariances and expectations are conditioned on information at time t.

Table 3

Monetary Model, $\gamma = 2.00$, $\beta = 0.95$

Subsample	$E_t R^q$ %	$E_t[R^q - R^0]$ %	$E_t r$ %	$E_t[r - R^0]$ %	$cov(S, \pi^{-1})$	β^q
1	7.538	.8463	6.673	−.01877	.00299	−.681
2	6.248	.4939	5.735	−.01861	.00319	−.371
3	6.403	.4466	5.938	−.01864	.00321	−.335
4	6.900	.6705	6.210	−.01869	.00309	−.502
5	7.239	.8531	6.367	−.01872	.00299	−.638
6	7.147	.7956	6.333	−.01871	.00302	−.594
7	6.751	.5848	6.147	−.01868	.00314	−.438
8	7.494	.9234	6.552	−.01875	.00296	−.689
9	6.894	.6578	6.217	−.01869	.00310	−.492
10	7.421	.9497	6.453	−.01873	.00294	−.709
11	7.449	.8964	6.534	−.01875	.00297	−.669
12	7.336	.7702	6.547	−.01875	.00303	−.575
13	6.956	.7003	6.237	−.01869	.00307	−.524
14	7.112	.7416	6.352	−.01872	.00305	−.555
15	7.095	.7278	6.348	−.01871	.00306	−.544

$var(R^s) = 0.012556$

*Each subsample is of length 10 and sample averages are reported. The parameter values are: $\theta_0 = -0.01382$, $\theta_1 = 0.519$, $\psi_1 = 0.0189$, $\delta_0 = 0.00705$, $\delta_1 = 0.105968$, $\eta_1 = 0$, $\sigma_u^2 = 0.00532$, $\sigma_v^2 = 0.003159$, and $\sigma_{uv} = -0.003247$. The time series model is (3.2); the initial state is $y_0 = 1$, $\ell n\ \lambda_{-1} = -0.0285$, $\ell n\ \psi_{-1} = 0.04237$. The variables in the columns are: conditional expected equity return, conditional expected risk premium, expected real return on the nominal bond, conditional inflation premium, conditional covariance of MRS with the inverse of gross inflation and the β-coefficient of the equity return.

Table 4

Real Model, $\gamma = 2.00$, $\beta = 0.95$

Subperiod	$E_tR^{\bar{q}}$ %	$E_t[R^{\bar{q}}-R^0]$ %	E_tR^s %	E_tR^0 %	cov(S,λ)	$\beta^{\bar{q}}$
1	6.127	.242	5.721	5.885	–.00211	–1.4795
2	5.847	.241	5.442	5.606	*	*
3	5.908	.241	5.503	5.667	*	*
4	6.017	.242	5.611	5.775	*	*
5	5.967	.241	5.561	5.725	*	*
6	6.040	.242	5.634	5.798	*	*
7	6.013	.242	5.607	5.771	*	*
8	6.123	.242	5.717	5.881	*	*
9	5.964	.241	5.558	5.722	*	*
10	6.085	.242	5.680	5.843	*	*
11	6.123	.242	5.717	5.880	*	*
12	6.121	.242	5.715	5.879	*	*
13	5.997	.242	5.591	5.755	*	*
14	6.047	.242	5.642	5.805	*	*
15	6.016	.242	5.610	5.774	*	*

var(R^s) = 0.00154

*Unchanged from previous value.

Each subsample is of length 10 and sample averages are reported. The formulas in Table 2 are used to compute the variables above; the parameters for the endowment process are are: $\alpha_0 = 0.00705$, $\alpha_1 = 0.105968$, $\alpha_2 = 0$, $\alpha_3 = 0$, and $\sigma_e^2 = 0.003159$. The endowment realization is identical to the endowment used in the computation reported in Table 3. The variables reported are: the conditional expected equity return, conditional risk premium, conditional return on the perfectly correlated asset, conditional covariance of MRS with the endowment shock, conditional covariance of the MRS with the equity price and the β-coefficient of the equity return.

Table 5

Monetary Model, $\gamma = 2.00$, $\beta = 0.95$

Subsample	E_tR^q %	$E_t[R^q-R^0]$ %	E_tr %	$E_t[r-R^0]$ %	$cov(S,\pi^{-1})$	β^q
1	6.745	1.0478	7.060	1.362	–.00857	–.790
2	7.046	.8032	7.612	1.369	–.00892	–.602
3	6.940	.5942	7.717	1.371	–.00924	–.444
4	7.214	.6930	7.894	1.373	–.00908	–.519
5	7.174	.9645	7.579	1.369	–.00868	–.724
6	7.239	.8325	7.779	1.372	–.00887	–.625
7	7.138	.8412	7.668	1.370	–.00868	–.631
8	6.732	.8803	7.216	1.364	–.00882	–.663
9	7.355	.9747	7.752	1.371	–.00866	–.730
10	6.795	.6509	7.513	1.368	–.00915	–.489
11	7.191	.7689	7.794	1.372	–.00897	–.576
12	6.274	.6977	6.938	1.361	–.00910	–.527
13	6.913	.8003	7.481	1.368	–.00893	–.603
14	6.831	.3271	7.877	1.373	–.00963	–.245
15	7.059	.7164	7.714	1.371	–.00905	–.538

$var(R^s) = 0.012556$

*The parameter values are identical to those reported in Table 5 with the important exception that $\sigma_{uv} = 0.003247$. The parameters are: $\theta_0 = -0.01382$, $\theta_1 = 0.519$, $\psi_1 = 0.0189$, $\delta_0 = 0.00705$, $\delta_1 = 0.105968$, $\eta_1 = 0$, $\sigma_u^2 = 0.00532$, $\sigma_v^2 = 0.003159$, and $\sigma_{uv} = 0.003247$. Each subsample is of length 10 and sample averages are reported. The formulas summarized by Table 1 are used in the computations.

References

Backus, David K.; Gregory, Allan W.; and Zin, Stanley E. 1987. Risk premiums in the term structure: Evidence from artificial economies, unpublished manuscript, Queen's University, Canada.

Campbell, John Y. 1986. Bond and stock returns in a simple exchange model. Quarterly Journal of Economics no. 407, 785–804.

______________. 1987. Stock returns and the term structure. Journal of Financial Economics 18, 373–99.

Fama, Eugene F. 1983. Inflation, Output, and Money. Journal of Business 55, no. 2, 201–31.

______________, and Gibbons, Michael R. 1983. Inflation, real returns, and capital investment. Journal of Monetary Economics 9, 297–93.

Grossman, S., and Shiller, R. J. 1981. The determinants of the variability of stock market prices. American Economic Review 71, 222–27.

Hansen, Lars Peter; Richard, Scott F.; and Singleton, Kenneth J. 1981. Econometric implications of the intertemporal asset pricing model, unpublished paper, Carnegie–Mellon University, Pittsburgh, PA.

______________, and Singleton, Kenneth J. 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica 50, 1269–86.

Labadie, Pamela A. 1988. Asset prices, stochastic inflation, and the equity premium, unpublished manuscript, Graduate School of Business, Columbia University, New York.

LeRoy, Stephen F. 1984. Nominal prices and interest rates in general equilibrium: Endowment shocks. Journal of Business 57, no. 2, 177–95.

______________. 1984. Nominal prices and interest rates in general equilibrium: Money shocks. Journal of Business 57, no. 2, 197–213.

Lucas, Robert E., Jr. 1978. Asset prices in an exchange economy. Econometrica 46, 1429–45.

____________. 1980. Equilibrium in a pure currency market. Economic Inquiry 18, 203–20.

____________. 1982. Interest rates and currency prices in a two–country world. Journal of Monetary Economics 10, 336–60.

Mehra, R., and Prescott, Edward C. 1985. The equity premium: A puzzle. Journal of Monetary Economics 15, 145–62.

Svennson, Lars E. O. 1985. Money and asset prices in a cash–in–advance economy. Journal of Political Economy 93, 919–14.

Stulz, Rene M. 1986. Asset pricing and expected inflation. Journal of Finance 41, 209–24.