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CURRENT REAL BUSINESS CYCLE THEORIES AND AGGREGATE LABOR MARKET FLUCTUATIONS

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ABSTRACT

In the 1930s, Dunlop and Tarshis observed that the correlation between hours worked and the return to working is close to zero. This observation has become a litmus test by which macroeconomic models are judged. Existing real business cycle models fail this test dramatically. Based on this result, we argue that technology shocks cannot be the sole impulse driving post-war U.S. business cycles. We modify prototypical real business cycle models by allowing government consumption shocks to influence labor market dynamics in a way suggested by Aschauer (1985), Barro (1981,1987), and Kormendi (1983). This modification can, in principle, bring the models into closer conformity with the data. Our results indicate that when aggregate demand shocks arising from stochastic movements in government consumption are incorporated into the analysis, and an empirically plausible degree of measurement error is allowed for, the model's empirical performance is substantially improved.

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1. Introduction

This paper assesses the quantitative implications of existing real business cycle (RBC) models for the time series properties of average productivity and hours worked. We find that the single most salient shortcoming of existing RBC models lies in their predictions for the correlation between average productivity and hours worked. Existing RBC models predict that this correlation is well in excess of .9. The actual correlation which obtains in the aggregate data is roughly zero.[1] These results lead us to incorporate aggregate demand shocks arising from stochastic movements in government consumption into the RBC framework. In addition, we investigate the impact of two types of measurement error on our analysis: (i) misalignment between standard measures of output and hours worked and (ii) classical measurement error in hours worked. In combination, these changes generate substantial improvements in the models' empirical performance.

The ability to account for the observed correlation between the return to working and hours worked is a traditional litmus test by which aggregate models are judged. For example, Dunlop (1938) and Tarshis' (1939) critique of the classical and Keynesian models was based on the implications of those models for the correlation between real wages and employment. Both models share the common assumption that real wages and hours lie on a stable downward sloped marginal productivity of labor curve.[2] Consequently, they predict, counterfactually, a strong negative correlation between real wages and hours worked. Modern versions of the Dunlop--Tarshis critique continue to play a central role in assessing the empirical plausibility of different business cycle models. For example, in discussing the Fischer (1977) sticky wage business cycle model, McCallum (1989,p.191) states

> As it happens, the main trouble with the Fischer model concerns its real wage behavior. In particular, to the extent that the model itself explains fluctuations in output and employment, these should be inversely related to real wage movements: output should be high,

> according to the model, when real wages are low. But in the actual U.S. economy there is no strong empirical relation of that type.

In remarks that are particularly relevant for RBC models, Lucas (1981,p.226) writes

> Observed real wages are not constant over the cycle, but neither do they exhibit consistently pro– or countercyclical tendencies. This suggests that any attempt to assign systematic real wage movements a central role in an explanation of business cycles is doomed to failure.

Existing RBC models fall prey to this (less well known) Lucas critique. In contrast to the classical and Keynesian models which *understate* the correlation between hours worked and the return to working, existing RBC models grossly *overstate* that correlation. This is because, according to existing RBC models, the *only* impulses generating fluctuations in aggregate employment are stochastic shifts in the marginal product of labor. Loosely speaking, the time series on hours worked and the return to working are modeled as the intersection of a stochastic labor demand curve with a fixed labor supply curve. It is therefore not surprising that these theories predict a strong positive correlation between the return to working and hours of work.[3]

There are at least two strategies for modeling the observed weak correlation between measures of the return to working and hours worked. The first is to consider models in which the return to working is unaffected by shocks to agents' environments, regardless of whether they correspond to aggregate demand or aggregate supply shocks. Pursuing this strategy, Blanchard and Fischer (1989, p. 372) argue that the key assumption of Keynesian macro models – *nominal wage and price stickiness – is motivated by the view* that aggregate demand shocks affect employment while leaving real wages unaffected. The second response is to simply abandon one shock models of the business cycle. The basic idea here is that aggregate fluctuations are generated by a variety of impulses. Under these circumstances the Dunlop–Tarshis observation imposes no restrictions per se on the response of real wages to any particular type of shock. But, given a particular structural model, it does impose restrictions on the relative frequency of different types of shocks.

This suggests that one strategy for reconciling existing RBC models with Dunlop–Tarshis type observations is to find measurable economic impulses that shift the labor supply function.[4] With impulses impacting on both the labor supply and demand functions there is no a priori reason for hours worked to display any sort of marked correlation with the return to working.

Candidates for such shocks include tax rate changes, shocks to the money supply, demographic changes in the labor force, and shocks to government spending. In this paper we focus on the latter type of shocks, namely changes in government consumption. By ruling out any role for government consumption shocks in labor market dynamics, existing RBC models implicitly assume that public and private consumption have the same impact on the marginal utility of private spending. Aschauer (1985) and Barro (1981, 1987) argue that when $1 dollar of additional public consumption drives the marginal utility of private consumption down by less than does $1 of additional private consumption, then shocks to government consumption in effect shift the labor supply curve outwards. Coupled with diminishing labor productivity, these type of impulses will, absent technology shocks, generate a negative correlation between hours worked and the return to working in RBC models.

In our empirical work we measure the return to working by the average productivity of labor rather than, for example, the real wage. We do this for two reasons. First, from an empirical point of view our results are not very sensitive to whether the return to working is measured using real wages or average productivity. Neither displays a strong positive correlation with hours worked. For this reason, it seems appropriate to refer to the low correlation between the return to working and hours worked as the Dunlop–Tarshis observation, regardless of whether the return to working is measured by the real wage or average productivity. Second, from a theoretical point of view it is well known that there are a variety of ways to support the quantity allocations emerging from RBC models. By using average productivity as our measure of the return to working we avoid imposing the assumption that the market structure is one in which real wages are equated to the marginal product of labor on a period–by–period basis. In addition, existing parameterizations of RBC models imply that the marginal and average productivity of labor are proportional to each other, so that the two are interchangeable for the calculations we perform.

Our empirical results indicate that incorporating government into the analysis does lead to some improvements in the model's performance. Interestingly, the impact of this perturbation is about as large as allowing for nonconvexities in labor supply of the type stressed by Hansen (1985) and Rogerson (1988). However, as long as we abstract from measurement error, this improvement is not sufficiently large so as to allow the model to account for the Dunlop–Tarshis observation. At the same time, taking measurement error into account substantially affects our results. Indeed, once measurement error and government are incorporated into the analysis, one cannot reject that a version of our model is consistent with the observed correlation between hours worked and average productivity, as well as the observed volatility of hours worked relative to average productivity. This is not the case if we account for measurement error, but exclude government from the analysis.

The remainder of this paper is organized as follows. In section 2 we describe a general equilibrium model which nests as special cases a variety of existing RBC models. In section 3 we present our econometric methodology for estimating and evaluating the empirical performance of the model. In section 4 we present our empirical results. Finally, in section 5 we offer some concluding remarks.

2. Two Prototypical Real Business Cycle Models

In this section we present two prototypical real business cycle models. The first corresponds to a stochastic version of the one sector growth model (see, e.g. Kydland and

Prescott [1980, p.174]). The second corresponds to a version of the model economy considered by Hansen (1985) in which labor supply is indivisible. In both cases, we relax the assumption implicit in existing RBC models that public and private spending have identical effects on the marginal utility of private consumption.

2.1 The Models

Consistent with existing RBC models we assume that the time series on the beginning of period t per capita stock of capital, k_t, private time t consumption c_t^p, and hours worked at time t, n_t, correspond to the solution of a social planning problem which can be decentralized as a Pareto optimal competitive equilibrium. The following problem nests both our models as special cases. Let N be a positive scalar which denotes the time t endowment of the representative consumer and let γ be a positive scalar. The social planner ranks streams of consumption services, c_t, leisure, $N-n_t$ and publicly provided goods and services, g_t, according to the criterion function:

$$(2.1) \quad E_0 \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \gamma V(N-n_t)\}.$$

Following Kormendi (1983), Aschauer (1985), and Barro (1981,1987) we suppose that consumption services are related to private and public consumption as follows:

$$(2.2) \quad c_t = c_t^p + \alpha g_t,$$

where α is a parameter which governs the sign and magnitude of the derivative of the marginal utility of c_t^p with respect to g_t.[5] Throughout, we assume that agents view g_t as an uncontrollable stochastic process. In addition, we suppose that g_t does not depend on the current or past values of the endogenous variables in the model.[6]

We consider two specifications for the function $V(\cdot)$. In the *divisible labor model*, $V(\cdot)$ is given by,

$$(2.3) \quad V(N-n_t) = \ln(N-n_t) \quad \text{for all t.}$$

In the *indivisible labor model*, $V(\cdot)$ is given by,

$$(2.3)' \quad V(N-n_t) = (N-n_t) \quad \text{for all t.}$$

There are at least two interpretations of specification (2.3)′. First, it may just reflect the assumption that individual utility functions are linear in leisure. The second interpretation builds on the assumption that there are indivisibilities in labor supply. Here individuals can either work some positive number of hours or not at all. Assuming that agents' utility functions are separable across consumption and leisure, Rogerson (1988) shows that a market structure in which individuals choose lotteries rather than hours worked will support a Pareto optimal allocation of consumption and leisure. The lottery determines whether individuals work or not. Under this interpretation (2.3)′ represents a reduced form preference ordering which can be used to derive the Pareto optimal allocation using a fictitious social planning problem. This is the specification used by Hansen (1985).

Per capita output, y_t, is produced using the Cobb–Douglas production function

$$(2.4) \quad y_t = (z_t n_t)^{(1-\theta)} k_t^{\theta},$$

where $0 < \theta < 1$ and z_t is an aggregate shock to technology which has the time series representation

$$(2.5) \quad z_t = z_{t-1}\exp(\lambda_t).$$

Here λ_t is a serially uncorrelated iid process with mean λ and standard error σ_λ. The aggregate resource constraint is given by

$$(2.6) \quad c_t^p + g_t + k_{t+1} - (1-\delta)k_t \leq y_t,$$

i.e., per capita consumption and investment cannot exceed per capita output.

At date 0 the social planner chooses contingency plans for $\{c_t^p, k_{t+1}, n_t: t \geq 0\}$ to maximize (2.1) subject to (2.3) or (2.3)', (2.4) – (2.6), k_0 and a law of motion for g_t. Because of the nonsatiation assumption implicit in (2.1) we can, without loss of generality, impose strict equality in (2.6). Substituting (2.2), (2.4) and this version of (2.6) into (2.1), we obtain the following *social planning problem*:

Maximize

$$(2.7) \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[(z_t n_t)^{(1-\theta)} k_t^\theta + (1-\delta)k_t - k_{t+1} + (\alpha-1)g_t\right] + \gamma V(N-n_t)\right\},$$

subject to k_0 given, a law of motion for g_t, and $V(\cdot)$ is given by either (2.3) or (2.3)', by choice of contingency plans for $\{k_{t+1}, n_t: t \geq 0\}$.

It is convenient to represent the social planning problem (2.7) in a way that all of the planner's decision variables converge in nonstochastic steady state. To this end we define the following detrended variables:

$$(2.8) \quad \bar{k}_{t+1} = k_{t+1}/z_t, \quad \bar{y}_t = y_t/z_t, \quad \bar{c}_t = c_t/z_t, \quad \bar{g}_t = g_t/z_t.$$

To complete our specification of agents' environment we assume that $\bar{g}_t$ evolves according to

$$(2.9) \quad \ln(\bar{g}_t) = (1-\rho)\ln(\bar{g}) + \rho\ln(\bar{g}_{t-1}) + \mu_t,$$

where $\ln(\bar{g})$ is the mean of $\ln(\bar{g}_t)$, $|\rho| < 1$ and μ_t is the innovation in $\ln(\bar{g}_t)$ with standard deviation σ_μ. Notice that g_t has two components, z_t and $\bar{g}_t$. Movements in the former give rise to permanent changes in the level of government consumption, whereas perturbations in the latter produce temporary changes in g_t. With this specification, the factors that give rise to permanent shifts in government consumption are the same as those which permanently enhance the economy's productive ability.

Substituting (2.8) into (2.7) we obtain the criterion function:

$$(2.10) \quad \kappa + E_0 \sum_{t=0}^{\infty} \beta^t r(n_t, \bar{k}_t, \bar{k}_{t+1}, \bar{g}_t, \lambda_t),$$

$$(2.11) \quad r(n_t, \bar{k}_t, \bar{k}_{t+1}, \bar{g}_t, \lambda_t) =$$

$$\left\{ \ln\left[n_t^{(1-\theta)} \bar{k}_t^{\theta} \exp(-\theta\lambda_t) + \exp(-\lambda_t)(1-\delta)\bar{k}_t - \bar{k}_{t+1} + (\alpha-1)\bar{g}_t \right] + \gamma V(N-n_t) \right\},$$

$\kappa = E_0 \sum_{t=0}^{\infty} \beta^t \ln(z_t)$,and $V(\cdot)$ is given by either (2.3) or (2.3)′. Consequently the original planning problem is equivalent to the problem of maximizing (2.10), subject to $\bar{k}_0$, (2.9), (2.11) and $V(\cdot)$ is given by either (2.3) or (2.3)′. Since κ is beyond the planner's control, it can be disregarded in solving the planner's problem.

The only case in which it is possible to obtain an analytical solution for the problem just discussed is when $\alpha = \delta = 1$ and the function $V(\cdot)$ is given by (2.3). This case is analyzed in, among other places, Long and Plosser (1982). For general values of α and δ

analytical solutions are not available. Here we use Christiano's (1988) log linear modification of the procedure used by Kydland and Prescott (1982) to obtain an approximate solution to our social planning problems. In particular, we approximate the optimal decision rules with the solution to the linear quadratic problem obtained when the function r in (2.11) is replaced by a function R which is quadratic in $\ln(n_t)$, $\ln(\bar{k}_t)$, $\ln(\bar{k}_{t+1})$, $\ln(\bar{g}_t)$ and λ_t. The function R is the second order Taylor expansion of $r[\exp(A_1),\exp(A_2),\exp(A_3),\exp(A_4),A_5]$ about the point $[A_1,A_2,A_3,A_4,A_5] = [\ln(n),\ln(\bar{k}),\ln(\bar{k}),\ln(\bar{g}),\lambda]$. Here n and $\bar{k}$ denote the steady state values of n_t and $\bar{k}_t$ in the nonstochastic version of (2.10) obtained by setting $\sigma_\lambda = \sigma_\mu = 0$.

It follows from results in Christiano (1988) that the decision rules which solve this problem are of the form:

$$(2.12) \quad \bar{k}_{t+1} = \bar{k}(\bar{k}_t/\bar{k})^{r_k}(\bar{g}_t/\bar{g})^{d_k}\exp[e_k(\lambda_t-\lambda)],$$

and

$$(2.13) \quad n_t = n(\bar{k}_t/\bar{k})^{r_n}(\bar{g}_t/\bar{g})^{d_n}\exp[e_n(\lambda_t-\lambda)].$$

In (2.12) and (2.13), r_k, d_k, e_k, r_n, d_n and e_n are scalar functions of the models' underlying structural parameters.[7]

To gain intuition for the role of $\bar{g}_t$ in aggregate labor market fluctuations it is useful to briefly discuss the impact of three key parameters, α, ρ and γ, on the equilibrium response of n_t to $\bar{g}_t$. This response is governed by the coefficient d_n. First, notice that when $\alpha = 1$, the only way in which c_t^p and g_t enter into the social planner's preferences and constraints is via their sum, $c_t^p + g_t$. Thus, exogenous shocks to g_t induce one–for–one offsetting shocks in c_t^p, leaving other variables like y_t, k_{t+1} and n_t unaffected. This implies that the coefficients d_n and d_k in the planner's decision rules for k_{t+1} and n_t both equal zero. Consequently, the absence of a role for g_t in existing RBC models can be rationalized by the assumption that $\alpha = 1$.

Second, consider the case when α is less than one. The limiting case of $\alpha = 0$ is particularly useful for gaining intuition. Here, government consumption is formally equivalent to a pure resource drain on the economy. Thus, agents respond to an increase in government consumption as if they had suffered a reduction in their wealth (as footnote 5 indicates, this does not imply that they have suffered a reduction in utility.) Since we assume that leisure is a normal good, d_n is positive, i.e. increases in $\bar{g}_t$ are associated with increases in n_t and decreases in y_t/n_t. Continuity suggests that d_n is decreasing in α. The same logic suggests that d_n is an increasing function of ρ. This is because the wealth effect of a given shock to $\bar{g}_t$ is increasing in ρ. For a formal analysis of the effects of government consumption in a more general environment than the one considered in this paper, see Aiyagari, Christiano and Eichenbaum (1989).

Finally, consider the impact of γ on aggregate labor market fluctuations. In several experiments we found that e_n and d_n were increasing in γ.[8] To gain intuition into this result it is useful to think in terms of a version of our model in which the gross investment decision rule is fixed exogenously. In this simpler model economy, labor market equilibrium is the result of the intersection of static labor demand and supply curves. It is straightforward to show that, given our assumptions regarding the utility function, the response of labor supply to a change in the return to working is an increasing function of γ, i.e., the labor supply curve becomes flatter as γ increases. Consequently, the equilibrium response of n_t to λ_t (which shifts the labor demand curve) is increasing in γ. This is consistent with the finding that in our model e_n is increasing in γ. With respect to d_n, it is straightforward to show that, in the static framework, the extent of the shift in the labor supply curve induced by a change in $\bar{g}_t$ is also an increasing function of γ. This is consistent with the finding that in our model, d_n is an increasing function of γ.

The fact that e_n and d_n are increasing in γ leads us to expect that the volatility of hours worked will also be an increasing function of γ. However, we cannot say a priori how larger values of γ will impact on the Dunlop–Tarshis correlation. This is because larger values of e_n drive that correlation up, but larger values of d_n drive it down.

3. Econometric Methodology.

In this section we accomplish three tasks. First, we describe our strategy for estimating the structural parameters of the model as well as various second moments of the data. Second, we describe our method for evaluating the models' implications for aggregate labor market fluctuations. Third, we describe the basic data set used in our empirical analysis.

While similar in spirit, our empirical methodology is quite different from the methods typically used to evaluate RBC models. Much of the existing RBC literature makes little use of formal econometric methods at either the stage when model parameters values are selected, or at the stage when the fully parameterized model is compared with the data. Instead a variety of informal techniques, often referred to as "calibration", are used. In contrast, we use a version of Hansen's (1982) Generalized Method of Moments (GMM) procedure at both stages of the analysis. Our estimation criterion is set up in a way so that, in effect, estimated parameter values succeed in equating model and sample first moments of the data. As it turns out, these values are very similar to the values employed in existing RBC studies. However, an important advantage of our GMM procedures is that we are able to quantify the degree of uncertainty in our estimates of the model's parameters. This turns out to be an important ingredient of our model evaluation techniques.

3.1 Estimation

In this subsection we discuss our estimation strategy. The parameters of interest can be divided into three groups. Let Ψ_1 denote the structural parameters of the model:

$$(3.1) \quad \Psi_1 = \{\delta, \theta, \gamma, \rho, \bar{g}, \sigma_\mu, \lambda, \sigma_\lambda\}.$$

The parameters N, β and α were not estimated. Instead we fixed N at 1369 hours per quarter, and set the parameter β so as to imply a 3% annual subjective discount rate, i.e. $\beta = (1.03)^{-.25}$. Two alternative values of α were considered: $\alpha = 0$ and $\alpha = 1$.

The second and third set of parameters, Ψ_2 and Ψ_3, correspond to various second moments of the data. Our measures of c_t^p, dk_t, k_t, y_t, $(y/n)_t$, and g_t all display marked trends, so that some stationary inducing transformation of the data must be adopted. The two sets of second moments correspond to the two different transformations which we employ. The first transformation is motivated by the fact that according to our models, the first difference of the logarithms of all the variables which enter into the analysis are stationary stochastic processes. The second transformation corresponds to the Hodrick and Prescott (HP) detrending procedure discussed in Hodrick and Prescott (1980) and Prescott (1986). Our use of the HP transformation is motivated by the fact that many authors, including most prominently Kydland and Prescott (1982, 1988), Hansen (1985) and Prescott (1986), have investigated RBC models using data which have been filtered in this manner. That the HP filter is a stationary inducing transformation for difference stationary stochastic processes follows directly from results in King and Rebelo (1988). This result is applicable because according to our model, the logarithms of c_t^p, dk_t, k_t, y_t, $(y/n)_t$, and g_t are all difference stationary stochastic processes.

Let Ψ_2 denote a vector of population moments corresponding to the output of our first stationary inducing transformation, i.e. growth rates of the data:

$$(3.2) \qquad \Psi_2 = \{\sigma_{c^p}/\sigma_y, \sigma_{dk}/\sigma_y, \sigma_n, \sigma_n/\sigma_{y/n}, \sigma_g/\sigma_y, \text{corr}(y/n,n)\},$$

where σ_x denotes the standard deviation of the growth rate of the variable x, x = {c^p, y, dk, n, y/n, g}, and corr(y/n,n) denotes the correlation between the growth rate of the returns to working and the growth rate of hours worked. To carry out inference regarding σ_n/σ_y and σ_y, we exploit the fact that these are exact functions of the elements of Ψ_2.[9]

Let Ψ_3 denote a vector of population moments of the HP filtered data:

$$(3.3)\quad \Psi_3 = \{(\sigma_{c^p}/\sigma_y)^{hp},(\sigma_{dk}/\sigma_y)^{hp},(\sigma_n)^{hp},(\sigma_n/\sigma_{y/n})^{hp},(\sigma_g/\sigma_y)^{hp},(\mathrm{corr}(y/n,n))^{hp}\}$$

The superscript hp denotes the fact that the population moment refers to HP filtered data. In carrying out inference regarding $(\sigma_n/\sigma_y)^{hp}$ and $(\sigma_y)^{hp}$, we exploit the fact that these are exact functions of the elements of Ψ_3.

The Unconditional Moments Underlying Our Estimator of Ψ_1

According to our model, $\delta = 1 + dk_t/k_t - k_{t+1}/k_t$. Let δ^* denote the unconditional mean of the time series $[1 + dk_t/k_t - k_{t+1}/k_t]$, i.e.

$$(3.4)\quad E\{\delta^* - (1 - dk_t/k_t - k_{t+1}/k_t)\} = 0.$$

We identify δ with a consistent estimate of the parameter δ^*.

The social planner's first order necessary condition for capital accumulation requires that the time t expected value of the marginal rate of substitution of goods in consumption equals the time t expected value of the marginal return to physical investment in capital. It follows that,

$$(3.5)\quad E\{\beta^{-1} - [\theta(y_{t+1}/k_{t+1}) + 1 - \delta]c_t/c_{t+1}\} = 0.$$

This is the moment restriction that underlies our estimate of θ.

The first order necessary condition for hours worked requires that, for all t, the marginal productivity of hours times the marginal utility of consumption equals the marginal disutility of working. This implies the condition $\gamma = (1-\theta)[y_t/n_t]/[c_t V'(N-n_t)]$ for all t. Let γ^* denote the unconditional expected value of the time series on the right hand side of the previous expression, i.e.

$$(3.6)\quad E\{\gamma^* - (1-\theta)(y_t/n_t)/[c_t V'(N-n_t)]\} = 0.$$

We identify γ with a consistent estimate of the parameter γ^*.

Fourth, consider the random variable $\lambda_t = \ln(z_t/z_{t-1}) = (1-\theta)^{-1}\Delta\ln(y_t) - \Delta\ln(n_t) - \theta(1-\theta)^{-1}\Delta\ln(k_t)$. Here Δ denotes the first difference operator. Under the null hypothesis of balanced growth $\lambda \equiv E\lambda_t$ equals the unconditional growth rate of output, μ_y. It follows that

$$(3.7)\quad \begin{aligned} &E[\Delta\ln(y_t) - \mu_y] = 0, \quad \text{and} \\ &E[(\lambda_t - \mu_y)^2 - \sigma_\lambda^2] = 0. \end{aligned}$$

Relation (3.7) summarizes the moment restrictions underlying our estimators of λ ($= \mu_y$) and σ_λ.

Our assumptions regarding the stochastic process generating government consumption imply the unconditional moment restrictions,

$$(3.8)\quad \begin{aligned} &E[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})] = 0, \\ &E[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})]\bar{g}_{t-1} = 0, \\ &E\{[\ln(\bar{g}_t) - (1-\rho)\ln(\bar{g}) - \rho\ln(\bar{g}_{t-1})]^2 - \sigma_\mu^2\} = 0. \end{aligned}$$

These moment restrictions can be used to estimate ρ, $\bar{g}$ and σ_μ.

Equations (3.4) – (3.8) consist of eight unconditional moment restrictions involving the eight elements of Ψ_1 and

$$(3.9)\quad X_{t+1} = [dk_t/k_t, k_{t+1}/k_t, k_t/k_{t-1}, c^p_{t+1}/c^p_t, y_{t+1}/k_{t+1}, y_t/k_t, y_{t-1}/k_{t-1}, \\ y_t/y_{t-1}, n_t, n_{t-1}, y_t/g_t, c^p_{t+1}/g_{t+1}, c^p_t/g_t, g_t/g_{t-1}].$$

With this notation we can summarize (3.4) – (3.8) as

$$(3.10)\quad E\{H_1[X_{t+1},\Psi^o_1]\} = 0, \quad \text{for all } t \geq 0,$$

where Ψ^o_1 is the true value of Ψ_1, and $H_1(\cdot,\cdot)$ is the 8 x 1 vector valued function whose elements are left hand sides of (3.4) – (3.8) before expectations are taken.

The Unconditional Moments Underlying Our Estimator of Ψ_2

Our model implies that the unconditional expected value of the growth rate of n_t equals zero. It follows that

$$(3.11)\qquad E\{\Delta\ln(n_t)^2 - \sigma_n^2\} = 0,$$

which can be used to estimate σ_n. The unconditional moments underlying our estimators of σ_{c^p}/σ_y, σ_{dk}/σ_y, σ_g/σ_y and $\sigma_n/\sigma_{y/n}$ can be written as

$$(3.12)\qquad E[(\Delta\ln(y_t) - \mu_y)^2(\sigma_x/\sigma_y)^2 - (\Delta\ln(x_t) - \mu_y)^2] = 0 \text{ for } x_t = [c^p_t, dk_t, g_t] \\ E\{[\Delta\ln(y/n)_t - \mu_y]^2(\sigma_n/\sigma_{y/n})^2 - \Delta\ln(n)_t{}^2\} = 0.$$

Here we have used the fact that, under balanced growth, y_t, c_t^p, dk_t, $(y/n)_t$ and g_t have the same unconditional growth rate, μ_y.

Finally, to estimate corr(y/n,n) we exploit the unconditional moment restriction,

$$(3.13) \qquad E\{[\sigma_n^2/(\sigma_n/\sigma_{y/n})]\mathrm{corr}(y/n,n) - [\Delta\ln(y/n)_t - \mu_y]\Delta\ln(n_t)\} = 0,$$

where again we have used the balanced growth restriction that the unconditional growth rate of $(y/n)_t$ equals μ_y.

Equations (3.11)– (3.13) consist of six unconditional moment restrictions involving the six elements of Ψ_2 as well as the elements of X_{t+1}. With this notation we can summarize (3.11) – (3.13) as

$$(3.14) \quad E\{H_2[X_{t+1},\Psi_2^o]\} = 0, \quad \text{for all } t \geq 0,$$

where Ψ_2^o is the true value of Ψ_2 and $H_2(\cdot,\cdot)$ is the 6 x 1 vector valued function whose elements are the left hand sides of (3.11) – (3.13) before expectations are taken.

The Unconditional Moments Underlying Our Estimator of Ψ_3

Data which are transformed via the HP filter are zero mean by construction. It follows that we can estimate the parameters of Ψ_3 by exploiting the unconditional moment restrictions,

$$(3.15) \qquad \begin{aligned} &E\{\tilde{y}_t^{\,2}[(\sigma_x/\sigma_y)^{hp}]^2 - (\tilde{x}_t))^2\} = 0 \quad x_t = [\tilde{c}_t^p, d\tilde{k}_t, \tilde{g}_t], \\ &E[\tilde{n}_t^2 - (\sigma_n^{hp})^2] = 0, \\ &E\{(\widetilde{y/n})_t^2[(\sigma_n/\sigma_{y/n})^{hp}]^2 - \tilde{n}_t^{\,2}\} = 0, \\ &E\{[\sigma_n^{hp}]^2/[(\sigma_n/\sigma_{y/n})^{hp}]\mathrm{corr}(y/n,n)^{hp} - (\widetilde{y/n})_t\tilde{n}_t\} = 0,. \end{aligned}$$

where the superscript ˜ denotes data which have been transformed using the HP filter. Equation (3.15) consists of six unconditional moment restrictions involving the six elements of Ψ_3 and the vector valued function $\tilde{X}_t = [\tilde{c}^p_t, \tilde{y}_t, \tilde{dk}_t, \tilde{g}_t, \widetilde{(y/n)}_t, \tilde{n}_t]$. With this notation we can rewrite (3.15) as

$$(3.16) \quad E\{H_3[\tilde{X}_t, \Psi_3^o]\} = 0, \quad \text{for all } t \geq 0,$$

where Ψ_3^o is the true value of Ψ_3 and $H_3(\cdot,\cdot)$ is the 6 x 1 vector valued function whose elements are the left hand sides of (3.15) before expectations are taken.

In order to discuss our estimator it is convenient to define the 20 x 1 parameter vector $\Psi = [\Psi_1\ \Psi_2\ \Psi_3]'$, the 20 x 1 vector valued function $H = [H_1\ H_2\ H_3]'$, and the data vector $F_{t+1} = [X_{t+1}, \tilde{X}_t]'$. With this notation the unconditional moment restrictions (3.10), (3.14) and (3.16), can be written as

$$(3.17) \quad E\{H[F_{t+1}, \Psi^o]\} = 0 \quad \forall\, t \geq 0,$$

for $\Psi = \Psi^o$, the true parameter vector. Let g_T denote the 20 x 1 vector valued function

$$(3.18) \quad g_T(\Psi) = (1/T)\sum_{t=0}^{T} H(F_{t+1}, \Psi),$$

which can be calculated given a sample on $\{F_t$: t=1,2,...T+1$\}$. All of our models imply that F_{t+1} is a stationary and ergodic stochastic process. Since $g_T(\cdot)$ is of the same dimension as Ψ_T, it follows from Hansen (1982) that the estimator Ψ_T defined by the condition, $g_T(\Psi_T) = 0$, is a consistent estimator of Ψ^o.

Let D_T denote the matrix of partial derivatives

(3.19) $$D_T = \frac{\partial g_T(\Psi_T)}{\partial \Psi'}$$

evaluated at Ψ_T. Then it follows from results in Hansen (1982) that a consistent estimator of the variance – covariance matrix of Ψ_T is given by

(3.20) $$\mathrm{Var}(\Psi_T) = [D_T]^{-1} S_T [D_T']^{-1}/T.$$

Here S_T is a consistent estimate of the spectral density matrix of $H(F_t, \Psi^0)$ at frequency zero.[10]

3.2 Testing

In this subsection we describe how a Wald–type test statistic described in Eichenbaum, Hansen and Singleton (1984) and Newey and West (1987) can be used to formally assess the plausibility of the models' implications for subsets of the second moments of the data. Our empirical analysis concentrates on assessing the model's implications for the labor market moments, [corr(y/n), $\sigma_n/\sigma_{y/n}$] and [corr(y/n)^{hp}, $(\sigma_n/\sigma_{y/n})^{hp}$].[11] Here we describe our procedure for testing the first set of moments. The procedure for testing the second set of moments is completely symmetric. More generally, the test procedure can be used for any finite set of moments.

Given a set of values for Ψ_1, our model implies particular values for [corr(y/n), $\sigma_n/\sigma_{y/n}$] in population. We represent this relationship via the function f that maps $\mathbb{R}^8$ into $\mathbb{R}^2$:

(3.21) $$f(\Psi_1) = [\mathrm{corr}(y/n,n), \sigma_n/\sigma_{y/n}].$$

The function $f(\cdot)$ is highly nonlinear in Ψ_1 and must be computed using numerical methods. Here we used the spectral technique used in Christiano and Eichenbaum (1989).

Let A be the 2 x 20 matrix composed of zeros and ones with the property

$$(3.22) \quad A\Psi = [\mathrm{corr}(y/n,n),\ \sigma_n/\sigma_{y/n}]'$$

and let

$$(3.23) \quad F(\Psi) = f(\Psi_1) - A\Psi.$$

Under the null hypothesis that the model is true

$$(3.24) \quad F(\Psi^o) = 0.$$

In order to test the hypothesis that $F(\Psi^o) = 0$, we require the asymptotic distribution of $F(\Psi_T)$ under the null hypothesis. Taking a first order Taylor series approximation of $F(\Psi_T)$ about Ψ^o yields,

$$(3.25) \quad F(\Psi_T) \cong F(\Psi^o) + F'(\Psi^o)[\Psi_T - \Psi^o]'.$$

It follows that a consistent estimator of $\mathrm{Var}[F(\Psi_T)]$ is given by

$$(3.26) \quad \mathrm{Var}[F(\Psi_T)] = [F'(\Psi_T)]\mathrm{Var}(\Psi_T)[F'(\Psi_T)]'.$$

An implication of results in Eichenbaum, Hansen and Singleton (1984) and Newey and West (1987) is that the test statistic

$$(3.27) \quad J = F(\Psi_T)'\mathrm{Var}[F(\Psi_T)]^{-1}F(\Psi_T)$$

is asymptotically distributed as a chi–square random variable with two degrees of freedom. This fact can be used to test the null hypothesis (3.24).

3.3 The Baseline Dataset

Here, we discuss the baseline dataset, which we use initially in our analysis. Later, in section 4.2, we modify the dataset in order to assess the impact of measurement error on our analysis. In all of our empirical work, private consumption, c_t^p, was measured as quarterly real expenditures on nondurable consumption goods plus services, plus the imputed service flow from the stock of durable goods. The first two measures were obtained from the Survey of Current Business. The third measure was obtained from the data base documented in Brayton and Mauskopf (1985). Government consumption, g_t, was measured by real government purchases of goods and services minus real government (federal, state and local) investment.[12] A measure of government investment was provided to us by John Musgrave of the Bureau of Economic Analysis. This measure is a revised and updated version of the measure discussed in Musgrave (1980). Gross investment, dk_t, was measured as private sector fixed investment plus real expenditures on durable goods plus government fixed investment. The capital stock series, k_t, was chosen to match the investment series. Accordingly, we measured k_t as the stock of consumer durables, producer structures and equipment, plus government and private residential capital plus government nonresidential capital. Gross output, y_t, was measured as c_t^p plus g_t plus dk_t plus time t inventory investment. Given our consumption series, the difference between our measure of gross output and the one reported in the Survey of Current Business is that ours includes the imputed service flow from the stock of consumer durables but excludes net exports. Our baseline measure of hours worked corresponds to the one constructed by

Hansen (1984) which is based on the household survey conducted by the Department of Labor Statistics. The data were converted to per capita terms using an efficiency weighted measure of the population. All series cover the period 1955,3 – 1983,4.[13]

4. Empirical Results

In this section we report our empirical results. The section is organized as follows. In subsection 4.1 we report results obtained using the baseline dataset described in Section 3.3. In subsection 4.2 we consider the impact of measurement error on our analysis.

4.1 Results for the Baseline Dataset

Table 1a reports our estimates of Ψ_1 along with standard errors for the different structural models. The coefficients of the equilibrium laws of motion for $\bar{k}_{t+1}$ and n_t corresponding to the estimated values of Ψ_1 are displayed in Table 2. One way to assess the plausibility of our estimates of Ψ_1 is to investigate their implications for various first moments of the data. To do this, we used the equilibrium laws of motion reported in Table 2 to simulate 1000 time series, each of length 113, the number of observations in our dataset. First moments were calculated on each synthetic data set. Table 3 reports the average value of these moments across synthetic data sets, as well as estimates of the corresponding first moments of the data. As can be seen, all of the models do extremely well on this dimension. Notice that the model predicts the same mean growth rates for c^p_t, k_t, g_t and y_t. This reflects the balanced growth property of our model. This restriction does not seem implausible given the point estimates and standard errors reported in Table 2. The model also predicts that the unconditional growth rate of n_t is zero. Again, this restriction seems reasonably consistent with the data.

Tables 4A and 4B display estimates of a subset of the second moments of the data as well as the analog model predictions. The first table reports results corresponding to HP filtered data, while the second table reports results obtained working with growth rates of the data. Since the results are qualitatively similar, we concentrate on Table 4A. All of the models do reasonably well at matching the estimated values of $(\sigma_{cP}/\sigma_y)^{hp}$, $(\sigma_{dk}/\sigma_y)^{hp}$, $(\sigma_g/\sigma_y)^{hp}$, and $(\sigma_y)^{hp}$. Interestingly, introducing government into the analysis, i.e. moving from $\alpha = 1$ to $\alpha = 0$, actually improves the performance of the models with respect to $(\sigma_{cP}/\sigma_y)^{hp}$ and $(\sigma_g/\sigma_y)^{hp}$, but has relatively little impact on their predictions for $(\sigma_{dk}/\sigma_y)^{hp}$ or $(\sigma_y)^{hp}$. In contrast, the models do less well at matching the volatility of hours worked relative to output. Not surprisingly, incorporating government into the analysis ($\alpha = 0$) generates additional volatility in n_t, as does allowing for indivisibilities in labor supply. Indeed, the quantitative impact of these two perturbations to the base model (divisible labor, $\alpha = 1$) is similar. Nevertheless, even when both effects are operative, the model still underpredicts the volatility of n_t relative to y_t. Similarly, allowing for nonconvexities in labor supply and introducing government into the analysis improves the model's performance with respect to the volatility of n_t relative to y_t/n_t. In fact the fourth model which incorporates both of these effects actually *overstates* the volatility of n_t relative to y_t/n_t.[14]

Next we consider the ability of the different models to account for the Dunlop–Tarshis observation. From Table 4A we see that the basic model (i.e., divisible labor, $\alpha = 0$) fails dramatically along this dimension. Introducing nonconvexities in labor supply has almost no impact on the model's prediction for this correlation. Introducing government into the analysis ($\alpha = 0$) does reduce the correlation between n_t and y_t/n_t. But, despite the improvement, the models with $\alpha = 0$ still substantially overstate the correlation between average productivity and hours worked.

Table 5 reports the results of implementing the diagnostic procedures discussed in section 3. Columns labeled HP and DIFF refer to results generated from HP filtered data and growth rates, respectively. The first three rows report results for the correlation between average productivity and hours worked. The second set of three rows report results for the relative volatility of hours worked and average productivity. The last row of the table labeled "J" reports the statistic for testing the joint null hypothesis that the model predictions for both corr(y/n,n) and $\sigma_n/\sigma_{y/n}$ (or corr(y/n,n)^{hp} and $(\sigma_n/\sigma_{y/n})^{hp}$) are true. As can be seen, this null hypothesis is overwhelmingly rejected for every version of the model, irrespective of whether growth rates or HP filtered data is used. Notice also that the t statistics associated with corr(y/n,n) and corr(y/n,n)^{hp} are, in every instance, larger than the corresponding t statistics associated with $\sigma_n/\sigma_{y/n}$ and $(\sigma_n/\sigma_{y/n})^{hp}$. This is consistent with our claim that the single most striking failure of the models lies in their implications for the Dunlop–Tarshis observation, rather than the relative volatility of hours worked and average productivity.

4.2 Measurement Error

There are at least two reasons to believe that the negative correlation between hours worked and average productivity reported in section 4.1 is spurious and reflects measurement error. One potential source of distortion lies in the fact that our base output measure covers more sectors than do our base hours data (see Appendix 1 of Christiano and Eichenbaum (1988)). In addition, the base hours data may suffer from classical measurement error. This type of measurement error can have a particularly important impact on estimates of corr(y/n,n) and corr(y/n,n)^{hp} because average productivity is constructed using the hours worked data.

Alignment Error

In order to investigate the quantitative impact of alignment error, we considered alternative measures of hours worked and the returns to working which do not suffer from this problem: output per hour of all persons in the non–agricultural business sector (CITIBASE mnemonic LBOUTU) and per capita hours worked by wage and salary workers in private non–agricultural establishments as reported by the Bureau of Labor Statistics (IDC mnemonic HRSPST). For convenience, we refer to this measure of n_t as establishment hours. With the new data, the estimated values of corr(y/n,n) and $\text{corr}(y/n,n)^{hp}$ are .21 and .16 with corresponding standard errors of .07 and .08. These results are consistent with the view that the negative correlations reported in Table 5 reflect, in part, alignment error. Interestingly, our estimates of $\sigma_n/\sigma_{y/n}$ and $(\sigma_n/\sigma_{y/n})^{hp}$ are also significantly affected by moving to the new data sets. These now equal 1.27 and 1.64 with corresponding standard errors of .13 and .16. So while the models' performance with respect to the Dunlop–Tarshis observation ought to be enhanced by moving to the new data set, it ought to deteriorate with respect to the relative volatility of hours worked and output per hour. Therefore, the net effect of the new dataset on overall inference cannot be determined a priori.

To assess the net impact on the models' performance, we reestimated the structural parameters and redid the diagnostic tests discussed in section 3. The new parameter estimates are reported in Table 1b. The results of our diagnostic tests are summarized in Table 6, which is the exact analog of Table 5. The data used to generate Tables 5 and 6 are the same, with two exceptions. First, in the calculations associated with the intratemporal Euler equation, i.e. the third element of $H(\cdot,\cdot)$, we used our new measure of average productivity, which is actually an index. This measure of average productivity was scaled so that the sample mean of the transformed index coincides with the sample mean of our measure of y_t divided by establishment hours. The second difference is that, apart from the calculations involving y_t/n_t, we measured n_t using establishment hours.

Notice that, for every single model and both stationary inducing transformations, the J statistics in Table 6 are lower than the corresponding entries in Table 5. Nevertheless, as long as government is not incorporated into the analysis, i.e. $\alpha = 1$, the models are still rejected at essentially the zero percent significance level. However this is no longer true when government is incorporated into the analysis, i.e. $\alpha = 0$. In particular, when we work with growth rates, we can no longer reject the divisible labor model at the one percent significance level. Even more dramatically, when we work with HP filtered data, we cannot reject the indivisible labor model at even the ten percent significance level.[15]

To understand these results, consider first the impact of the new data set on inference regarding the correlation between hours worked and average productivity. Comparing the $\alpha = 0$ models in Tables 5 and 6 we see a dramatic drop in the t statistics. There are two principal reasons for this improvement. The most obvious is that $\widehat{\text{corr}}(y/n,n)$ and $\widehat{\text{corr}}(y/n,n)^{hp}$ are positive in the new data set (.21 and .16) while they are negative in the base data set (–.71 and –.20). In this sense the data have moved towards the model. Second, the new values of $\hat{\Psi}_1$ generate smaller values for corr(y/n,n) and $\text{corr}(y/n,n)^{hp}$. For example in the indivisible labor model ($\alpha = 0$), $\text{corr}(y/n,n)^{hp}$ drops from .737 to .575. In part, this reflects the new values of $\hat{\rho}$ and $\hat{\gamma}$. Consider $\hat{\rho}$ first. With the baseline set, $\hat{\rho}$ is .96 (after rounding) for all of the models. In the new data set, $\hat{\rho}$ is .98 (after rounding) for all the models. As we emphasized in section 2, increases in ρ are associated with decreases in the correlation between y_t/n_t and n_t.[16] Next, consider $\hat{\gamma}$. With the new data set the estimates of γ are consistently larger than we obtained with the old data set.[17] For example, in the indivisible labor model ($\alpha = 0$), $\hat{\gamma}$ was .0037, while now $\hat{\gamma} =$ .0046. As we noted in section 2, the impact of a change in γ on corr(y/n,n) and $\text{corr}(y/n,n)^{hp}$ cannot be determined a priori. As it turns out, the increase in $\hat{\gamma}$ contributes to a drop in these statistics.[18]

We now examine the impact of the new data set on inference regarding the relative volatility of hours worked and average productivity. Comparing Tables 5 and 6 we see that in all cases but one, the t statistics drop. In the exceptional case, i.e. the divisible labor model with $\alpha = 0$, the change is very small. There are three factors which influence the change in these t statistics. First, the point estimates of $\sigma_n/\sigma_{y/n}$ and $(\sigma_n/\sigma_{y/n})^{hp}$ are larger with the new dataset. Other things equal, this hurts the empirical performance of all the models, except the indivisible labor model with $\alpha = 0$. Second, these statistics are estimated less precisely with the new data set. Other things equal, this contributes to a reduction in the t statistics. Finally, the new parameter estimates lead to an increase in each model's implied values of $\sigma_n/\sigma_{y/n}$ and $(\sigma_n/\sigma_{y/n})^{hp}$. For example, the value of $(\sigma_n/\sigma_{y/n})^{hp}$ implied by the indivisible labor model with $\alpha = 0$ rises to 1.437 from 1.348. In part this reflects the new values of $\hat{\rho}$ and $\hat{\gamma}$. For example, the value of $(\sigma_n/\sigma_{y/n})^{hp}$ implied by the baseline indivisible labor model ($\alpha = 0$) with ρ increased to .98 is 1.396. The analog experiment with γ increases the value of this statistic to 1.436.

Classical Measurement Error in Hours Worked

Recall that we have two different measures of hours worked, BLS establishment hours and the baseline measure (i.e., Gary Hansen's measure of hours worked.) Denote these two time series by n_t^e and n_t^h, respectively. Let $\overset{*}{n}_t$ denote true hours worked at time t. We assume, as does Prescott (1986a), that the measurement error in these two time series are independently and identically distributed, and orthogonal to each other as well as to the logarithm of true hours worked, so that

$$\text{(4.1)} \qquad \begin{aligned} \ln n_t^e &= \ln \overset{*}{n}_t + v_t^e \\ \ln n_t^h &= \ln \overset{*}{n}_t + v_t^h. \end{aligned}$$

It follows that

$$\text{(4.2)} \qquad \sigma_{v^e}^2 = .5\{\sigma_{\Delta n^e}^2 - \mathrm{cov}[\Delta \ln(n_t^e), \Delta \ln(n_t^h)]\} \quad \text{and}$$

$$\sigma^2_{v^h} = .5\{\sigma^2_{\Delta n^h} - \text{cov}[\Delta\ln(n^e_t), \Delta\ln(n^h_t)]\},$$

where $\sigma^2_{\Delta n^e}$ and $\sigma^2_{\Delta n^h}$ denote the variance in the growth rates of n^e_t and n^h_t, respectively, while $\sigma^2_{v^e}$ and $\sigma^2_{v^h}$ denote the variance of v^e_t and v^h_t.

We can estimate $\sigma^2_{v^e}$ and $\sigma^2_{v^h}$ by replacing the objects to the right of the equalities in (4.2) by their sample counterparts. We map this estimator into our GMM framework in order to take into account the impact of sampling uncertainty in $\sigma^2_{v^e}$ and $\sigma^2_{v^h}$ on our model diagnostics. The unconditional moment restrictions associated with (4.2) are,

$$(4.3) \qquad \begin{aligned} &E\{\sigma^2_{v^h} - .5[\Delta\ln(n^h_t)]^2 - .5\Delta\ln(n^e_t)\Delta\ln(n^h_t)\} = 0 \\ &E\{\sigma^2_{v^e} - .5[\Delta\ln(n^e_t)]^2 - .5\Delta\ln(n^e_t)\Delta\ln(n^h_t)\} = 0. \end{aligned}$$

In redoing the empirical analysis underlying Table 5 (Table 6) we added the left hand side of the first (second) equation in (4.3), before expectations are taken, to our specification of the function $H(\cdot,\cdot)$ and added $\sigma^2_{v^h}$ ($\sigma^2_{v^e}$) to our specification of Ψ_1.

Next, we show how measurement error impacts on the remaining unconditional moment conditions which define our estimator of Ψ_1. To do this, we let v_t denote v^h_t or v^e_t, depending on whether measurement error is being incorporated into the new version of Table 5 or Table 6. The corresponding variance of the measurement error is denoted by σ^2_v.

Obviously, if hours are mismeasured then so will the Solow residual, z_t. Let $\overset{*}{z}_t$ denote the true Solow residual at time t. Relation (4.1) and the definition of $\overset{*}{z}_t$ imply that

$$(4.5) \qquad z_t = \overset{*}{z}_t - v_t.$$

It follows that the second equation in (3.7) must be replaced by

$$(4.6) \qquad E[(\lambda_t-\mu_y)^2-\sigma_\lambda^2-2\sigma_v^2]=0.$$

Since z_t is mismeasured, we must also modify (3.8), the unconditional moment restrictions used to estimate ρ, σ_μ, and $\bar{g}$. Given our model of measurement error, we now have the restrictions

$$(4.7) \qquad \begin{aligned} &E[\ln(\bar{g}_t)-(1-\rho)\bar{g}-\rho\ln(\bar{g}_{t-1})]=0 \\ &E\{[\ln(\bar{g}_t)-(1-\rho)\bar{g}-\rho\ln(\bar{g}_{t-1})]\ln(\bar{g}_{t-1})+\rho\sigma_v^2\}=0 \\ &E\{[\ln(\bar{g}_t)-(1-\rho)\bar{g}-\rho\ln(\bar{g}_{t-1})]^2-\sigma_u^2-(1+\rho^2)\sigma_v^2\}=0. \end{aligned}$$

The last unconditional moment condition which involves n_t is the one which defines our estimator of γ. Under our assumptions regarding the nature of the measurement error, this estimator remains valid, to a first approximation. Consider, for example, the case of the divisible labor model. Under our assumptions the expected value of our estimator of γ, $E(1-\theta)(y_t/c_t)[(N/n_t)-1]$, equals

$$E(1-\theta)(y_t/c_t)[(N/n_t^*)-1]+(1-\theta)E(y_t/c_t)(N/n_t^*)[\exp(-v_t)-1]$$

$$=\gamma^*+(1-\theta)E(y_t/c_t)(N/n_t^*)E[\exp(-v_t)-1]$$

$$\cong\gamma^*+(1-\theta)E[(y_t/c_t)(N/n_t^*)]\sigma_v^2/2.$$

The equality in the above expression exploits the fact that, by definition, $\gamma^*=E(1-\theta)(y_t/c_t)[(N/n_t^*)-1]$, and makes use of our independence assumptions on v_t. The last relation makes use of the approximation $\exp(-v_t)\cong 1-v_t+.5v_t^2$. It follows that the bias in our estimator of γ is roughly $(1-\theta)E[(y_t/c_t)(N/n_t^*)]\sigma_v^2/2$. The same logic indicates that the bias in the indivisible labor model equals $(1-\theta)E[(y_t/c_t)(1/n_t^*)]\sigma_v^2/2$. Relative to the point estimates provided below, these biases are negligible.

Using the methodology discussed above, we reestimated all the models, thus generating eight new sets of parameter estimates. The first four were obtained using the baseline dataset. Here, establishment hours are incorporated into the analysis in order to estimate estimate σ_{vh}. The results of the corresponding diagnostic tests are reported in Panel A of Table 7. The second four sets of parameter estimates were obtained using the alignment corrected data set. Here, the baseline hours data where incorporated into the analysis in order to estimate σ_{ve}. In both cases the only parameter estimates which were significantly affected are those of σ_{ϵ}, σ_{μ} and ρ. In the baseline dataset

$$(\hat{\sigma}_{\epsilon}, \hat{\sigma}_{\mu}, \hat{\rho}) = \begin{array}{l} (.018, .020, .96), \text{ without measurement error} \\ (.014, .017, .98), \text{ with measurement error,} \end{array}$$

for all models. With the alignment corrected dataset,

$$(\hat{\sigma}_{\epsilon}, \hat{\sigma}_{\mu}, \hat{\rho}) = \begin{array}{l} (.012, .016, .98), \text{ without measurement error, } \alpha = 0,1, \\ (.010, .014, .98), \text{ with measurement error, } \alpha = 1, \\ (.011, .014, .98), \text{ with measurement error, } \alpha = 0, \end{array}$$

for the divisible and indivisible labor models. The estimated standard error of $\hat{\sigma}_{\epsilon}$ and $\hat{\sigma}_{\mu}$ is always .001, while the estimated standard error of $\hat{\rho}$ is .03 in all cases. Not surprisingly, taking measurement error into account reduces our estimates of σ_{ϵ} and σ_{μ}, but increases our estimate of ρ. The estimated values of σ_{ve} and σ_{vh} are .0041 (.0006) and .0087 (.0009), respectively, where numbers in parentheses denote standard errors. Evidently, our baseline measure of hours suffer more from measurement error than does the establishment measure. For example, our estimates imply that roughly 80 percent of the standard deviation of the growth rate in the baseline measure of hours worked can be attributed to measurement error. The corresponding figure for establishment hours worked is 58 percent.

We now consider the results of our formal diagnostic tests. Comparing Panel A of Table 7 with Table 5, we see that allowing for measurement error in the baseline dataset has a dramatic impact on the models' implications for the observed values of corr(y/n,n) and corr(y/n,n)^{hp}. For example in the base model, (divisible labor, $\alpha = 1$), the predicted values of corr(y/n,n)^{hp} and corr(y/n,n) go from .951 and .960 to –.145 and –.638 respectively. The corresponding t statistics drop from 10.56 and 25.18 to .48 and 1.39, respectively. In this sense, measurement alone resolves the Dunlop–Tarshis puzzle. However, the J statistic reveals that all of the models are rejected at very high significance levels. To see why, notice that the models overpredict corr(y/n,n) and corr(y/n,n)^{hp}, but with one exception, they underpredict $\sigma_n/\sigma_{y/n}$ and $(\sigma_n/\sigma_{y/n})^{hp}$. At the same time, the correlation between $F_1(\hat{\Psi})$ and $F_2(\hat{\Psi})$ for the different models lies between .4 and .8. (Here, F is the 2 by 1 dimensional vector function $F \equiv [F_1\ F_2]'$ defined in [3.23].) This is why the J statistics assign low probability to these estimates: We conclude that measurement error alone does allow these models to account for the Dunlop–Tarshis observation and the relative volatility of hours worked and average productivity when each statistic is considered individually. But measurement error alone does not account for their joint behavior.

It is precisely on the joint behavior of these statistics that the models with government do somewhat better. Setting $\alpha = 0$ increases the predicted values for the relative volatility of hours worked and average productivity. This effect is particularly dramatic in the indivisible labor model where the model actually overpredicts $(\sigma_n/\sigma_{y/n})^{hp}$. In conjunction with the low t statistics, this produces a value for the J statistic according to which the model cannot be rejected at even the thirty percent significance level.

The combined impact of correcting for alignment and classical measurement error can be seen by comparing Table 5 and Panel B of Table 7. In all cases, the reported J statistic falls by a factor of three or more. To assess the impact of classical measurement error, conditioning on having adjusted for alignment error, compare Table 6 with Panel B of Table 7. With two exceptions, incorporating classical measurement error into the analysis substantially improves the performance of the models. Here, the indivisible labor models with government cannot be rejected at even the ten percent significance level, regardless of whether we work with growth rates or HP filtered data. Finally, to see the impact of incorporating government into the analysis when both types of measurement error are dealt with, consider the reported J statistics in Panel B of Table 7. In every case, moving from $\alpha = 1$ to $\alpha = 0$ substantially improves the performance of the model. Indeed, when we work with growth rates, neither model can be rejected at the three percent significance level. Working with HP filtered data yields more ambiguous conclusions. Here, the indivisible labor model cannot be rejected at the thirty percent significance level, but the divisible labor model can be rejected at the one percent significance level. We conclude that once measurement error is taken into account, incorporating government into the analysis substantially alters inference about the plausibility of the models.

5. Concluding Remarks

Existing RBC theories assume that the only source of impulses to post war US business cycles are exogenous shocks to technology. We have argued that this feature of RBC models generates a strong positive correlation between hours worked and average productivity. Unfortunately, this implication is grossly counterfactual, at least for the post war US. This leads us to conclude that there must be other quantitatively important shocks driving fluctuations in aggregate U.S. output. This paper focused on assessing the importance of shocks to government consumption. Our results indicate that when aggregate demand shocks arising from stochastic movements in government consumption are incorporated into the analysis, and measurement error is allowed for, the model's empirical peformance is substantially improved.

We wish to emphasize two important caveats about our empirical results. First, we have implicitly assumed that public and private capital are perfect substitutes in the aggregate production function. A number of authors, including most prominently Aschauer (1989), have argued that this assumption is empirically implausible. To the extent that these authors are correct, and to the extent that public investment shocks are important, our assumption makes it easier for our model to account for the Dunlop–Tarshis observation. This is because these kinds of shocks impact on the model in a manner very similar to technology shocks, so that they contribute to a positive correlation between hours worked and productivity. Second, we have implicitly assumed that all taxes are lump sum. We chose this strategy in order to isolate the role of shocks to government consumption per se.

We leave to future research the important task of incorporating distortionary taxation into our framework. It is not clear what impact distortionary taxes would have on our model's ability to account for the Dunlop–Tarshis observation. Recent work by Braun (1989) and McGratten (1989) indicates that randomness in marginal tax rates enhances the model on this dimension. On the other hand, some simple dynamic optimal taxation arguments suggest the opposite. For example, suppose that it is optimal for the government to immediately increase distortionary taxes on labor in response to an increase in government consumption that is persistent. This would obviously mitigate the positive employment effect of an increase in government consumption, thus hurting the model's ability to account for the Dunlop–Tarshis observation. Suppose, however, that it is optimal for the government to increase taxes with a lag. We suspect that this would enhance the model's empirical performance.

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Footnotes

[1] This finding is closely related to McCallum's (1989) observation that existing RBC models generate grossly counterfactual predictions for the correlation between average productivity and output.

[2] In Keynes' own words: "Thus I am not disputing this vital fact which the classical economists have (rightly) asserted as indefeasible. In a given state of organisation, equipment and technique, the real wage earned by a unit of labour has a unique (inverse) correlation with the volume of employment." (Keynes [1964,p.17].)

[3] Although Prescott (1986a) and Kydland and Prescott (1982) never explicitly examine the hours/real wage correlation implication of the RBC, Prescott (1986a) nevertheless implicitly acknowledges that failure to account for the Dunlop—Tarshis observation is *the* key remaining deviation between "economic theory" and observations. He states (p.21): "The key deviation is that the empirical labor elasticity of output is less than predicted by theory." Denote the empirical labor elasticity by η. By definition, $\eta \equiv \text{corr}(y,n)\sigma_y/\sigma_n$, where corr(i,j) is the correlation between i and j, σ_i is the standard deviation of i, y is log detrended output and n is log hours. Simple arithmetic yields $\text{corr}(y-n,n) = [\eta-1](\sigma_n/\sigma_{y-n})$. If—as Prescott claims—the magnitude of σ_n / σ_{y-n} in the RBC is empirically accurate, then saying that the RBC overstates η is equivalent to stating that it overstates corr(y—n,n). In Prescott's model corr(y—n,n) is exactly the same as the correlation between real wages and hours worked. (Also, under log detrending, y—n is log detrended productivity.)

[4] An alternative strategy is pursued by Bencivenga (1987), who allows for shocks to labor suppliers' preferences. Shapiro and Watson (1988) also allow for unobservable shocks to the labor supply function.

[5] We can generalize the criterion function (2.1) by writing it as $\ln(c_t) + \gamma V(T-n_t) + \phi(g_t)$, where $\phi(\cdot)$ is some positive concave function. As long as g_t is modeled as an exogenous stochastic process, the presence of such a term has no impact on the competitive equilibrium. However, the presence of $\phi(g_t) > 0$ means that agents do not necessarily feel worse off when g_t is increased. The fact that we have set $\phi(\cdot) \equiv 0$ reflects our desire to minimize notation, not the view that that the optimal level of g_t is zero.

[6] Under this assumption, g_t is isomorphic to an exogenous shock to preferences and endowments. Consequently, existing theorems which establish that the competitive equilibrium and the social planning problem coincide are applicable.

[7] Christiano (1987a;1988,ftn 9,18) discusses the different properties of the the log—linear approximation which we use here and linear approximations of the sort used by Kydland and Prescott (1982).

[8] The statements in the text about the relation between (e_n,d_n) and γ are based on the following experiments involving the divisible and indivisible labor models with $\alpha = 0$. For the divisible labor model we computed the decision rule parameters in (2.12) and (2.13) at two sets of model parameter values. First we set the model parameters to their baseline values reported in the relevant column in Table 1a. The associated decision rule parameters are reported in the relevant column in Table 3. Second, we perturbed the baseline parameter values by setting $\gamma = 5.15$. With these new parameter values, $\bar{k} = 9820.22$, $r_k = .95$, $\bar{g} = 190.81$, $d_k = -.0017$, $e_k = -.95$, $\lambda = .0040$, $n = 266.26$, $r_n = -.42$, $d_n = .20$, $e_n = .42$. We also computed two sets of decision rules for the indivisible labor ($\alpha = 0$) model. The first is reported in the relevant column of Table 3, and is based on the baseline parameter values reported in the relevant column of Table 1a. The second is based on perturbing the baseline parameter values by setting $\gamma = .0046$. The decision rule parameters corresponding to this are $\bar{k} = 9874.80$, $r_k = .94$, $\bar{g} = 190.81$, $d_k = .0023$, $e_k = -.94$, $\lambda = .0040$, $n = 267.74$, $r_n = -.61$, $d_n = .28$, $e_n = .61$. In each experiment we found that e_n and d_n increased with γ.

[9] Let b and c denote the fourth and sixth elements of Ψ_2, respectively. Then, after some algebraic manipulation, $\sigma_n/\sigma_y = b/\sqrt{(1+2cb+b^2)}$.

[10] Let $S_o = \sum_{k=-\infty}^{\infty} E[H(F_{t+k+1},\Psi^o)][H(F_{t+1},\Psi^o)]'$ denote the true spectral density matrix of $H(F_t,\Psi^o)$ at frequency zero. Proceeding as in Hansen (1982) we can estimate S_0 by replacing the population moments in the previous expression by their sample counterparts evaluated at Ψ_T. In order to guarantee that our estimate of S_0 is positive definite we use the damped truncated covariance estimator discussed in Eichenbaum and Hansen (1988). The results we report were calculated by truncating after 6 lags.

[11] Our formal test does not include σ_n/σ_y and $(\sigma_n/\sigma_y)^{hp}$ because these are exact function of [corr(y/n), $\sigma_n/\sigma_{y/n}$] and [corr(y/n)^{hp}, $(\sigma_n/\sigma_{y/n})^{hp}$], respectively (see footnote 9.)

[12] It would be desirable to include in g_t a measure of the service flow from the stock of government owned capital, since government capital is included in our measure of k_t. Unfortunately we know of no existing measures of that service flow. This contrasts with the case of household capital, for which there exist estimates of the service flow from housing and the stock of consumer durables. The first is included in the official measure of consumption of services, and the second is reported in Brayton and Mauskopf (1985).

[13] For further details on the data, see Christiano (1987b).

[14] These results differ in an important way from those in Hansen (1985). Using data processed using the HP filter, he reports that the indivisible labor model with $\alpha = 1$ implies a value of $(\sigma_n/\sigma_{y/n})^{hp}$ equal to 2.7 (see Hansen [1985], Table 1.) This exceeds the corresponding empirical quantity by over 220%. Our version of this model ($a = 1$) underpredicts $(\sigma_n/\sigma_{y/n})^{hp}$ by over 20%. The reason for the discrepancy is that Hansen chooses to model innovations to technology as having a transient effect on z_t, whereas we assume its effect is permanent. Consequently the intertemporal substitution effect of a shock to technology is considerably magnified in Hansen's version of the model.

[15] Interestingly, despite the small t statistics associated with the indivisible labor model ($\alpha = 0$), the J statistic computed using growth rates is large. This is because the estimated correlation between $F_2(\hat{\Psi})$ and $F_2(\hat{\Psi})$ is −.51. At the same time $F_1(\hat{\Psi})$ is .4 while $F_2(\hat{\Psi})$ is .2. Because of the negative correlation between these statistics, the J statistic, which is computed under the null hypothesis that both are zero, assigns very low probability to this outcome. The principal reason why this correlation is negative has to do with the important role played by the sampling uncertainty in ρ. When ρ is high, the model correlation between y_t/n_t and n_t is low, but the relative volatility of n_t and y_t/n_t is high. In fact, the correlation between $f_1(\hat{\Psi})$ and $f_2(\hat{\Psi})$ is −.95. As it turns out, for the model under consideration, this correlation is the key empirical determinant of the correlation between $F_1(\hat{\Psi})$ and $F_2(\hat{\Psi})$.

[16] Consistent with this, the value of corr(y/n,n)^{hp} that emerges from the baseline indivisible labor model ($\alpha = 0$) with ρ increased to .98 equals .644.

[17] To see why the new data set generates a higher value of $\hat{\gamma}$, it is convenient to concentrate on the divisible labor model. The parameter $\hat{\theta}$ is invariant to which data set or model is used. In practice, our estimator of $\hat{\gamma}$ is approximately

$$\hat{\gamma} \cong \frac{(1-\hat{\theta})}{c/y}\left[\frac{N}{n}-1\right],$$

where c/y denotes the sample average of $(c_t^p + \alpha g_t)/y_t$, and N/n denotes the sample average of N/n_t. Obviously, $\hat{\gamma}$ is a decreasing function of n. The value of n with our baseline set is 320.4, and the implied value of n/N is .23. In the new data set, n = 257.7 and the implied value of n/N is .19. Our estimates of γ are different from the one used by Kydland and Prescott (1982). This is because they deduce a value of γ based on the assumption that n/N = .33. In defending this assumption, Prescott (1986b,p.15) states: "Ghez and Becker (1975) find that the household allocates approximately one–third of its productivity time to market activities and two–thirds to nonmarket activities." We cannot find any statement of this sort in Ghez and Becker (1975).

[18] For example, the value of $\text{corr}(y/n,n)^{hp}$ that emerges from the baseline indivisible labor model ($\alpha = 0$) with γ increased to .0046 equals .684 (see footnote 8 for more details about this computational experiment.)

Table 1a

Model Parameters Estimates (Standard Errors)
Generated by Baseline Dataset[1]

	Divisible Labor (α=1)	Indivisible Labor (α=1)	Divisible with Gov't (α=0)	Indivisible with Gov't (α=0)
T	1369	1369	1369	1369
δ	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)
β	$1.03^{-0.25}$	$1.03^{-0.25}$	$1.03^{-0.25}$	$1.03^{-0.25}$
θ	0.339 (0.006)	0.339 (0.006)	0.344 (0.006)	0.344 (0.006)
γ	2.99 (0.03)	0.00285 (0.00003)	3.92 (0.05)	0.00374 (0.00005)
λ	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)
σ_ϵ	0.018 (0.001)	0.018 (0.001)	0.018 (0.001)	0.018 (0.001)
$\bar{g}$	186.0 (10.74)	186.0 (10.74)	190.8 (7.09)	190.8 (7.09)
ρ	0.96 (0.028)	0.96 (0.028)	0.96 (0.029)	0.96 (0.029)
σ_μ	0.020 (0.001)	0.020 (0.001)	0.021 (0.001)	0.021 (0.001)

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[1] Standard errors are reported only for estimated parameters. Other parameters were set a priori.

Table 1b

Model Parameters (Standard Errors) Estimated on Alignment–Corrected Data Set[1]

	Divisible Labor (α=1)	Indivisible Labor (α=1)	Divisible with Gov't (α=0)	Indivisible with Gov't (α=0)
δ	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)	0.0210 (0.0003)
θ	0.339 (0.006)	0.339 (0.006)	0.344 (0.006)	0.344 (0.006)
γ	3.92 (0.03)	0.00353 (0.00003)	5.15 (0.05)	0.00463 (0.00005)
λ	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)	0.0040 (0.0015)
σ_ϵ	0.012 (0.0008)	0.012 (0.0008)	0.012 (0.0008)	0.012 (0.0008)
$\bar{g}$	144.9 (22.30)	144.9 (22.30)	148.9 (19.65)	148.9 (19.65)
ρ	0.98 (0.03)	0.98 (0.03)	0.98 (0.03)	0.98 (0.03)
σ_μ	0.016 (0.001)	0.016 (0.001)	0.016 (0.001)	0.016 (0.001)

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[1] Standard errors are reported only for estimated parameters. Other parameters were set a priori.

Table 2

Decision Rule Parameters of Baseline Models

$$k_{t+1} = z_t\bar{k}[(k_t/z_{t-1})/\bar{k}]^{r_k}[(g_t/z_t)/\bar{g}]^{d_k}\exp[e_k(\lambda_t-\lambda)]$$

$$n_t = n[(k_t/z_{t-1})/\bar{k}]^{r_n}[(g_t/z_t)/\bar{g}]^{d_n}\exp[e_n(\lambda_t-\lambda)]$$

$$z_t/z_{t-1} = \exp(\lambda_t)$$

$$\log g_t = \log z_t + (1-\rho)\log \bar{g} + \rho[\log g_{t-1} - \log z_{t-1}] + \mu_t$$

	Divisible Labor ($\alpha=1$)	Indivisible Labor ($\alpha=1$)	Divisible with Gov't ($\alpha=0$)	Indivisible with Gov't ($\alpha=0$)
$\bar{k}$	11,113.4	11,062.75	11,614.36	11,569.25
r_k	0.95	0.94	0.95	0.94
$\bar{g}$	186.0	186.0	190.8	190.8
d_k	0.0	0.0	−0.0020	0.0020
e_k	−0.95	−0.94	−0.95	−0.94
λ	0.0040	0.0040	0.0040	0.0040
n	315.52	314.08	314.91	313.68
r_n	−0.30	−0.49	−0.38	−0.59
d_n	0.0	0.0	0.15	0.23
e_n	0.30	0.49	0.38	0.59

Table 3
Selected First Moment Properties, Baseline Models

	Models[1]				
	Divisible Labor	Indivisible Labor	Divisible with Gov't	Indivisible with Gov't	U.S.[2] Data (1955.4–1983.4)
c^p_t/y_t	0.56	0.56	0.56	0.56	0.55
	(0.012)	(0.012)	(0.010)	(0.010)	(0.003)
g_t/y_t	0.177	0.178	0.176	0.177	0.177
	(0.007)	(0.007)	(0.006)	(0.006)	(0.003)
dk_t/y_t	0.260	0.260	0.264	0.264	0.269
	(0.009)	(0.010)	(0.009)	(0.009)	(0.002)
k_{t+1}/y_t	10.54	10.54	10.68	10.68	10.62
	(0.268)	(0.260)	(0.307)	(0.293)	(0.09)
n_t	315.60	314.24	315.19	314.12	320.2
	(3.01)	(4.09)	(4.47)	(5.74)	(1.51)
$\Delta\log c^p_t$	0.0040	0.0040	0.0040	0.0040	0.0045
	(0.0017)	(0.0016)	(0.0016)	(0.0016)	(0.0007)
$\Delta\log y_t$	0.0040	0.0040	0.0040	0.0040	0.0040
	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0014)
$\Delta\log k_t$	0.0040	0.0040	0.0040	0.0040	0.0047
	(0.0015)	(0.0016)	(0.0015)	(0.0016)	(0.0005)
$\Delta\log g_t$	0.0040	0.0040	0.0040	0.0040	0.0023
	(0.0019)	(0.0019)	(0.0019)	(0.0019)	(0.0017)
$\Delta\log n_t$	0.1E–04	0.2E–04	0.1E–04	0.1E–04	0.0002
	(0.0002)	(0.0003)	(0.0003)	(0.0005)	(0.0013)

---

[1] Numbers are averages, across 1,000 simulated data sets of length 113 observations each, of the sample average of the corresponding variable in the first column. Numbers in parentheses are the standard deviation, across data sets, of the associated statistic.

[2] Empirical averages, with standard errors.

Table 4a

Second Moment Properties After HP Detrending Models Estimated Using Baseline Dataset

	Models[2]				
Statistic[1]	Divisible Labor (α=1)	Indivisible Labor (α=1)	Divisible with Gov't (α=0)	Indivisible with Gov't (α=0)	U.S.[3] Data (1955.4–1983.4)
σ_{c^p}/σ_y	0.57 (0.085)	0.53 (0.076)	0.49 (0.049)	0.46 (0.05)	0.44 (0.027)
σ_{dk}/σ_y	2.33 (0.16)	2.45 (0.17)	2.11 (0.16)	2.24 (0.17)	2.24 (0.062)
σ_n/σ_y	0.36 (0.004)	0.50 (0.006)	0.46 (0.02)	0.62 (0.03)	0.86 (0.060)
$\sigma_n/\sigma_{(y/n)}$	0.54 (0.01)	0.96 (0.03)	0.79 (0.07)	1.36 (0.14)	1.21 (0.11)
σ_g/σ_y	1.76 (0.24)	1.55 (0.21)	1.66 (0.20)	1.44 (0.16)	1.15 (0.23)
σ_y	0.020 (0.0026)	0.023 (0.003)	0.021 (0.003)	0.025 (0.003)	0.019 (0.001)
corr(y/n,n)	0.95 (0.014)	0.92 (0.022)	0.81 (0.058)	0.73 (0.074)	–0.20 (0.11)

---

[1] All of the statistics in this table are computed after first logging and then detrending the data using the Hodrick–Prescott (HP) method. σ_i is the standard deviation of variable i detrended in this way. corr(x,w) is the correlation between detrended x and detrended w.

[2] Average of corresponding statistics in column 1, across 1,000 simulated data sets each of length 113. Number in parentheses is the associated standard deviation.

[3] Results for U.S. data. Numbers in parentheses are associated standard errors, computed as discussed in the text.

Table 4b

Second Moment Properties After Log First Differencing
Models Estimated Using Baseline Dataset

	Models[2]				
Statistic[1]	Divisible Labor $(\alpha=1)$	Indivisible Labor $(\alpha=1)$	Divisible with Gov't $(\alpha=0)$	Indivisible with Gov't $(\alpha=0)$	U.S.[3] Data (1955.4–1983.4)
σ_{c^P}/σ_y	0.55	0.51	0.47	0.44	0.46
	(0.049)	(0.05)	(0.04)	(0.04)	(0.039)
σ_{dk}/σ_y	2.35	2.48	2.12	2.26	1.93
	(0.15)	(0.17)	(0.14)	(0.15)	(0.095)
σ_n/σ_y	0.36	0.51	0.46	0.62	1.30
	(0.003)	(0.004)	(0.013)	(0.014)	(0.14)
$\sigma_n/\sigma_{(y/n)}$	0.55	1.00	0.80	1.41	0.98
	(0.010)	(0.025)	(0.039)	(0.083)	(0.05)
σ_g/σ_y	1.76	1.54	1.66	1.43	1.29
	(0.12)	(0.10)	(0.10)	(0.08)	(0.13)
σ_y	0.016	0.018	0.017	0.019	0.011
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
corr(y/n,n)	0.97	0.95	0.84	0.77	–0.71
	(0.016)	(0.023)	(0.030)	(0.040)	(0.07)

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[1] In this table, c, dk, y, y/n, n refer to the first difference of the log of the indicated variable. Then, σ_i is the standard deviation of variable i and corr(ℓ,P) is the correlation between ℓ and P.

[2] Average of corresponding statistics in column 1, across 1,000 simulated data sets each of length 113. Number in parenthesis is the associated standard deviation.

[3] Results for U.S. data. Numbers in parentheses are standard errors, computed as discussed in the text.

Table 5: Diagnostic Results for Baseline Models[1]

			Models[3]							
Statistic	U.S. Data[2]		Divisible Labor		Indivisible Labor		Divisible with Govt.		Indivisible with Govt.	
	–HP–	–DIFF–	–HP–	–DIFF–	—HP—	—DIFF—	—HP—	—DIFF—	–HP–	–DIFF–
corr(y/n,n)	–.20	–.71	.951	.960	.915	.940	.818	.826	.737	.759
	(.11)	(.07)	(.11)	(.07)	(.11)	(.07)	(.14)	(.11)	(.15)	(.12)
	–	–	[10.56]	[25.18]	[10.23]	[24.94]	[7.10]	[13.97]	[6.17]	[12.08]
$\sigma_n/\sigma_{(y/n)}$	1.21	.98	.543	.548	.959	.985	.785	.791	1.348	1.386
	(.11)	(.05)	(.11)	(.05)	(.11)	(.05)	(.12)	(.05)	(.12)	(.06)
	–	–	[5.87]	[8.11]	[2.13]	[.12]	[3.67]	[3.63]	[1.16]	[7.14]
J	–	–	168.84	1004.33	119.29	712.54	62.18	202.32	41.46	261.60
	–	–	{0}	{0}	{0}	{0}	{0}	{0}	{0}	{0}

Notes:

[1] All results are based on data detrended by the Hodrick–Prescott filter (HP) or the log first difference filter (DIFF), as indicated.

[2] Point estimates based on U.S. data of the statistic in the first column. These numbers are taken directly from tables 4a and 4b. Number in ( ) is the associated standard error estimate.

[3] Number not in parentheses is the value of the statistic in the first column implied by the indicated model at its estimated parameter values. Number in ( ) is the standard error of the discrepancy between the statistic reported above and its associated sample value, reported in the corresponding U.S. Data column. For the "DIFF" columns, this standard error is computed by taking the square root of the appropriate diagonal element of (3.26). The number in [ ] is the associated t statistic. The J statistic is computed using (3.27). For the "HP" columns these standard errors, t and J statistics are computed using the analogue of equations (3.26) and (3.27), valid when the data have been transformed by the HP filter. The number in { } is the probability that a Chi–square with 2 degrees of freedom exceeds the reported value of the associated J statistic.

Table 6: Diagnostic Results for Alignment Corrected Data Set[1]

	Models[3]									
Statistic	U.S. Data		Divisible Labor		Indivisible Labor		Divisible with Govt.		Indivisible with Govt.	
	–HP–	–DIFF–	–HP–	–DIFF–	—HP—	—DIFF—	—HP—	—DIFF—	–HP–	–DIFF–
corr(y/n,n)	.16 (.08)	.21 (.07)	.946 (.08) [9.43]	.958 (.07) [10.50]	.915 (.08) [9.02]	.940 (.07) [10.25]	.659 (.22) [2.30]	.668 (.21) [2.25]	.575 (.22) [1.84]	.593 (.22) [1.79]
$\sigma_n/\sigma_{(y/n)}$	1.64 (.16) –	1.27 (.13) –	.605 (.16) [6.45]	.612 (.12) [5.28]	.959 (.16) [4.23]	.985 (.12) [2.30]	.951 (.18) [3.75]	.960 (.15) [2.14]	1.437 (.19) [1.07]	1.476 (.15) [1.35]
J	–	–	131.35 {0}	139.24 {0}	100.53 {0}	111.12 {0}	14.55 {.0007}	6.40 {.041}	3.48 {.176}	9.91 {.007}

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[1] See notes to Table 5.

Table 7: Impact of Measurement Error on Diagnostics

Statistic	U.S. Data		Models^3							
			Divisible Labor		Indivisible Labor		Divisible with Govt.		Indivisible with Govt.	
	–HP–	–DIFF–	–HP–	–DIFF–	—HP—	—DIFF—	—HP—	—DIFF—	–HP–	–DIFF–
PANEL A: Results Based on Baseline Dataset										
corr(y/n,n)	–.20	–.71	–.145	–.638	.010	–.544	–.098	–.596	–.015	–.508
	(.11)	(.07)	(.11)	(.05)	(.11)	(.06)	(.12)	(.05)	(.12)	(.06)
			[.48]	[1.39]	[1.86]	[2.69]	[.86]	[2.10]	[1.46]	[3.19]
$\sigma_n/\sigma_{(y/n)}$	1.21	.98	.766	.897	.978	.996	.907	.959	1.225	1.113
	(.11)	(.05)	(.11)	(.05)	(.11)	(.05)	(.11)	(.05)	(.13)	(.06)
			[4.13]	[1.59]	[2.06]	[.34]	[2.68]	[.40]	[.12]	[2.39]
J			22.62	15.61	16.11	17.15	10.14	11.07	2.20	10.15
			{0}	{.0024}	{.0003}	{.0002}	{.0064}	{.0039}	{.335}	{.0055}
PANEL B: Results Based on Alignment Corrected Data Set										
corr(y/n,n)	.16	.21	.326	–.254	.429	–.132	.238	–.222	.232	–.162
	(.08)	(.07)	(.21)	(.22)	(.19)	(.23)	(.21)	(.21)	(.21)	(.20)
			[.80]	[2.11]	[1.43]	[1.47]	[.36]	[2.06]	[.33]	[1.80]
$\sigma_n/\sigma_{(y/n)}$	1.64	1.27	.709	.830	.969	.993	.986	.996	1.365	1.249
	(.16)	(.13)	(.15)	(.11)	(.16)	(.12)	(.17)	(.12)	(.19)	(.16)
			[6.30]	[4.10]	[4.21]	[2.24]	[3.80]	[2.26]	[1.40]	[.14]
J			45.70	18.22	28.90	5.17	14.55	6.82	2.09	4.13
			{0}	{.0001}	{0}	{.075}	{.0007}	{.033}	{.35}	{.127}

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