Discussion Paper 21

Institute for Empirical Macroeconomics
Federal Reserve Bank of Minneapolis
250 Marquette Avenue
Minneapolis, Minnesota 55480

December 1989

# COMPARISONS OF ALTERNATIVE IDENTIFICATION SCHEMES FOR THE U.S. REAL GNP-UNEMPLOYMENT LEVEL CORRELATION: SENSITIVITY ANALYSIS

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ABSTRACT

The paper employs three different types of identifying restrictions to calculate the impulse responses for the trivariate series composed of the U.S. unemployment level, real GNP and the money stock. The first two are the zero restrictions, arising from the assumption of the delayed information pattern available in forming a money reaction function. The third assumes a particular simplified structural model. The paper shows that the impulse response patterns are generally insensitive to these alternative specifications. Similar exercises are carried out for the bivariate series composed of the U.S. and the unemployment level.

*The author gratefully acknowledges discussions with C. Sims. Discussions with R. Fiorito have also been quite helpful.

Keywords: Impulse responses, identification, sensitivity analysis, state space innovation models.

This material is based on work supported by the National Science Foundation under Grant No. SES-8733451. The Government has certain rights to this material.

*Introduction*

In the recent U.S. literature, the U.S. unemployment rates time series have been modeled as the univariate time series, and also in conjunction with other macroeconomic series. See Evans (1987), Blanchard and Quah (1988), and others. In modeling the unemployment rate with the real GNP, Blanchard and Quah imposed a constraint that demand shocks have no permanent effect on unemployment rates to achieve the just identified condition for the bivariate series.

This paper discusses modeling the U.S. unemployment level together with one or two additional macroeconomics times series to evaluate robustness of the impulse response behavior of the models that are identified in three alternative ways. In this sense, this paper is in the same spirit as Bernanke (1986) in which he attempted to draw out implications of alternative identification schemes. We use the delayed information pattern of the central bank in our identifying schemes which is similar to that used by Sims (1986), and examine the implications of a simplified structural model on the shapes of the impulse responses. More specifically, we model the unemployment level, the real GNP, and the money stock as the three components of the vector-valued series by imposing three conditions to determine nine elements of the 3 × 3 matrix which connects structural shocks to the time series innovation vector by $e_t = D_0\eta_t$ where $e_t$ is the innovations vector $\eta_t$ is structural shocks with $D_0 = \varphi_0^{-1}\theta_0$ in a structural model for $y_t$ where $\varphi(L)y_t = \theta(L)\eta_t$, and $\varphi_0$ and $\theta_0$ are 3 × 3 constant matrices in the matrix polynomials $\varphi(L)$ and $\theta(L)$.

As a further step in evaluating the robustness of the time profiles of impulse responses (dynamic multiplier) of the estimated models, we then drop the money stock from the time series and estimate two bivariate models. Impulse response behavior of these bivariate models roughly mirror those for the trivariate models of the two retained variables. We take this as a further piece of evidence supporting the claim that qualitative features of dynamic multiplier relationships of the components of the series are robust, in particular between the real GNP and unemployment level in the U.S.A.

Although the components are all apparently trending we do not impose a constraint that a common unit root exists in our model parameter estimation, but allow for a possibility that the components share a dynamic mode which is close to but not exactly equal to that of the unit root, i.e., we treat the series as sharing a nearly integrated process which is described in Chan (1988), for example.[1] We apply the two-step procedure outlined in Aoki (1989) to estimate such models. Further details on statistical properties of this procedure is available in Aoki (1990).

*Models*

The 84 quarterly observations from 1965 to 1985 on the U.S. real GNP, unemployment level and M2 have been used to estimate a trivariate model by the two-step procedure outlined in Aoki (1989). Even though we do not impose the unit root, we eliminate effects of non-zero initial conditions by scaling each observation by its first data value so that the logarithms of three series all start with zero.

We first model the slowest dynamic mode by a scalar state variable $\tau_t$

(1) $$y_t = C\,\tau_t + w_t$$

$$\tau_{t+1} = \rho\tau_t + bw_t$$

where $y_t$ is $3 \times 1$ and the matrix C is $3 \times 1$ which disaggregates the common slow mode represented by $\tau_t$ into the three components of the data vector, i.e., the components of the matrix C show how the common slow movement represented by $\{\tau_t\}$ is shared by the three components of $y_t$. The residuals $w_t$ are highly correlated and is further modeled by

(2) $$w_t - \bar{w} = H\,z_t + e_t,$$

$$z_{t+1} = F\,z_t + G\,e_t,$$

where $\bar{w}$ is the sample mean of $\{w_t\}$.

The joint effect of (1) and (2) are to express $y_t$ as a sum of three terms

$$y_t = C\,\tau_t + H\,z_t + e_t$$

where $\tau_t$ and $z_t$ evolve dynamically as

$$\begin{bmatrix} \tau_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & bH \\ 0 & F \end{bmatrix} \begin{bmatrix} \tau_t \\ z_t \end{bmatrix} + \begin{bmatrix} b \\ G \end{bmatrix} e_t.$$

The dimension of the vector $z_t$ is chosen to be two.

The impulse response (dynamic multiplier) to the innovation is

$$y_t = \sum_{\tau=0}^{\infty} M_{t-\tau} e_\tau \qquad , M_0 = I$$

where

$$M_{k+1} = (C \quad H) \begin{bmatrix} \rho & bH \\ 0 & F \end{bmatrix}^k \begin{bmatrix} b \\ G \end{bmatrix} \qquad , k = 0, 1, \ldots$$

is the impulse response matrix which shows the affect of shock k + 1 period earlier on the current observation, or the effect of current shock on the data vector k + 1 periods later.

Since the vector $e_t$ is related to the structural shocks by $e_t = D_0 \eta_t$, the impulse responses to structural shocks are easily calculated by postmultiplying the matrix $(b'G')'$ by $D_0$, once $D_0$ is specified. We next describe several alternatives in selecting the matrix $D_0$.

*Alterntive Identification Schemes*

Since the algebraic relation

$$\text{(3)} \qquad \text{cov } e_t = \Delta = D_0 D_0'$$

where cov $\eta_t = I$ is assumed without loss of generality, is not enough to uniquely determine $D_0$, we examine the effects of alterative choices of $D_0$ on general shapes of the multipliers.

With y = (u realgnp m)′,

$$\Delta = \begin{bmatrix} .345 & -.046 & -.144 \\ x & .058 & .049 \\ x & x & .253 \end{bmatrix} \times 10^{-2}$$

See Appendix for the parameter values of the model.

One way to solve $D_0$ uniquely in (3) is to assume that the structural shock to m is exogenous, i.e., the money reaction function does not depend on the contemporaneous values of shocks to u and y. This assumption is similar to the one used by Sims (1986).

Given that the (3,1) and (3,2) elements of $D_0$ are zero, there are six ways for introducing another zero elements into $D_0$. Of the six, two, i.e., $(D_0)_{13} = 0$ and $(D_0)_{23} = 0$, are eliminated because the patterns imply that either $\Delta_{13} = 0$ or $\Delta_{23} = 0$. These contradict the time series evidence which shows that none of the elements of $\Delta$ is zero.

We first examine the two cases out of the remaining four possibilities, those in which the diagonal elements of $D_0$ are not zero. We are then left with two possibilities:

$$D_0 = \begin{bmatrix} .513 & 0 & -.287 \\ -.036 & .218 & .097 \\ 0 & 0 & .503 \end{bmatrix} \tag{4}$$

and

$$D_0 = \begin{bmatrix} .506 & -.082 & -.287 \\ 0 & .221 & -.097 \\ 0 & 0 & .503 \end{bmatrix}. \tag{5}$$

Additional insights may be gained by examining the patterns of zero in the inverse of $D_0$.

In (4)

$$(6)\qquad D_0^{-1} = \begin{bmatrix} 1.949 & 0 & 1.112 \\ .322 & 4.587 & -.701 \\ 0 & 0 & 1.988 \end{bmatrix},$$

and (5) has

$$(7)\qquad D_0^{-1} = \begin{bmatrix} 1.576 & .733 & 1.269 \\ 0 & 4.525 & .873 \\ 0 & 0 & 1.988 \end{bmatrix}.$$

Eq. (6) implies that

$$(8)\qquad u_t = -.57M_t + .52\eta_{1t} + \cdots$$

$$(9)\qquad 4.6y_t - .88M_t = \eta_{2t} - .17\eta_1 + \cdots$$

Eq. (7) implies that

$$u_t = -.57M_t + .51\eta_{1t} - .08\eta_{2t} + \cdots$$

$$(10)\qquad 4.5y_t = -.87M_t + \eta_{2t} + \cdots$$

Eq. (10) shows that $\eta_{1t}$ is absent unlike in (9). Eq. (9) and (10) show m shocks affect $y_t$ in opposite direction. Since (10) seems counterintuitive, the pattern $D_0$ of (5) is questionable. It is dropped from further consideration.

If we eliminate m, (6) implies that

$$(11)\qquad 1.55u_t + 4.59y_t = \eta_{2t} + .90\,\eta_{1t} + \cdots$$

The coefficient 4.59/1.55 = 3.2 between $u_t$ and $y_t$ may be interpreted as corresponding to that of the Okun's relation.

The multiplier profiles with (4) are shown in Figure 1 through 3.

The remaining two possibilities are

$$D_0 = \begin{bmatrix} 0 & .513 & -.286 \\ .217 & -.035 & .097 \\ 0 & 0 & .503 \end{bmatrix} \tag{12}$$

with

$$D_0^{-1} = \begin{bmatrix} .317 & 4.608 & -.712 \\ 1.945 & 0 & 1.110 \\ 0 & 0 & 1.988 \end{bmatrix} \tag{13}$$

and

$$D_0 = \begin{bmatrix} .014 & .303 & -.503 \\ .220 & 0 & .097 \\ 0 & 0 & .503 \end{bmatrix} \tag{14}$$

with

$$D_0^{-1} = \begin{bmatrix} 0 & 4.353 & -.879 \\ 3.300 & -.204 & 3.400 \\ 0 & 0 & 1.988 \end{bmatrix} \tag{15}$$

With the matrix $D_0$ of (12), the relation between $u_t$ and $m_t$ is as in (8) except for the fact that $\eta_{2t}$ appears instead of $\eta_{1t}$

$$u_t = -.57m_t + .52\eta_{2t} + \cdots$$

and

$$(16) \qquad 1.57u_t + 4.61\ y_t = \eta_{1t} + .33\eta_{2t} + \cdots$$

Eq. (16) is roughly comparable with (11). The coefficient in the Okun's relation is a little smaller, $4.61/1.57 = 2.94$ and the role of $\eta_{1t}$ and $\eta_{2t}$ are reversed. If $u_t$ is eliminated we obtain $4.6y_t - .89m_t = \eta_{1t} - .167\eta_{2t} + \ldots$ which is similar to (9) except for the exchange between $\eta_{1t}$ and $\eta_{2t}$. Therefore (4) and (12) are roughly the same except for relabeling of $\eta_{1t}$ and $\eta_{2t}$.

From (15), we obtain

$$3.3u_t + 17.34y_t = \eta_{2t} + 3.88\eta_{1t} + \cdots$$

$$m_t = 5.16y_t - 1.14\eta_{1t} + \cdots$$

The coefficient $17.34/3.3 = 5.25$ in the Okun's relation is higher than in (16).

Except for the relabeling there are then three distinct $D_0$ matrices (4), (12), and (14). $E_q$. (14) implies $u_t$ responds about 1.8 times more to $m_0$ shocks, while the response of y is more complex with the $D_0$ in (4).

Using the $D_0$ of (12), the impulse responses to the structural shock vector are shown in Figure 4 through 6. The responses with the $D_0$ of (14) are in Figure 7 through 9. Observe that Figure 4 ~ 6 are very similar to Figure 7 ~ 9 (except for magnitudes). Figure 1 is similar to Figure 6, Figure 2 to Figure 4, and Figure 3 to Figure 6 except for the fact that the m and y profiles are reversed.

The last scheme imposes zero in the (1,2) position of $D_0^{-1}$, i.e., $u_t$ is not contemporaneously affected by $m_t$. It may come from a simplified structural model[2] composed of aggregate supply, demand relations, and money supply,

$$u_t = -\alpha y_t + \eta_{1t}$$

$$y_t = -\beta u_t + \gamma m_t + \eta_{2t}$$

and

$$m_t = \eta_{3t}.$$

This produces the inverse of the matrix $D_0$

$$D_0^{-1} e_t = \eta_t$$

with (dropping the assumption that cov $\eta = I$)

$$D_0^{-1} = \begin{bmatrix} 1 & \alpha & 0 \\ \beta & 1 & -\gamma \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\text{cov } \eta_t = \text{diag}(d_1^2 \ \ d_2^2 \ \ d_3^2).$$

Solving the set of six nonlinear algebraic equations for the six variables $\alpha$, $\beta$, $\gamma$, $d_i^2$, i = 1, 2, 3, we obtain

$$\alpha = 2.966, \quad \beta = -.609, \quad \gamma = .5387$$

$$d_1^2 = .586, \quad d_2^2 = .169, \quad d_3^2 = .253.$$

The matrix $D_0$ is

$$D_0 = \begin{bmatrix} .356 & -1.057 & -.570 \\ .217 & .356 & .019 \\ 0 & 0 & 1 \end{bmatrix}$$

Note that there is no third zero in $D_0$ unlike the previous two cases.

The impulse response time profiles with this D matrix are shown in Figure 10 ~ 12. Figure 10 is similar to Figure 5. Figure 11 is close to Figure 7 and Figure 12 to Figure 9.

*Bivariate Models*

Eq. (12) implies $D_0 = \begin{bmatrix} 0 & x \\ x & x \end{bmatrix}$ when the m series is dropped. When we eliminate the money stock series, (4) implies a lower triangular $D_0$, i.e., $D_0 = \begin{bmatrix} x & 0 \\ x & x \end{bmatrix}$. Eq. (14) implies an upper triangular $D_0$, $D_0 = \begin{bmatrix} x & x \\ x & 0 \end{bmatrix}$ for the bivariate series $y = (u, \text{realGNP})'$ response profiles. The impulse responses with $D_0$ corresponding to the Cholesky decomposition for (u,realGNP)′ and for (real GNP,u)′ are shown in Figure 13 through 14. Figure 13 and 15 are similar. They both resemble Figure 8 and Figure 10 except that u has a better defined minimum in the bivariate case. Figure 14 and 16 are similar to each other and to Figure 4 and 7. Appendix 2 summarizes the estimated state space model.

These differences are, however, minor compared with overall agreements of all the impulse response patterns.

The lower triangular version is

$$D_0 = \begin{bmatrix} .538 & 0 \\ -.0568 & .296 \end{bmatrix}.$$

Then the impulse responses to the first structural shock are similar to the u- and y- time profiles in Figure 9. Those to the second shock are similar to those in Figure 8. See Figure 13 and 14, respectively.

The matrix

$$D_0 = \begin{bmatrix} 0 & .538 \\ .296 & -.057 \end{bmatrix}$$

produces the same patterns of the impulse responses, except for the relabeling.

Therefore, only one more distinct $D_0$ exist, $D_0 = \begin{bmatrix} .527 & -.1011 \\ 0 & .3082 \end{bmatrix}$. The impulse responses are drawn in Figure 15 and 16.

*Concluding Discussions*

This short note finds that the impulse response time profile (dynamic multiplier profile) are robust with respect to some specific assumptions on structural models.

Although this finding is specific to the three-dimensional macroeconomic model and cannot be taken two literally, this exercise is useful because there are similar exercises in the literature for one or too dimensional macroeconomic time series involving the unemployment rate or the real GNP, as described in Introduction.

These all use the unemployment rate, presumably to avoid dealing with "trending" series. We deal directly with the level of unemployment and our procedure for dealing with apparently trending time series may be of wider applicability than any specific finding of this report.

Footnotes

[1]Blanchard and Quah imposes a unit root in the real GNP. The elements of the matrix C shown in Appendix 1 are all significantly nonzero. These elements show how the dynamic mode with eigenvalue of about .986 is shared by the three component series.

[2]This is suggested by R. Fiorito.

Appendix 1 Trivariate Balanced State Space Innovation Model

The model parameters are

$$\rho = .986, \qquad b = (.756, .369, .326)$$

$$F = \begin{bmatrix} .869 & -.123 \\ .154 & .776 \end{bmatrix}, \qquad \lambda(F) = .822 + j\,.130,\ .822 - j\,.130$$

$$C = \begin{bmatrix} .828 \\ .296 \\ .777 \end{bmatrix}, \qquad H = \begin{bmatrix} .111 & -.015 \\ -.175 & -.041 \\ -.064 & -.003 \end{bmatrix}$$

$$G = \begin{bmatrix} 4.820 & -4.648 & 6.078 \\ -.8709 & 1.871 & -17.792 \end{bmatrix}$$

Appendix 2 Bivariate Balanced State Space Innovation Model

$\rho = .988, \quad b = (1.274, .0824)$

$$F = \begin{bmatrix} .847 & -.191 \\ .180 & .741 \end{bmatrix}, \qquad \lambda(f) = .794 + j\ .178,\ .794 - j\ .178$$

$$C = \begin{bmatrix} .844 \\ .287 \end{bmatrix}, \qquad H = \begin{bmatrix} .042 & -.363 \\ -1.039 & -.184 \end{bmatrix}$$

$$G = \begin{bmatrix} .534 & -.833 \\ .934 & .340 \end{bmatrix}, \qquad \Delta = \begin{bmatrix} .288 & -.0305 \\ \alpha & .0911 \end{bmatrix} \times 10^{-2}$$

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Figure 1

1st Shock

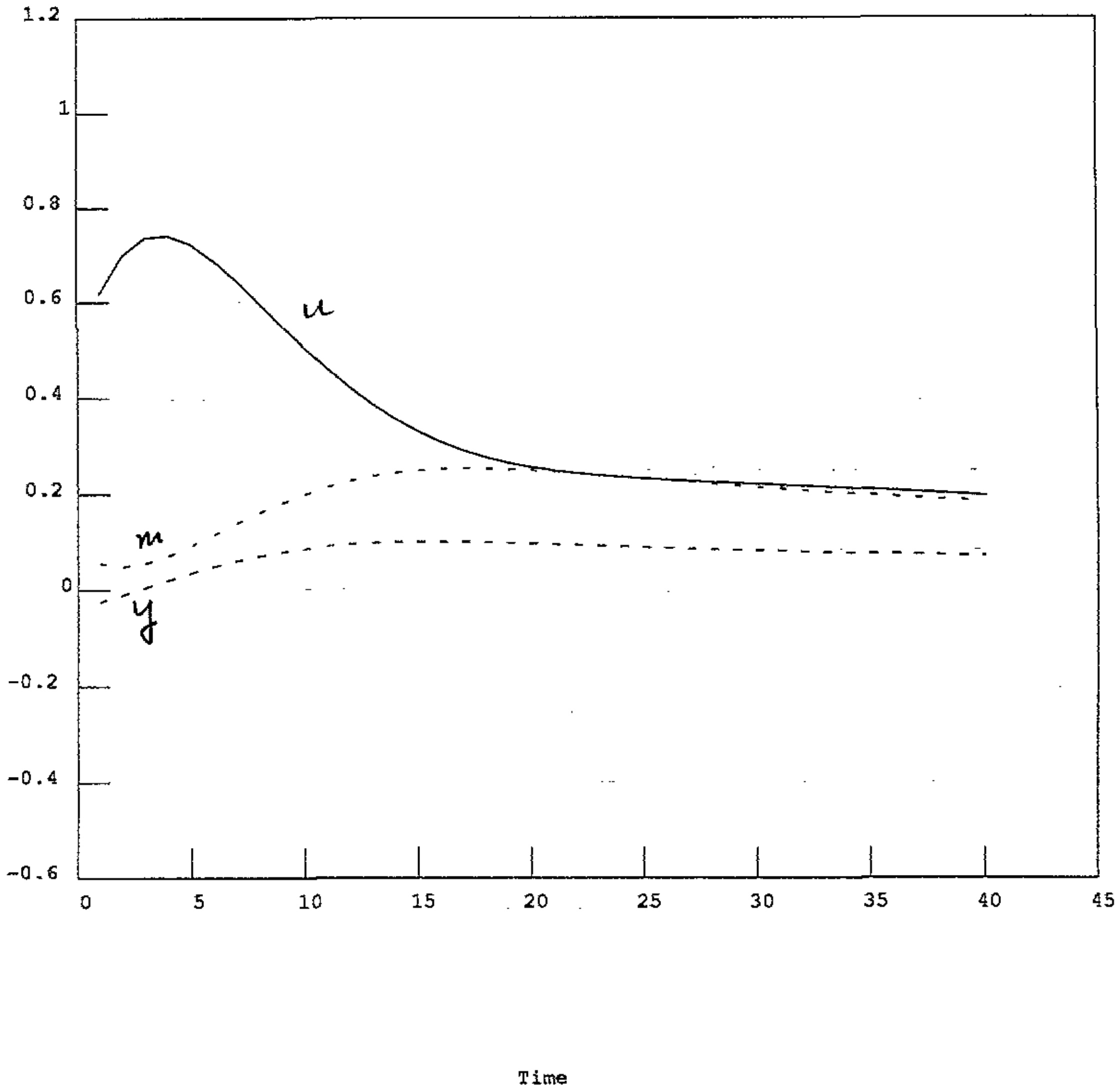

Figure 2

Response to 2nd Shock

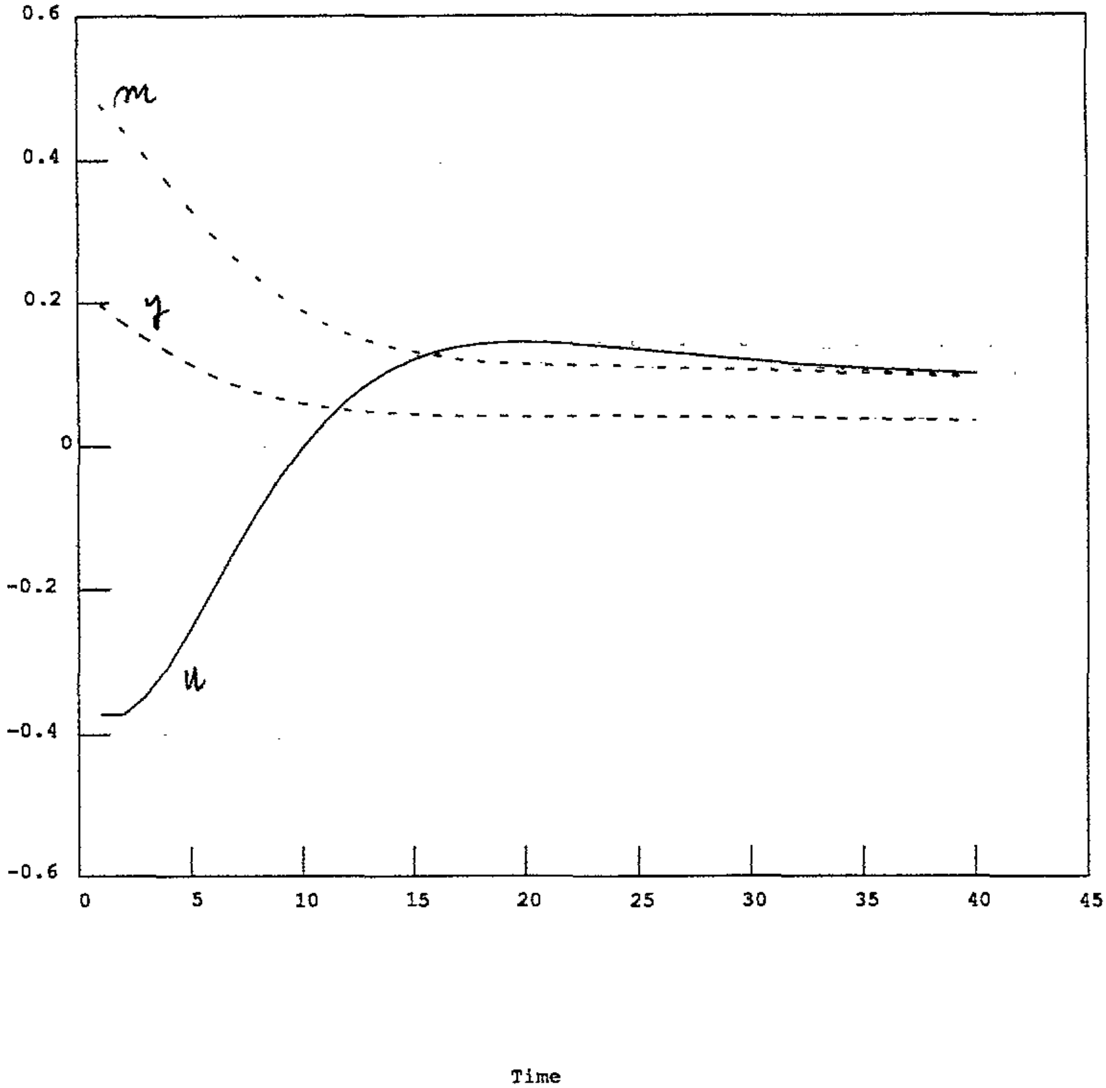

Figure 3

Response to 3rd Shock

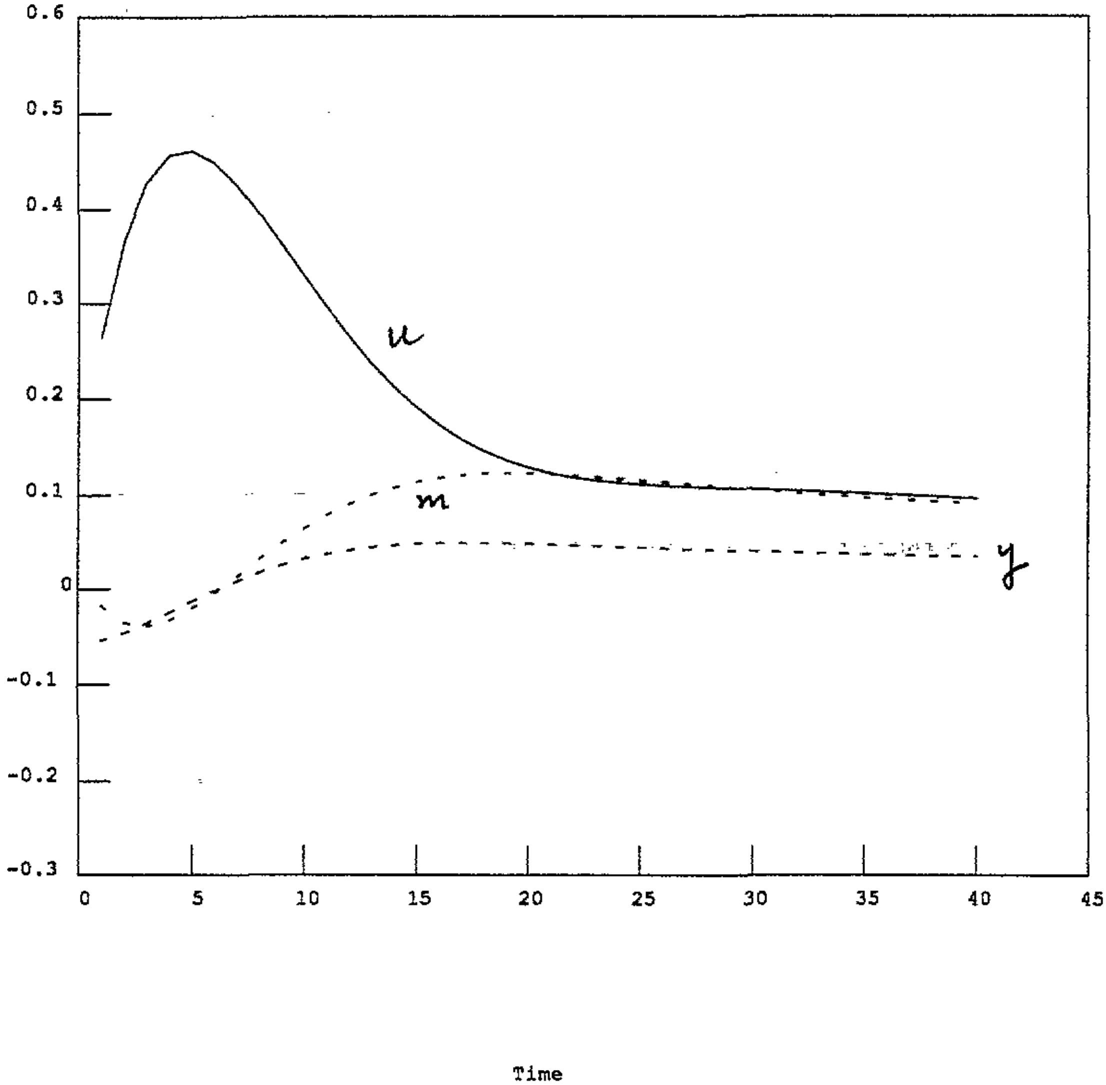

Figure 4

Response to 1st Shock with $D_0$ in (12)

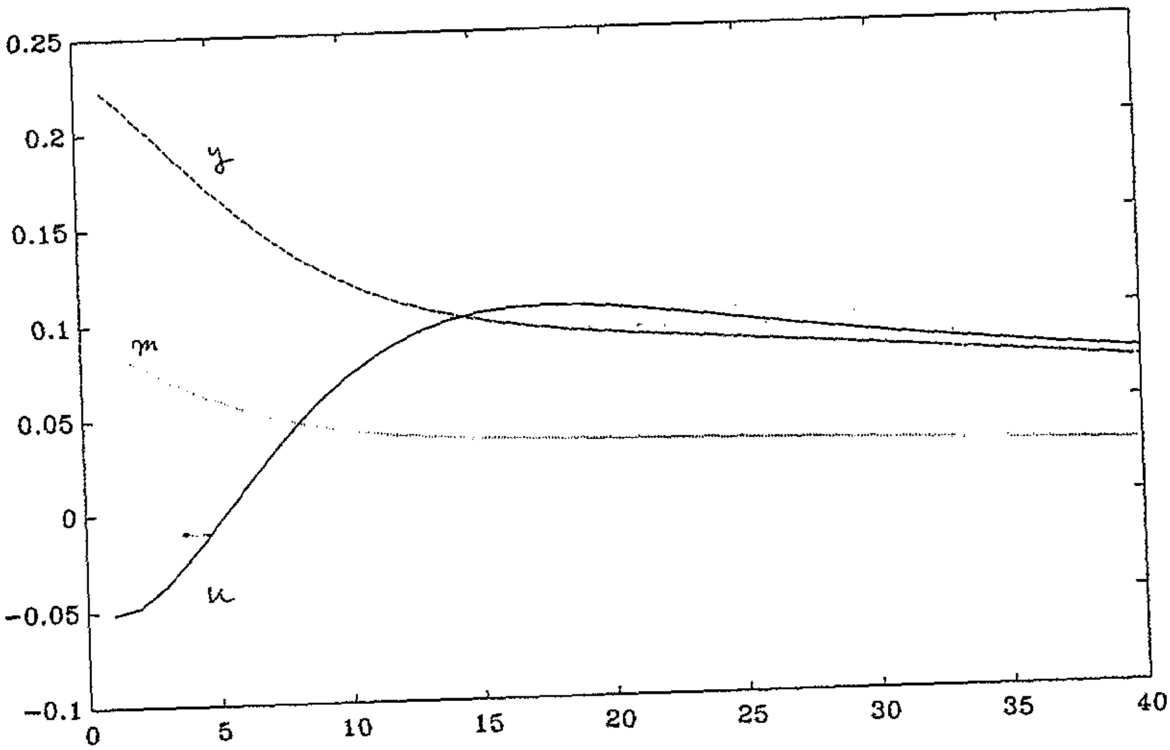

Figure 5

Response to 2nd Shock with $D_0$ in (12)

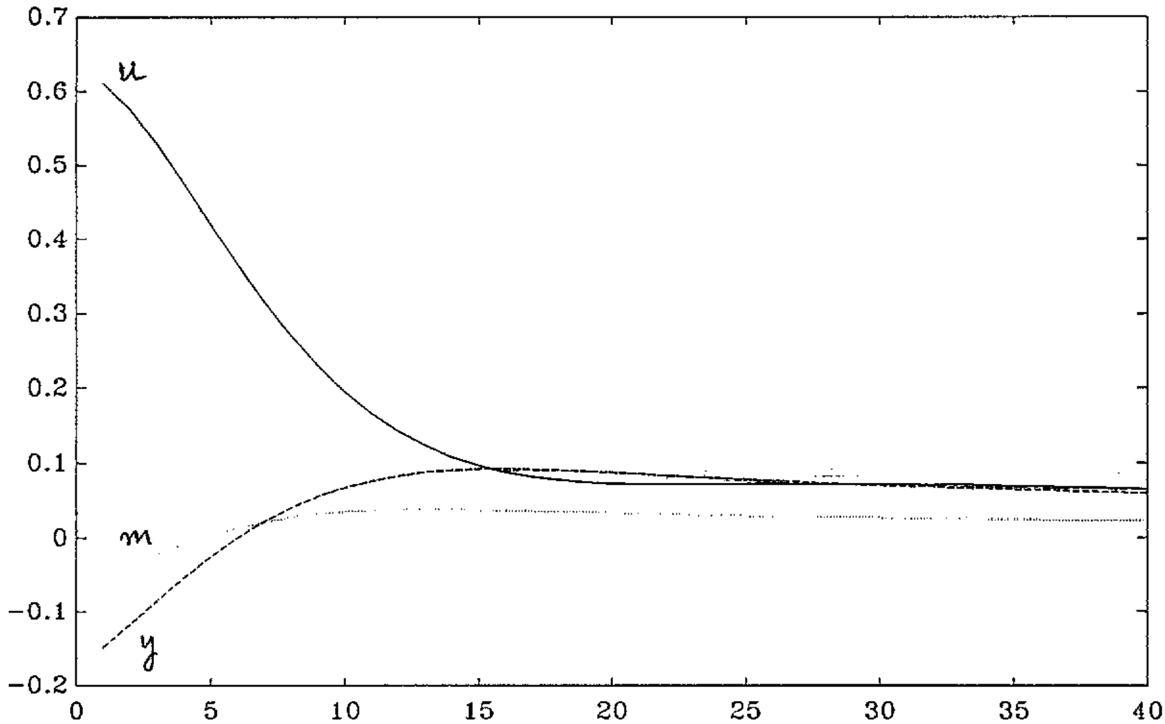

Figure 6

Response to 3rd Shock with $D_0$ in (12)

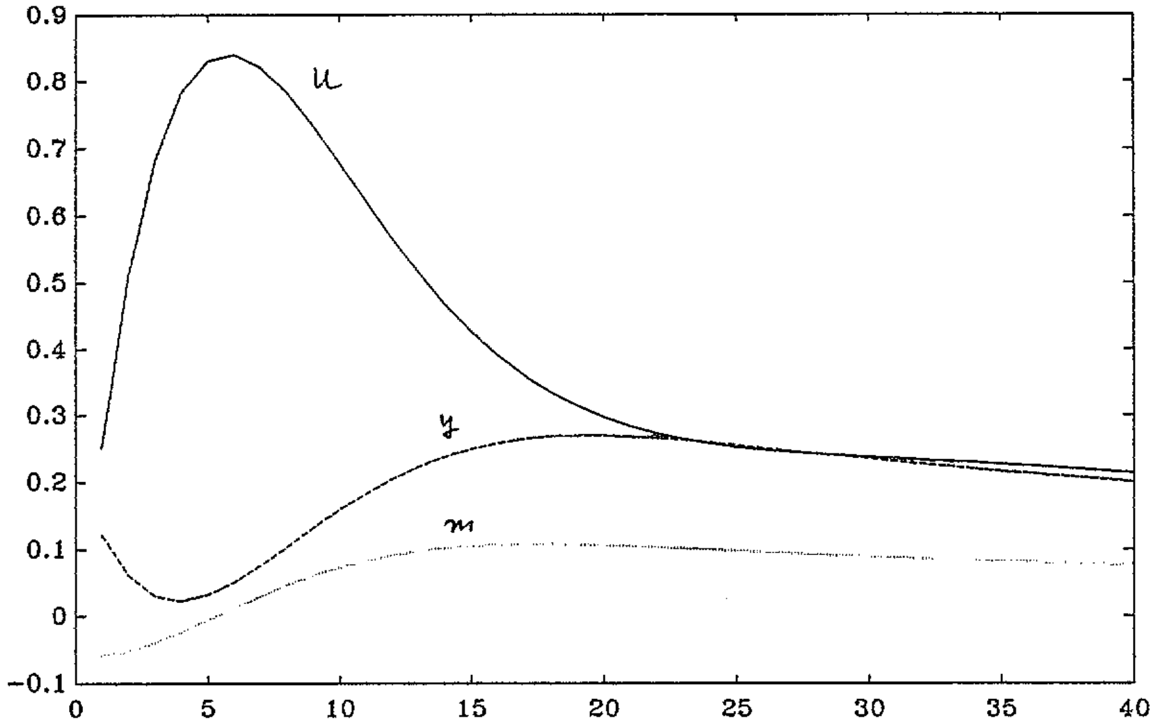

Figure 7

Response to 1st Shock with $D_0$ in (14)

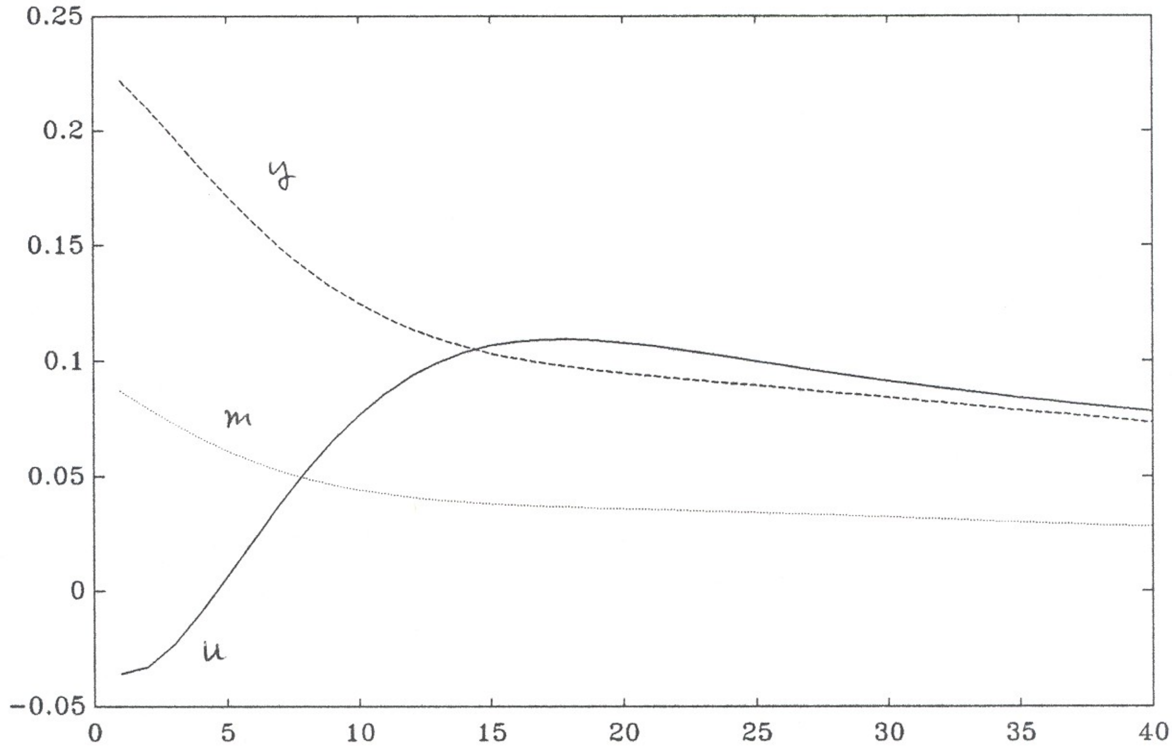

Figure 8

Response to 2nd Shock with $D_0$ in (14)

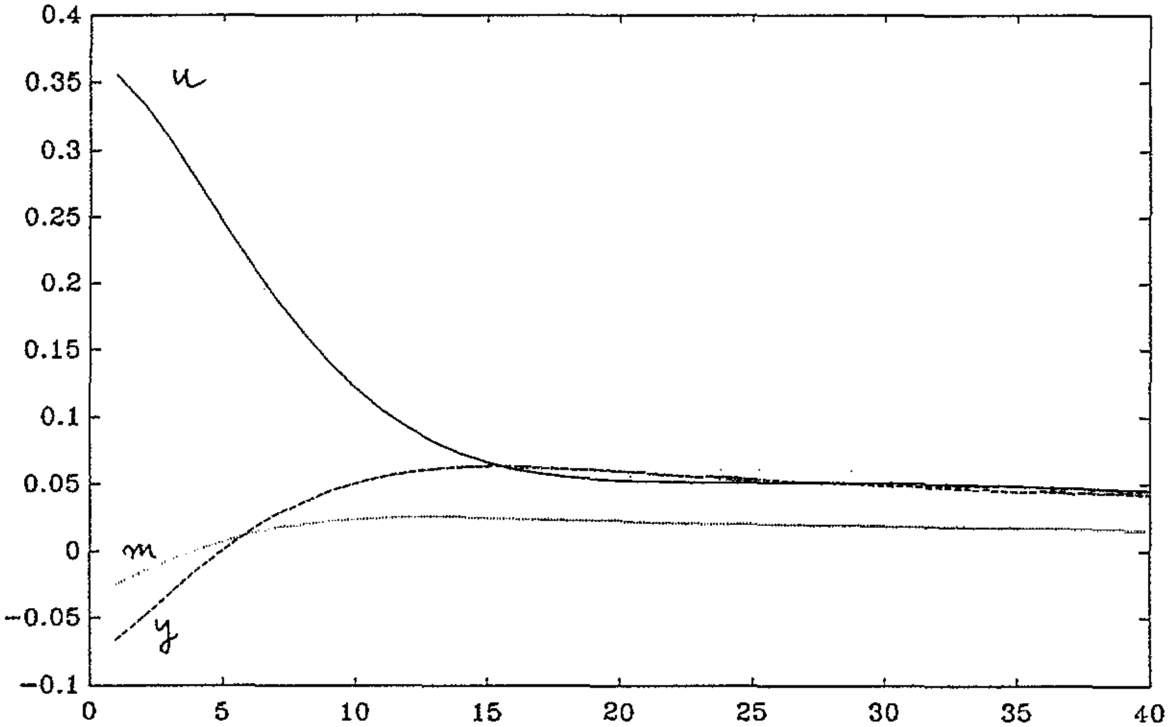

Figure 9

Response to 3rd Shock with $D_0$ in (14)

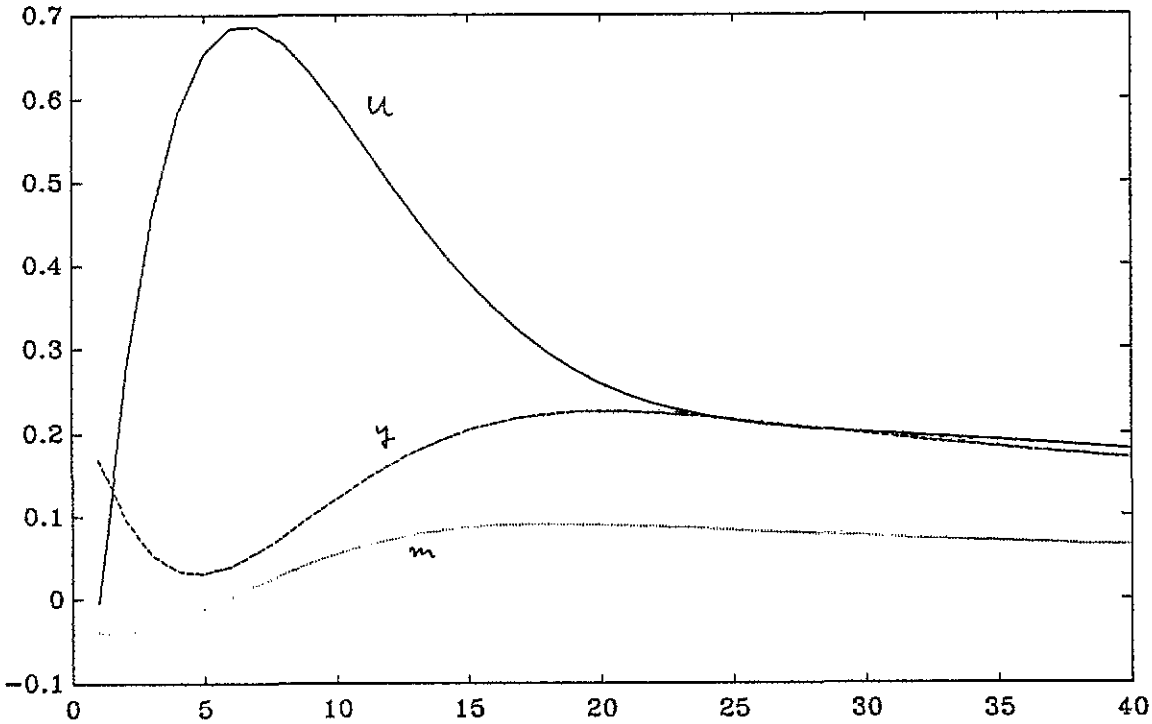

Figure 10

Response to 1st Shock

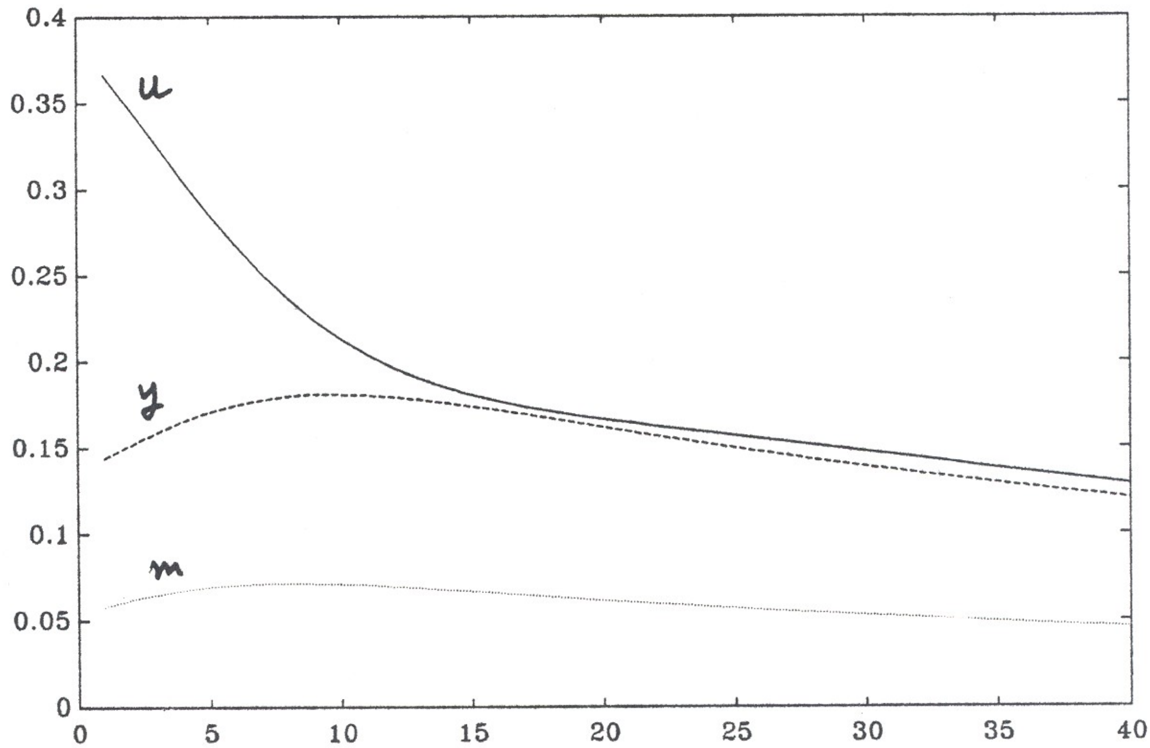

Figure 11

Response to 2nd Shock

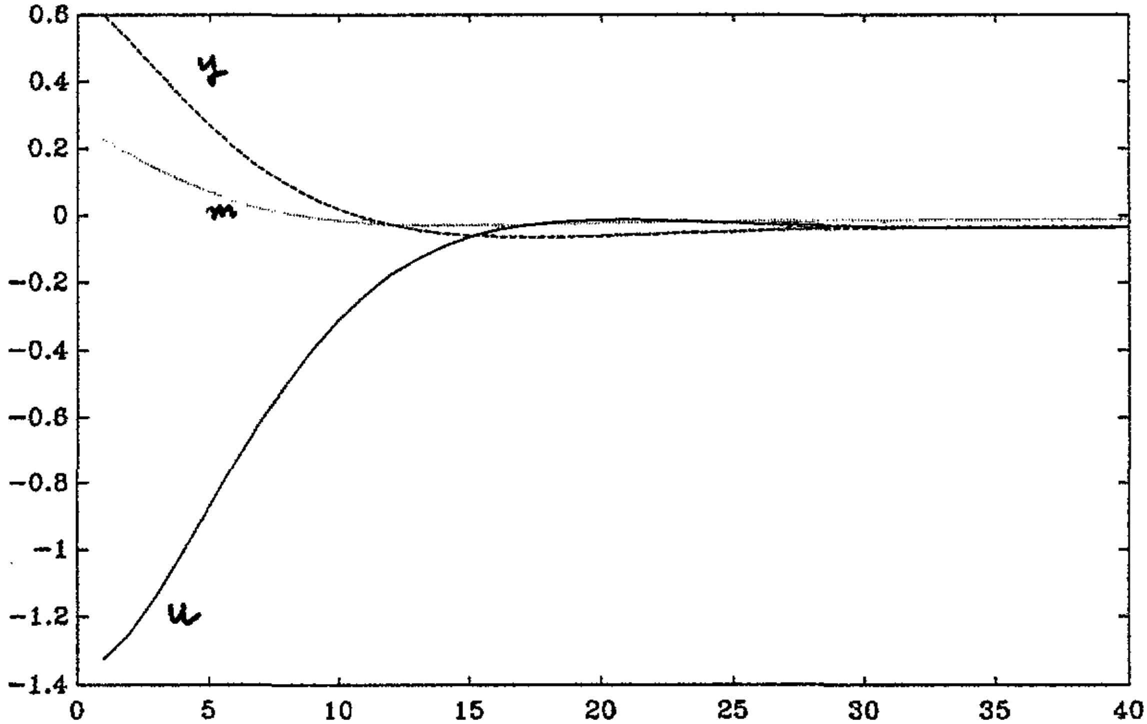

Figure 12

Response to 3rd Shock

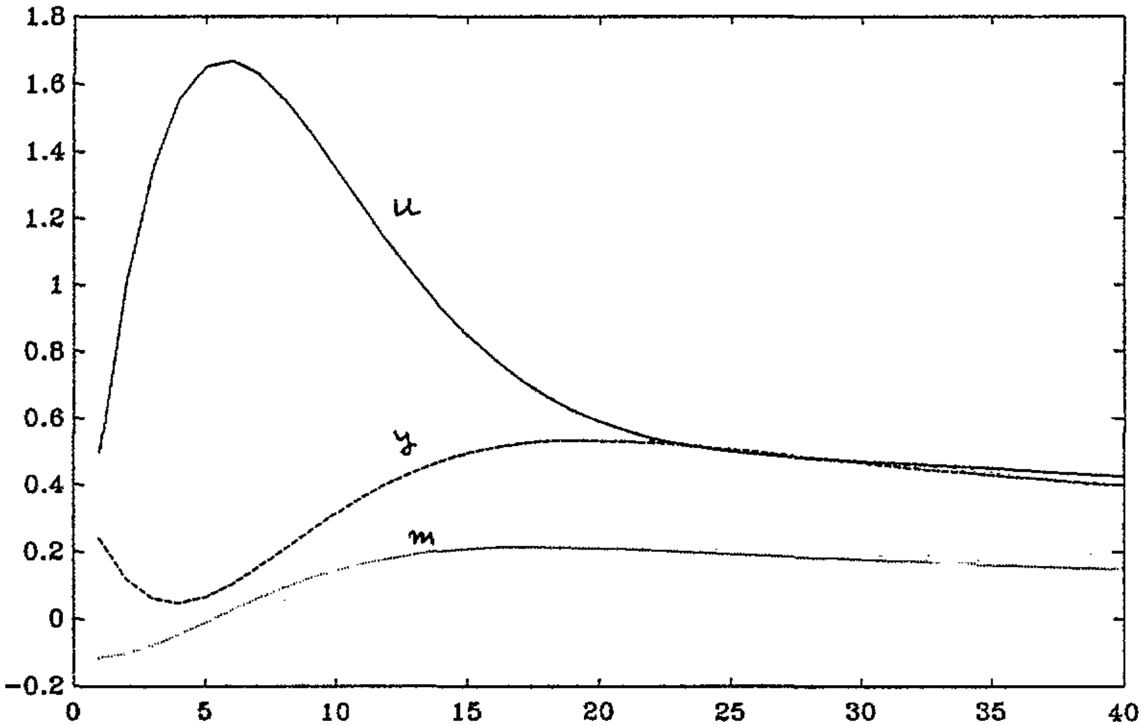

Figure 13

Response to 1st Shock

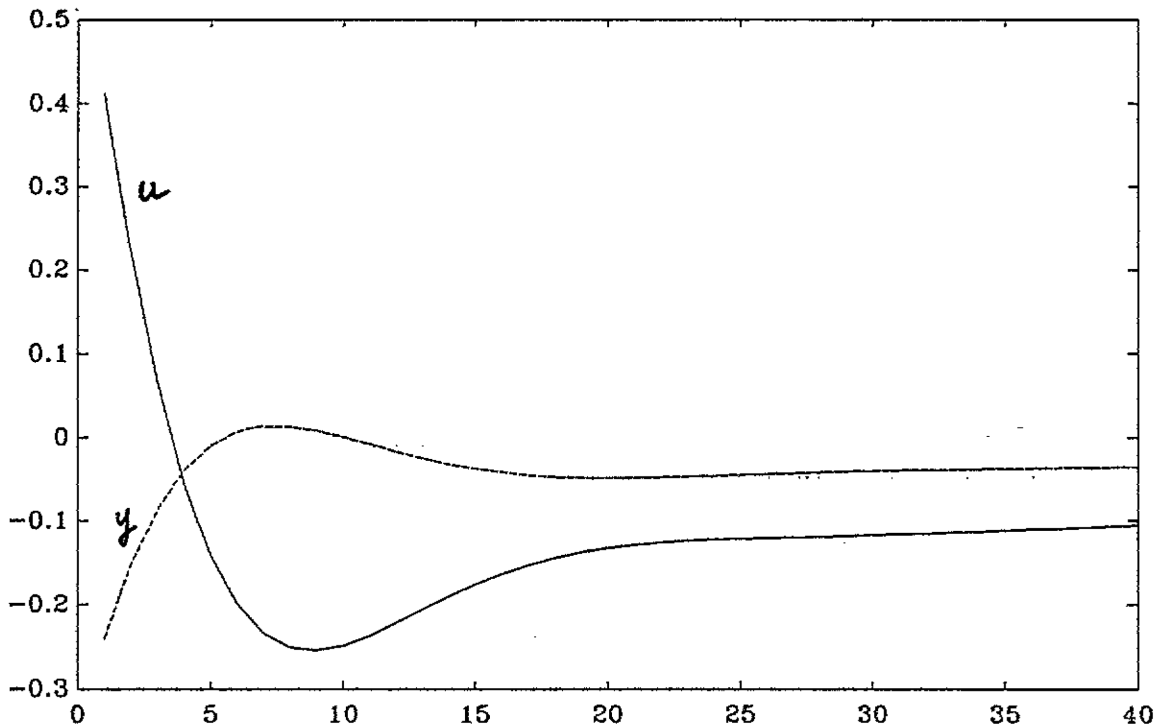

Figure 14

Response to 2nd Shock

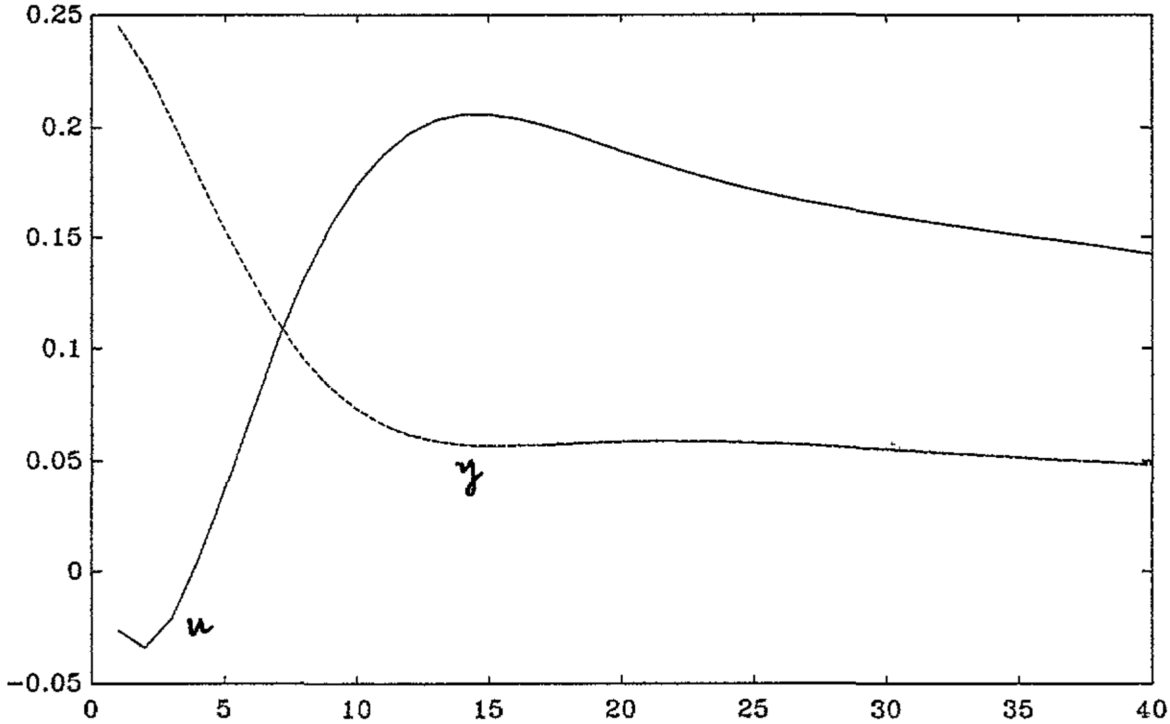

Figure 15

Response to 1st Shock

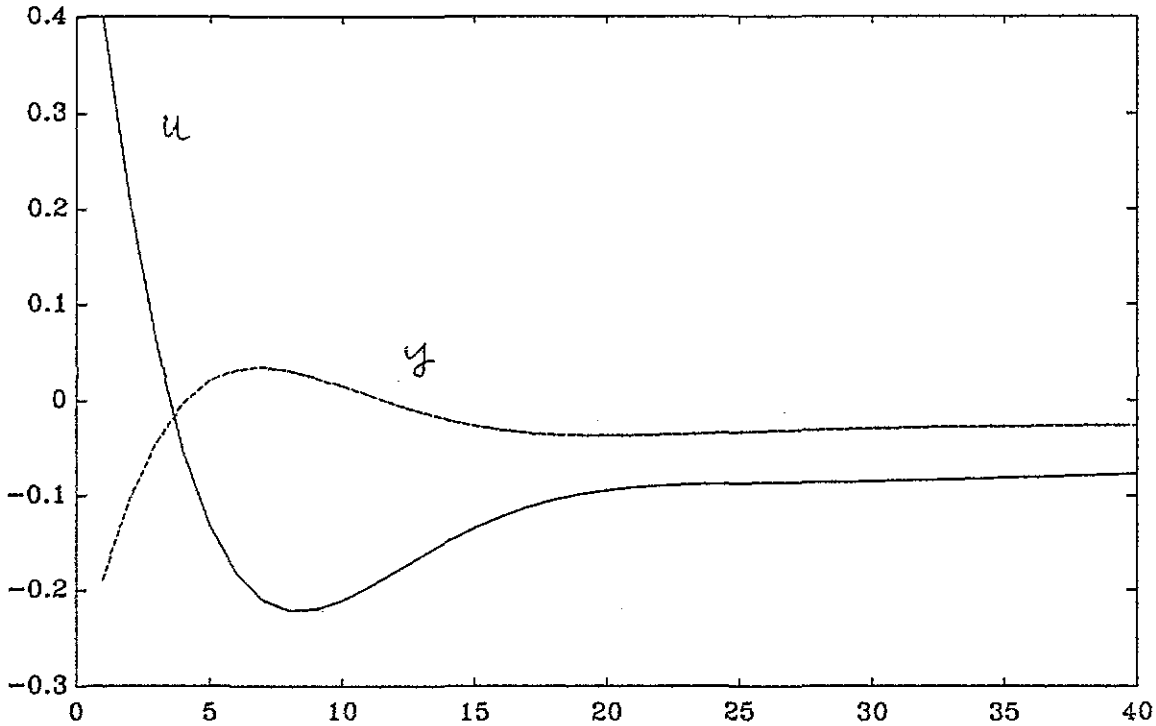

Figure 16

Response to 2nd Shock