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STRUCTURAL CHANGES IN THE REAL GNP INTERDEPENDENCE OF THE U.S., WEST GERMANY, AND JAPAN DURING THE PERIOD 1970-1986

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ABSTRACT

The paper first locates quarters in the early 1970s at which the covariance matrices of the innovation vectors have shifted for the real GNPs of the USA, West Germany and Japan treated as univariate series. The paper then exhibits differences in the impulse response time profiles of the two models estimated from the data primarily before and after the break as a concise summary of the changes in dynamic interactions of the three real GNPs.

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Keywords: Regime shift, state space models, innovations, likelihood ratio test, dynamic multiplier, impulse response.

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Introduction

This short note first applies the state space modeling method for "trending" time series, i.e., time series with roots close to one, to locate possible breaks in the real GNPs and money stock series of the USA, West Germany, and Japan by the likelihood ratio test. The real GNP series are first treated separately as univariate series to locate a likely quarter at which some characteristics of data generating process has shifted.

The paper then jointly treats the three real GNP series as a trivariate series. The focus in this part of the paper is to examine the differences in the structure of dynamic interdependence by the time profiles of impulse responses (dynamic multipliers), rather than pinpointing the quarter of structural shifts.

A variety of methods have been proposed in literature to detect sudden (or gradual) changes in parameters of data generating processes, see Andrew and Fair (1988), Goldfeld and Quandt (1976), Lo and Newey (1985) in the econometric literature and Wilsky and Jones (1976) and Basseville et al. (1987) in the systems literature. We are interested in detecting structural changes in vector-valued macroeconomic time series, such as money stocks and real GNP. Most of the method are for univariate series although some extension for vector-valued series are available.

In the context of sate space innovation modeling of time series, the innovation vectors, $e_t = y_t - y_{t|t-1}$, where $y_{t|t-1}$ is the orthogonal projection of y_t onto the manifold spanned by its own past data, are modeled as approximately normally distributed with mean zero and sample covariance matrix Δ. The joint probability distribution of $y_1, \ldots, y_T$ has only Δ as the parameters when the innovation representation is used.

A parameter shift in data generating process manifest itself then as changes in the covariance matrix Δ of the innovation vector.[1] Given that a single shift in the covariance matrix has occurred in a sample period, we can adopt the method of Goldfeld and Quandt (1976) to locate the time instant which is the most likely instant of the parameter shift by maximizing the joint likelihood function over the sample period.

Suppose that t_c is the instant of the parameter shift so that cov $e_t = \Delta_1$, for $t \leq t_c$ bus cov $e_t = \Delta_2$ $t > t_c$.

The joint likelihood function is

$$L(y_1 ... y_T | t_c) = \text{const} \ |\Delta_1|^{-t_c/2} |\Delta_2|^{(T-t_c)/2} .$$

$$\exp - 1/2 \ \text{tr}(\Delta_1 S_1 + \Delta_2 S_2)$$

where

$$S_1 = \Sigma_{t=1}^{t_c} e_t e_t'$$

and

$$S_2 = \Sigma_{t=t_{c+1}}^{T} e_t e_t' .$$

The regime shift is identified with the t_c which maximizes the joint likelihood function.

Univariate Series

Episode in the late sixties and early seventies such as the demise of the Brettonwood accord and the oil shocks tell us that a regime shift is likely during a period spanning late '60s and early '70s. The procedure outlined above is applied to the quarterly U.S. money stock data from the first quarter 1947 to the second quarter 1982. The total of 141 data points is split into two periods and separate mode are estimated for each subperiods to calculate the joint likelihood function. This data produces the fourth quarter of 1970 as the most likely quarter in which the U.S. monetary regime has shifted. The method is then applied to the U.S. real GNP series. It produces the first quarter of 1971 as the mostly likely shift point for the quarterly U.S. real GNP series based on 152 data points from the first quarter 1949. Note that the shift in the real GNP series occurred one quarter later than that for the M2 series.

Since both money stock and real GNP series are apparently trending with (near) unit roots, we apply the two-step modeling procedure outlined in Aoki (1989, 1990) to separately model the largest eigenvalue which is near one (slowest dynamic model).

For example, for the U.S. real GNP series, the model before the break is

$$y_t = .482\ \tau_t + (.779\ \ .144)\ z_t + e_t$$

where

$$\begin{bmatrix} \tau_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} .984 & 1.606 & .296 \\ 0 & .429 & -.569 \\ 0 & .569 & .484 \end{bmatrix} \begin{bmatrix} \tau_t \\ z_t \end{bmatrix} + \begin{bmatrix} 2.063 \\ -.585 \\ .402 \end{bmatrix} e_t$$

with $\quad \mathrm{cov}\ e_t = .931 \times 10^{-4}.$

The model after the break is

$$y_t = .238\ \tau_t + (.598\ \ .021)\ z_t + e_t$$

$$\begin{bmatrix} \tau_{t+1} \\ z_{t+1} \end{bmatrix} = \begin{bmatrix} .974 & 2.512 & .087 \\ 0 & .612 & -.337 \\ 0 & .337 & .956 \end{bmatrix} \begin{bmatrix} \tau_t \\ z_t \end{bmatrix} + \begin{bmatrix} 4.20 \\ -.402 \\ .064 \end{bmatrix} e_t$$

with $\quad \mathrm{cov}\ e_t = .114 \times 10^{-3}.$

Note that the statistic $T(\hat{\rho}-1)$ is about -1.5 for the largest eigenvalue, and the largest eigenvalues may or may not be equal to 1. We treat both series as not random walks, but rather as nearly integrated series as in Chan (1988).

When we examine the real GNP and money stock series of Japan and West Germany on the assumption that there is a simple break, the method of Goldfeld and Quandt applied to the estimated state space innovation models place the breaks during 1971 well before the episode of the first oil shock.

Three Real GNPs

Our preliminary analysis of the three real GNP series of the U.S., West Germany and Japan as univariate series indicate that they are likely to have individually experienced shifts in the parameters of the data generating process somewhere in 1970-1971. This section treat them as a trivariate series. It would be useful to break the data at or about the first quarter 1972. Since the data set at our disposal covers the period from the second quarter 1965 to the fourth quarter 1985, a total of 83 data points, this choice would leave only about 30 data points for the initial period. To increase the data points of the first subperiod to about 40 to balance the magnitudes of statistical errors in the two models for the two superiods, we break the data at 1974 and estimate two models for the two subperiods 1965.2-1974.2 and 1974.1-1985.4. We are now more interested in learning what differences if any are exhibited by the two models rather than locating the break point exactly. To this end, we evaluate the differences in the time paths of the impulse responses, i.e., dynamic multiplier profiles implied by the two models. The greater the discrepancies measured somehow, the more significant are the consequences of the structural shift.

In order to calculate impulse responses we need to identify the matrix D which relates the time series innovation vector, e_t, with the shocks, n_t, in the structural model,

$$e_t = Dn_t.$$

See Bernanke (1986) or Sims (1986) for some ways for identification.

As explained in these references, the matrix D may be thought of as $\varphi_0^{-1}\theta_0$ in the structural model $\varphi(L)y_t = \theta(L)n_t$, where y_t is the three-dimensional vector composed of the real GNP of the U.S., West Germany, and Japan in that order, and φ_0 and θ_0 are the 3×3 constant matrices in the lag polynomination matrices $\varphi(L)$ and $\theta(L)$ respectively. The covariance matrix cov n_t is normalized to be $I_3 \times 10^{-4}$. Thus,

$$\text{(1)} \qquad \text{cov } e_t = \Delta = DD' \times 10^{-4}.$$

There are nine elements in D and there are six elements in Δ which are estimated. Thus, we need three additional conditions to uniquely specify the matrix D. Once the matrix D is specified, the multiplier profits are given by $\Theta\Phi^k\Psi D$ k = 0, 1, . . . where the matrices appear in the innovation model

$$y_t = \Theta\, \chi_t + e_t$$

$$\chi_{t+1} = \Phi\, \chi_t + \Psi\, e_t$$

which are estimated as in Aoki (1988) by the two step procedure series since y_t is "trending."

One way to identify the matrix D is to use the Wold causal chain structure, i.e., to use the Cholesky decomposition. Instead, we use the decomposition into a common shock and uncorrelated[2] country specific shocks, i.e.,we model e_t by

$$e_t = \mu\, \nu_t + \begin{bmatrix} d_1 & n_J \\ d_2 & n_G \\ d_3 & n_J \end{bmatrix}_t$$

where ν, n_U, n_G and n_J all have variance 1×10^{-4} and mutually uncorrelated, i.e.,

$$\Delta = [\mu\, \mu' + \mathrm{diag}(d_1^2\; d_2^2\; d_3^2)] \times 10^{-4}.$$

The elements of the vector μ indicate how the common shock impinges on the three countries. The set of six algebraic relations in (1) can be solved for the three components of μ and d_i, i = 1, 2, 3 uniquely in general.

Then the multiplier profile $\Theta\Phi^k\Psi\mu$, k = 0, 1, . . . show how the three real GNP respond to a common shock and the profiles $\Theta\Phi^k\psi_i$, i = 0, 1, 2, 3 where ψ_i is the i-th column vector of the matrix Ψ show how the three real GNP respond to a shock originating in one country only. Note that d_1, d_2, and d_3 affects the relative magnitudes but not the shapes nor the timings of peaks and troughs in the multiplier time profiles, if any.

The parameters of the estimated models are as follows:

The First Period Model

$$B = \begin{bmatrix} .182 & .084 & -.065 \\ .207 & -.280 & -.105 \\ .502 & -.120 & .106 \end{bmatrix}, \qquad \Psi = \begin{bmatrix} 1.898 & -.0318 & 1.372 \\ 1.038 & -2.279 & 0.884 \\ -4.376 & 0.427 & 1.381 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} .971 & .000 & .010 \\ 0 & .9412 & -.132 \\ 0 & .147 & .869 \end{bmatrix}, \qquad \mu = \begin{bmatrix} .267 \\ .995 \\ .483 \end{bmatrix},$$

$d_1 = 1.078$, $d_2 = .660$, $d_3 = 1.093$.

The Second Period Model

$$B = \begin{bmatrix} .148 & .212 & -.051 \\ .120 & .157 & .070 \\ .282 & -.029 & -.061 \end{bmatrix}, \qquad \Psi = \begin{bmatrix} -.233 & 1.413 & 3.017 \\ 5.266 & 1.142 & -1.550 \\ -6.145 & 8.435 & -3.181 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} .966 & .009 & -.007 \\ 0 & .805 & -.052 \\ 0 & .213 & .783 \end{bmatrix}, \qquad \mu = \begin{bmatrix} .687 \\ .774 \\ .481 \end{bmatrix},$$

$d_1 = .701$, $d_2 = .652$, $d_3 = .172$.

(When the whole period is modeled jointly, then $\mu = (.485, .891, .522)'$ and $d_1 = .896$, $d_2 = .894$ and $d_3 = .904$).

Even before we examine the multiplier profiles, some differences are clearly evident. The eigenvalues of Φ of the second model is slower, for example. The common shock affects

West Germany more than the other two in each period, but less so in the second period. The country specific shocks of the U.S. and Japan are smaller in the second period. The U.S. economy is more exposed to a common shock in the latter period, while the Japanese exposure remains about the same. In the first period the U.S. is more immune to the common shock than in the second. This tendency is more pronounced in Japan.

The differences in the dynamic interactions in the two periods are clearly visible from the multiplier in Figure 1 ~ 8. Figures 1 and 2 show responses to a common shock in the lst and 2nd period. The model dimension is three in both periods, i.e., dim $\tau_t = 1$, dim $z_t = 2$ where τ_t is the state variable with the slowest decay, and the vector z_t is for the shorter-run dynamics. Figures 3 ~ 5 are for the first period and Figures 6 ~ 8 are for the second. The solid lines are for the U.S. responses. Figure 5 and Figure 8 show that a Japanese shock has more pronounced effect on U.S in the second period than the first while the effect on Germany remain about the same in both periods. The U.S. shocks affects the West Germany and Japan with opposite signs in the two periods. Japan is less affected by the German shock in the second period, while the opposite is true for the U.S. The Japanese shock affects the U.S. more in the second period than the first.

Concluding Remarks

A state space modeling method for apparently nonstationary time series and the resulting impulse responses are used to concisely portray the qualitative differences in the interaction characteristics of the three real GNPs which apparently took place in early 1970s.

Footnotes

[1]We need not adopt ad hoc assumptions on the breaks of "slope" or intercept points of the time series.

[2]A related work by Gerlach and Klock (1988) covers a period roughly comparable with the second subperiod. They jointly model six real GNPs with VAR. Our model is equivalent to vector ARMA.

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response to a common shock of m1.1+2.2 from 65.2

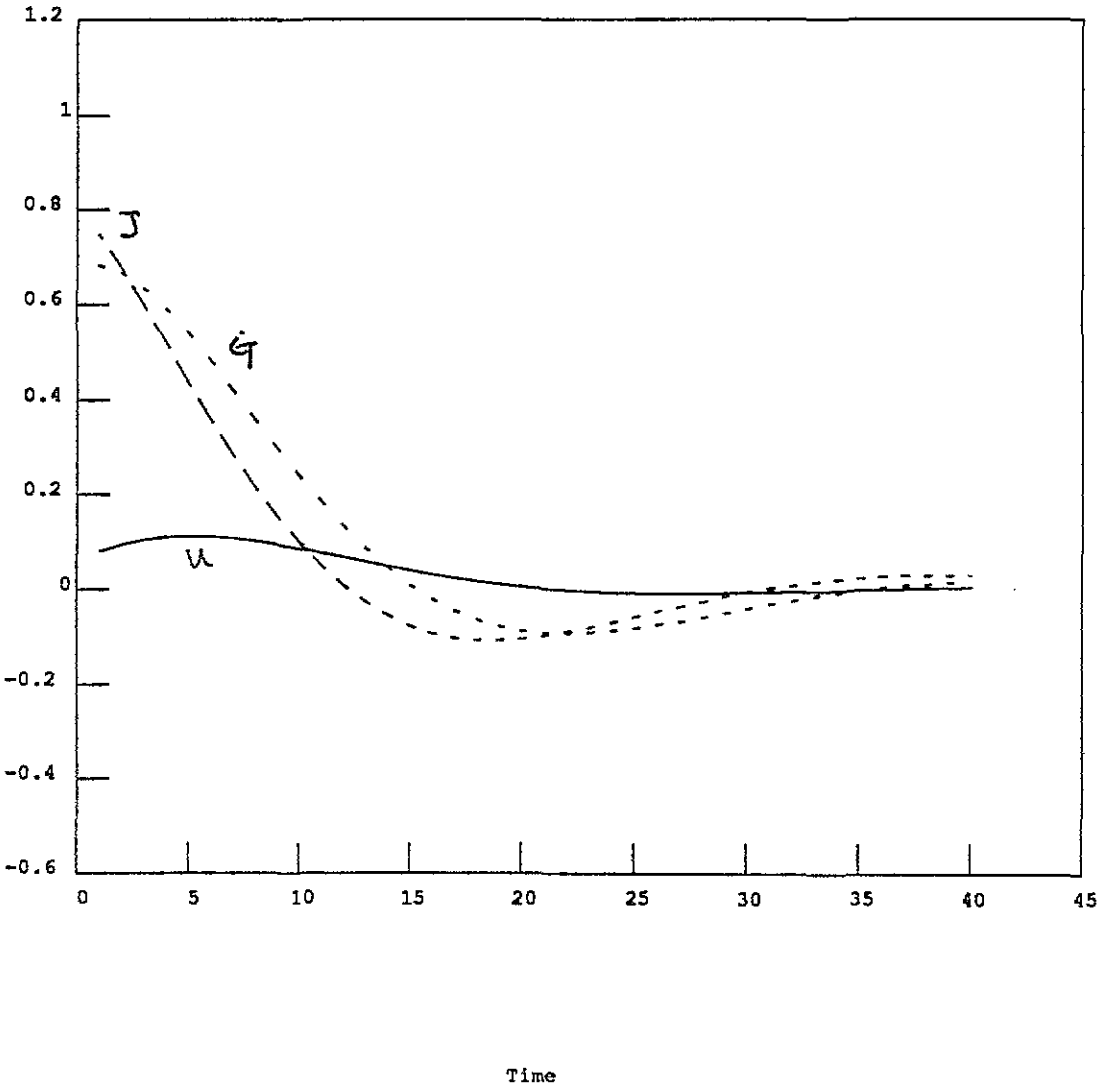


Fig 1

response to a common shock of m1.1+2.2 from 74.1

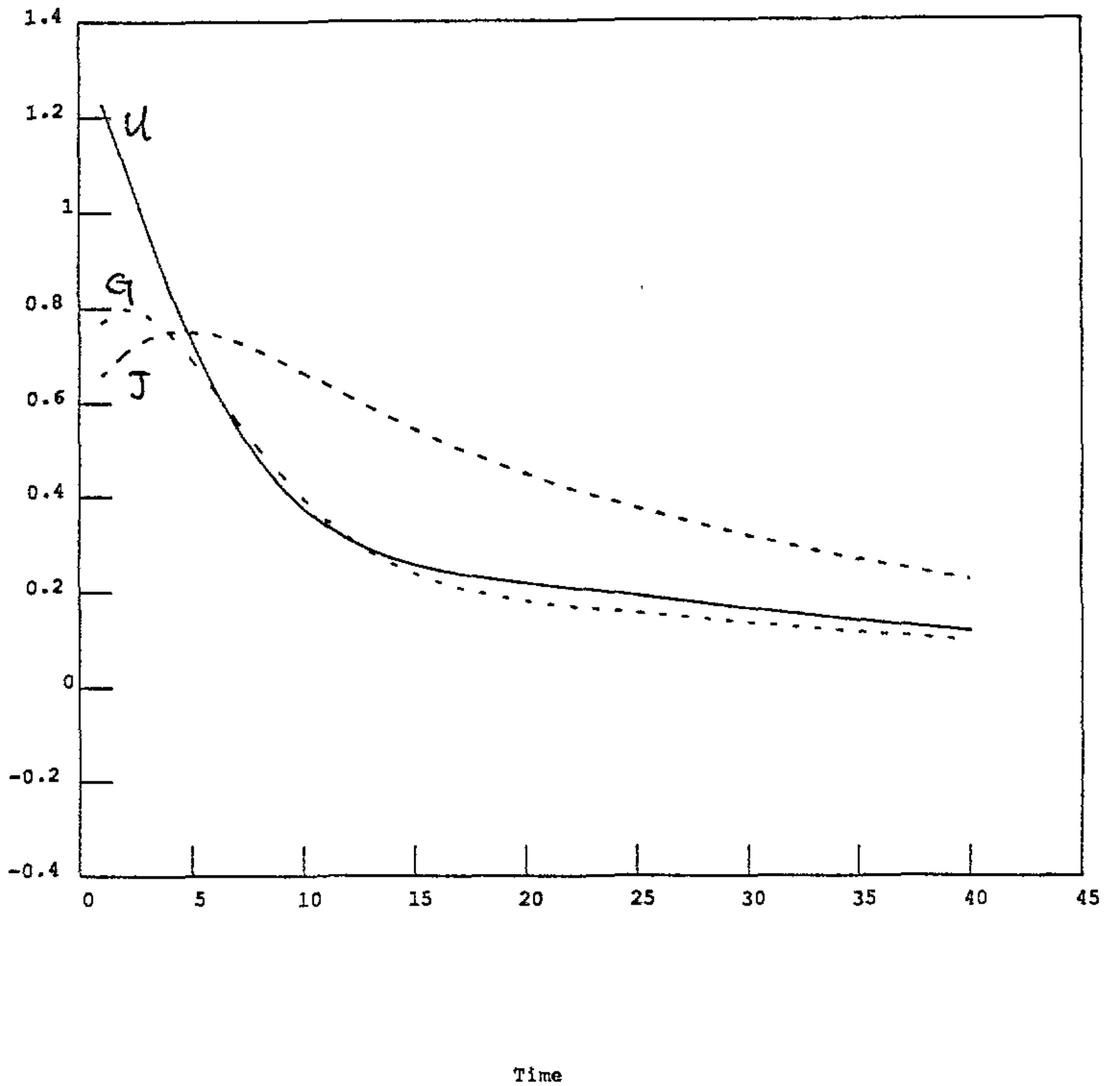


Fig 2

response to us shock earlier period

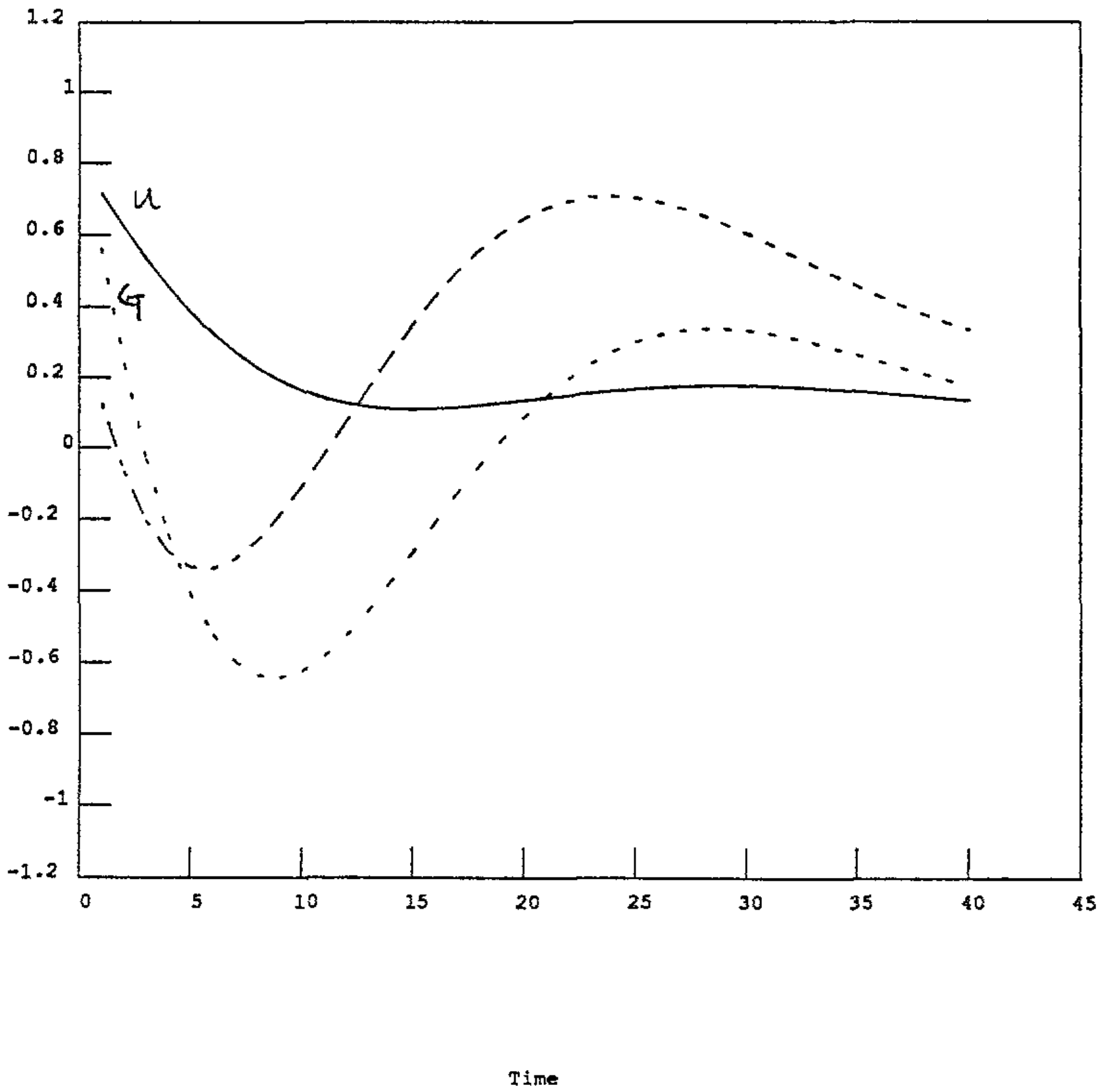


Fig 3

response to german shock earlier period

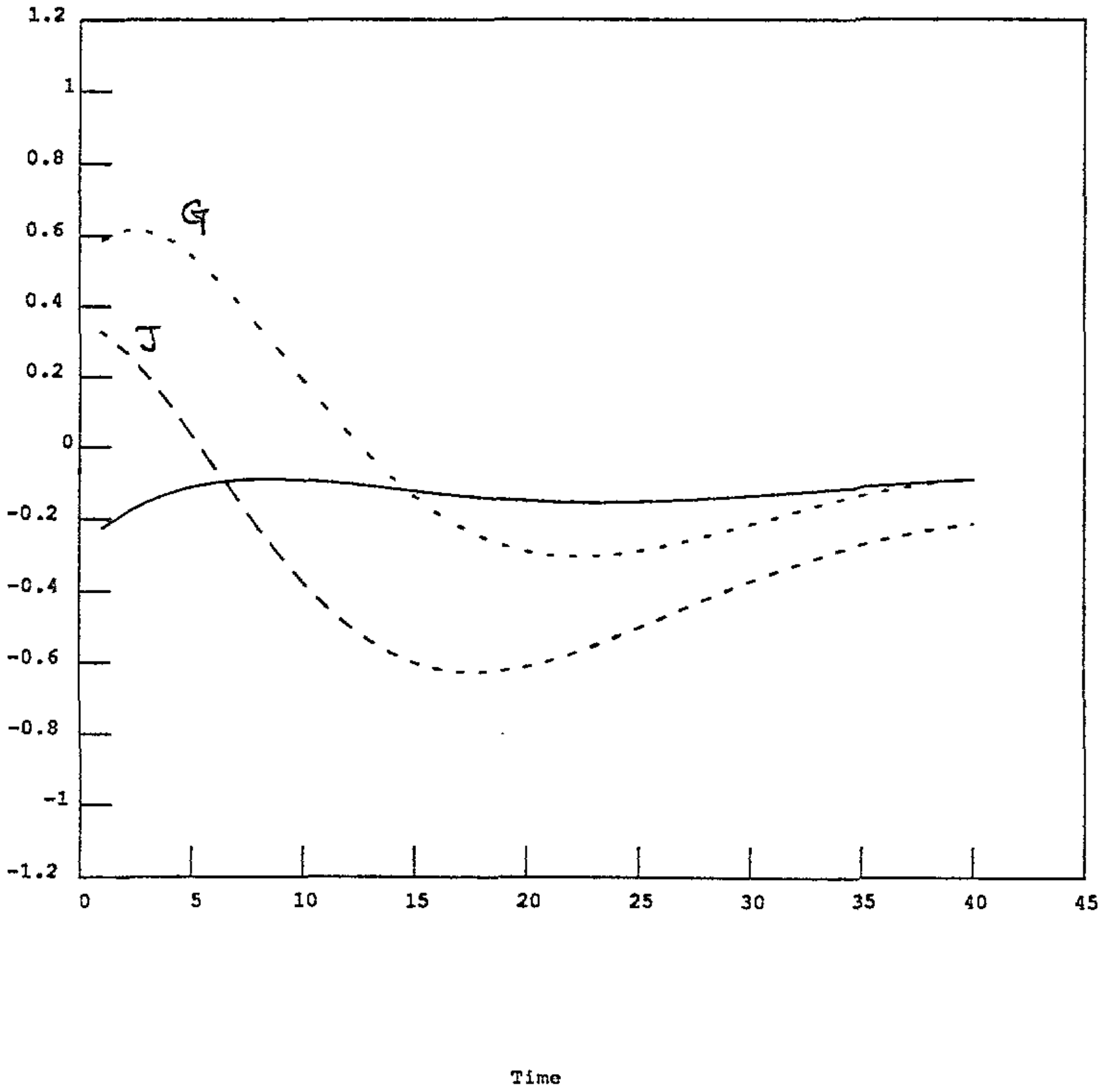
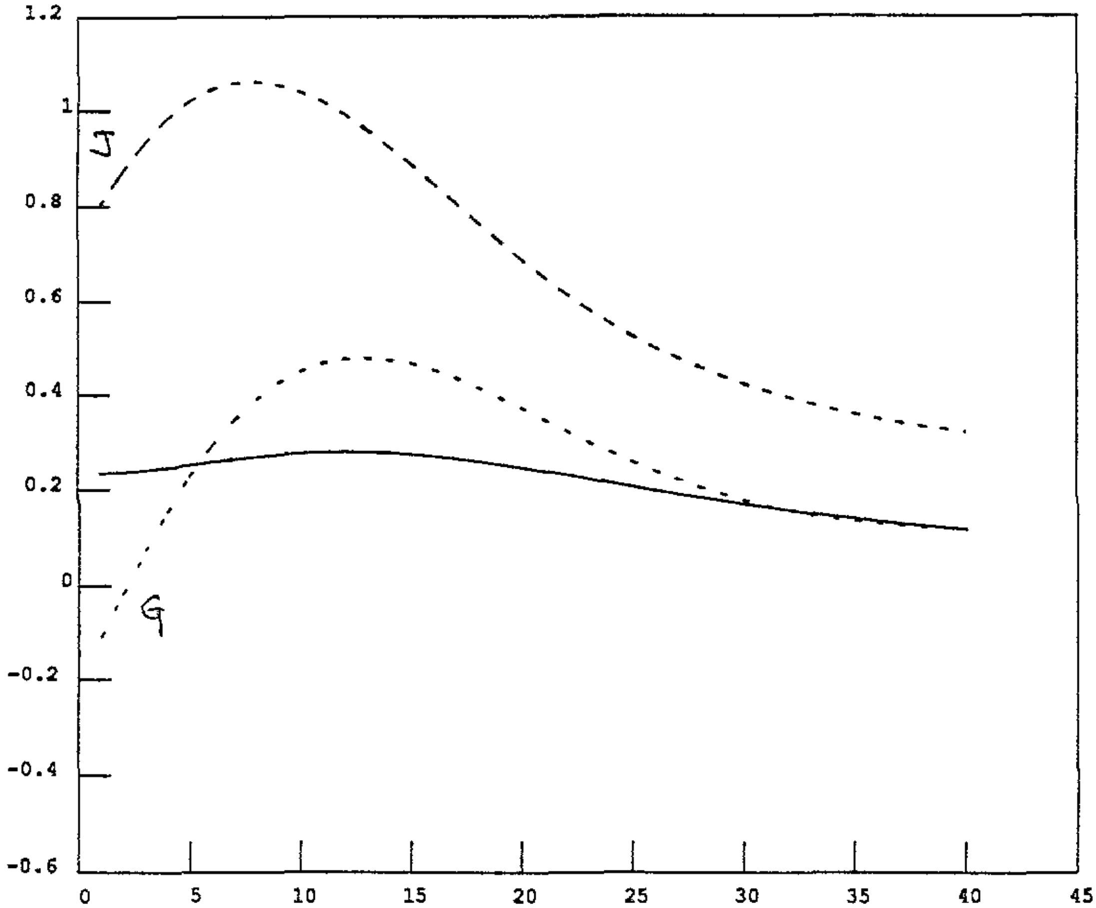


Fig 4

response to a japanese shock earlier period



Time

Fig 5

response to us shock later period

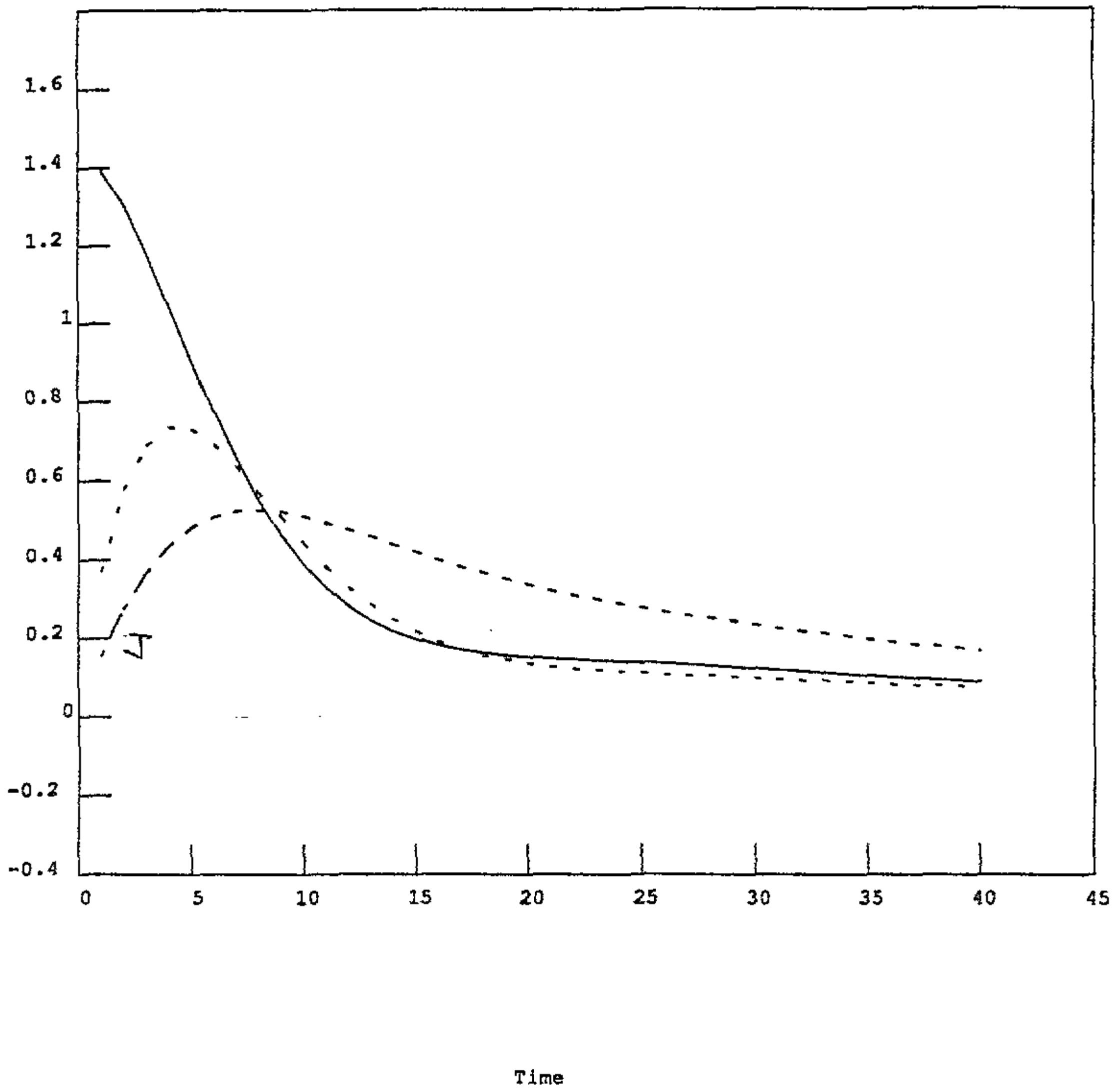
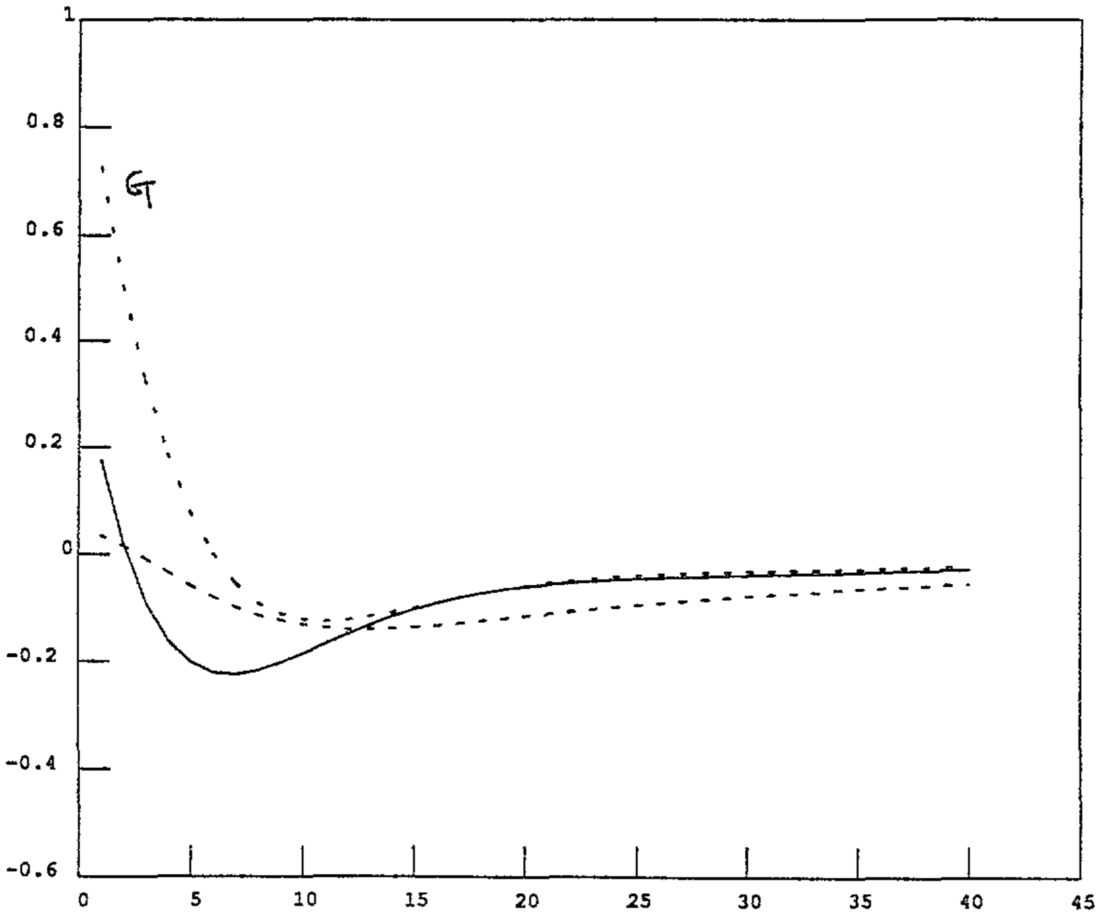


Fig 6

response to a german shock later period



Time

Fig 7

response to japanese shock later period

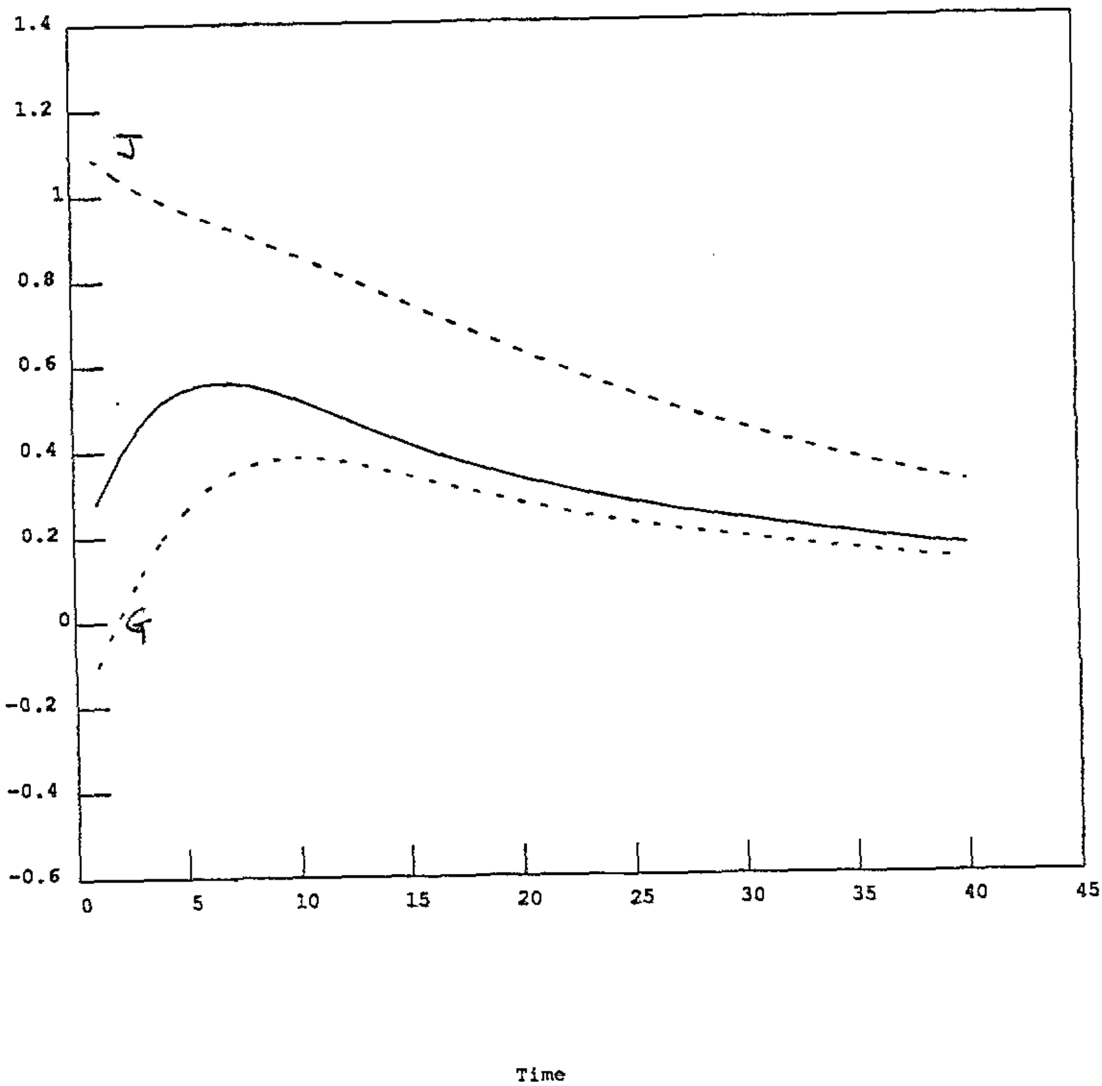


Fig 8