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HOURS AND EMPLOYMENT VARIATION IN BUSINESS CYCLE THEORY

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ABSTRACT

Previous business cycle models have made the assumption that all the variation in the labor input is either due to changes in hours per worker or changes in number of workers, but not both. In this paper, both vary. We think this is a better model for estimating the contribution of Solow technology shocks to aggregate fluctuations. We find that about 70 percent of U.S. postwar cyclical fluctuations are induced by variations in the Solow technology parameter.

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Introduction

In previous studies (Kydland and Prescott, 1982 and 1988a), we estimate the importance of variations in the Solow technology parameter as a source of aggregate fluctuations. We find that they were a major source accounting for over half of the fluctuations in the output of the American economy in the post-Korean-War period. These conclusions are based upon the study of model economies with the property that all workers work the same number of hours in equilibrium and that there is no variation in the number employed. Hansen (1985) studied a growth economy with the Rogerson (1988) labor indivisibilities. In his environment, individuals are constrained each period to either work some fixed number of hours or not at all. By construction, it is the number employed rather than the hours worked per employed person that varies. In such worlds, the aggregate willingness of people to intertemporally substitute leisure is considerably higher than that of the individuals whose behavior is being aggregated. For the Hansen economy, fluctuations exceeded those experienced by the U.S. economy in the post-Korean-War period.

We know that both the hours per worker and the number of workers employed vary. In this paper we present a computable general equilibrium structure in which both the hours a plant is operated and the number of employees that operate it are choice variables. We think that this is a better model to assess the importance of various shocks as a source of aggregate fluctuations. We calibrate the model economy to national income and product account and household survey data and we use it to assess the importance of variations in the Solow (1957) technology parameter. Our estimate is that their contribution is approximately 70 percent of the total. This is larger than the estimate we obtained previously but it is significantly smaller than that obtained by Hansen (1988). It would be interesting to know whether the findings of Braun (1988), Chang (1988), Christiano and Eichenbaum (1988), and McGratten (1988) regarding the importance of public finance shocks would be altered if they included this structure in their models.

In the spirit of the Hansen economy, ours has a nonconvexity in the consumption possibility sets of the households but, the nonconvexity is nowhere near as extreme as the Rogerson indivisibility constraint. In our economy, agents spend time in commuting to and from work. They can allocate to market activities any number of hours subject only to the time-endowment constraint. Hansen and Sargent (1988) study a similar problem in their straight-time and overtime model. In their economy, agents choose one of three time allocations at each date. These choices are either not to work, to work regular time, or to work regular plus overtime. In that model, during the overtime period fewer workers use the same capital stock. Consequently, the capital-labor ratio is larger than that during the regular time period. In our model economy, output of a plant is the number of hours it is operated times a constant-returns-to-scale production function with capital and the number of workers as inputs operating the plant. Both the number of workers operating the plant and the number of hours the plant is in operation can be varied. We think that this construction better conforms to micro observations.

In our model economy the utilization rate of capital is proportional to the number of hours the plants are operated. The capital utilization rate therefore varies. In this paper we examine whether abstracting from this fact seriously biases the Prescott (1986) estimate of the innovation variance of the Solow technology parameter process. We find the bias is small, but not insignificant. It results in a reduction in our estimate of the contribution of Solow technology shocks to business cycle fluctuations.

An additional feature of our model is that resources are utilized whenever agents move between the household sector and the market sector. The amount of resources used varies across individuals. The nature of the equilibrium is such that those with lower transfer costs are the first to be moved. Total resources used for this purpose turn out to be a convex function of the number moved. The economy behaves as if there were a stand-in household that experiences costs of adjusting its employment as assumed by Sargent (1979). By being explicit about the microfoundations of these so-called aggregate adjustment costs, there is some hope of deducing their size by examining micro observations. We find that the magnitude of the parameter for which the relative fluctuations in hours per worker and the number of workers of the model economy match those of the postwar American economy is not implausibly large. Given this parameter value, the total costs of moving people between sectors are less than one hundredth of a percent of GNP on average.

By introducing heterogeneity of agents we are following Rogerson (1987). A key difference, however, is that, in our model, resources are used up in changing the level of employment while, in his, costs are an increasing function of the number employed. Cho and Cooley (1988) examine the implications of using a modified version of the Rogerson construct to study the empirical elasticity of labor supply responses to temporary changes in the real wage.

The paper is organized as follows: Section 1 specifies the economic environment. Section 2 represents it as an economy in the sense of Arrow-Debreu-McKenzie and carries out the aggregation. In Section 3, we calibrate the economy to the national income and product account and survey data. The experiments based on the model economies are outlined in Section 4. In Section 5, we examine the cyclical behavior of these model economies. The final section contains summary and conclusions.

1. The Economic Environment

Preferences

There are a large number of ex-ante identical agents and these agents have measure one. An agent's utility function is

$$E \sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t),$$

where c_t is consumption at date t, ℓ_t is leisure, and $\beta \in (0,1)$ is the subjective time discount factor. The function U has the form

$$U(c,\ell) = \frac{G(c,\ell)^{1-\gamma} - 1}{1 - \gamma},$$

where $\gamma > 0$, $\gamma \neq 1$, and G is a CES function whose parameters will be determined as part of the calibration.

An individual's time endowment in each period is one. The amount of labor allocated to the market, however, is not $1 - \ell$. Letting h be hours of labor services,

$$\ell(h) = \psi - h \text{ for } 0 < h \leq \psi < 1,$$

while

$$\ell(0) = 1.$$

The function $\ell(h)$ is discontinuous at zero. The reason for this discontinuity is that time $1 - \psi$ is required for commuting to work every period that the individual is employed.

Each period, agents are indexed by a parameter ξ which is identically and independently distributed both over time and over agents.[1] Random variable ξ determines the amount of the composite output good that is required to move an individual of type ξ between the household sector and the market sector. More precisely, if, for any individual, $h_{t-1} > 0$ and $h_t = 0$ or if $h_{t-1} = 0$ and $h_t > 0$, a cost is incurred. We assume ξ is uniformly distributed on [0,1]. The size of the moving cost is proportional to ξ with a different constant depending upon the direction of the move. The relative size of these constants will be selected in such a way that "adjustment costs" of changing aggregate employment are symmetric whenever last period's employment rate is equal to the average employment rate.

Technology

A given agent working h hours and using k units of capital produces

$$a = zhk^{1-\theta}$$

units of some intermediate good. This good is an input to a constant-returns-to-scale aggregated CES production function along with inventory services y. This production function is denoted F(a,y). Output is used either for consumption c, investment i, or for moving people between sectors m. In particular,

$$m + c + i \leq \left[(1-\sigma)a^{-\nu} + \sigma y^{-\nu}\right]^{-1/\nu} \equiv F(a,y),$$

where $1/(1+\nu)$ is the elasticity of substitution between a and y and σ the share parameter of inventory services.

Investment i_t is the sum of inventory investment, $y_{t+1} - y_t$, and investment in plant and equipment. Time is required to build new k_t. Letting s_{jt} for $j = 1, \ldots, J$ be the number of units of capital j periods from completion, the laws of motion of the capital stocks are

$$k_{t+1} = (1-\delta)k_t + s_{1t}$$

and

$$s_{j,t+1} = s_{j+1,t} \quad \text{for } j = 1, 2, \ldots, J - 1.$$

The fraction of value put in place in each stage is denoted ϕ_j. Consequently, total investment in period t is

$$i_t = (y_{t+1} - y_t) + \sum_{j=1}^{J} \phi_j s_{jt}.$$

The number of time periods required to build new capital and the pattern of value added over the construction period are parameters that must be calibrated.

The shocks to technology are the sum of two independent components

$$z_{1,t+1} = 0.95\, z_{1t} + \varepsilon_{1t}$$

and

$$z_{2,t+1} = \varepsilon_{2t},$$

where $z_t = z + z_{1,t+1} + z_{2,t+1}$. The means of ε_{1t} and ε_{2t} are zero. Parameter z is the mean of the $\{z_t\}$ process. Observed at the beginning of period t is $z_t + \varepsilon_{3t}$, where ε_{3t} is a measurement error. All shocks are normally distributed and independent. For the Kalman filter analysis of this structure, see Kydland and Prescott (1982).

2. Aggregation

At time zero, all agents are identical. Using the competitive theory with lotteries of Prescott and Townsend (1984) as extended by Prescott and Rios-Rull (1988), all agents receive the same distribution of date- and event-contingent consumption-leisure pairs but possibly different realizations of the lottery. The competitive equilibrium is the Pareto optimum that maximizes the sum of agents' utilities. This fact is exploited in developing our algorithm for computing the equilibrium.

At a given point in time, agents differ in terms of their current moving-cost parameter ξ and of their previous-period employment state. Given that the ξ are identically and independently distributed both over time and across individuals, the aggregate state variables must include only the measure of agents employed the previous period, the value of the technology parameter, a set of sufficient statistics for forecasting future values of this parameter, and the aggregate stocks of capital.

We address two issues in this section: the size of aggregate moving costs, given the number of people to be moved, and the distribution of consumption and leisure across agents in each period, given the aggregate per capita variables.

Aggregate Moving Costs

Current-period employment is n and last-period employment e. If $n > e$, measure $n - e$ of people must be moved from the household sector to the market sector. Those with the smallest ξ are moved first. We assume that the cost of moving a ξ type from the household sector to the market sector is $\alpha\xi$. If $n < e$, measure $e - n$ must be moved from the market sector to the household sector. The cost of moving a ξ type from the market sector to the household sector is $\alpha\xi$.

The total moving costs when $n > e$ are

$$m = \alpha \int_0^{n-e} \xi \, \frac{f(\xi)d\xi}{1 - e}.$$

Similarly, when $n < e$ aggregate moving costs are

$$m = \alpha \int_0^{e-n} \xi \, \frac{f(\xi)d\xi}{e}.$$

The moving-cost function is convex and has value zero if $e = n$. Given this, we locally approximate the function using a quadratic function. We denote the quadratic aggregate moving costs as $M(e,n) = \alpha(n-e)^2$.

Distribution of Consumption and Leisure

In the remainder of this section we have to make a distinction between population means and individual values. Capital letters denote population means of the corresponding variable.

Let $x(B)$ be the measure of people who consume c, work h hours and use k units of capital for (c,h,k) belonging to measurable set B. Since we need a linear space for standard competitive analysis, the measures are signed measures. The planner's problem is

$$R(I,K,Y,N) = \max_{x \geq 0} \int U[c,\ell(h)]dx$$

subject to

$$\int c\ dx + I + M(E,N) \leq F(z \int hk^{(1-\theta)}dx, Y)$$

$$\int dx = 1$$

$$\int k\ dx \leq K$$

$$\int I_{\{h>0\}}dx = N.$$

For technical reasons we impose the constraint that individual consumption is bounded above by some number as is the amount of capital used by an individual. This results in the space over which x is defined being the Borel sigma algebra of a compact metric space. In equilibrium, both of these constraints are nonbinding.

For the production functions and utility structure of the CES variety, the solution to this programming problem is to assign people to at most two (c,h,k)-triples. One of these points has both $h = 0$ and $k = 0$ and some level of consumption $c = c_0$. The other point is denoted (c_1,h_1,k_1). Letting n be the measure or fraction of people assigned (c_1,h_1,k_1), then $k_1 = K/n$ because it is optimal to assign all capital to workers. Measure or population fraction $1 - n$ are assigned $(c_0,0,0)$. For the formal analysis, see Hornstein and Prescott (1989).

For some values of the parameters of our CES preference and technology structures, the optimal n is one and all people work. But, for our calibrated model economy, this is not the case.

This program has a maximum given that the constraint set is compact and the objective function continuous in the weak* topology. Further, the objective is concave and its constraint set jointly concave in the decision variables x and N and in the constraint variables E, K, and I. Consequently, the value of the program is concave and continuous in E, K, and I.

An implication of this analysis is that the following more restricted social optimum problem can be considered:

$$\max E \sum \beta^t[(1-n_t)U(c_{0t},1) + n_tU(c_{1t},\psi-h_t)]$$

subject to

$$c_t = (1-n_t)c_{0t} + n_tc_{1t},$$

and to the constraints of Section 1.

3. Steady State and Calibration

A steady state for the deterministic version of this economy is its rest point when the variances of the shocks are zero. These steady state values of the model aggregates are also the means of the quadratic approximation of the model economies. The purpose of the calibration is to choose the parameter values for which the steady state values of the model aggregates are approximately equal to the averages of corresponding variables for the U.S. postwar economy. Given that we normalize aggregate quarterly per capita output to one and that we are calibrating our model economy to U.S. data, we choose the investment share of output to be one quarter ($i = 0.25$). Consequently, the consumption share is three-quarters ($c = 0.75$). Other parameter values that are chosen to be approximately equal to U.S. averages are the inventory stock to quarterly output ratio ($y = 1.0$), quarterly real interest rate ($r = 0.01$), the fraction of the working-age population who work ($n = 0.75$), and the fraction of productive time that working people work ($h = 0.44$). We abstract from growth in our economy. Justification is provided by Hansen (1988), who shows that, provided investment shares are equal, variations in the average rate of exogenous technological change do not affect business cycle accounting.

The first step in the calibration is then to choose the elasticities of substitution between inputs in both the household and business sector. First, we consider the households. Over the last few decades, the real wage increased two to three times, while hours of work per household remained essentially constant.

Kydland (1984) formally demonstrates that the unitary-elasticity case of a CES utility function is the one consistent with this observation. We therefore choose the form of the current-period utility function to be

$$U(c_t, \ell_t) = \frac{[c_t^{\mu} \ell_t^{1-\mu}]^{1-\gamma} - 1}{1 - \gamma},$$

where 0 <hn μ < 1 and $\gamma > 0$ but different from one. Values of γ close to one correspond to using a logarithmic utility function.

A technology constraint is

$$m_t + c_t + i_t \leq \left[(1-\sigma)(z_t h_t n_t^{\theta}\, k_t^{1-\theta})^{-\nu} + \sigma y_t^{-\nu}\right]^{-1/\nu},$$

where

$$i_t = \textstyle\sum_{j=1}^{J} \phi_j s_{jt} + y_{t+1} - y_t,$$

and where $0 < \theta < 1$, $0 < \sigma < 1$, and $\nu > 0$. Steady state m_t is zero since, when the economy is in its steady state, employment is constant and no workers have to be moved between sectors. The fraction ϕ_j of total resources put in place at stage j of the project is 1/J for all j. We choose J = 3. Obviously, some projects take longer than three quarters and others less, but this value appears to be a reasonable compromise. There is little evidence that the time to build varies over the cycle.

The elasticity of substitution between capital and labor inputs is unity for our production technology. The empirical studies that led us to this choice are the ones that led Auerbach and Kotlikoff (1987, p. 52) to make the same choice. They are Nerlove (1967), and Berndt and Christensen (1973). The recent study by Jorgenson, Gollop, and Fraumeni (1987, p. 341) is consistent with the earlier ones in finding the elasticity to be near one. On the other hand, our knowledge about the elasticity between the inventory stock and the remaining composite input is more ambiguous. An event which caused the relative price of the two to move considerably would give us a sharp estimate of its magnitude. Unfortunately, this is yet to happen. In the meantime, our view is that the elasticity of substitution, $1/(\nu+1)$, is rather small and that therefore ν is significantly greater than zero. We choose $\nu = 3$.

We consider the household's and the firm's problems separately, in both cases taking prices as given. We first take hours per period h as a given, and we derive the first-order conditions with respect to n, k, and y in the case of the firm, and with respect to c_0, c_1, and n in the case of the stand-in household. In equilibrium, the value of h must be such that the marginal product of working h hours equals the negative of the ratio of marginal utilities with respect to hours and consumption.

Rental Prices of Capital

The price of newly produced capital is

$$q = \sum_{j=1}^{J} \phi_j (1+r)^{j-1}.$$

This is the value of the resources used up to produce one unit of new k in terms of the same-date consumption good. Consequently, the rental price of capital is

$$u_k = (r+\delta)q.$$

The prices of the capital goods in process, s_1 and s_2, are

$$q_1 = \phi_3(1+r) + \phi_2$$

and

$$q_2 = \phi_3.$$

Real gross investment in plant and equipment at date t, using steady state prices, is then

$$q_1(s_{1,t+1}-s_{1t}) + q_2(s_{2,t+1}-s_{2t}) + q\big(k_{t+1}-(1-\delta)k_t\big).$$

In a steady state allocation, the first two terms are zero and the last is simply $q\delta k$. Steady state GNP is therefore $c + q\delta k$. Finally, the rental price of inventories is

$$u_y = r$$

since inventories do not depreciate.

The Firm's Problem

The firm rents capital and inventories. Its rental prices are u_k and u_y, respectively. These steady state prices are given by the expressions derived above. Abstracting from growth, the steady-state real interest rate, r, equals the rate of time preference, $(1-\beta)/\beta$. The quarterly wage per worker depends on the number of hours h worked in that period. We denote it by w_h to indicate this dependence. Every period, the firm maximizes the value of its output minus the cost of the inputs,

$$F(zhn^{\theta}k^{1-\theta},y) - u_k k - u_y y - w_h n,$$

We take the price of output to be one. The units to measure output are chosen so that steady-state output is one. Then, from the production function,

$$(zhn^{\theta}k^{1-\theta})^{-\nu} = 1 - \sigma y^{-\nu}.$$

From this equation we obtain

$$z = [hn^{\theta}k^{1-\theta}(1-\sigma y^{-\nu})^{1/\nu}]^{-1}.$$

The condition $F_y = u_y$ yields

$$\sigma = u_y y^{\nu+1}.$$

Similarly, equating the marginal product of k to u_k implies

$$1 - \theta = u_k k/(1-\sigma y^{-\nu}).$$

Finally,

$$w_h = wh = F_a(a,y)\theta zhk^{1-\theta}n^{\theta-1}.$$

Wage rate w is a parameter of the household's problem.

To summarize, in this part of the calibration, technology parameters z, θ, and σ, and preference parameter β, are selected so that the steady state r, k, y, and F(a,y) have the specified values, given the specified values of i, h, and n. On the other hand, we use independent evidence to select J, the number of periods required to construct new productive capital, the ϕ_j, which are the fractions of value added at each of the J stages of production, and ν, which determines the elasticity of substitution between inventory stocks and the composite of the other inputs.

The Household's Problem

The household's problem treats the steady state values of prices and capital stocks parametrically. The maximization problem faced by the stand-in household, given its steady state capital income b, is

$$\max \sum_{t=0}^{\infty} \beta^t [(1-n_t)U(c_{0t},1) + n_t U(c_{1t},\psi-h_t)]$$

subject to

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} [(1-n_t)c_{0t} + n_t c_{1t}] \le \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} [wh_t n_t + b].$$

The maximization is over $\{h_t, n_t, c_{0t}, c_{1t}\}_{t=0}^{\infty}$. Given that $\beta = 1/(1+r)$ and that the first-order conditions have been shown to be necessary for an optimum, the optimal values of the variables are date independent. Consequently, we drop the time subscript.

The problem can then be simplified to

$$\max_{c_0, c_1, h, n \ge 0} [(1-n)U(c_0,1) + nU(c_1,\psi-h)]$$

subject to

$$(1-n)c_0 + nc_1 \le whn + b.$$

The per-period utility function is

$$U(c,\ell) = \frac{[c^{\mu}\ell^{1-\mu}]^{1-\gamma} - 1}{1 - \gamma}.$$

Steady state net capital income is

$$b = ry + rqk + rq_1\delta k + rq_2\delta k,$$

that is, the interest rate times each of the values of the four capital stocks, y, k, s_1, and s_2.

We still have to determine parameters γ, μ, and ψ. We choose $\gamma = 2.0$. This is larger than the value of 1.5 that we use in our previous research. The problem with $\gamma = 1.5$ is that the resulting calibrated value of ψ exceeds one. This would be inconsistent with the theory. With $\gamma = 2.0$, the consumption of the unemployed is about 75 percent of the consumption of the employed. For smaller values, the difference is less, with the difference approaching zero as γ approaches one.

The parameters μ and ψ are selected so that the optimal h is 0.44 and the optimal n is 0.75. For this purpose we use the four first-order conditions and the budget constraint. The resulting values are $\mu = 0.33$ and $\psi = 0.99$. With these values, steady state consumption of the employed is $c_1 = 0.80$, while it is $c_0 = 0.59$ for those who are not employed. Necessarily, the resulting value of per capita consumption, $c = (1-n)c_0 + nc_1$, is 0.75, the consumption share of output.

The remaining parameters to be calibrated are the variances, σ_i^2, for i = 1, 2, 3, of the shocks. The variance of the highly persistent shock, σ_1^2, is set equal to the variance of the Solow residuals for the postwar U.S. economy as estimated by Prescott (1986). This value is 0.0076^2. The ratios of the remaining two variances to the first are set equal to the values used in Kydland and Prescott (1982,1988a).

The parameter and steady state values for our model economy are listed in Table 1.

4. Experiments

To specify our model economy fully we need to choose a value for the moving-cost parameter α. In the first experiment we choose $\alpha = 0$. This value corresponds to zero cost of moving a person into the market sector or out of it. We find that, for this economy, virtually all the variation in the aggregate hours of labor is in the number of workers employed and almost none in the hours per employed person. The economy behaves very much like the Hansen (1985) economy. In both economies, fluctuations in aggregate output are approximately as large as those for the U.S. economy in the 1954-1988 period and the aggregate willingness of agents to intertemporally substitute leisure is very high. An important difference, however, is that the amplitude of fluctuations induced by a given variation in the Solow technology parameter is not as large as it is for the Hansen economy. One reason for this difference is that our agents are more risk averse than Hansen's (our γ is 2.0 while his is 1.0). Another reason is that, in our economy, a period of three quarters of a year rather than one is required to build new capital.

We conclude that both this economy and Hansen's overestimate the amount of fluctuations induced by Solow technology shocks. This failure of hours per worker to vary led us to introduce costs of moving people between the household and the market sector. The issue to be addressed then is what value to choose for the moving-cost parameter α. We select $\alpha = 0.5$. This value is associated with a ratio of the variation in employment to the variation in hours per employee that is a little larger than that for the U.S. economy. This is as it should be given the nature of our abstraction. Even when the time period is a quarter of a year, there is considerable temporal aggregation. With temporal aggregation, some of the hours-per-worker variation over the period reflects variation in employment over the subperiods.

With $\alpha = 0.5$, the average aggregate moving costs are less than one-hundredth of a percent of average GNP. This is not a large number. At the microlevel, if the increase in employment is two percent in a quarter, then the cost of moving one additional person to the market sector is $100. We do not consider this number to be unreasonable.

To summarize, Economy I has no moving costs (α=0). Economy II has what we consider to be reasonable moving costs, with $\alpha = 0.5$. We use this economy to estimate the importance of Solow technology shocks. For comparison purposes, a third economy is also examined. In this economy, the moving costs are so large (α=500) that virtually all the variation occurs in hours per worker and none in the number of workers. Our view is that this economy underestimates the magnitude of fluctuations induced by technology shocks.

5. Cyclical Behavior of the Model Economies

Lucas (1977, p. 9) defines the business cycle phenomena as the regularities of the comovements of the cyclical components of aggregate time series. This definition is not complete until the method for calculating the cyclical component of a time series is specified. The method we use is to subject each time series to a common linear transformation. This transformation filters out low-frequency movements in the data. Consequently, the statistics that we are labeling the cyclical components, change little if some slowly varying function is added to a time series prior to its transformation. If the added component is a linear trend, the cyclical component series does not change at all. For details of the method, see Kydland and Prescott (1982, fn. 15).

The particular time series that we examine are chosen to resemble those of the augmented neoclassical growth model when both the consumption-savings decision and the market-time-allocation decision are endogenized. The statistics that we consider are autocorrelations of output, percentage standard deviations for all the variables and their correlations with GNP, including leads and lags. They describe the strength of the comovements with output, the phase shifts in the comovements and the relative amplitude of fluctuation. The autocorrelations of real output describe persistence of fluctuations.

This we found to be a very useful summary organization of data from the point of view of the theory. These statistics have two desirable properties. First, they are insensitive to the very low-frequency movements that can arise from any number of factors from which we abstract. There are, of course, other statistics that are insensitive to these low-frequency movements--in particular, the first two moments of the first differences of the time series. This transformation, however, has the undesirable feature that much of the power at the business cycle frequency is eliminated. A more serious problem with first differences is that what is of concern is the magnitude of the deviation from trend and not the rate of change of for example real output and employment. Two time series can have very different variances of deviations and yet have the same variation of rates of change. In short, our second reason for using deviations is that, from the point of view of the theory, the deviations are the quantities of interest.

For purposes of comparison, we present in Table 2 statistics for the cyclical components of U.S. aggregate time series for the 138-quarter period 1954:1-1988:2. The cyclical component is defined in exactly the same way for the U.S. data as for the model economies, that is, the cyclical components for the U.S. data and for each simulation of the model are summarized by the same statistics.

For each of the three values of α, 50 independent samples are drawn. For each sample of 138-quarter length, the cyclical components are calculated and the same set of statistics computed as for the U.S. data. For each statistic we report the averages and standard deviations of the 50 samples. These are estimates of the means and standard deviations for the sampling distributions of the statistics for the model economies and can be compared with the statistics for the U.S. economy in Table 2. The outcomes of the three experiments are reported in Tables 3.1-3.3.

Findings

The key question motivating this and our previous studies is what fraction of U.S. postwar business cycles can be accounted for by technological shocks, also commonly referred to as Solow residuals. For the economy with no moving costs and with the variance of the highly persistent technology shock calibrated to correspond in size to Solow residuals for the U.S. economy, the standard deviation of cyclical GNP is almost as large as that for the U.S. data. In the economy with reasonable moving cost, and therefore with variation in both employment and hours per worker, technology shocks induce a variance of cyclical output that is about 75 percent as large as in the data.

In our model economy, the capital utilization rate varies. The estimate of the technology-shock variance that we use was computed under the assumption of no variation in the capital utilization rate. An issue is whether estimating this parameter under the incorrect assumption that the capital utilization rate is constant seriously biases our conclusions. We address this issue as follows. For the 50 simulations, we estimate the variances of the technology shock while incorrectly treating the capital utilization rate as a constant. The mean estimate is 0.0079^2 when in fact the true value for the model economy is 0.0076^2. This finding leads us to reduce our estimate of how variable the U.S. economy would have been if Solow technology shocks were the only source of fluctuations from 75 percent as variable to 70 percent as variable.

Total hours for the model economy varies less than output by a greater margin than in the U.S. data. Furthermore, the correlation between output and labor productivity is 0.89 for the model economy and only 0.51 for the U.S. using data on hours from the household survey and 0.31 using data from the establishment survey. If the Solow technology shocks accounted for virtually all of the fluctuations, this would be bothersome. This, however, is not our finding. We find that the Solow technology shocks account for about 70 percent of postwar business cycles. Given this, figure, if the correlation between output and labor productivity for the U.S. data were close to one, the theory would be in trouble. Our estimate of the importance of technology shocks implies that over a quarter of the cycle is accounted for by other factors. These other factors, which do not alter the production functions, induce output and productivity fluctuations that are of opposite signs. This is an implication of the law of diminishing returns and the fact that cyclically the capital stock varies little. Consequently it is comforting that the correlation between productivity and output is smaller for the U.S. economy than it is for the model economy.

Another reason why the correlation between hours and productivity should be lower for the U.S. economy than for the model economy is that, cyclically, aggregate hours is not that good a measure of the labor input for the U.S. economy. As documented in Kydland and Prescott (1988b), in the PSID panel for the 1969-82 period, the aggregate quality-weighted labor input varies only three-quarters as much as does aggregate hours. This difference arises because those with less human capital, on average, have significantly greater cyclical variation in hours of employment than those with more human capital.

With no moving costs, all the aggregate hours variation is the result of changes in the number of workers and hours per worker does not fluctuate. For the economy with $\alpha = 0.5$, hours per worker varies considerably. Employment lags the cycle while hours per worker leads slightly and productivity leads the cycle. This is also the case for the U.S. data (see Table 2).

6. Summary and Conclusions

We have developed a computable general equilibrium structure in which both the hours a plant is operated and the number of employees can be varied. This, we think, is a better structure for assessing the contribution of shocks, of whatever origin, to aggregate fluctuations.[2] We use this theory to estimate the importance of Solow technology shocks and we find that they are a major contributor. We find that, if they were the only source of shocks, the variance of aggregate fluctuations would be about 70 percent as large as the corresponding one for the U.S. data.

In the aggregate, leisure is more substitutable than at the individual level.[3] In this sense, the economy behaves as if there were indivisibilities.[4] It has been suggested that the indivisibilities of Hansen (1985) and Rogerson (1988) were ad hoc. Our framework provides a theoretical foundation for their approach.

Another innovation is a microbased theory of aggregate workforce adjustment costs. Without modest costs associated with individuals moving into and out of the market sector, there are virtually no variations in hours per worker. With these adjustment or moving costs, hours per worker leads output as is indeed the case for aggregate U.S. time series.

Footnote

[1]We will be using the Uhlig (1987) law of large numbers for a continuum of identical and independent random variables.

[2]Cooley and Hansen (1988) introduce money via a cash-in-advance constraint. Greenwood, Hercowitz, and Huffman (1988) permit the utilization rate of capital to vary. Hansen (1988) introduces positive growth. Danthine and Donaldson (1989) introduce an efficiency-wage construct. In all these cases, the quantitative nature of fluctuations induced by technology shocks changed little. Backus, Kehoe, and Kydland (1989) introduce interaction between domestic and foreign technology shocks and study the implications for foreign trade and for the comovements of the key output components in the U.S. and abroad. It will be interesting to know whether this feature affects the amount of fluctuations accounted for by such shocks.

[3]Our intertemporal elasticity of substitution of leisure is higher than the estimates by some micro labor economists (e.g. Altonji 1986). Hall (1988) convincingly argues that these estimated values cannot be interpreted as short-run elasticities, and that a much larger value is likely for that elasticity.

[4]In fact there is an indivisibility. An individual cannot work one-half of a thirty-hour week and one-half of a fifty-hour week. Workweeks of different lengths are different factors of production.

Table 1

Values used in the experiments

Parameter values		Steady states	
Technology:		GNP	1.000
θ	0.643	c	0.750
σ	0.010	c_0	0.594
ν	3.000	c_1	0.802
δ	0.025	i	0.750
ϕ_1	0.333	k	10.000
ϕ_2	0.333	y	1.000
ϕ_3	0.333	h	0.440
		n	0.750
Preferences:			
β	0.990		
μ	0.326		
γ	2.000		
ψ	0.993		
Shock variances:			
σ_1^2	0.760^2		
σ_2^2	0.154^2		
σ_3^2	0.760^2		

Table 2

Cyclical behavior of the U.S. economy:
Deviations from trend of key variables, 1954:1-1988:2[a]

Variables x	Std. dev.	Cross-correlation of output with										
		x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
Output Components												
Gross National Product	1.74%	-0.03	0.15	0.38	0.63	0.85	1.00	0.85	0.63	0.38	0.15	-0.03
Consumption Expenditures	1.27	0.25	0.41	0.56	0.71	0.81	0.81	0.66	0.45	0.22	-0.01	-0.20
Services & Nondurable Goods	0.86	0.20	0.38	0.53	0.67	0.76	0.76	0.63	0.47	0.28	0.07	-0.10
Durable Goods	5.08	0.25	0.38	0.50	0.65	0.74	0.77	0.60	0.37	0.10	-0.14	-0.32
Fixed Investment Expenditures	5.51	0.09	0.26	0.44	0.65	0.83	0.90	0.81	0.60	0.35	0.08	-0.14
Capital Stocks												
Nonresidential Capital[b]	0.62	-0.58	-0.61	-0.58	-0.48	-0.31	-0.08	0.16	0.39	0.56	0.66	0.70
Equipment[b]	0.99	-0.57	-0.58	-0.53	-0.41	-0.22	0.02	0.26	0.47	0.62	0.70	0.71
Structures[b]	0.37	-0.45	-0.51	-0.55	-0.53	-0.44	-0.29	-0.10	0.09	0.25	0.38	0.45
Total Nonfarm Inventories	1.68	-0.37	-0.33	-0.23	-0.06	0.18	0.49	0.72	0.82	0.81	0.71	0.54
Hours and Productivity												
Hours (Household Survey)	1.50	-0.11	0.05	0.23	0.44	0.68	0.86	0.86	0.75	0.60	0.38	0.18
Hours Per Worker	0.56	0.08	0.21	0.35	0.48	0.64	0.69	0.58	0.43	0.29	0.11	-0.03
Civilian Employment	1.08	-0.19	-0.04	0.14	0.36	0.61	0.82	0.89	0.82	0.67	0.47	0.26
Hours (Establishment Survey)	1.69	-0.23	-0.07	0.14	0.39	0.67	0.88	0.92	0.81	0.64	0.42	0.21
GNP/Hours (Household Survey)	0.90	0.12	0.23	0.35	0.49	0.51	0.51	0.21	-0.03	-0.26	-0.33	-0.35
GNP/Hours (Establishment Survey)	0.84	0.41	0.46	0.49	0.53	0.43	0.31	-0.08	-0.32	-0.49	-0.51	-0.49

[a]Data Source: Citibase

[b]For the period 1954:1-1984:2

Table 3.1

Cyclical behavior of economy with no moving costs[a]

Variables x	Std. dev.	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
		Cross-correlation of output with										
Output	1.68%	-0.07	0.08	0.26	0.44	0.71	1.00	0.71	0.44	0.26	0.08	-0.07
	(0.18)	(0.13)	(0.12)	(0.10)	(0.08)	(0.05)	(0.00)	(0.05)	(0.08)	(0.10)	(0.12)	(0.13)
Consumption	0.69	-0.17	-0.03	0.15	0.35	0.65	0.97	0.77	0.55	0.38	0.22	0.08
	(0.08)	(0.11)	(0.11)	(0.10)	(0.08)	(0.05)	(0.01)	(0.05)	(0.09)	(0.11)	(0.14)	(0.15)
Fixed Investment	4.58	-0.05	0.06	0.23	0.39	0.60	0.90	0.83	0.50	0.18	0.05	-0.04
	(0.51)	(0.13)	(0.13)	(0.11)	(0.09)	(0.07)	(0.02)	(0.03)	(0.06)	(0.10)	(0.11)	(0.12)
Capital Stock	0.43	-0.46	-0.45	-0.45	-0.37	-0.22	-0.09	0.15	0.53	0.59	0.54	0.64
	(0.07)	(0.09)	(0.08)	(0.07)	(0.07)	(0.07)	(0.07)	(0.06)	(0.03)	(0.04)	(0.06)	(0.06)
Inventory Stock	1.21	-0.12	-0.03	0.16	0.30	0.47	0.78	0.66	0.21	0.20	0.24	-0.02
	(0.10)	(0.09)	(0.09)	(0.07)	(0.06)	(0.09)	(0.05)	(0.05)	(0.12)	(0.10)	(0.11)	(0.12)
Hours	1.27	-0.03	0.10	0.28	0.41	0.63	0.95	0.71	0.41	0.22	0.03	-0.12
	(0.12)	(0.13)	(0.13)	(0.11)	(0.09)	(0.07)	(0.01)	(0.05)	(0.07)	(0.08)	(0.10)	(0.11)
Hours Per Worker	0.00											
Employment	1.27	-0.03	0.10	0.28	0.41	0.63	0.95	0.71	0.41	0.22	0.03	-0.12
	(0.12)	(0.13)	(0.13)	(0.11)	(0.10)	(0.07)	(0.01)	(0.05)	(0.07)	(0.08)	(0.10)	(0.11)
Productivity (output/hours)	0.61	-0.13	0.02	0.14	0.34	0.63	0.77	0.46	0.35	0.26	0.17	0.08
	(0.05)	(0.10)	(0.08)	(0.07)	(0.07)	(0.05)	(0.04)	(0.09)	(0.11)	(0.13)	(0.13)	(0.12)

[a]These are the means of 50 simulations, each of which was 138 periods long. The numbers in parentheses are standard deviations.

Table 3.2

Cyclical behavior of economy with moving costs[a]

		Cross-correlation of output with										
Variables x	Std. dev.	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
Output	1.52%	-0.04	0.12	0.30	0.50	0.75	1.00	0.75	0.50	0.30	0.12	-0.04
	(0.17)	(0.13)	(0.12)	(0.10)	(0.08)	(0.05)	0.00	(0.05)	(0.08)	(0.10)	(0.12)	(0.13)
Consumption	0.63	-0.16	0.00	0.19	0.40	0.68	0.97	0.81	0.61	0.43	0.27	0.12
	(0.08)	(0.11)	(0.11)	(0.09)	(0.07)	(0.04)	(0.01)	(0.04)	(0.08)	(0.11)	(0.14)	(0.15)
Fixed Investment	4.20	-0.02	0.10	0.27	0.44	0.64	0.89	0.84	0.55	0.23	0.06	-0.03
	(0.49)	(0.13)	(0.13)	(0.11)	(0.09)	(0.06)	(0.03)	(0.03)	(0.06)	(0.10)	(0.11)	(0.12)
Capital Stock	0.40	-0.50	-0.49	-0.46	-0.37	-0.22	-0.06	0.18	0.51	0.62	0.60	0.66
	(0.07)	(0.09)	(0.07)	(0.07)	(0.08)	(0.07)	(0.07)	(0.06)	(0.03)	(0.03)	(0.05)	(0.06)
Inventory Stock	1.04	-0.11	0.01	0.18	0.35	0.55	0.80	0.69	0.30	0.22	0.25	0.05
	(0.09)	(0.09)	(0.09)	(0.07)	(0.06)	(0.07)	(0.04)	(0.05)	(0.11)	(0.11)	(0.12)	(0.13)
Hours	0.92	-0.05	0.08	0.26	0.44	0.66	0.93	0.87	0.65	0.44	0.22	0.02
	(0.11)	(0.14)	(0.14)	(0.13)	(0.10)	(0.07)	(0.02)	(0.03)	(0.05)	(0.08)	(0.10)	(0.12)
Hours Per Worker	0.24	0.15	0.25	0.36	0.42	0.52	0.66	0.23	-0.16	-0.31	-0.39	-0.41
	(0.02)	(0.13)	(0.11)	(0.09)	(0.07)	(0.06)	(0.04)	(0.03)	(0.03)	(0.05)	(0.06)	(0.07)
Employment	0.81	-0.10	0.02	0.19	0.37	0.59	0.86	0.92	0.80	0.59	0.36	0.15
	(0.11)	(0.13)	(0.14)	(0.14)	(0.12)	(0.08)	(0.03)	(0.02)	(0.04)	(0.06)	(0.09)	(0.12)
Productivity (output/hours)	0.74	-0.01	0.15	0.30	0.48	0.71	0.89	0.46	0.21	0.08	-0.02	-0.10
	(0.06)	(0.11)	(0.09)	(0.08)	(0.07)	(0.05)	(0.01)	(0.08)	(0.11)	(0.12)	(0.12)	(0.12)

[a]These are the means of 50 simulations, each of which was 138 periods long. The numbers in parentheses are standard deviations.

Table 3.3

Cyclical behavior of economy with very large moving costs[a]

Variables x	Std. dev.	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
		Cross-correlation of output with										
Output	1.33% (0.14)	-0.06 (0.13)	0.09 (0.12)	0.25 (0.10)	0.44 (0.08)	0.71 (0.05)	1.00 0.00	0.71 (0.05)	0.44 (0.08)	0.25 (0.10)	0.09 (0.12)	-0.06 (0.13)
Consumption	0.52 (0.06)	-0.18 (0.11)	-0.04 (0.11)	0.14 (0.09)	0.35 (0.07)	0.64 (0.05)	0.97 (0.01)	0.78 (0.05)	0.56 (0.08)	0.39 (0.11)	0.24 (0.14)	0.11 (0.15)
Fixed Investment	3.79 (0.41)	-0.05 (0.12)	0.07 (0.13)	0.22 (0.11)	0.38 (0.09)	0.58 (0.07)	0.86 (0.03)	0.84 (0.03)	0.52 (0.06)	0.18 (0.10)	0.02 (0.11)	-0.05 (0.11)
Capital Stock	0.34 (0.06)	-0.46 (0.09)	-0.46 (0.08)	-0.44 (0.07)	-0.37 (0.07)	-0.23 (0.07)	-0.08 (0.07)	0.16 (0.06)	0.51 (0.03)	0.62 (0.03)	0.57 (0.06)	0.62 (0.06)
Inventory Stock	0.97 (0.07)	-0.10 (0.09)	0.00 (0.09)	0.16 (0.07)	0.31 (0.07)	0.51 (0.08)	0.79 (0.04)	0.66 (0.05)	0.23 (0.11)	0.13 (0.11)	0.20 (0.12)	0.01 (0.12)
Hours	0.43 (0.04)	-0.02 (0.13)	0.10 (0.13)	0.26 (0.12)	0.41 (0.10)	0.62 (0.07)	0.93 (0.02)	0.75 (0.04)	0.45 (0.07)	0.23 (0.08)	0.05 (0.10)	-0.10 (0.11)
Hours Per Worker	0.43 (0.04)	-0.02 (0.13)	0.10 (0.14)	0.27 (0.12)	0.42 (0.10)	0.63 (0.07)	0.92 (0.02)	0.74 (0.04)	0.44 (0.07)	0.22 (0.08)	0.04 (0.10)	-0.11 (0.11)
Employment	0.01											
Productivity	0.94 (0.10)	-0.07 (0.12)	0.08 (0.11)	0.24 (0.09)	0.44 (0.08)	0.71 (0.05)	0.98 (0.00)	0.66 (0.06)	0.42 (0.09)	0.25 (0.11)	0.10 (0.12)	-0.03 (0.13)

[a]These are the means of 50 simulations, each of which was 138 periods long. The numbers in parentheses are standard deviations.

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