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# Fluctuating Risk in an Aggregated *Ss*-Model

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ABSTRACT

Using U.S. data it is shown that as the stock market goes into a period of high volatility, nondurables consumption is unaffected but durables consumption falls substantially. It is argued that a plausible explanation for this is that consumers face irreversibilities when adjusting their durables stock. They will thus apply *Ss*-type rules with bandwidths with widths that vary over time as in Hassler (1996). To quantify the aggregate implications of such behavior an aggregated irreversible investment model for consumer durables is estimated on U.S. It is found that a shift to higher risk leads to a simultaneous widening of individual *Ss*-bands which causes demand to fall substantially. This effect diminishes over time but is substantial also after a year.

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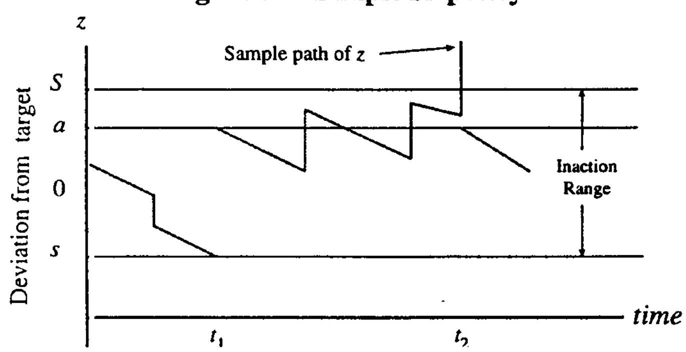
## 1. Introduction

In a series of papers following Schwert (1989) it has been noted that financial volatility varies counter-cyclically over the business cycle. In this paper I will follow this line of research by focusing on the relation between financial volatility and consumption. To get an indicator of financial volatility I will estimate a Hamilton (1989, 1989) regime switching model for the S&P500 index. The model will allow me to distinguish two risk-states, one with high financial volatility and one with low, as well as the conditional probability that the market is in the high risk state. As a point of departure I will show that the growth of non-durables consumption is uncorrelated with the risk state. When the stock market enters a spell of high volatility, non-durables consumption hardly change at all. Purchases of durables, on the other hand, fall significantly also when taking into account that purchases of durables represent stock-adjustments rather than changes in consumption. In a regression of durables purchases, the high risk state enters with a *t*-statistic of minus 2.6.

I will argue that a reasonable interpretation of these findings is that consumers face transaction costs associated with adjusting their stocks of durables. They will thus use *Ss*-rules with widths that depend on the degree of uncertainty. A graphical representation of an *Ss*-rule is given in *Figure* 1. The vertical axis represent the percentage deviation of the current stock of durables from some target level, that would be chosen if no transaction costs existed. The target stock moves over time, due to for example, wealth and price shocks. This together with depreciation imply that the deviation from the target changes over time also when no adjustments is undertaken. Adjustments only occur when the deviation "hits" one of the triggers $S$ and $s$. A stock adjustment is then done so that the deviation is brought back to some interior point in the band, here denoted by $a$. The drawn sample path shows adjustments at time $t_1$ and $t_2$.

That irreversibility creates a link between risk and investments is well known. McDonald and Siegel (1986) emphasize that that the decision when to execute an investment plan is strongly affected by the level of uncertainty. Pindyck (1991) also argues that the level of risk may be more important than taxes and interest rates for aggregate investments[1]. Also Bentolila and Bertola (1990) consider the effects of different levels of uncertainty in a model of hiring and firing costs. Eberly (1994) shows that *Ss* rules well describes car purchases of US households. She finds cross-section evidence that those households who face higher income risk tolerate larger deviations of their durables stocks from a target level before adjusting. In this paper I will, however, focus on time series variation in risk and aggregate demand. To do this I will construct an aggregated *Ss*-model of durables demand where the level of risk is allowed to fluctuate. I will build on Hassler (1996) where I construct an *Ss* model for individual behavior where risk stochastically switches between two levels.

**Figure 1 A Simple *Ss* policy**

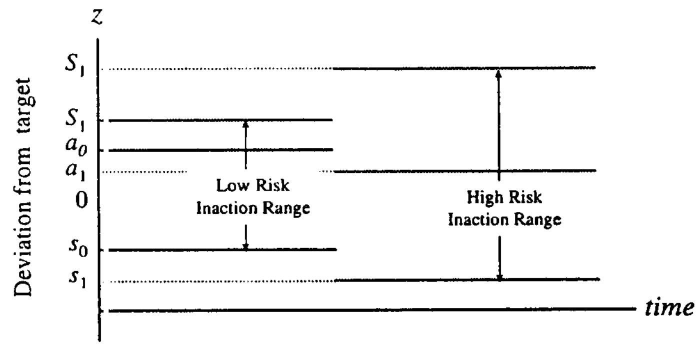



Introducing a stochastic risk level by allowing the flow of information about future earnings to fluctuate stochastically between two levels gives an optimal policy that can be described by two *Ss* bands as shown in *Figure* 2. Whenever risk is low the deviation is kept within a narrow band given by $S_0s_0$. When risk is high a wider band, $S_1s_1$ is applied. In Hassler (1996) is shown that the widening of the band is larger the shorter the high risk period is expected to be. Even though the long run effect on demand by an increase in risk is likely to be small, the results in Hassler (1996) suggest that temporary shifts in uncertainty may have give rise to large swings in demand.

[1] For firms it is, however, not necessarily the case that increased uncertainty decreases investment. The value of waiting increases with uncertainty, which tends to delay investment. But if the first derivative of the profit function with respect to capital is convex, also the value of installed capital increases in uncertainty and this works in the opposite direction. Similar results would follow if consumers are non-prudent, i.e., has concave marginal utility. For a discussion of this see Caballero (1991).

The idea that durables demand may be sensitive to the level of uncertainty due to the presence of irreversibilities may arguably have dramatic implications. Romer (1990) suggests that the stock market collapse of 1929 caused a radical increase in consumers' uncertainty that lead them to delay purchases of durables. The consecutive fall in aggregate demand was large enough to be a critical factor behind the Great Depression.

**Figure 2 An *Ss* policy with 2 Levels of Risk**



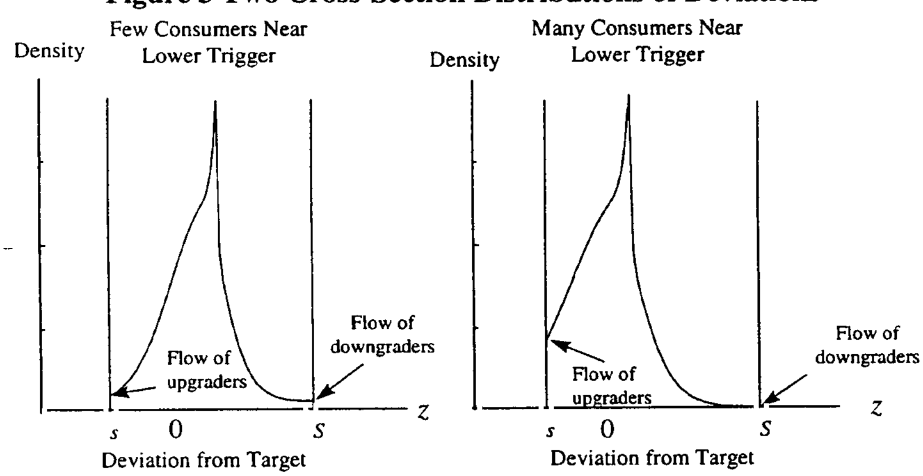

One obvious problem in the empirical implementation of *Ss*-models is that the target stock is unobservable. In this paper this problem is handled by noting that the irreversibility model only applies to durables, not non-durables. Under suitable assumptions, this makes it possible to use consumption of non-durables to identify the wealth shocks that cause the target to switch. Also changes in precautionary savings, which would affect the target stock, can be controlled for by using non-durables consumption.

The model described in the previous section is set up to describe the behavior of an individual consumer. To estimate it directly one would need panel data with relatively high frequency. High frequency is necessary since shifts in uncertainty is predicted to have large but temporary effects on demand. Using yearly data thus seems unlikely to be a fruitful way to test the implications of variations in risk on demand. High frequency panel data on durables purchases are, however, not available. I will thus have to work out the models implications for aggregate purchases, for which we can find monthly data.

Going from the model of individual behavior in Hassler (1996) to an aggregate model is far from straight forward and that step will constitute the major part of this paper. It is immediately clear that the effects shifts in uncertainty (as well as aggregate wealth shocks and depreciation) have on purchases will depend on the cross section distribution of the agents' deviations from their respective target stocks. To illustrate that, consider *Figure* 3 which shows two typical cross-sectional distributions of deviations from the target stock. The left panel illustrates a situation where relatively few consumer are close to the lower trigger ($s$). Depreciation cause the ones closest to that trigger to hit it, creating a small flow of purchases relative to the case in the right panel where the density is much higher close to the lower trigger.

A positive aggregate wealth shock would shift the whole distribution leftwards since everybody's target stocks have increased and thus made the deviation between the target and the current stock more negative. This would cause a large (small) amount of consumers to hit the lower trigger in the right (left) panel and thus create a large (small) amount of purchases. Similarly, a reduction in the width of the *Ss* band will have larger effects on durables purchases in the right panel than in the left. In order to confront an *Ss* model with aggregate data it is thus necessary to keep track of how the distribution of deviations evolve over time. Caballero and Engel (1993) has developed methods to do this in the case of constant risk levels. In this paper I will adapt their work to the present issue of fluctuating risk.

**Figure 3 Two Cross-Section Distributions of Deviations**




In section 2 I identify periods of high risk. As already noted. I assume that risk is stochastic and follows a two-state Markov process. Following the method devised by Hamilton (1988, 1989) I estimate the probability that the US economy is in a high risk state for each month between 1959 and 1992. In Section 3 I construct an aggregate irreversibility model of *Ss*-type. I show that risk should enter the model by increasing the inaction range when risk is high. A shift to high risk causes agents to delay purchases, which leads to a sharp fall in purchases. An shift from the low to the high risk state has an average effect of -8% on durables. The effect is persistent, most of the negative effect is still present after a year if the high risk states should persist that long. I make some concluding remarks.

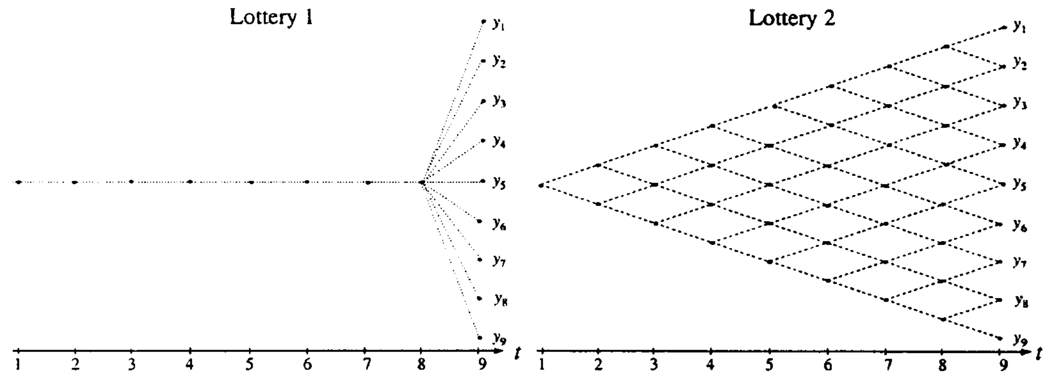
## 2. Estimation of Changes in Risk

Before estimating the regime switching model on the stock market data I need to motivate the use of financial volatility despite the fact that most household hold only small amounts of stock market shares. From the theoretical literature on irreversibility models we get some guidance as to what is the relevant risk measure to use. The important risk measure is the rate of flow of information. The condition which determines the triggers in the *Ss* policy is the value of waiting should equal the temptation to adjust (Hassler, 1996a). The latter is the increase in the flow of utility that can be achieved by adjusting and is not directly affected by risk. The former, however, is the expected value of the information flow – i.e., the value of new relevant information that the consumer expects to receive in the immediate future. Risk that is resolved far away into the future is less important than risk that is resolved soon.

To fix ideas, think of two lotteries, depicted in *Figure* 4. Both lotteries have equal distributions over the final outcome. The difference is, however, that in the second lottery risk is resolved gradually. The second lottery is thus characterized by a continuous flow of information about the final outcome – the prediction about the final result gradually becomes increasingly precise. Contrast this with lottery 1, in which the information flow is zero during the first 8 periods.

There is an empirically important difference between the two risk concepts. Consider a consumer who participates in lottery 2. Each period the consumer receives more information about the final outcome. For each draw he will then change his level of consumption. A participant in the first will, on the other hand, not receive any information during the first 8 periods. His consumption should thus not fluctuate over time (unless, of course, there are other sources of risk). Similarly, if the lottery represents stochastic profits of a firm, the share price will fluctuate over time in lottery two but be constant during the first 8 periods in lottery 1. This means that there should be a direct link between the current flow of information and the current volatility of observable variables. Long run risk, like uncertainty over the final outcome in the lottery example, may, on the other hand, have no relation to current volatility.

**Figure 4 Two Lotteries**




The two preceding paragraphs suggest that;

- the current information flow can be measured as the current volatility of some observable variable, and
- that the irreversibility mechanism should be particularly sensitive to the current information flow. When the information flow is high, the tendency to delay purchases of durables should be stronger than otherwise.

I thus identify periods of high information flow as periods of high current volatility. The relevant stochastic variable to use would be wealth, including the expected present value of future income. Such a measure is, however, not directly observable. Alternative proxies could be current income or production if they are assumed to be random walks or financial wealth, e.g., a stock market index. The stock market is supposedly quite efficient in processing the continuous flow of information. Particularly high frequency shocks may then be more easily visible at the stock market than if I would use other variables. I will thus examine the evolution of a stock market index to try to find periods of high risk.

The typical household does not own much public stock and a large portion of its wealth is expected future labor income.[2] Despite this fact shifts in the volatility of the stock market may very well be good indicators of shifts in the volatility of household wealth. Fluctuations in risk may be due to variations in the volatility of a stochastic trend common to both firm values and household wealth, for example technology shocks. In this case the variances of household wealth and the stock market, as well as their levels, have a positive correlation. There is also evidence of a positive relationship between financial and macroeconomic volatility. Schwert (1989) reports that financial volatility significantly helps to predict future volatility in industrial production for the period 1891 to 1987

A positive correlation between the level of the stock market and household wealth is, however, not necessary for their volatilities to be positively correlated. Another potential source of volatility is variations in the share of total income going to labor. Such share variations would tend to give negative correlations between the value of firms and human capital. Nevertheless, increased share volatility will increase the volatility of the stock market as well as of human capital. We may also think of cases in which the levels are uncorrelated while the variances are positively correlated.

It is certainly possible that no association exists between stock market and aggregate household wealth volatility. This, however, seems to be an unlikely knife edge case. A maintained hypothesis is thus that periods of high stock market volatility tend to coincide with periods of high volatility of aggregate household wealth.[3]

## 2.1 Stochastic State Model

I assume that the economy switches stochastically between two risk states, $s_t = 0$ or 1. When $s_t = 1$ risk, i.e., i.e., the current information flow, is higher. The current state of the economy,

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[2] For an early discussion of this, see Roll (1977).

[3] Note that it is this dating of high risk periods which is the aim of the state estimation discussed in this section. An estimation of the *levels* of volatility of household wealth requires more information than what I need for this purpose.

$s_t$, is not observable. By specifying a stochastic process for wealth we may, however, compute the probability that the economy is in a particular state, conditional on a series of realized wealth innovations. Aggregate wealth, $w_t$, is assumed to follow a generalized random walk. I allow for state shifts to be associated with shifts in volatility as well as with shifts in level and drift;

$$\Delta \ln w_t = \mu_0 + (\mu_1 - \mu_0)s_t + \mu_2 \Delta s_t + \left(\lambda_0 + (\lambda_1 - \lambda_0)s_t\right)\vartheta_t \tag{1}$$

where $\Delta$ is the first difference operator and $\vartheta$ is a sequence of i.i.d. normal innovations. The drift of the log of wealth is thus $\mu_0$ in state 0 and $\mu_1$ in state 1.

A shift in the discount rate due to a state shift may shift the present discounted value of future income. This motivates the third right hand side term of (1). If the state shifts from 0 to 1 the level shifts by $\mu_2$ and if the state shifts from 1 to 0 by -$\mu_2$. The standard deviation of the wealth innovations in absence of state shifts is $\lambda_0$ in state 0 and $\lambda_1$ in state 1.

As in Hassler (1996) the state is assumed to follow a first-order Markov process with a transition matrix;

$$\begin{bmatrix} 1-\gamma_0 & \gamma_0 \\ \gamma_1 & 1-\gamma_1 \end{bmatrix} \tag{2}$$

If the current state is $S$ the probability of a state shift is thus $\gamma_S$. The value of the current state is not observed by the econometrician but by observing $\{w_\tau\}_1^t$ we may draw inference about the likelihood $P(s_t = S_t \mid \{W\}_1^t)$. Hamilton (1988, 1989) shows how to do this in a similar setup. Hamilton's method is recursive; given $P(s_{t-1} = S_{t-1} \mid \{W\}_1^{t-1})$ and $W_t$, we may compute $P(s_t = S_t \mid \{W\}_1^t)$. We may also "back-cast" and estimate $P(s_{t-k} = S_{t-k} \mid \{W\}_1^t)$ for $k \geq 1$. The "back-casting" may improve the accuracy in the state estimation by using more information. The parameters of the model are estimated by maximizing the likelihood function implied by (1) and (2).

## 2.2 State Estimation Results

Using 396 monthly observations of the S&P 500 stock market index, covering the period from January 1959 to January 1992, I have estimated the parameter of the model. The estimates together with estimated standard errors are given in *Table* 1.[4] There we find clear evidence of the existence of periods of higher financial volatility. The point estimate is that the standard deviation of the S&P 500 index increases with 1.49 percentage points from 1.81 to 3.30% per month in the high risk state. The relative increase in volatility is over 80% and it is strongly significant; the $z$-value for a null hypothesis of equal risk in both states is 4.14.

We also see that high-risk periods seem to be associated with lower than average stock returns; the mean growth rate is estimated to be negative in state 1. A shift to higher (lower) volatility is on average also associated with negative (positive) shocks to the value of the stock market index. This result is also reported in the finance literature, e.g., by French et al. (1987). That study finds evidence for a negative relation between unexpected increases in stock market volatility and realized stock returns. The mechanism may be that an unexpected increase in volatility increases the expected future volatility since volatility is persistent. This increases the discount rate for future cash flows which reduces stock prices. The effect of a shift to the high risk state would thus be an immediate fall in stock prices, i.e., $\mu_2 < 0$.

French et al. (1987) find weak evidence for a positive association between predicted volatility and ex ante returns. Haugen et al. (1991) find that monthly returns are lower (higher) during the second four weak period subsequent to a fall (increase) in volatility. In the high risk state expected volatility is high, which thus should give a positive $(\mu_1\text{-}\mu_0)$. On the other hand, as long as the state does not shift back, realized volatility is higher than expected, tending to mitigate this positive effect. The negative sign of $(\mu_1\text{-}\mu_0)$ is nevertheless somewhat surprising.

The probability of a state shift is fairly low in both states. This implies that the level of risk is persistent. We also see that the probability of a state switch is higher when the risk is high. This implies that periods of high risk on average are shorter than periods of low risk. The expected time to next state shift is approximately $1/\gamma_0 \approx 28.3$ months in state 0 while it is

[4] The covariance matrix of the parameters is estimated as the inverse of the estimated likelihood Hessian.

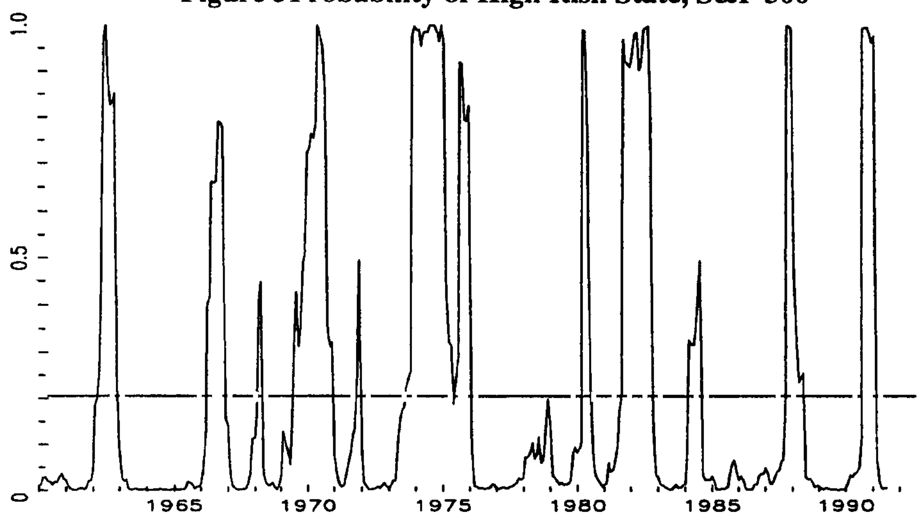
$1/\gamma_1 \approx 7.4$ months in state 1. The shorter high risk periods imply that the economy is in the low risk state most of the time. The unconditional probability of state 1 is 0.207.

**Table 1 State Model Parameters**

| | S&P 500 | | |
|---|---|---|---|
| | Estimated value %/month | Asymptotic St. Dev. | *z*-value ≠ from 0 |
| $\gamma_0$ | 3.53 | 4.67 | 0.76 |
| $\gamma_1$ | 13.53 | 1.39 | 9.73 |
| $\mu_0$ | 0.91 | 0.18 | 4.94 |
| $(\mu_1-\mu_0)$ | -1.91 | 0.71 | -2.71 |
| $\mu_2$ | -5.24 | 1.21 | -4.35 |
| $\lambda_0$ | 1.81 | 0.10 | 17.69 |
| $(\lambda_1-\lambda_0)$ | 1.49 | 0.36 | 4.14 |

Using the estimated parameters I calculate the series of conditional likelihoods. The conditional likelihoods using S&P 500 are plotted in *Figure* 5. For 105 of the 396 S&P500 observations the conditional likelihood of state 1 is higher than the unconditional likelihood, which is depicted as the horizontal line in the graph. For non-durables the analogous event occurs 133 times.

**Figure 5 Probability of High Risk State, S&P 500**




In Figure 5 we see that the longest periods of high likelihoods of the high risk state occur around the years 1970, 1974 and 1982. We also have periods of high likelihoods of the high risk state in 1962, 1966, 1980, 1987 and 1990. The two latter appear to be due to the stock market crashes. A large fall in the stock market will increase the likelihood of state 1, both since $\mu_2$ is negative and since a higher variance increases the likelihood of large realizations. It also seems reasonable that the degree of uncertainty was high immediately after these stock market crashes. After these two periods we see that the likelihood of state 0 recovers quickly.

Figure 5 indicates a relationship between high risk periods and recessions. Such a relationship is also reported in the finance literature. Schwert (1989) shows that volatility generally has been higher during months that have NBER classified as recession. For the period 1859 to 1987 the standard deviation of monthly stock returns was 61 % higher during recession and between 1953 and 1987 68% higher.

### 2.3 Regressions of Consumption against State Probabilities

In the introduction I claimed that high risk states appear to have little or no effect on non-durables consumption while durables purchases are strongly affected. To give some support for this let me run the following regression

$$\begin{aligned} \frac{\Delta c_t}{c_{t-1}} &= \alpha_0 + \sum_{s=1}^{n} \alpha_s \frac{\Delta c_{t-s}}{c_{t-s-1}} + \beta_1 \Delta P_t + \varepsilon_t, \\ \frac{\Delta c_t}{c_{t-1}} &= \alpha_0 + \sum_{s=1}^{n} \alpha_s \frac{\Delta c_{t-s}}{c_{t-s-1}} + \beta_1 \Delta P_t + \beta_2 \Delta P_{t-1} + \varepsilon_t, \end{aligned} \tag{3}$$

where $c_t$ denotes nominal non-durables consumption taken from Citibase and $P$ is the high risk state probability back-casted 6 periods. The regression is estimated on monthly data between 1969:1 and 1991:6 and the results are presented in *Table* 2.

**Table 2 Regression Results**

| | $\beta_1$ x 100 | $\beta_2$ x 100 | *F test* $\beta_1$=$\beta_2$=0 | *DW* |
|---|---|---|---|---|
| **Non-Durables** | 0.00 (0.00) | | | 2.00 |
| | -0.02 (-0.08) | 0.19 (0.74) | 0.760 | 2.00 |
| **Durables** | | | | |
| | -1.13 (-2.56) | | | 2.53 |
| | -1.04 (-2.36) | -0.91 (-2.08) | 0.006 | 2.00 |

To compare the previous results with the effects of the risk-state on durables purchases we have to recognize that durables purchases represent stock adjustments rather than consumption flows. As a point of departure I will assume that the individual has Cobb-Douglas preferences over non-durables and the services of durables and that these services are proportional to the stock of the durable. Given this, the consumer would in absence of transaction costs spend a constant share of his income on non-durables and rental costs of the durables stocks. This then implies that the durables stock $k_t$ satisfies

$$\begin{aligned} p^c c &= \pi p^d (r+\delta)k \\ \Rightarrow k &= \frac{p^c c}{\pi p^d (r+\delta)} \end{aligned} \tag{4}$$

where $p^c$ and $p^d$ are the prices of durables and non-durables, $\pi$ is a factor of proportionality, and $r$ and $\delta$ are the interest and depreciation rates. Now by noting that purchases of durables in real terms equals $k_t - (1-\delta)k_{t-1}$, substituting from (4) and allowing time varying prices we get[5]

$$\begin{aligned} d_t &\equiv k_t - (1-\delta)k_{t-1} \\ &= \phi\left(p_t^c / p_t^d\right)c_t - (1-\delta)\phi\left(p_t^c / p_t^d\right)c_{t-1} \end{aligned} \tag{5}$$

where $\phi \equiv (\pi(r+\delta))^{-1}$. Taking first differences of (5) and dividing by $c_{t-1}$ to reduce heteroschasticity and allowing some more lages we arrive at the following regression model

[5] See also Mankiw (1982) who derive a similar equation for durables.

$$\frac{\Delta d_t}{c_{t-1}} = \alpha_0 + \sum_{s=1}^{n} \alpha\left(p_t^c / p_t^d\right)c_{t-1} + \beta_1 \Delta P_t + \upsilon_t,$$
$$\frac{\Delta d_t}{c_{t-1}} = \alpha_0 + \sum_{s=1}^{n} \alpha_s \frac{\Delta c_{t-s}}{c_{t-1}} + \beta_1 \Delta P_t + \beta_2 \Delta P_{t-1} + \upsilon_t. \tag{6}$$

The estimation results are presented in *Table* 2. We see that the coefficients on the probability of the risk-state are significantly negative. The sum of the coefficients in the last line is -1.95% corresponding to -5.79% of the average ratio of durables purchases to non-durables purchases. This implies that as a high risk state is entered, durables purchases fall by 5.79% more than what would be predicted from the stock adjustments calculated from price movements and non-durables purchases. As noted in the introduction, this finding is the main motivation for estimating an *Ss*-model with switching bands, which will be the purpose of the next section.

## 3. An Aggregated *Ss*-model

The model derived in this section draws on the work by Caballero (1993), Caballero and Engel (1993, 1994) and Hassler (1996). The consumers in the model derive utility from individual stocks of durables. Each agent has a target level for her stock that evolves stochastically over time.[6] The stock continuously depreciates at rate $\delta$. Due to transaction costs it is not optimal to continuously compensate for the depreciation and possible changes in the target level. Instead the individual consumer lets her stock of durables deviate within an inaction range (*Ss*-band) without adjusting. When a band limit (trigger) is hit this triggers adjustment to the target level. So far the assumptions are standard for *Ss* models. The following two assumptions are, on the other hand, non-standard.

First; in standard *Ss*-models the agent is inactive with probability one until a band limit is hit when she adjusts with probability one. Following Caballero and Engel (1993) I instead assume that the probability of adjustment is increasing smoothly in the absolute value of the

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[6] In Grossman and Laroque (1990) a model with financial investments where the agents derive utility from durable stocks that are costly to adjust it is found that the target stock of durables should be proportional to wealth.

relative deviation from the target level. This feature is captured by a hazard function with the absolute relative deviation as one of its arguments. Caballero and Engel (1993) motivate the hazard function by an assumption that the transaction cost has an idiosyncratic and stochastic component. They derive hazard functions from different distributional assumptions about the transaction cost.

Second; I assume that increases in risk affect the individual's inaction band. In Hassler (1996) I show that it is optimal to increase the width of the inaction band in a model where risk stochastically fluctuates between two levels. In the current setting, increases in the inaction band are formalized by including the current level of risk, or rather the probability that the current state is high risk, in the hazard function. The aim of this section is then to estimate the effect of risk on the hazard function and quantify the implications on aggregate demand.

The model explicitly aggregates the individuals.[7] The effect of an aggregate shock on adjustments and thus aggregate purchases depend on the current distribution of individuals over their *Ss* bands. A positive aggregate shock may, for example, have unusually large effects on demand if many agents are near their lower trigger. Aggregate shocks change the distribution of agents over their *Ss* bands so the current distribution, and thus the sensitivity of demand to aggregate shocks, is a function of the history of aggregate shock. By keeping track of how the distribution of deviations evolves we may describe the relation between aggregate shocks and aggregate demand at any point in time.

### 3.1 The model

The model has (infinitely) many consumers, each with a stock of durables $K_{t,i}$ with a target level $K^*_{t,i}$. If the consumer decides to adjust its durable stock it chooses $K^*_{t,i}$ by definition. Define the relative deviation of each individual's stock of durables as

$$z_{t,i} = \ln \frac{K_{t,i}}{K^*_{t,i}}. \tag{7}$$

[7] Bertola and Caballero (1994) construct and test a model of aggregate firm investment when investment is irreversible.

We define $f(z,t)$ as the cross section density function at the end of each period $t$, i.e., the share of consumers with relative deviation $z$ at the end of period $t$. In each period idiosyncratic and aggregate shocks occur, the durables stocks depreciate and the individual households may adjust their durable stocks. This is described by the following the following four steps. Each step changes the distribution of relative deviations and the resulting densities are given superscripts to the corresponding step.

*Step 1. Idiosyncratic Shock*

Each individual is hit by an individual multiplicative wealth shock $\exp(\upsilon_{t,i})$ with distributed as $N(0,\sigma_\upsilon)$.[8] The wealth shock is assumed to shift the target stock proportionally so $z_{t,i}$ falls by $\upsilon_{t,i}$. This causes the distribution of relative deviations to change;

$$f^1(z,t) = \int_{-\infty}^{\infty} f(x,t-1)\phi(z-x,\sigma_\upsilon)dx \tag{8}$$

where $\phi$ is the normal density.

*Step 2. Aggregate Shock*

The aggregate shock shifts the target stock of durables and $z$ with $\varepsilon_t$ and $-\varepsilon_t$ for all consumers.

$$f^2(z,t) = f^1(z+\varepsilon,t) \tag{9}$$

*Step 3. Depreciation*

The durables stock depreciates with at a rate $\delta$, which cause an equivalent fall in $z$.

$$f^3(z,t) = f^2(z+\delta,t) \tag{10}$$

*Step 4. Adjustments*

With a probability that is given by the hazard function $h(z,p_t)$, individuals with deviation $z$ adjust there stocks, so

$$f(z,t) = \begin{cases} \left(1-h(z,p_t)\right)f^3(z,t) & \forall z \neq 0 \\ f^3(z,t) + \int_{-\infty}^{\infty} h(x,p_t)f^3(x,t)dx & z = 0 \end{cases} \tag{11}$$

[8] In reality, also idiosyncratic risk is likely to fluctuate over time. Allowing this in the model, however, adds a parameter that is hard to estimate or find a proxy for.

The hypothesis to test is that $h_p(z,p_t)<0$, i.e., the adjustment probability for a given deviation from the target stock decreases in the probability that the economy is in the high risk state. We should note that (11) implies that $h(z,p_t)=1 \Rightarrow f(z,t)=0$. There is thus upper and lower limits for $z$ ($S$-$s$) that never are crossed, given that the hazard function eventually reach unity.[9]

By approximating $z_{t,i} \approx (K_{t,i}-K^*_{t,i})/K_{t,i}$ and using (11) we may calculate the amount of net purchases the model predicts

$$\hat{Y}_t = \int_{-\infty}^{\infty} h(z,p_t)(-z)f^3(z,t)\overline{K}_{t,z}dz \tag{12}$$

where $\overline{K}_{t,z}$ is the average durable stock of agents with relative deviation $z$ at time $t$. Assume further that $\overline{K}_{t,z}$ is (approximately) independent of $z$[10]. We may then write (approximately)

$$\begin{aligned}\hat{Y}_t &= \int_{-\infty}^{\infty} f^3(z,t)\overline{K}_{t,z}dz\int_{-\infty}^{\infty} h(z,p_t)(-z)f^3(z,t)dz \\ &= K_t\int_{-\infty}^{\infty} h(z,p_t)f^3(z,t)(-z)dz\end{aligned} \tag{13}$$

where $K_t$ is the aggregate durables stock in the economy.

### 3.2 Estimation strategy

The data used in the estimation all come from Citibase. I use monthly time series of purchases of durables, cars, non-durables and prices covering the period 1959:01 to 1992:01. First I need to compute the series of aggregate shocks. Here I follow the example by Caballero (1993). I assume that the log of the target stock of durables is a linear function of the log of non-durables consumption ($c$) and the relative price between durables and non-durables ($\pi$).

$$k^* = [1 \quad c \quad \pi]\phi \tag{14}$$

where $\phi$ is a parameter vector to estimate. Now use the definition of the relative deviation;

[9] In continuous time, the density may be strictly larger than 0 for all finite deviations given that the hazard function is finite.

[10] This amounts to assuming that knowing the durable stock of an individual conveys no (non-negligible) information about her position in the $Ss$ band. This requires a substantial amount of heterogeneity among the individuals target stocks.

$$k_t \equiv k_t^* + z_t = [1 \quad c \quad \pi]\phi + z_t. \tag{15}$$

We know from the model that $z$ is a stationary variable. By assuming that $k$, $c$ and $\pi$ are integrated of order 1 (15) defines a cointegrating relationship.[11] By using the dynamic OLS estimation method described by Stock and Watson (1993)[12] I estimate $\phi$. The aggregate shocks are then estimated as

$$\hat{\varepsilon}_t = \hat{k}_t^* - \hat{k}_{t-1}^* = (1-L)\left[1 \quad c_t \quad \pi_t\right]\hat{\phi} \tag{16}$$

where $L$ is the lag operator. The series $k_t$ is constructed by using integrating purchases using the depreciation rate $\delta$, estimated as described below.

To conclude the model I need a specification for the hazard function. I choose to postulate an increasing hazard function, similar to Caballero and Engel (1994)[13], but allowing for risk level dependency.

$$h(z, p_t) = \left((\alpha_0 - \alpha_1 p_t)z\right)^2. \tag{17}$$

We can here interpret $\alpha_0$ as a bandwidth parameter, high values of $\alpha_0$ implies frequent and small adjustments. $\alpha_1$ is a shift parameter; a positive $\alpha_1$ implies that the adjustments are delayed if the probability of high risk increases. The quadratic assumption is ad-hoc and may be seen as an approximation. It is also an implicit assumption about the distribution of transaction costs. If each individual each period is assigned a transaction cost, drawn from a constant distribution, we can in principle compute the probability that an individual with deviation $z$ will get a transaction cost sufficiently small to induce him to adjust for a given value of $p$. In practice, however, solving such a model is beyond the scope of this paper.[14]

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[11] To test this I use the augmented Dickey-Fuller test (ADF), including lags up to the last that is significant at 5%. I can not reject non-stationarity on 10% significance regardless of whether intercept and/or time trends are included.

[12] The method is to include first differences of the RHS variables at some number of leads and lags as regressors. I chose to use 4 leads and 20 lags.

[13] They use a "semi-structural" hazard of the form $h(z)=1-\exp(-\alpha_0-\alpha_1 z^2)$ where $\alpha_0$ is estimated to zero.

[14] An alternative would be to assume heterogeneity among consumers with respect to transaction costs. This, however, would make the model intractable. With this assumption we would need to keep track of, not only of the density of individuals over their *Ss* bands, but also the distribution of types in each moment. For a given distribution of consumer over the *Ss* band, the flow of purchases and its sensitivity to shocks will be different if the high transaction cost types are concentrated at the band ends than if not. See Caballero and Engel (1993) for a more thorough discussion about this.

The distribution of $z$ is discretized in 251 equal steps, $z_1 \cdots z_{251}$. The hazard function always reach unity at or before $z=\pm(\alpha_0-\alpha_1)^{-1}$ which thus gives a maximum support for the density $f(z,t)$. This density, however, in practice turns out to be non-negligible on a substantially smaller interval. I have thus reduce the interval $[-(\alpha_0-\alpha_1)^{-1}, (\alpha_0-\alpha_1)^{-1}]$ by multiplying by a constant $k$. This does not change the estimates much but may increase the precision by making the steps $z_n$-$z_{n-1}$ smaller. The constant $k$ is chosen by starting from unity and in steps of 0.1 reducing it as long as $\sum_{k=1}^{12} f(z_k,t) < 0.001$. The density in the upper region of the interval is always much smaller due to depreciation. When depreciation and shocks are non-integer multiples of $z_n$-$z_{n-1}$ linear interpolations are used. Lastly I have constructed the series of durables stocks by integrating purchases using the estimated depreciation rate $\delta$. The starting value of the durables stock has been set to $\exp(k_0^*)$ using relation (14).

Starting from an initial distribution $f(z,0)$, $\varepsilon_1$ and $k_1$, and parameter values I apply the steps 1 to 4 described above. This gives me $\hat{Y}_1$ from (13) and $f(z,1)$ from (8) through (11). Repeating until the last observation at time $T$ gives a series $\{\hat{Y}\}_1^T$. The last issue is how to get $f(z,0)$. Since it is difficult to compute the ergodic distribution I start with a rectangular distribution. Steps 1 to 4 above, with aggregate shocks set to zero, are then been applied 144 times. The resulting distribution is used as initial distribution. To reduce the impact of this approximation I have furthermore excluded the first 6 years of observations on $\hat{Y}_t$ from the estimation of the parameters. I have also checked that the results are insensitive to the length of this exclusion.

Due to computational resource restrictions it has been necessary to somewhat limit the number of parameters to estimate. For the idiosyncratic risk I have thus used the estimate in MaCurdy (1982). Using the PSID panel data set he estimates the stochastic process for the logarithm of yearly household earnings to $\Delta y_t = \upsilon_t - 0.411\upsilon_{t-1} - 0.106\upsilon_{t-2}$ with $\sigma_\upsilon^2 = 0.054$. This implies a monthly standard deviation of permanent earnings equal to $(1-0.411-0.106)\sigma_\upsilon/\sqrt{12} \approx 0.0324$ which is used as the idiosyncratic risk in (8). The remaining parameters; the band width, the shift parameter and the depreciation rate are chosen to minimize the negative of the concentrated log likelihood function net of constants

$$L\left(\cdot\middle|\alpha_0,\alpha_1,\delta\right)=\frac{T-72}{2}\ln\sum_{73}^{T}\frac{1}{T-72}\left(\frac{Y_t-\hat{Y}_t(\alpha_0,\alpha_1,\delta)}{K_t(\alpha_0,\alpha_1,\delta)}\right)^2 \tag{18}$$

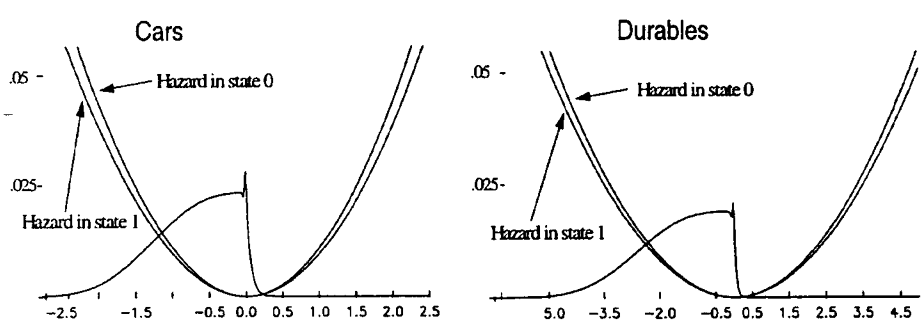
with a covariance matrix

$$V\left(\hat{\alpha}_0,\hat{\alpha}_1,\hat{\delta}\right)=\frac{\partial^2 L\left(\cdot\middle|\hat{\theta}\right)}{\partial\theta\partial\theta'}. \tag{19}$$

### 3.3 Results

The average distributions $\bar{f}(z)\equiv T^{-1}\sum_t f(z,t)$ together with estimated hazards in the two risk states are shown in *Figure* 6. The average densities imply that the average size of the durable stocks over the whole sample $\left(=\sum_k e^{z_k}\bar{f}(z_k)\right)$ is 55% of the value for return point for cars and 34% for durables. The estimated parameters are shown in *Table* 3. We find that the three parameters $\alpha_0$, $\alpha_1$, and $\delta$ are estimated fairly precisely. In particular, the band shift parameter $\alpha_1$ is positive and significant. This implies that the likelihood of adjustment for a given deviation from the target durables stock is lower when the risky state is likely. Changes in the value of $\sigma_\upsilon$ had very small effects on the estimates of $\alpha_1$and its precision. Lower (higher) $\sigma_\upsilon$, however, tended to increase (reduce) the estimate of $\alpha_0$ and $\delta$.

**Figure 6Average Densities and Hazard Functions**




The fall in purchases associated with a shift from low to high risk depends on the current distribution of deviations. One way to express the magnitude of the fall in demand for purchases is to calculate the average distribution in the sample and then compare the demand that would result for the to risk levels. For cars I find that if the low risk hazard function is applied to the average distribution we get net demand equivalent to 1.308% of the stock per month. If instead the high risk hazard is applied net demand falls to 1.14%. This means that demand falls by 12.3%. For the broader aggregate of durables, demand falls from 2.18% to 2.00%, i.e., by 8.66%.

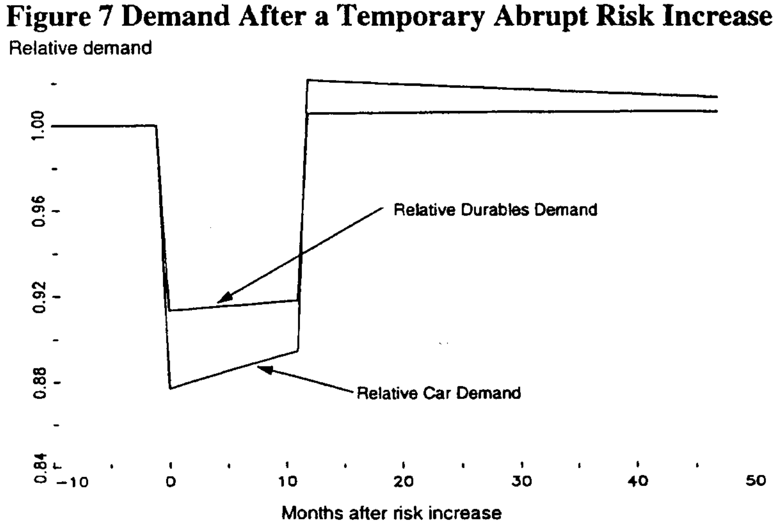
**Table 3 Parameter Estimates**

| | Car purchases | | Durables purchases | |
|---|---|---|---|---|
| | | z-value* | | z-value |
| $\alpha_0$ | 0.105 | 12.54 | 0.047 | 13.44 |
| $\alpha_1$ | 0.0067 | 4.38 | 0.0021 | 5.08 |
| $\delta * 12$ | 0.092 | 10.06 | 0.203 | 13.76 |
| $L(\cdot\mid\alpha_0,\alpha_1,\delta)$ | -1758.7 | - | -1752.9 | |

* Parameter estimate divided by its standard deviation estimated from (19)

The dynamic pattern of the effects of a temporary abrupt shift to high risk implied by the model is shown in *Figure 7*. There I show the result from a simulation of the path of demand if no aggregate shocks occur. The simulation starts with a period of zero probabilities of the high risk state. This period is set long enough for the distribution and thus demand to settle down to a constant. This constant is normalized to unity. At time 0 I let the probability of the high risk state go to 1 and stay there for 12 months. After that the probability goes back to 0.

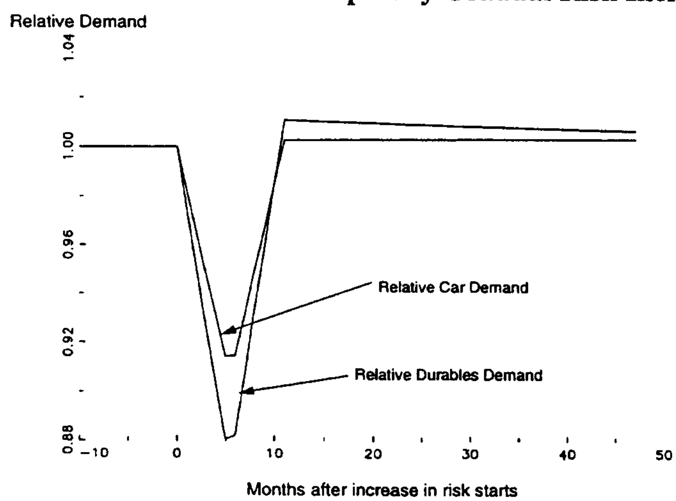
**Figure 7 Demand After a Temporary Abrupt Risk Increase**



In Figure 7 we see that demand falls drastically when the shock to risk occurs. Immediately it begins to pick up but it takes substantial time for demand to recover. When risk shifts back demand increases to a level above unity. The demand then returns slowly to the pre-shock value of unity. In *Figure* 8 I let the probability of the high risk state build up linearly from $t$=0 until $t$=6 when it reaches unity. It then returns to zero gradually between $t$=6 and $t$=12. Here we see a more gradual fall in demand that eventually (at $t$=6) reach approximately the same amplitude as in Figure 7.

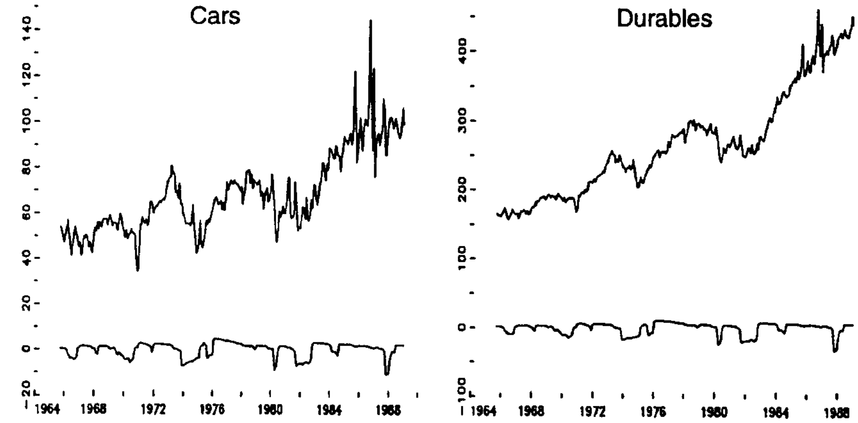
The results depicted in Figure 7 and 8 show that demand may be substantially affected by variations in risk. As important as the large amplitudes are the strong sluggishness. A shock to risk may have effects on demand that are substantial for periods well over a year.

**Figure 8 Demand After a Temporary Gradual Risk Increase**




In *Figure* 9 I depict how much of the variation in car and durable purchases that the model attributes to risk shifts. This has been computed in the following way. Instead of using the estimated state probabilities I set the probability of the high risk state to zero for the whole sample period and let the model predict purchases. The difference between the predictions with estimated probabilities and zero probabilities may be interpreted as what the model attributes to risk fluctuation. This difference and actual purchases are plotted in Figure 9. We find that the variation in what is attributed to risk is non-negligible compared to the actual series.

**Figure 9Purchases; Actual and Attributed to Risk Shifts**




Drops in purchases during 1973, 1980, 1981 and 1987 appear to be substantially related to risk increases. Shifts in demand attributed to risk shifts are, however, quite infrequent and do thus not contribute to a large share of the variation over the full sample period. This feature, infrequent and large, shifts in demand is probably due to the assumption that risk only takes two values; high and low. A more realistic assumption of a continuous level of risk may give more frequent shifts.

## 4. Concluding Remarks

The results in the previous section support the hypothesis that variations in aggregate risk may be of importance for the volatility of durables demand. We furthermore find that this result can be given an interpretation in a model of aggregate behavior where the realistic assumption of adjustment costs at the micro level is taken seriously. The volatility of the fluctuations the model predicts is of a non-negligible magnitude. When the economy enters a high risk period demand for durables fall sharply. The method used to distinguish high risk periods emphasize such sharp shifts in risk. It is possible that other, more smooth, fluctuations in risk exist and should be captured with other methods.

Many questions are certainly left unanswered in this paper. It is, for example, hard to know how sensitive the results are to the assumption about the target stock of durables in (14) . Temporary fluctuations in the relative price of durables could create a "speculative" motive for purchases that could be correlated with financial volatility. Similarly, non-separability between durables and non-durables could be of importance. It is also difficult to assess the importance of the functional form of the hazard function, in particular its constancy over time. These and other issues warrant further research on the empirical relevance of the irreversibility model. If high frequency data on individual behavior becomes available one would like to a micro model like in Eberly (1994) to allow stochastic fluctuations in the risk level.

Despite these questions, a line of research which takes the aggregate implications of microeconomic *Ss*-behavior seriously seems possible and important in order to get an understanding of the particular relationship that appears to exist between risk and durables purchases.

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