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## **Household Production and Taxation in the Stochastic Growth Model**

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## 1. Introduction

In this paper, we estimate the parameters of a dynamic general equilibrium model of the U.S. economy that includes an explicit household production sector. We use the estimates to investigate two issues. First, we analyze how well the model accounts for business cycles. Second, we use the model to analyze the effects of tax changes.

The inclusion of an explicit household production sector is important for both of these issues, since a key common element concerns individuals' willingness to substitute in and out of market activity. For example, Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991) show that once home production is included, the performance of simple real business cycle models can improve along several dimensions. Unfortunately, the extent to which home production matters in these models depends critically on some parameters about which we have little independent information, including elasticities of substitution between household and market variables in utility and production functions. These same parameters are likely to be important for the effects of tax changes. In the endogenous growth literature, for example, it is well known that the results of tax policy hinge crucially on similar parameters (see Stokey and Rebelo (1993)). These parameters are difficult to calibrate based on long-run growth observations.

Therefore, following McGrattan (1992), we obtain maximum likelihood estimates. Our findings are as follows. There is a large and statistically significant elasticity of substitution between the market and home produced consumption goods. The estimated elasticity is lower than that used in Benhabib, Rogerson, and Wright (1991), and so agents are less willing to substitute between the home and market than assumed in that paper; however, our estimates imply that stochastic shocks to the production functions in the two sectors are not highly correlated, and so there is a greater incentive to substitute between the home and market than assumed in that paper. Overall, the estimated model does a good job of accounting for the standard set of moments on which business cycle theorists typically concentrate. Perhaps surprisingly, key parameter estimates are not sensitive to transforming the data by removing a "trend" before estimation.<sup>1</sup>

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<sup>1</sup> The model includes exogenous geometric growth. Given the nature of the data, including our constructed tax series, it seemed essential to incorporate measurement error into the specification. We also tried Hodrick-Prescott filtering the time series before estimation. This affects the estimate of the

To illustrate the significance of home production for evaluating changes in tax policy, we carry out several fiscal experiments using our estimated parameters. We contrast these predictions with those implied by the estimates of McGrattan (1992), who analyzed a model without home production. We find significant differences for both changes in allocations and welfare. For example, reducing the capital income tax rate from 57 percent to zero and keeping the budget balanced requires increasing the labor income tax from 23 percent to 31 percent. This policy change increases output 13.7 percent and increases welfare by an amount equivalent to 10.6 percent of market consumption. By comparison, the model without home production predicts that the labor income tax would have to increase to about 36.5 percent for budget balance, with output increasing by 21.3 percent and welfare increasing by 14.7 percent.

## 2. The Model

The model is a stochastic growth model with an explicit household sector and government expenditures financed by distorting taxes on market labor and market capital. The details of the specification follow.

There is a continuum of identical agents with preferences defined over stochastic processes for consumption and leisure given by:

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{(c_t^b \ell_t^{1-b})^{1-\sigma}}{1-\sigma}$$

where  $c_t$  is an aggregate of three types of consumption at time  $t$ : private market consumption ( $c_{mt}$ ), private nonmarket consumption ( $c_{nt}$ ), and public consumption ( $c_{gt}$ ). There are two steps involved in going from the three consumptions to the aggregator  $c_t$ . First, the two private consumptions are combined using a CES aggregator:

$$c_{pt} = \{a_2 c_{mt}^{b_2} + (1 - a_2) c_{nt}^{b_2}\}^{\frac{1}{b_2}}.$$

The private consumption aggregate is then combined with public consumption once again using a CES aggregator:

$$c_t = \{a_1 c_{pt}^{b_1} + (1 - a_1) c_{gt}^{b_1}\}^{\frac{1}{b_1}}.$$

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growth process, of course, and also the measurement error processes, but it does not substantially alter the estimates of the economic parameters, including the key elasticities of substitution.

Leisure at time  $t$  is defined by

$$\ell_t = \bar{h} - h_{mt} - h_{nt}$$

where  $h_{mt}$  and  $h_{nt}$  are market and nonmarket hours respectively.

There are two technologies, one each for the market and home sector, each of which improves at a geometric rate over time and is subject to a stationary shock. The market technology is specified by:

$$y_t = \{a_4 k_{mt}^{b_4} + (1 - a_4)(\mu^t e^{s_{mt}} h_{mt})^{b_4}\}^{\frac{1}{b_4}},$$

where  $k_{mt}$  is the market capital stock in period  $t$ ,  $\mu$  is the (trend) growth rate, and  $s_{mt}$  is the market technology shock. Feasibility requires that

$$c_{mt} + i_t + c_{gt} \leq y_t$$

in each period, where  $i_t$  is period  $t$  investment. We assume that government consumption in period  $t$  is given by a stochastic process given by:

$$c_{gt} = \alpha_t y_t,$$

where  $\alpha_t$  is a random variable. An important distinction between the market and non-market (or home) sectors is that capital can only be produced in the market, although it is used as an input in both sectors. Hence, the home technology is specified as

$$c_{nt} = \{a_3 k_{nt}^{b_3} + (1 - a_3)(\mu^t e^{s_{nt}} h_{nt})^{b_3}\}^{\frac{1}{b_3}}$$

where  $k_{nt}$  is home sector capital,  $\mu$  is as above, and  $s_{nt}$  is the home sector shock. It is assumed that total capital at time  $t$  can be allocated arbitrarily between the two sectors, hence the constraint:<sup>2</sup>

$$k_{mt} + k_{nt} \leq k_t.$$

Total capital evolves according to

$$k_{t+1} = (1 - \tilde{\delta})k_t + i_t$$

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<sup>2</sup> This specification assumes that capital in place can be moved in the next period to a different sector. But, in equilibrium, capital is not often reallocated between sectors.

where  $\tilde{\delta}$  is the depreciation rate, implicitly assumed to be independent of the sector in which the capital is used.

We will compute the competitive equilibrium for the above economy in the presence of distorting taxes, and hence it is necessary to specify the nature of the tax system, in particular the budget constraints of individual agents and the government. Let  $\tau_{kt}$  and  $\tau_{ht}$  denote the (stochastic) tax rates on income from market capital and market labor respectively,  $r_t$  be the rental rate for market capital, and  $w_t$  be the market wage rate. In each period there is also be a lump sum transfer  $T_t$  from the government to private agents to ensure that the government budget constraint is satisfied in a period by period sense. Hence, the government budget constraint is:<sup>3</sup>

$$T_t + \alpha_t y_t = \tau_{kt} r_t k_{mt} + \tau_{ht} w_t h_{mt} - \tilde{\delta} \tau_{kt} k_{mt}$$

whereas the budget constraint for the private individual is:

$$c_{mt} + i_t \leq (1 - \tau_{kt}) r_t k_{mt} + (1 - \tau_{ht}) w_t h_{mt} + \tilde{\delta} \tau_{kt} k_{mt} + T_t.$$

The transfer payments  $T_t$  are treated entirely as a residual, i.e. they are assumed to take on whatever value is dictated by the government budget constraint, given the realization of spending and tax rate shocks and the private choices of capital and hours. Note also that we have not included a tax on household capital. Thus, we are implicitly assuming that residential property taxes are offset by tax savings from deducting property tax payments and interest payments for home mortgages.<sup>4</sup>

The final aspect of the economy to be specified is the stochastic environment. At this point we present a general specification that is restricted in a later section. Let  $z_t = (\alpha_t, s_{mt}, s_{nt}, \tau_{kt}, \tau_{ht})$  be the vector of exogenous stochastic variables at time  $t$ . We write:

$$\gamma(L)z_t = \gamma_0 + \gamma_\epsilon \epsilon_t \tag{2.1}$$

as a VAR representation of this process, with  $\gamma(L) = I - \gamma_1 L - \dots - \gamma_q L^q$ ,  $E\epsilon_t = 0$ ,  $E\epsilon_t \epsilon_t' = I$ .

<sup>3</sup> This specification assumes that the government balances its budget each period and never issues debt. Although unrealistic, this greatly simplifies the analysis.

<sup>4</sup> In Section 4 we consider the effects of including a tax on  $k_n$ .

As mentioned earlier, we compute the competitive equilibrium for the above economy. With  $\mu > 1$  the competitive equilibrium entails many variables with a component that grows at the rate  $\mu$ , and hence that are not stationary. In what follows it is important that the series be stationary, and a simple transformation of the above economy produces this result. The transformed economy is identical to that described above with three exceptions. First  $\mu$  takes on the value of 1 in both production functions. Second, the discount rate is written as  $\beta$  with  $\tilde{\beta}\mu^{b(1-\sigma)}$ , and third, the law of motion for capital accumulation is written as

$$k_{t+1} = (1 - \delta)k_t + i_t/\mu,$$

where the depreciation rate  $\delta$  is now given by  $\delta = 1 - (1 - \tilde{\delta})/\mu$ . In what follows we treat the transformed economy as the prime unit of analysis. The equilibrium for the original economy is easily obtained from the equilibrium for the transformed economy. For  $y_t, k_{mt}, i_t, c_{nt}, c_{mt}, c_{gt}$ , one simply adds a geometric trend with growth rate  $\mu$ . The other series (i.e. time allocations) are unaffected.

### 3. Estimation

#### 3.1. Estimation Procedure

We use the same procedure as McGrattan (1992), and refer the reader there for more details. Here we provide a brief overview. Following Kydland and Prescott (1982), the decision functions are computed using a linear quadratic approximation in the neighborhood of the deterministic steady state, although the presence of distorting taxes makes the procedure somewhat more complicated.<sup>5</sup> This results in the mapping

$$x_{t+1} = Ax_t + \eta_t. \tag{3.1}$$

where  $x_t = (k_t, z_t, z_{t-1}, \dots, z_{t-q}, k_{mt}, h_{mt}, i_t)$ . The elements of  $A$  in (3.1) are nonlinear functions of the preference and technology parameters,  $\Gamma = (\beta, b, \sigma, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, \delta, \mu)$ , and the parameters of the matrices in  $\gamma(L)$ . The elements of  $\eta$  are linear functions of  $\epsilon$ . We allow for the possibility that all variables are measured with error, yielding on measurement equation

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<sup>5</sup> See the Appendix for the derivation of the approximate decision functions.

$$\zeta_t = Cx_t + \omega_t \quad (3.2)$$

where  $\zeta = [k, k_n, i, c_g, y, \tau_k, \tau_h]$  and  $\omega$  is a vector of measurement errors.<sup>6</sup> Note that  $c_m$  and  $k_m$  are redundant and hence not included in  $\zeta$ . Additionally we assume

$$\omega_t = D\omega_{t-1} + \nu_t, \quad E\nu_t = 0, E\nu_t\nu_t' = \Omega. \quad (3.3)$$

Following Harvey (1981), we formulate our empirical model in state space form using (3.1), (3.2), and (3.3),

$$\begin{aligned} x_{t+1} &= Ax_t + \eta_t \\ \bar{\zeta}_t &= \bar{C}x_t + \bar{\nu}_t. \end{aligned} \quad (3.4)$$

System (3.4) is obtained by differencing (3.2) so that  $\bar{\zeta}_t = \zeta_{t+1} - D\zeta_t$ ,  $\bar{C} = CA - DC$ , and  $\bar{\nu}_t = C\eta_t + \nu_t$ . If the disturbances,  $\eta_t$  and  $\bar{\nu}_t$ , are normal, then estimation involves maximizing the following Gaussian likelihood function

$$L(\Theta) = -T \ln 2\pi - .5T \ln |V| - .5 \sum_{t=1}^T u_t' V^{-1} u_t$$

where  $u_t = \bar{\zeta}_t - E[\bar{\zeta}_t | \bar{\zeta}_{t-1}, \dots]$  is the innovation process for  $\zeta$  and  $V = Eu_t u_t'$  and  $\Theta$  are the parameters of preferences and technologies ( $\Gamma$ ), the autoregressive process for  $z$  ( $\gamma(L), \Sigma$ ), and the measurement process ( $D, \Omega$ ). Harvey (1981) shows how one can construct the innovations by applying a Kalman filter to system (3.4).

### 3.2. Data

All data are for the years 1947-1987. The data for investment, government purchases, and market output are taken from the *National Income and Product Accounts*. Investment is defined to be fixed investment and consumer durables. Market output is the sum of investment, government consumption, and private market consumption. Private market consumption consists of consumption of nondurables and services excluding the service flow from the housing stock. We exclude the product from the housing stock because we interpret this as nonmarket production.

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<sup>6</sup> Given large differences in tax rates across studies, the possibility of measure error is particularly important.

The market capital stock is the net stock of private nonresidential structures and equipment. The nonmarket capital stock is the net stock of private residential capital and consumer durables. The capital series are obtained from the *Survey of Current Business*.

The market hours series is total manhours for all industries and all employees taken from the Bureau of Labor Statistics' household survey. The seasonally adjusted monthly series is aggregated to obtain a quarterly measure. The total hours  $\bar{h}$  is set equal to 1134 which is the measure of discretionary time used by Hill (1985).

Annual tax rates on labor and capital are constructed using the definition of Joines (1981). (See Appendix B for the actual series.) The two data sources for these measures are the *National Income and Product Accounts* and the *Statistics of Income* series of the Internal Revenue Service. The constructed tax rate on labor corresponds to Joines' definition of MTRL1. The constructed tax rate on capital corresponds to Joines' definition of MTRK1 with property tax payments excluded.

All variables are in per-capita, real terms. Capital stocks and tax rates, which are annual series, are interpolated to obtain a quarterly series.

### 3.3. Empirical Results

Table 1a<sup>7</sup> reports estimates and standard errors for the parameters of the utility and production functions. The estimates for  $a_1$ ,  $b_1$ ,  $\sigma$ ,  $\delta$ ,  $\beta$ , and  $\mu$  are similar to those found in McGrattan (1992). The parameter that weights private and public consumption in household's utility,  $a_1$ , hits its upper bound of 1 and was therefore constrained during estimation. This estimate is consistent with utility functions that are separable in public and private consumption. With  $a_1 = 1$ ,  $b_1$  is not identified and can therefore be set arbitrarily. The value of the risk aversion parameter,  $\sigma$ , is 5.27 but has a large standard error. The estimate for depreciation given in Table 1a is 0.022 with a small standard error. This value is pinned down by observations on total capital and investment and is consistent with most estimates of quarterly depreciation [3],[10],[11]. The value of  $\beta$  is estimated to be 0.991 which is less than 1 but not significantly. The estimate of the growth rate,  $\mu$ , is 1.0054 with a small standard error. This value is in the range of the sample growth rates

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<sup>7</sup> Results are reported for two data sets. We label the tables or figures with a number and the letter "a" for the data detrended by geometric growth case and "b" for the case that the data was pre-filtered.



for the capital stocks, output, and investment.

As mentioned earlier, a key parameter governing the interaction between household production and market activity is  $b_2$ , which determines the elasticity of substitution between market and nonmarket consumption ( $1/(1 - b_2)$ ). This elasticity measures the willingness of agents to substitute between market and nonmarket goods. If  $b_2 = 1$ , then  $c_m$  and  $c_n$  are perfect substitutes. If  $b_2 = 0$ , the function is Cobb-Douglas and the goods are complements. Benhabib, Rogerson, and Wright (1991) show that if  $k_n$  is relatively small,  $b_2 = 0$  implies that distinguishing between leisure and home production has no implications for market variables. As shown in Table 1a, the estimate of  $b_2$  is 0.385 with standard deviation 0.145. This estimate is significantly different from zero and indicates that there is significant willingness on the part of households to substitute between market and nonmarket goods.

Suppose that we parameterize the model as follows:  $\sigma = 1$ ,  $b_2 = 0$ ,  $b_3 = 0$  and all of the coefficients on  $s_{nt}$  or its innovation are set equal to zero in the equations for government consumption, market technology, and tax rates. There are 15 restrictions for this new parameterization. These restrictions imply that market variables are not affected by changes in the home sector. We test the hypothesis that this set of constraints holds against the alternative unconstrained specification. We find that we can easily reject the hypothesis when applying the likelihood ratio test. The probability that a  $\chi^2(15)$  falls below the computed likelihood ratio test statistic is approximately 1.

The weight on market consumption in the utility function,  $b$  is 0.448 with standard error 0.121. Benhabib, Rogerson, and Wright (1991) back this parameter out from estimates of the means of  $h_m$  and  $h_n$ . Their estimates are derived from the time-use survey [6] and imply that  $h_n/h_m \approx 0.85$ . Here we did not put any direct restrictions on the mean of  $h_n$ . Our estimate implies a mean value of  $h_n$  equal to be 140, which is about half of the mean for market hours.

Next we consider estimates of the market production function. First note that  $b_4$  is not significantly different from zero, indicating that the market production function is approximately Cobb-Douglas. The weight on capital in market production,  $a_4$ , has a value of 0.234 with standard error 0.120. Although this estimate is consistent with others

[3],[11], the addition of taxes and home production could imply an estimate different from the model without taxes or the home sector. This result is due to the fact that with taxes on capital, the value of  $a_4$  must be increased to generate a given stock of capital. In a model without home production, McGrattan (1992) finds an estimate of 0.4. On the other hand, the addition of the home sector should decrease  $a_4$  because a large fraction of total capital is assumed to be allocated to the nonmarket sector. These two factors imply an estimate of  $a_4$  in the range of estimates found with taxes and home production excluded.

Now we turn to a discussion of the estimates of the technology shocks. The estimates and standard errors for the parameters of the vector autoregression are given in Figure 1a. If the autoregressive process is completely unrestricted, the estimates of coefficients on government consumption, market technology, and tax rates in the home technology equation are tiny and insignificantly different from zero. Therefore, we restricted these coefficients to be zero and let next period's shock to the home technology depend only on its own lags. These restrictions help to identify the parameters of the home technology process. Because we do not have time series for home consumption and hours worked, the home technology process cannot be completely identified. For example, since the consumption of the home good and the labor input in home production are latent, the mean of  $s_{nt}$  is not identifiable.<sup>8</sup> Therefore, to avoid singularities of the information matrix, the third element of  $\gamma_0$  of equation (2.1) is set equal to zero when searching over the parameter space for an optimum. In reporting the standard errors, the variance of the error of the home technology equation,  $var(\epsilon_{3t})$ , is also restricted. Excluding the (3,3) element of  $\gamma_\epsilon$  before computing the standard errors significantly affects the the standard errors for  $a_2$ ,  $a_3$ , and several covariances between the error term in the home technology shock equation and other error terms. For example, if  $\gamma_\epsilon(3,3)$  is included in the parameter vector when standard errors are computed, then the standard errors for  $a_2$  and  $a_3$  are 0.299 and 0.190,

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<sup>8</sup> This can be argued as follows. Start with a parameterization  $\bar{\Theta}$ . Following the procedure of Appendix A, the choice of  $\bar{\Theta}$  determines the steady state values for total capital, market capital, market hours, hours in home production (by equation (A3)). Steady states for all other variables follow from these four. Consider next a parameterization  $\hat{\Theta}$ , which is assumed to be different from  $\bar{\Theta}$  in the choice of  $b$ ,  $a_2$ , and the parameters governing the autoregressive process for  $z$  ( $\gamma$ ). The “ $\hat{\cdot}$ ” parameters are chosen in a way that the mean of  $s_n$  is changed but the means of all observable series are unaltered. In particular,  $\gamma_0(3)$  is perturbed with the remaining elements in  $\gamma$  altered to leave  $\bar{\alpha}$ ,  $\bar{s}_m$ ,  $\bar{\tau}_k$ , and  $\bar{\tau}_h$  as before. The parameters  $a_2$  and  $b$  are perturbed in such a way that the solution of (A3) for  $\bar{k}_m$ ,  $\bar{h}_m$ , and  $\bar{k}$  is unaltered.

respectively, while the standard errors for the remaining utility and production parameters are similar in magnitude to those reported in Table 1a. We suspect that these differences indicate that not all of second moments can be identified without observations on  $c_n$  or  $h_n$ . Since the standard errors do not change significantly if we eliminate any other covariances before computing standard errors, we report the case with only the (3,3) element of  $\gamma_\epsilon$  restricted.

The correlation between market and home shocks can be computed using the estimates from Figure 1. This correlation is of interest because it indicates the frequency with which individuals are presented an incentive to substitute between market and nonmarket production. The estimated correlation is -0.186. Note that this correlation is much smaller than the number used in Benhabib, Rogerson, and Wright (1991), indicating significantly more incentive for substitution between the two sectors.

The results from Figure 1a indicate that the process  $z$  is highly persistent. The estimation procedure attempts to capture the low-order frequencies evident in aggregate time series either through  $\gamma(L)$  or through the measurement error processes. This is especially true for the labor tax rate series which has a significant upward trend. The upward trend in labor tax rates causes a near unit root estimate of the tax rate process or a near unit root estimate of its measurement error. The estimates of the measurement error process are reported in Figure 2b. The measurement errors on output and government consumption are assumed to be zero. Only hours of work and the two tax rates are assumed to have serially correlated measurement errors. Because there are trends in the labor tax rates and hours of work over the sample, the coefficients of  $D$  are high.

### 3.4. Alternative Procedure

In this section we present estimates derived from using data which have been filtered and assume that  $\mu = 1$ . In order to make our results comparable to much of the current business cycle literature, we use the filter described in Hodrick and Prescott (1980) and Prescott (1986). Their filter fits a smooth curve through the data and, thus, eliminates low frequencies.

We include the results for the filtered cases because there are several problems with the model specification of Section 2. We assumed that all variables that grow do so at the

same geometric rate. While this may be a correct assumption, we have a short sample and growth rates do not match exactly. We have assumed that tax rates are stationary but over our particular sample, the constructed series have trends. The tax rate on labor has increased over the post-war sample and the tax rate on capital has decreased. Therefore, the filtered case gives us a check on the robustness of our structural parameters (*e.g.* those of utility and production). We can check to see if the key parameters are sensitive to our assumptions about the low frequencies in the data.

Table 1b and Figures 1b and 2b report the parameter estimates assuming that the true series are those that have been Hodrick- Prescott filtered. The estimates of preferences and technology are similar to those of Table 1a. In fact, the changes in point estimates are less than one standard deviation. The major difference between the two cases are the estimates of the autoregressive process and the measurement equations (Figures 1 and 2). Because we have filtered out the low-frequencies of the data in the second case, we do not have the same difficulties with near unit-root processes.

In the case of filtered data, the correlation between the technology shocks,  $s_n$  and  $s_m$ , is -0.926. The negative correlation implies that agents have an opportunity to substitute between home and market. Given the incentives to substitute between the market and nonmarket sectors, home production can have a big effect even if agents' willingness to substitute between market and nonmarket goods is small ( $b_2$  much less than 1).

### 3.5. Discussion

Table 2a and Table 2b report the standard deviations and correlations with output of the series predicted by the model and their analogues from the data. Both the model and data series are logged for computation of standard deviations. The column heading ' $C\hat{x}$ ' implies that  $CE[x_t | \bar{\zeta}_{t-1}, \bar{\zeta}_{t-2}, \dots, \bar{\zeta}_1, \hat{x}_1]$  is used to obtain the statistics, where  $\hat{x}_1$  is an estimate of the initial state. The first eight variables under the column 'z' are the observed series. Market capital is derived from total capital and capital in the home. The series for consumption is output less investment and government consumption. The series  $i_m$  is investment in nonresidential structures and durable equipment for the U.S. The series  $i_n$  is investment in residential structures and consumer durables. Both investment series were obtained from the *National Income and Product Accounts*. The series for the model

are constructed from  $k_{mt+1} - (1 - \delta)k_{mt}$  and  $k_{nt+1} - (1 - \delta)k_{nt}$ , respectively.

A comparison of the columns for the model and the data in Table 2a, indicates a good match for the capital stocks, output, investment, consumption, and government purchases. Although it is not reported, the first moments match up well with the exception of the tax on capital. The sample mean in the data is around 0.5 while the model predicts a rate near 0.7. In the case of investment and hours, the model predicts too much variation. One factor may be that these series are still highly persistent when geometrically detrended. The estimation procedure attempts to fit these features of the data but may miss the higher frequencies in investment and hours. Also reported are correlations of the various series with output. Note that investment in consumer durables and residential structures is procyclical for both the data and the model.

In Table 2b, standard deviations and correlations are reported for data that has been filtered first. A comparison of Table 2a and Table 2b indicates substantial differences in the magnitudes of the variances. In fact, the model now underpredicts the variation in hours of work in the market. Another important difference is the correlation of home investment and output; this correlation is now negative.

#### 4. Fiscal Policy Experiments

We analyze the effects of three tax experiments: reducing both labor and capital tax rates to zero, reducing the capital income tax rate to zero while increasing the labor income tax rate in order to maintain a balanced budget, and increasing the tax on residential capital. We do not argue that these are especially relevant, or imminent, policy changes. Rather, the results serve to illustrate the different predictions of models with and without household production.

Table 3 reports percent changes for output, consumption, investment, hours worked, and capital in both the home and market sectors, plus a variable  $\Delta$  that measures the welfare consequences of the policy change in terms of market consumption. For example, if  $\Delta = 0.01$  then the agents would be equally well off after the indicated policy change if their market consumption were reduced by 1 percent of its new steady state value, all else being the same. There are three columns, corresponding to a base case with  $\tau_k = 0.57$  and

$\tau_h = 0.23$ , a case with zero taxes, and a case with  $\tau_k = 0$  and  $\tau_h$  set at the indicated level in order to keep revenue constant. In the base case, government spending is set to balance the budget.

Based on our parameter estimates, the effect of eliminating distorting taxation altogether is quite sizable: output increases by 43 percent, consumption of market goods increases by 47 percent, investment increases by 83 percent, market hours increase by 22 percent, the stock of market capital more than doubles, and the ratio of market to household capital increases from 1.3 to 2.2. In the home sector, consumption of goods decreases by 1.4 percent, hours of work decreases by 20 percent, and the capital stock increases by 34 percent. Thus, there is a significant shift of labor from home to market production but an increase in capital in both sectors. In terms of welfare, the policy change is worth 22.1 percent of market consumption.

A model that ignores the household sector has very different predictions. For example, consider the model and parameter values of McGrattan (1992), who does not include a household sector. Experiment 1 now yields an increase in both output and market consumption of 58 percent, an increase in both investment and market capital (i.e., total capital) of 133 percent, and an increase in market hours of 22 percent. Thus, we find a much larger response of market consumption, investment, and output when we exclude the household sector. The increase in market hours is the same for the two models, but because home hours change when household production is explicitly modeled, the implication for leisure is not the same. In terms of welfare, we find that the policy change is worth 27.8 percent of market consumption.

Now consider the effect of eliminating the capital tax and raising the labor tax to keep revenue constant. Given our parameter estimates, the labor tax rate would have to increase from 0.23 to 0.31 in order to maintain a balanced budget. This is accompanied by a 13.7 percent increase in output, a 6 percent increase in market consumption, a 45 percent increase in investment, a 77 percent increase in the market capital stock, and a 3 percent decrease in market hours. The welfare gain is 10.6 percent of market consumption.

In contrast, the model without home production predicts that reducing  $\tau_k$  to 0 requires increasing  $\tau_h$  to 0.36 to keep revenue constant. Output increases by 21 percent, market

consumption increases by 8.6 percent, investment and the capital stock both increase by 79 percent, and market hours fall by 6.2 percent. The welfare gain is 14.7 percent of consumption. What is striking about this experiment is that even though there is little substitution between market and home activity in the home production economy, we still find very different implications for market variables.

For the final fiscal experiment, we consider adding a tax on household capital, an experiment that cannot be conducted with McGrattan's (1992) model. We consider replacing the individual budget constraint with

$$c_m + i \leq (1 - \tau_k)rk_m + (1 - \tau_h)wh_m + \delta\tau_k k_m + T - \tau_p k_n$$

where  $\tau_p$  is the residential property tax. Table 4 reports percent changes for output, consumption, investment, hours worked, and capital in both the home and market sectors, plus a variable  $\Delta$  that measures the welfare consequences of the policy change in terms of market consumption. For the base case, the residential property tax is set equal to zero and the other fiscal variables are set equal to the sample averages in the U.S. data, e.g.,  $\tau_k = 0.57$ ,  $\tau_h = 0.23$  and  $g = 584$ .<sup>9</sup> Steady state values are then calculated for economies with  $\tau_p = 0.01$ , 0.02, and 0.03.<sup>10</sup> In Table 4, the percent changes are changes in the variables relative to the base case. Note that, with the exception of nonmarket hours of work, all variables have lower steady state values in the cases with  $\tau_p > 0$  than in the base case. Thus, individuals are substituting labor for capital in household production. But there is no increase in market production in response to the increased tax on the home capital. For example, market output in the economy with  $\tau_p = 0.01$  is 13 percent below that in the economy without a property tax. Market consumption and investment are 12.4 and 25.5 percent below their base case levels. Market capital falls 39.3 percent while market hours fall 1.6 percent. These results are due in part to the fact that home capital is produced in the market. In fact, there is a larger decline in market production and consumption than home production and consumption. For higher tax rates, we see similar patterns but larger changes. Note, however, that the differences between allocations for

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<sup>9</sup> The spending process is detrended with a geometric trend  $\mu = 1.0054$  before a sample average is computed.

<sup>10</sup> Jorgenson and Yun (1991) use a rate equal to 0.01.

$\tau_p = 0$  and  $\tau_p = 0.01$  are larger than those for the economies with  $\tau_p = 0.02$  and  $0.03$ . On the other hand, the welfare costs increase almost linearly for the three examples (e.g.,  $\Delta = .147, .279$ , and  $.400$ ).

## 5. Conclusions

The objective of this paper was to evaluate the importance of home production in models of aggregate economic activity, by obtaining maximum likelihood estimates of a stochastic growth model with an explicit household sector. The parameters that are most important for the hypothesis that home production affects predictions of market activity are the elasticity of substitution between output of the two sectors, the correlation between stochastic shocks to home and market technologies, and parameters of the home production function. For these parameters, there is little microeconomic or long-run evidence.

Our parameter estimates suggest that there is a significant elasticity of substitution between home and market goods. Estimated correlations of the home and market technology shocks also suggest that there is an opportunity for such substitution. Furthermore, when we reestimate the model with restrictions that imply market activity is not affected by changes in the home sector, we find a significant difference in the likelihood values for the constrained and unconstrained specifications.

We compare the predictions of the model estimated here to that of McGrattan (1992), who does not include a household sector. We show that including home production significantly affects the model's predictions for certain fiscal experiments. The movement of resources between different activities in response to changes in tax rates depends critically on the opportunities for substituting between taxed and nontaxed activities and on individuals' willingness to substitute.

One fiscal experiment that cannot be analyzed in a model without a household sector is an increase in the residential property tax rate. The model predicts significant differences between economies with a 0 percent rate and a 1 percent rate. For example, the differences in market output and market consumption are around 13 percent. The difference in home capital is around 10 percent.

The fiscal experiments that we conduct are intended to illustrate the potential biases



due to abstracting from home production. However, there may be other experiments for which home production can play an important role. In such cases, the estimates reported in this paper may prove to be useful.

## Appendix A

In this appendix, the approximate linear decision rules are derived analytically for the model of Section 2 with general functions for aggregate consumption, private consumption, and production. The optimization problem to be solved in this case is

$$\max_{\{\hat{k}_{mt}, \hat{h}_{mt}, \hat{h}_{nt}, \hat{i}_t\}} E \left[ \lim_{T \rightarrow \infty} \sum_{t=0}^{T-1} \beta^t u(\hat{c}_t, \bar{h} - \hat{h}_{mt} - \hat{h}_{nt}) | \hat{k}_0, z_0, k_{m0}, h_{m0}, h_{n0} \right] \quad (A1)$$

subject to

$$\begin{aligned} \hat{c}_t &= c(\hat{c}_{pt}, \alpha_t y_t) \\ y_t &= y(k_{mt}, h_{mt}, s_{mt}) \\ \hat{c}_{pt} &= c_p(\hat{c}_{mt}, \hat{c}_{nt}) \\ \hat{c}_{mt} &= (\hat{k}_{mt} - \tau_{kt} \hat{k}_{mt} + \tau_{kt} k_{mt}) \partial y(k_{mt}, h_{mt}, s_{mt}) / \partial k_{mt} \\ &\quad + (\hat{h}_{mt} - \tau_{ht} \hat{h}_{mt} + \tau_{ht} h_{mt}) \partial y(k_{mt}, h_{mt}, s_{mt}) / \partial h_{mt} \\ &\quad + \delta \tau_{kt} (k_{mt} - \hat{k}_{mt}) - \alpha_t y(k_{mt}, h_{mt}, s_{mt}) - \hat{i}_t \\ \hat{c}_{nt} &= c_n(\hat{k}_t - \hat{k}_{mt}, \hat{h}_{nt}, s_{nt}) \\ \hat{k}_{t+1} &= (1 - \delta) \hat{k}_t + \hat{i}_t / \mu \\ z_{t+1} &= \gamma_0 + \gamma_1 z_t + \dots + \gamma_q z_{t-q} + \epsilon_{t+1}, \quad E \epsilon_t = 0, E \epsilon_t \epsilon'_t = \Sigma \\ \underline{z}_{t+1} &= \gamma_z \underline{z}_t + \epsilon_{t+1}, \quad E \epsilon_t = 0, E \epsilon_t \epsilon'_t = \Sigma_\epsilon \\ \underline{z}_t &= [z'_t, z'_{t-1}, \dots, z'_{t-q}, 1]' \\ z_t &= [\alpha_t, s_{mt}, s_{nt}, \tau_{kt}, \tau_{ht}]' \end{aligned} \quad (A2)$$

with per-capita levels of home capital and labor,  $\{k_{mt}, h_{mt}\}_{t=0}^{\infty}$  assumed given. Side conditions for this problem are given by  $\hat{k}_{mt} = k_{mt}$ ,  $\hat{h}_{mt} = h_{mt}$  and  $\lim_{t \rightarrow \infty} \beta^{\frac{1}{2}} k_t = 0$  and are imposed on the solution to the household's optimization problem.

To obtain approximate linear decision functions for the household, the utility function is first replaced by a quadratic expansion. Following Kydland and Prescott (1982), a second order Taylor expansion around the vector  $\bar{X} = [\bar{k}, \bar{z}', \bar{k}_m, \bar{h}_m, \bar{k}_n, \bar{h}_n, \mu \delta \bar{k}]'$  is used where  $\bar{z}$  satisfies  $\gamma(1)\bar{z} = \gamma_0$  and  $\bar{k}_m, \bar{h}_m, \bar{h}_n, \bar{k}$  satisfy

$$\begin{aligned} \bar{c}_{p1} [(1 - \bar{\tau}_k) \bar{y}_1 + \delta \bar{\tau}_k] - \bar{c}_{p2} \bar{c}_{n1} &= 0 \\ \bar{u}_1 \bar{c}_1 \bar{c}_{p1} (1 - \bar{\tau}_h) \bar{y}_2 - \bar{u}_2 &= 0 \\ \bar{u}_1 \bar{c}_1 \bar{c}_{p2} \bar{c}_{n2} - \bar{u}_2 &= 0 \\ \beta \bar{c}_{p2} \bar{c}_{n1} + \mu (1 - \beta (1 - \delta)) \bar{c}_{p1} &= 0 \end{aligned} \quad (A3)$$

where subscripts indicate partial derivatives and “-” implies the function is evaluated at steady state values (e.g.  $\bar{c}_{p1} = \partial c_p(\bar{c}_m, \bar{c}_n) / \partial \bar{c}_m$ ,  $\bar{c}_m = (1 - \bar{\alpha}) y(\bar{k}_m, \bar{h}_m, \bar{s}_m) - \mu \delta \bar{k}$ , and  $\bar{c}_n = c_n(\bar{k} - \bar{k}_m, \bar{h}_n, \bar{s}_n)$ ). The elements of the vector  $\bar{X}$  are the steady state values for  $X_t = [\hat{k}_t, z'_t, k_{mt}, h_{mt}, \hat{k}_{mt}, \hat{h}_{mt}, \hat{h}_{nt}, \hat{i}_t]'$  when  $\Sigma = 0$ . The equations (A3) are therefore derived from the first order conditions in the nonstochastic case.

Given the steady state values  $\bar{X}$ , the second order expansion of the utility function is derived as follows:

$$\begin{aligned} u(\hat{c}, \bar{h} - \hat{h}_m - \hat{h}_n) &= U(X) \quad (\text{for } \hat{c} \text{ as defined above}) \\ &\approx U(\bar{X}) + \frac{\partial U}{\partial X} \Big|_{\bar{X}} (X - \bar{X}) + \frac{1}{2} (X - \bar{X})' \frac{\partial^2 U}{\partial X^2} \Big|_{\bar{X}} (X - \bar{X}) \\ &= X' \left[ e(U(\bar{X})) - \frac{\partial U}{\partial X} \Big|_{\bar{X}} \bar{X} + \frac{1}{2} \bar{X}' \frac{\partial^2 U}{\partial X^2} \Big|_{\bar{X}} \bar{X} \right] e' \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \left( e \frac{\partial U}{\partial X} \Big|_{\bar{X}} + \frac{\partial U}{\partial X} \Big|_{\bar{X}} e' - e \bar{X}' \frac{\partial^2 U}{\partial X^2} \Big|_{\bar{X}} - \frac{\partial^2 U}{\partial X^2} \Big|_{\bar{X}} \bar{X} e' + \frac{\partial^2 U}{\partial X^2} \Big|_{\bar{X}} \right) X \\
& = X' \begin{pmatrix} Q & W \\ W' & R \end{pmatrix} X,
\end{aligned} \tag{A4}$$

where  $e$  is a  $13 \times 1$  vector with  $e(7) = 1$ ,  $e(j) = 0$ ,  $j \neq 7$ ,  $Q$  is  $9 \times 9$ ,  $R$  is  $4 \times 4$ , and  $W$  is  $9 \times 4$ .

With the quadratic function (A4) used in place of the return function of (A1), the optimization problem can be restated as: choose streams of capital in the market sector, hours in the market sector, hours in the home sector, and investment  $(\{\hat{k}_{mt}, \hat{h}_{mt}, \hat{h}_{nt}, \hat{i}_t\}_{t=0}^{\infty})$  to maximize  $\lim \sum_0^{T-1} \beta^t U(X_t)$  subject to restrictions (A2) and sequences for per-capita hours and capital. To solve this constrained optimization problem in the finite-horizon case, form the Lagrangian

$$\mathcal{L}(\{X_t\}, \{p_t\}) = \sum_{t=0}^{T-1} \beta^t U(X_t) - p_{t+1}' \begin{bmatrix} \hat{k}_{t+1} - (1-\delta)\hat{k}_t - \hat{i}_t/\mu \\ \gamma(L)z_{t+1} - \gamma_0 \end{bmatrix}$$

where the sequence  $\{p_t\}$  are Lagrange multipliers. Differentiating the Lagrangian with respect to decision variables  $\hat{d}_t = (\hat{k}_{mt}, \hat{h}_{mt}, \hat{h}_{nt}, \hat{i}_t)'$ ,  $t = 0, \dots, T-1$ , capital  $\hat{k}_t$ ,  $t = 1, \dots, T$ , and the first element of the multiplier vector  $p_{1t}$ ,  $t = 1, \dots, T$ , and imposing  $\hat{k}_t = k_t$ ,  $\hat{k}_{mt} = k_{mt}$ ,  $\hat{h}_{mt} = h_{mt}$ ,  $\hat{h}_{nt} = h_{nt}$ , and  $\hat{i}_t = i_t$  gives

$$\begin{aligned}
d_t &= -(R + W_2' \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix})^{-1} \left( W_1' \begin{bmatrix} k_t \\ z_t \end{bmatrix} + [0 \ 0 \ 0 \ p_{1t+1}/\mu]' \right) \\
k_t &= (1-\delta)k_{t-1} + [0 \ 0 \ 0 \ 1/\mu] d_{t-1} \\
p_{1t} &= \beta(1-\delta)p_{1t+1} + \beta[q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16}, q_{17}] \begin{bmatrix} k_t \\ z_t \end{bmatrix} + \beta[q_{18} + w_{11}, q_{19} + w_{12}, w_{13}, w_{14}] d_t \\
p_{1T} &= 0
\end{aligned} \tag{A5}$$

where  $d_t = (k_{mt}, h_{mt}, h_{nt}, i_t)'$ ,  $r_{ij}$  is the  $(i, j)$  element of  $R$ ,  $q_{ij}$  is the  $(i, j)$  element of  $Q$ , and  $w_{ij}$  is the  $(i, j)$  element of  $W = [W_1', W_2']'$ . These matrices are functions of partial derivatives of  $u$  and  $f$  evaluated at the steady state values.

If the decision variables are substituted into the difference equation for  $k_t$  and  $p_{1t}$ , then these equations can be written in the following form

$$\begin{bmatrix} k_{t+1} \\ p_{1t} \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} k_t \\ p_{1t+1} \end{bmatrix} + \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} z_t$$

which is solved jointly for  $(k_t, p_{1t+1}, t = 0, \dots, T-1, k_T)$  subject to the given value for  $k_0$ , the sequence  $\{z_t\}_{t=0}^T$  (with  $z_{t+1} = \gamma_0 + \gamma(L)z_t$ ,  $z_0$  given), and  $p_{1T} = 0$ . A solution for this difference equation takes the form

$$p_{1t} = \alpha_{0t} k_t + \alpha'_{it} z_t, \quad t = 1, \dots, T \tag{A6}$$

with  $\alpha_{0T} = 0$ ,  $\alpha_T = 0$  and

$$\begin{aligned}
\alpha_{0t} &= \psi_{21} + \frac{\psi_{11} \psi_{22} \alpha_{0t+1}}{1 - \psi_{12} \alpha_{0t+1}} \\
\alpha'_{it} &= \Phi_2 + \frac{\psi_{22} (\alpha'_{i+1} \gamma_z + \alpha_{0t+1} \Phi_1)}{1 - \psi_{12} \alpha_{0t+1}}
\end{aligned} \tag{A7}$$

which by construction satisfies the difference equation and boundary conditions. The infinite time solution is found by taking limits of the first order conditions as  $T$  approaches infinity. This amounts to finding

$\lim_{t \rightarrow \infty} \alpha_{0t}$  and  $\lim_{t \rightarrow \infty} \alpha_t$ . These limits can be found directly by computing the roots of the two equations in (A7). Note that there are two roots for  $\alpha_0$  that satisfy the quadratic equation

$$\psi_{12}\alpha_0^2 + (\psi_{11}\psi_{22} - 1)\alpha_0 + \psi_{21} = 0.$$

To ensure that  $\lim_{t \rightarrow \infty} \beta^{\frac{t}{2}} k_t = 0$ , it must be the case that the root chosen implies  $\psi_{11}/(1 - \psi_{12}\alpha_0) < 1$ . However, since there are tax distortions, the two values for  $\psi_{11}/(1 - \psi_{12}\alpha_0)$  are not necessarily reciprocal pairs. There may be more than one stable root or no stable roots.

Using the solution for  $p_{1t}$  in (A6), the decision rule for next period capital is

$$k_{t+1} = \frac{\psi_{11}}{1 - \alpha_0\psi_{12}} k_t + \frac{\Phi_1 + \psi_{12}\alpha'\gamma_z}{1 - \alpha_0\psi_{12}} z_t.$$

For the decision functions, substitute

$$p_{1t+1} = \frac{\alpha_0\psi_{11}}{1 - \alpha_0\psi_{12}} k_t + \frac{\alpha_0\Phi_1}{1 - \alpha_0\psi_{12}} z_t$$

into the equation for  $d_t$  in (A5).

*Example.* Consider the specification with  $a_1 = 1$ ,  $\mu = 1$ ,  $q = 1$ ,  $\gamma_1$  diagonal, and

$$\begin{aligned} u(c, \ell) &= \ln(c) + b\ell \\ c_p &= \{a_2 c_m^{b_2} + (1 - a_2) c_n^{b_2}\}^{\frac{1}{b_2}} \\ c_n &= \{a_3 k_n^{b_3} + (1 - a_3)(e^{s_n} h_n)^{b_3}\}^{\frac{1}{b_3}} \\ y &= \{a_4 k_m^{b_4} + (1 - a_4)(e^{s_m} h_m)^{b_4}\}^{\frac{1}{b_4}}. \end{aligned}$$

In this case, the matrices  $\Psi$  and  $\Phi$  of the difference equation (A6) are given by

$$\begin{aligned} \psi_{11} &= 1 - \delta + (1 - \bar{\alpha})\bar{y}/\bar{k}_m \\ \psi_{22} &= 1 + \beta(1 - \bar{\tau}_k)\bar{y}_2\bar{h}_m/\bar{k}_m \\ \psi_{21} &= 0 \\ \psi_{12} &= \left\{ 1 + \frac{\bar{c}_{p2}\bar{c}_n}{\bar{c}_{p1}\bar{c}_m(1 - b_2)} \right\} \bar{c}_m^2 - (1 - \bar{\alpha})(1 - \bar{\tau}_h)\frac{\bar{y}_2^3}{b\bar{y}_{22}} - \{(1 - \bar{\tau}_k)\bar{y}/\bar{k}_m + \delta\bar{\tau}_k\} \\ &\quad \{(1 - \bar{\alpha})\bar{y}/\bar{k}_m\zeta_1 + \frac{b_2\bar{c}_m\bar{k}_n}{1 - b_2}\} - b_2\bar{c}_m\bar{k}_n/(1 - b_2)(1 - \bar{\alpha})\bar{y}/\bar{k}_m \\ \Phi &= \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} -\bar{y} & -b\kappa_3 & -b\kappa_2 & \frac{b\kappa_1(\bar{y}_1 - \delta)}{\bar{y}_2(1 - \bar{\tau}_h)} & \frac{b\kappa_3}{1 - \bar{\tau}_h} & 0 \\ 0 & \frac{b\beta\bar{h}_m(1 - \bar{\tau}_k)}{\bar{k}_m(1 - \bar{\tau}_h)} & 0 & -\frac{b\beta(\bar{y}_1 - \delta)}{\bar{y}_2(1 - \bar{\tau}_h)} & -\frac{b\beta\bar{h}_m(1 - \bar{\tau}_k)}{\bar{k}_m(1 - \bar{\tau}_h)^2} & 0 \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned} \kappa_1 &= (1 - \bar{\alpha})\frac{\bar{y}}{\bar{k}_m}\zeta_1 + \frac{b_2\bar{k}_n\bar{c}_m}{1 - b_2} \\ \kappa_2 &= (1 - \bar{\alpha})\frac{\bar{y}}{\bar{k}_m}\zeta_2 - \frac{b_2\bar{h}_n\bar{c}_m}{1 - b_2} \\ \kappa_3 &= \frac{(1 - \bar{\tau}_k)\bar{h}_m}{(1 - \bar{\tau}_h)\bar{k}_m}\kappa_1 + \frac{(1 - \bar{\alpha})\bar{y}_2^2}{b\bar{y}_{22}} \\ \zeta_j &= \bar{k}_n^2 \frac{\bar{c}_{n1}^{j-1}\bar{c}_{n2}^{4-j}}{b\bar{c}_{n22}\bar{c}_n^2} + (-\bar{h}_n)^{j-1}(-\bar{k}_n)^{3-j} \left( 1 + \frac{\bar{c}_{p1}\bar{c}_m}{\bar{c}_{p2}\bar{c}_n(1 - b_2)} \right), \quad j = 1, 2, 3 \end{aligned}$$

Note also that if the root of the quadratic equation (A9) is chosen to ensure  $\lim_{t \rightarrow \infty} \beta^{\frac{t}{2}} k_t = 0$ , then  $\alpha_0 = (1 - \psi_{11} \psi_{22}) / \psi_{12}$ . This value of  $\alpha_0$  is used to determine  $\alpha$  (from (A8)) and the optimal investment rule (A10).

Substitution of the shadow prices into the decision functions gives the decision function  $(d_t - \bar{d}) = (A - BC)(x_t - \bar{x})$  where  $d_t = [k_{mt}, h_{mt}, h_{nt}, i_t]'$ ,  $x_t = [k_t, z_t]'$ ,

$$A = \begin{bmatrix} 1 & 0 & -\frac{\bar{h}_m b \zeta_1 (1 - \bar{\tau}_k)}{k_m (1 - \bar{\tau}_h)} & -b \zeta_2 & \frac{b \zeta_1 (\bar{y}_1 - \delta)}{\bar{y}_2 (1 - \bar{\tau}_h)} & \frac{\bar{h}_m b \zeta_1 (1 - \bar{\tau}_k)}{k_m (1 - \bar{\tau}_h)^2} \\ \frac{\bar{h}_m}{k_m} & 0 & -\bar{h}_m - \frac{\bar{y}_2}{\bar{y}_{22}} - \frac{\bar{h}_m^2 b \zeta_1 (1 - \bar{\tau}_k)}{k_m^2 (1 - \bar{\tau}_h)} & -\frac{\bar{h}_m b \zeta_2}{k_m} & \frac{\bar{h}_m b \zeta_1 (\bar{y}_1 - \delta)}{k_m \bar{y}_2 (1 - \bar{\tau}_h)} & \frac{\bar{h}_m^2 b \zeta_1 (1 - \bar{\tau}_k)}{k_m^2 (1 - \bar{\tau}_h)^2} + \frac{\bar{y}_2 / \bar{y}_{22}}{1 - \bar{\tau}_h} \\ 0 & 0 & -\frac{\bar{h}_m b \zeta_2 (1 - \bar{\tau}_k)}{k_m (1 - \bar{\tau}_h)} & -\bar{h}_n - b \zeta_3 & \frac{b \zeta_2 (\bar{y}_1 - \delta)}{\bar{y}_2 (1 - \bar{\tau}_h)} & \frac{\bar{h}_m b \zeta_2 (1 - \bar{\tau}_k)}{k_m (1 - \bar{\tau}_h)^2} \\ \frac{(1 - \alpha) \bar{y}}{k_m} & -\bar{y} & -b \kappa_3 & -b \kappa_2 & \frac{b \kappa_1 (\bar{y}_1 - \delta)}{\bar{y}_2 (1 - \bar{\tau}_h)} & \frac{b \kappa_3}{1 - \bar{\tau}_h} \end{bmatrix}$$

and  $B$  and  $C$  are  $4 \times 1$  and  $1 \times 6$  vectors, respectively, with elements

$$B_1 = \frac{\bar{h}_m}{k_m} \bar{y}_2 \zeta_1 (1 - \bar{\tau}_k) + \bar{y}_2 (1 - \bar{\tau}_h) (\zeta_2 - \bar{k}_n / b)$$

$$B_2 = \frac{\bar{h}_m}{k_m} \left( \frac{\bar{h}_m}{k_m} \bar{y}_2 \zeta_1 (1 - \bar{\tau}_k) + \bar{y}_2 (1 - \bar{\tau}_h) (\zeta_2 - \bar{k}_n / b) \right) + \frac{\bar{y}_2^2 (1 - \bar{\tau}_h)}{b \bar{y}_{22}}$$

$$B_3 = \frac{\bar{h}_m}{k_m} \bar{y}_2 \zeta_2 (1 - \bar{\tau}_k) + \bar{y}_2 (1 - \bar{\tau}_h) (\zeta_3 + \bar{h}_n / b)$$

$$B_4 = -\psi_{12}$$

$$C_1 = \psi_{12}^{-1} (\psi_{22}^{-1} - \psi_{11})$$

$$C_{j+1} = \frac{1 - \psi_{11} \psi_{22}}{\psi_{12} \psi_{22} (\psi_{11} - \gamma_{1,jj})} \Phi_{1j} + \frac{\gamma_{1,jj}}{\psi_{22} (\psi_{11} - \gamma_{1,jj})} \Phi_{2j}, \quad j = 1, \dots, 5.$$

These rules can be used to determine the effect of our key parameters. For example, if  $a_2 = 1$ , private consumption is comprised only of market consumption.<sup>11</sup> Since  $\zeta_j = 0$ ,  $j = 1, 2, 3$  in this case, all coefficients in the equation for market capital are 0 with the exception of the first which is 1 (e.g.  $k_{mt} = k_t$ ). Similarly, the coefficients in the equation for hours of work at home are 0 since no hours will be allocated. If  $a_2 > 0$  but  $b_2 = 0$ , then the market and home goods are complements and private consumption is given by  $c_p = c_m^{a_2} c_n^{1-a_2}$ . If, in addition, home consumption is given by a Cobb-Douglas form,  $c_n = k_n^{a_3} (e^{s_n} h_n)^{1-a_3}$ , then  $\zeta_2 = 0$ ,  $\zeta_3 = -\bar{h}_n / b$ , and  $h_{nt} = \bar{h}_n$  for all  $t$ . With  $\zeta_2 = 0$  and  $b_2 = 0$ , the shock to home technology does not affect any market decisions. ■

<sup>11</sup> This is the case analyzed by McGrattan (1992).

## Appendix B

The effective tax rates for labor and capital used to estimate the model are given in Table B. The data sources for these series are *Statistics of Income, Individual Income Tax Returns* (Sources of Income and Taxable Income, all returns) and *Social Security Bulletin* (Tables 2a,4b). The rates are constructed using the definitions of Joines (1981), series MTRK1, MTRL1. One important difference between  $\tau_{k,t}$  of Table B and MTRK1 in Joines is the treatment of property taxes. MTRK1 is the sum of a proportional tax on income that is not specific to capital or labor, a proportional tax on income that is specific to capital, and a nonproportional tax on income that is specific to capital. The proportional tax on income that is specific to capital is simply tax receipts from capital divided by income from capital. We exclude property taxes from both tax receipts and Joines' measure of income which includes indirect business taxes.

$t$	$\tau_{kt}$	$\tau_{ht}$	$t$	$\tau_{kt}$	$\tau_{ht}$	$t$	$\tau_{kt}$	$\tau_{ht}$
1947	62.8	20.0	1961	61.6	21.9	1975	55.8	24.9
1948	54.8	17.2	1962	57.3	22.0	1976	56.9	25.0
1949	52.4	17.2	1963	57.3	22.4	1977	54.3	25.6
1950	64.2	18.3	1964	55.2	21.2	1978	53.4	25.6
1951	66.9	19.8	1965	53.4	20.5	1979	53.3	26.3
1952	63.9	20.8	1966	53.4	21.1	1980	54.5	27.9
1953	64.4	21.0	1967	54.4	21.4	1981	50.2	28.4
1954	61.1	19.4	1968	59.8	23.0	1982	49.6	27.4
1955	59.8	19.6	1969	60.4	24.1	1983	47.5	26.6
1956	61.5	20.2	1970	55.4	24.6	1984	46.6	26.1
1957	60.4	20.9	1971	57.7	23.3	1985	47.8	26.1
1958	60.7	20.7	1972	57.3	23.5	1986	52.9	25.9
1959	60.4	21.3	1973	57.0	24.0	1987	53.8	25.7
1960	60.6	21.6	1974	59.3	25.1			

Table B. Effective Tax Rates for Capital and Labor

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Function	Parameter Estimates
$u(c, \ell) = \frac{(c^b \ell^{1-b})^{1-\sigma}}{1-\sigma}$	$b = .448, \sigma = 5.27$ (.121) (6.84)
$c = \{a_1 c_p^{b_1} + (1 - a_1) c_g^{b_1}\}^{\frac{1}{b_1}}$	$a_1 = 1.0, b_1 = 0$
$c_p = \{a_2 c_m^{b_2} + (1 - a_2) c_n^{b_2}\}^{\frac{1}{b_2}}$	$a_2 = .485, b_2 = .385$ (.0591) (.145)
$c_n = \{a_3 k_n^{b_3} + (1 - a_3)(e^{s_n} h_n)^{b_3}\}^{\frac{1}{b_3}}$	$a_3 = .210, b_3 = .200$ (.0810) (.165)
$y = \{a_4 k_m^{b_4} + (1 - a_4)(e^{s_m} h_m)^{b_4}\}^{\frac{1}{b_4}}$	$a_4 = .234, b_4 = .0525$ (.120) (.381)
$\tilde{k} = (1 - \delta)k + i/\mu$	$\delta = .0223, \mu = 1.0054$ (.000311) (.000433)
$\beta$	$\beta = .991,$ (.0150)

Table 1a. Parameters of Preferences and Technology (Geometric Trend)

Function	Parameter Estimates
$u(c, \ell) = \frac{(c^b \ell^{1-b})^{1-\sigma}}{1-\sigma}$	$b = .427, \sigma = 5.29$ (.158) (7.62)
$c = \{a_1 c_p^{b_1} + (1 - a_1) c_g^{b_1}\}^{\frac{1}{b_1}}$	$a_1 = 1.0, b_1 = 0$
$c_p = \{a_2 c_m^{b_2} + (1 - a_2) c_n^{b_2}\}^{\frac{1}{b_2}}$	$a_2 = .477, b_2 = .285$ (.118) (.184)
$c_n = \{a_3 k_n^{b_3} + (1 - a_3)(e^{s_n} h_n)^{b_3}\}^{\frac{1}{b_3}}$	$a_3 = .158, b_3 = .247$ (.0704) (.196)
$y = \{a_4 k_m^{b_4} + (1 - a_4)(e^{s_m} h_m)^{b_4}\}^{\frac{1}{b_4}}$	$a_4 = .228, b_4 = .106$ (.114) (.200)
$\tilde{k} = (1 - \delta)k + i/\mu$	$\delta = .0224, \mu = 1.0$ (.000114)
$\beta$	$\beta = .989$ (.00435)

Table 1b. Parameters of Preferences and Technology (Data Filtered)



$$z_{t+1} = (\alpha \quad s_m \quad s_n \quad \tau_k \quad \tau_h)'_{t+1} = \begin{pmatrix} -0.000542 & .108 & 0 & .0455 & .0193 \\ (.0358) & (.0895) & & (.0145) & (.0157) \end{pmatrix}$$

$$+ \begin{pmatrix} 1.56 & .0151 & -.00336 & -.0917 & .514 \\ (.115) & (.0512) & (.00642) & (.307) & (.311) \\ .356 & .880 & -.0146 & .159 & -.0817 \\ (.186) & (.0952) & (.0163) & (.596) & (.605) \\ 0 & 0 & .954 & 0 & 0 \\ & & (.0787) & & \\ .0182 & -.0505 & .0108 & 1.68 & .106 \\ (.0584) & (.0526) & (.00605) & (.129) & (.116) \\ -.0754 & -.0252 & .00338 & .0432 & 1.70 \\ (.0347) & (.0201) & (.00253) & (.109) & (.104) \end{pmatrix} z_t$$

$$+ \begin{pmatrix} -.598 & -.00890 & .00431 & .0975 & -.540 \\ (.113) & (.0514) & (.00674) & (.284) & (.324) \\ -.202 & .0560 & -.000639 & -.136 & -.0101 \\ (.170) & (.102) & (.0168) & (.573) & (.635) \\ 0 & 0 & .00558 & 0 & 0 \\ & & (.0700) & & \\ .00663 & .0435 & -.0105 & -.714 & -.166 \\ (.0570) & (.0516) & (.00621) & (.126) & (.124) \\ .0889 & .0173 & -.00538 & -.0388 & -.740 \\ (.0305) & (.0211) & (.00246) & (.104) & (.112) \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} .00407 & 0 & 0 & 0 & 0 \\ (.000591) & & & & \\ .00210 & .0113 & 0 & 0 & 0 \\ (.00120) & (.00222) & & & \\ .0396 & -.0422 & .0763 & 0 & 0 \\ (.0100) & (.0188) & & & \\ -.000150 & .00125 & -.00101 & .000877 & 0 \\ (.000407) & (.000711) & (.000474) & (.000286) & \\ -.0000204 & .000446 & -.000367 & .000303 & .00119 \\ (.000206) & (.000261) & (.000160) & (.000358) & (.000217) \end{pmatrix} \epsilon_t$$

Figure 1a. Vector Autoregression for  $z$  (Geometric Trend)

$$\zeta_t = \zeta_t^* + \omega_t, \quad \omega_t = D\omega_{t-1} + \eta_t$$

$$D_{ii} = (0, 0, .991, 0, 0, 0, .990, .986) \\ (.0144) \quad (.0123) \quad (.0146)$$

$$\Omega_{ii} = (21.4, 39.1, 6.75, 3253, 0, 0, 4.26e-5, 1e-7) \\ (9.15) (26.8) (.691) (511) \quad (4.49 \times 10^{-6}) \quad (2.29 \times 10^{-7})$$

Figure 2a. Measured ( $\zeta$ ) and Actual Data ( $\zeta^*$ ) (Geometric Trend)

$$z_{t+1} = (\alpha \quad s_m \quad s_n \quad \tau_k \quad \tau_h)'_{t+1} = \begin{pmatrix} -.0243 & .347 & 0 & .107 & .0603 \\ (.0497) & (.174) & & (.0754) & (.0371) \end{pmatrix}$$

$$+ \begin{pmatrix} 1.35 & .0245 & -.00770 & -.0563 & .557 \\ (.0834) & (.0599) & (.0459) & (.122) & (.399) \\ .366 & .729 & -.0304 & .0545 & .180 \\ (.286) & (.142) & (.159) & (.406) & (.985) \\ 0 & 0 & .967 & 0 & 0 \\ & & (.210) & & \\ .0658 & -.0709 & .0190 & 1.49 & .639 \\ (.116) & (.107) & (.0779) & (.196) & (.644) \\ -.0350 & -.0588 & -.0463 & .0171 & 1.56 \\ (.0581) & (.0466) & (.0210) & (.0991) & (.225) \end{pmatrix} z_t$$

$$+ \begin{pmatrix} -.532 & .00118 & .00714 & -.0188 & -.203 \\ (.0695) & (.0524) & (.0428) & (.138) & (.367) \\ -.0848 & .00496 & -.0238 & .178 & -.0692 \\ (.190) & (.112) & (.161) & (.430) & (.942) \\ 0 & 0 & .0254 & 0 & 0 \\ & & (.206) & & \\ .0580 & .0473 & -.0294 & -.634 & -.708 \\ (.122) & (.103) & (.0724) & (.200) & (.542) \\ .0810 & .0161 & .0357 & .0168 & -.683 \\ (.0625) & (.0372) & (.0190) & (.0948) & (.220) \end{pmatrix} z_{t-1}$$

$$+ \begin{pmatrix} .00265 & 0 & 0 & 0 & 0 \\ (.000322) & & & & \\ -.000121 & .00824 & 0 & 0 & 0 \\ (.000952) & (.00285) & & & \\ .00184 & -.00535 & .0177 & 0 & 0 \\ (.00375) & (.00953) & & & \\ .000401 & .00145 & -.000848 & .00224 & 0 \\ (.000564) & (.000493) & (.000936) & (.000670) & \\ .000167 & .0000712 & -.000282 & .000629 & .00000353 \\ (.000271) & (.000321) & (.000405) & (.000491) & (.0682) \end{pmatrix} \epsilon_t$$

Figure 1b. Vector Autoregression for  $\mathbf{z}$  (Data Filtered)

$$\zeta_t = \zeta_t^* + \omega_t, \quad \omega_t = D\omega_{t-1} + \eta_t$$

$$D_{ii} = (.938, .901, .648, .782, .790, .773, .832, .501) \\ (.0747)(.0929) (.0478) (.104) (.203) (.292) (.303) (.187)$$

$$\Omega_{ii} = (1106, 674, 4.36, 19.7, 11.9, 30.1, 1.63 \times 10^{-5}, 1.13 \times 10^{-7}) \\ (183) (206) (.434) (8.32) (6.87) (23.9) (2.69 \times 10^{-6}) (1.98 \times 10^{-7})$$

Figure 2b. Measured ( $\zeta$ ) and Actual Data ( $\zeta^*$ ) (Data Filtered)

$z$	Data ( $\zeta$ )		Model ( $C\hat{x}$ )	
	std(ln( $z$ ))	corr( $z, y$ )	std(ln( $z$ ))	corr( $z, y$ )
$k$	4.08	0.67	4.07	0.66
$k_n$	4.94	0.62	4.94	0.61
$h_m$	3.09	0.32	4.04	0.98
$i$	7.18	0.44	9.04	0.74
$c_g$	20.23	0.62	20.17	0.61
$y$	9.24	1.00	9.21	1.00
$t_k$	8.24	0.76	3.85	0.71
$t_h$	12.76	-0.72	5.69	-0.70
$k_m$	3.38	0.69	3.34	0.68
$c_m$	8.23	0.91	6.89	0.82
$i_m$	7.92	0.40	13.23	0.62
$i_n$	9.57	0.31	14.01	0.52

Table 2a. Means and Standard Deviations for Data and Model (Geometric Growth)

$z$	Data ( $\zeta$ )		Model ( $C\hat{x}$ )	
	std(ln( $z$ ))	corr( $z, y$ )	std(ln( $z$ ))	corr( $z, y$ )
$k$	0.37	0.64	0.34	0.26
$k_n$	0.53	0.63	0.53	0.34
$h_m$	1.52	0.69	0.94	0.99
$c_g$	5.62	0.52	5.34	0.55
$i$	5.06	0.62	4.84	0.57
$y$	1.94	1.00	1.94	1.00
$t_k$	3.11	0.12	1.82	0.49
$t_h$	2.53	0.25	2.40	0.26
$k_m$	0.35	0.45	0.34	-0.01
$c_m$	1.21	0.85	1.29	0.93
$i_m$	5.26	0.63	9.95	0.47
$i_n$	5.75	0.63	8.52	0.00

Table 2b. Means and Standard Deviations for Data and Model (Data Filtered)

Series	(a) Lump-Sum Tax		(b) No Capital Tax	
	With Household Sector	Without Household Sector	With Household Sector	Without Household Sector
$y$	42.7	57.8	13.7	21.3
$c_m$	46.6	56.7	6.0	8.6
$i$	82.9	133.0	44.6	79.1
$k_m$	123.7	133.0	77.4	79.1
$h_m$	22.2	22.1	-3.0	-6.2
$c_n$	-1.4		2.2	
$k_n$	33.7		5.1	
$h_n$	-19.7		0.4	
$\Delta$	0.221	0.278	0.106	0.147

Table 3. Percent changes between case (a)  $\tau_k = 0$ ,  $\tau_h = 0$  or case (b)  $\tau_k = 0$ ,  $\tau_h = 0.31$  (with household production)  $\tau_h = 0.365$  (without household production) and the base case ( $\tau_k = 0.57$ ,  $\tau_h = 0.23$ ).

Series	Tax on Residential Capital ( $\tau_p$ )		
	0.01	0.02	0.03
$y$	-12.8	-20.5	-25.8
$c_m$	-12.4	-20.8	-26.9
$i$	-25.5	-38.5	-46.5
$k_m$	-39.3	-57.7	-68.1
$h_m$	-1.6	-2.0	-1.9
$c_n$	-1.8	-3.4	-4.7
$k_n$	-10.1	-17.1	-22.3
$h_n$	3.8	6.3	8.1
$\Delta$	0.147	0.279	0.400

Table 4. Percent changes between cases with  $\tau_p > 0$  and base case ( $\tau_p = 0$ ).