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Understanding 100 Years of the Evolution of Top Wealth Shares in the U.S.: What is the Role of Family Firms?

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Abstract

We use a simple random growth model to study the role of changing dynamics of family firms in shaping the evolution of top wealth shares in the United States over the course of the past century. Our model generates a time path for top wealth shares. The path is remarkably similar to those found by Saez and Zucman (2016) and Gomez (2019) when the volatility of idiosyncratic shocks to the value of family firms is similar to that found for public firms by Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016). We also show that consideration of family firms contributes not only to overall wealth inequality but also to considerable upward and downward mobility of families within the distribution of wealth. We interpret our results as indicating that improving our understanding of how families found new firms and eventually diversify their wealth is central to improving our understanding of the distribution of great wealth and its evolution over time.

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1 Introduction

Research by Saez and Zucman (2016); Smith, Zidar, and Zwick (2020); and others indicates that the share of total wealth held by the wealthiest Americans has changed substantially over the past 100 years. Top wealth shares have followed a U-shaped pattern — they were high in the 1920s, fell to a low around 1980, and have risen substantially since then (see Figure 1). Wealth based on dynastic family ownership of large firms both private and public is a striking feature of capitalist economies worldwide. In this paper we ask, what is the role of family firms in shaping the dynamics of top wealth shares in the United States over the past century?

Families with a concentrated ownership position in a single family firm constitute a large fraction of the wealthiest families in America. Owing to the concentration of their portfolios in the shares of a single firm, these families are subject to highly volatile idiosyncratic shocks to their wealth. Research by Herskovic et al. (2016) and others indicates that the volatility of idiosyncratic shocks to firm value, at least for publicly traded firms, has changed substantially over the past century. The idiosyncratic volatility of firm value has also followed a U-shaped pattern over the past century — the volatility of idiosyncratic shocks to firm value was high in the 1920s, it reached a low in the early 1950s, and has risen substantially since then (see Figure 2). We explore the hypothesis that the evolution of the shares of wealth held by families in the top percentiles of the distribution of wealth of the United States over the last 100 years can be accounted for by changes in the volatility of idiosyncratic shocks to the value of family firms over this time period.

We present a purely statistical model of the evolution of the distribution of wealth similar to that in Champernowne (1953); Luttmer (2016); Gabaix, Lasry, Lions, and Moll (2016); and Benhabib, Bisin, and Luo (2017) that adds consideration of the role of family firms in shaping the distribution of wealth. We find that when we feed into the model a time series for the volatility of idiosyncratic shocks to firm value similar to that observed by Herskovic et al. (2016) for public firms over the past 100 years, our model generates a time path for top wealth shares over this time period

similar to that found by Saez and Zucman (2016) and Gomez (2019) (see Figure 8). Of particular interest is the observation that the transition dynamics of the model lead to a roughly 30 year lag between the minimum point of firm volatilities and the minimum point of top wealth shares. Likewise, the model implies a roughly 30 year lag between the increase in firm volatilities observed in the stock market and increases in top wealth shares. This lag emerges endogenously from the dynamics of the model.

We also explore our model's implications for the mobility of families within the overall distribution of wealth once family firms are taken into account. Building on the work of Gabaix et al. (2016), Luttmer (2016), and Gomez (2019), we establish analytically a tight link between our model's quantitative success in explaining the transition dynamics of top wealth shares and its implications for the degree of wealth mobility experienced by families starting out at the bottom of the distribution of wealth, as well as its implications for the rate at which families starting out with great fortunes dissipate their wealth.

A continuum of family dynasties that live forever and are subject to both common and idiosyncratic shocks to wealth populates our model. The family dynasties in our model have one of two types. The vast majority are what we term the *diversified* type. These dynasties hold portfolios of assets that are subject to a relatively small amount of idiosyncratic risk. A small minority of dynasties are what we term the *family firm* type. These dynasties hold portfolios of assets subject to a large amount of idiosyncratic risk. We interpret these dynasties as holding a concentrated position in a single firm, which we term a *family firm*. We focus on family firms over and above the traditional notion of entrepreneurship (see, for example, Cagetti and De Nardi (2009) and Quadrini (2009)) because we allow for the possibility that these dynasties may maintain a concentrated ownership position in the family firm for multiple generations.

Family dynasties in our model switch type at random. We interpret a dynasty's switch from holding a diversified portfolio to a concentrated position in a family firm

as the founding of a new family business by a previously diversified dynasty. We interpret a dynasty's switch from holding a concentrated position in a family firm to a diversified portfolio as occurring when the firm's founding family sells the bulk of its ownership stake and diversifies its portfolio. We interpret these switching probabilities as standing in for an underlying economic process in which entrepreneurs and their families found new firms and hold a concentrated position in them while these firms are young. Eventually, they sell out and diversify their wealth.

The quantitative success of our model in accounting for the evolution of top wealth shares in the United States over the past 100 years rests heavily on our assumption that those dynasties with a concentrated investment in a single family firm face a great deal of idiosyncratic risk in the returns to their wealth — a level of idiosyncratic risk on par with that found for individual publicly traded stocks in Herskovic et al. (2016). It is also considerably higher than levels found in available studies of the idiosyncratic risk in returns to wealth even for wealthy families in panel data.[1] Our model can be reconciled with these data on the idiosyncratic volatility of family wealth for wealthy families, however, because of our assumption that our model economy is populated by two types of families facing different levels of idiosyncratic risk in returns to wealth. Thus, our model predicts that the distribution of idiosyncratic returns to wealth for families at any initial level of wealth is a mixture of two distributions of returns. This feature is decisive for reconciling the data in Gomez (2019) on the average standard deviation and excess kurtosis of innovations to wealth for the Forbes 400 with the large amount of idiosyncratic volatility faced by family firms in the calibration of our model.

We derive both analytical and numerical results regarding the connections between the mobility of families within the distribution of wealth and the transition dynamics of top wealth shares. We derive analytical results regarding the link beetween wealth mobility and the transition dynamics of top wealth shares in a simplified version of our model in which all families are identical. We refer to this simplified version of our

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[1]See, for example Bach, Calvet, and Sodini (2018); Fagereng, Guiso, Malcrino, and Pistaferri (2020); and Gomez (2019).

model as the *one-type* version of our model.[2] In particular, we show analytically that in our one-type model, there is a tight connection between the speed of transition of top wealth shares and the degree of upward wealth mobility for families starting at the bottom of the distribution of wealth. Moreover, in our one-type model, given a level of volatility of idiosyncratic risk in returns to wealth, there is a direct connection between the model's implications for top wealth shares in the steady-state and the rate at which dynasties that start high up in the distribution of wealth dissipate their fortunes. Specifically, holding fixed the model's implications for top wealth shares, the higher the volatility of idiosyncratic shocks to wealth is, the faster the dissipation of great fortunes relative to aggregate wealth is. We provide an extension of our analytical transition results to the two-type version of our model in the Appendix.

Motivated by these analytical results, for our one-type model, we examine the quantitative implications of our full model which includes family firms for both upward and downward wealth mobility. We find that after 50-70 years, the probability that a family that started at the bottom of the distribution of wealth ends up in the very top wealth percentiles approaches quite closely to the unconditional probability that any family is in these very top wealth percentiles. We also consider our model's implications for the dissipation of great fortunes. We compare the implications of our model in this dimension with data in Gomez (2019) on the level of turnover in the Forbes 400 from the period 1983 to the present and find that the two match well.

We show that policy changes that speed up the rate at which families with concentrated ownership positions in family firms sell those positions and diversify their portfolios would also have a powerful impact on the equilibrium distribution of wealth. We illustrate this point with an experiment. We speed up the rate at which dynasties switch from the family firm to the diversified type and find that holding all other parameters fixed, our model implies a dramatic reduction in top wealth shares.

We interpret our results as indicating for researchers going forward that improving

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[2]Our model implies a stationary distribution of wealth across dynasties in the long run because we assume that there is a minimum level of the idiosyncratic component of dynastic wealth that serves as a reflecting barrier as in Gabaix et al. (2016) andChampernowne (1953).

our understanding of how families found new firms and eventually diversify their wealth is central to improving our understanding of the distribution of great wealth and its evolution over time.[3]

Our paper follows a large literature on the dynamics of the distribution of income and wealth. To allow for an analytical comparison of our results with those in the literature, particularly those in Gabaix et al. (2016), we keep the model very simple. In particular, motivated by the results in Benhabib et al. (2017), we abstract from idiosyncratic risk in labor earnings' role in shaping the wealth distribution in our model and focus instead on the role of idiosyncratic risk in the returns to wealth in leading to the large shares of wealth held by those in the very top wealth percentiles as observed in the data. Benhabib, Bisin, and Luo (2019) and Hubmer, Krusell, and Smith (Forthcoming) study much richer models than ours, and they look to account for the distribution of wealth across a broad range of wealth percentiles. Our work differs from theirs in that we focus our attention on the evolution shares of wealth held by families in wealth percentiles above the top 1%, including a very top percentile corresponding to the Forbes 400. The model in Aoki and Nirei (2017) is most similar to ours in that they have only a small fraction of agents exposed to a large amount of idiosyncratic risk due to the concentration of their portfolios in the shares of a single firm. They study the evolution of top incomes shares in the United States in response to changes in tax rates. Jones and Kim (2018) also use a related model with creative destruction to study the evolution of top incomes shares in the United States in response to changes in tax rates.

Our paper is organized as follows. In section 2, we review the key data we use in calibrating and evaluating our model. In section 3, we present our model. In section 4, we present analytical results in the one-type version of our model. In section 5, we present the quantitative results from our full model. In section 6, we conclude. In the appendix in section provide an extension of our analytical transition result in the two-type version of our model and our procedure for mapping our calibration moments to the parameters of the model.

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[3]See Bertrand and Schoar (2006) for a discussion of the economics of family firms.

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2 Review of Key Data

In this section we review the data that we target with our model. We also review some of the data available on the prevalence of family firms among top firms in the United States using several alternative definitions of such firms.

Data on top wealth shares We ask whether our model can reproduce the evolution of top wealth shares in the United States over the past century. We target specifically the data on these shares reported in Saez and Zucman (2016). We also compare the model's implications with data on the evolution of the wealth share of the Forbes 400 since the early 1980s, as reported in Gomez (2019) and Zheng (2019). In figure 1, taken from Smith et al. (2020), we show the data on the evolution of various alternative estimates of the wealth share of the top 0.1% over the past century.
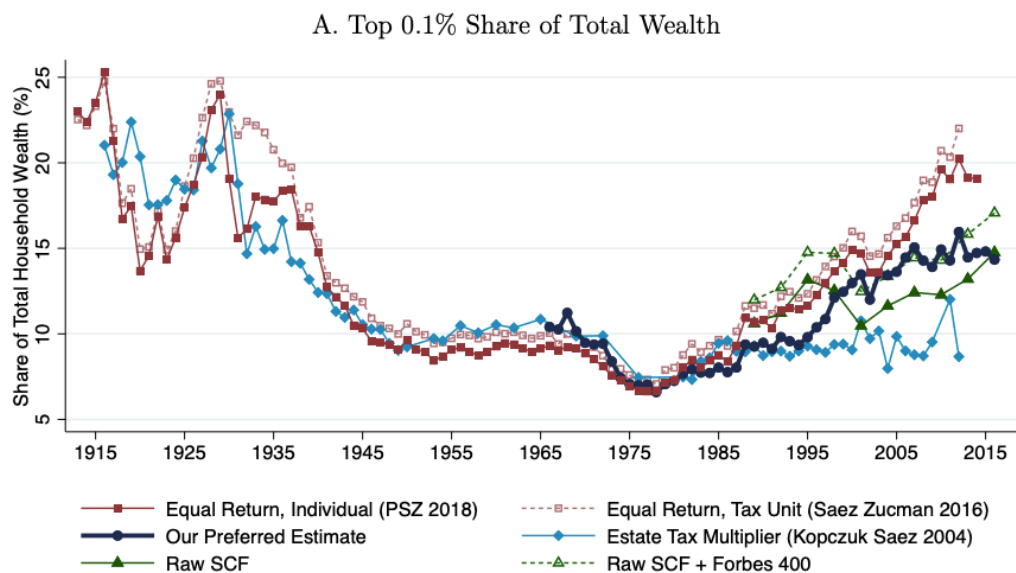
Figure 1: Evolution of the wealth share of the top 0.1% over the past century as shown in Smith et al. (2020)

As shown in this figure, the share of wealth held by the top 0.1% of the wealth distribution has followed a distinctive U-shape: this share was high in the 1920s, it reached a minimum around 1980, and has risen since then.

This U-shaped pattern of top wealth shares extends to very top percentiles of the wealth distribution. In table 1, we show data on a variety of top wealth shares at various dates, as reported in Saez and Zucman (2016). Gomez (2019) and Zheng (2019) also report a large increase in the share of wealth held by the Forbes 400 from the early 1980s to the present — its share increase by a factor of 3.5 over this time period.

% of total Household Wealth

Year	top 1%	top 0.1%	top 0.01%	top 0.00025%
1925	43.1	18.6	6.7	
1950	30.5	10.6	2.8	
1975	24.7	7.6	2.4	
1982	25.7	9.4	3.3	.83
2000	34.1	16.0	6.9	3.04
2012	41.8	22.0	11.2	3.09

Table 1: Top wealth shares from Saez and Zucman (2016)

Vermuelen (2018) reports on the tail coefficient of the distribution of top wealth implied by these top wealth shares.[4] In the calibration of the initial steady-state of our two-type model described below, we target a top tail coefficient of the distribution of gross assets of 1.5.

Data on wealth volatility   When we calibrate our model, we target the volatility of innovations to wealth for very wealthy individuals from one year to the next. Gomez (2019) reports on the moments of annual changes in the logarithm of wealth

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[4]It is common to model the right tail of the distribution of wealth as if it were the right tail of a Pareto distribution. The shape coefficient of a Pareto distribution fit to the right tail of the distribution of wealth is referred to as the tail coefficient of the wealth distribution.

for members of the Forbes 400. We reproduce these moments from his paper in table 2. In our calibration of our model, we aim to match the volatility and excess kurtosis of innovations to the logarithm of wealth reported in this table.

Forbes 400	Volatility	Skewness	Excess Kurtosis
1983-2016	0.27	-0.35	4.70

Table 2: Summary statistics on log wealth growth of Forbes 400 from Gomez (2019) table 2.

Two recent papers use more comprehensive panel data on wealth, drawn from Scandanavian wealth tax files, to study the distribution of innovations to wealth. Bach et al. (2018) report data on the idiosyncratic volatility of gross wealth for households in Sweden. They find that the volatility of innovations to wealth for the very wealthy is considerably higher than for those lower down in the wealth distribution. They report that the idiosyncratic volatility of returns on gross wealth range from 1.6% per year in the bottom decile to 6% per year for the top 10%-5%, 13.4% per year for the top 1%-0.5%, and 34.7% per year for the top 0.01%.

Fagereng et al. (2020) report on the volatility of innovations to wealth drawing from Norwegian data. In their table 3, they report a standard deviation of 22% for pre-tax returns to wealth of and a standard deviation of 15% for after-tax returns. Consistent with the findings of Gomez (2019), for both measures of returns, they report a large kurtosis of the distribution of returns.

Data on the evolution of the idiosyncratic volatility of firm values   The only factor driving changes in top wealth shares in the baseline simulations of our model is changes in the idiosyncratic volatility in firm value experienced by families with a concentrated ownership position in a single firm. To calibrate the time path of this idiosyncratic volatility in firm value, we rely on the estimates reported in Herskovic et al. (2016) on the evolution of the average idiosyncratic volatility of individual publicly traded stocks in the United States since 1926. Figure 2, reproduced from that paper, shows their result.

Of particular interest is the observation that the volatility of the idiosyncratic component of equity returns since 1926 also follows a U-shaped pattern, but the minimum of this U-shape occurs in the early 1950s, some 25-30 years before the minimum in top wealth shares reported on above. We show below that our model reproduces quite well this lag between the dynamics of firm volatility and the dynamics of top wealth shares.

Panel B: Average Volatility

Figure 2: Evolution of the average of the volatility of the idiosyncratic component of individual stock returns since 1926 as reported in Herskovic et al. (2016)

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Data on the prevalence of large family firms   Central to our model is the assumption that a minority of families hold concentrated stakes in a single firm.

We refer to these families as owning family firms. Here we review evidence on the prevalence of family firms among the wealthiest families and largest firms in the United States over the past 100 years.

In a precursor to the work of Saez and Zucman (2016), Lundberg (1937) used tax data made public in 1924 to estimate the income and wealth at that date of what he termed "America's 60 Families". He highlights the role of concentrated ownership in family firms as the source of these large fortunes.[5]

Further evidence of the importance of family firms in pre-WWII U.S. top fortunes is available in Goldsmith (1940). In 1937, the U.S. Congress commissioned Raymond Goldsmith to lead a study of the question of who owned the shares of the top 200 non-financial firms traded on the New York Stock Exchange. In Chapter 7 of this volume, Goldsmith organizes the data on share ownership by family interest group and reports three striking observations. Of the interest groups represented by America's wealthiest families, Goldsmith makes the following observations:

1. "Each interest group shows a strong tendency to keep its holdings concentrated in the enterprise in which the family fortune originated."

2. "Only a few of the large fortunes represented among the 20 largest record shareholdings appear to be on the way to a diversified state."

3. "None of the three largest family interest groups seem to be in this state [...] the Mellon group is now in the third generation, while the Rockefeller and Dupont groups are mainly in the second and partly in the third generation. Most of the other interest groups encountered in the study are also of the second or third generation".

We take these observations as motivation for our assumptions regarding undiversified portfolio holdings' prevalence and persistence across multiple generations for

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[5]Lundberg (1937) also highlights the role of intermarriage between heirs to large fortunes in concentrating wealth. Consideration of family demographics would be an interesting extension of our model.

many of the wealthiest dynasties in our model.

The available evidence suggests that this pattern of concentration of family wealth in a family firm over multiple generations persists today in the United States even for some very large, publicly traded firms.[6]

Anderson and Reeb (2003) report that family firms constitute over 35% of the S&P 500, with founding families owning on average 18% of the equity of the firm. Villalonga and Amit (2006) report that family control is prevalent in 37% of the firms in the Fortune 500.[7] More recently, two public lists of the largest family firms worldwide that corroborate these findings have been created.

Ernst & Young, in cooperation with the University of St. Gallen, has constructed a family business index.[8] This index is a worldwide database of the 500 largest family owned companies (both public and private) by revenue. To qualify as a family firm, the family must be in the second generation or more and have ownership of at least 32% of the shares of the firm. This index lists 122 U.S. firms, 82 of which are private and 40 of which are public.

Credit Suisse has constructed a list of the 1000 largest public family owned firms worldwide in Klerk, Kersley, Bhatti, and Vair (2018). To qualify as a family firm, the founding family must have ownership or voting rights totalling over 20%. This list includes 121 US Firms. Of these 121 U.S. Firms, over 80 are in the second or greater generation of family ownership.

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[6]See Smith, Yagan, Zidar, and Zwick (2019) for a study of the importance of business income in top incomes in the U.S. today. Here, we are focused on the prevalence of family ownership even of very large firms, both private and public, over multiple generations.

[7]Note that the Fortune 500 is a ranking of firms by sales. It includes both public and private firms.

[8]Available at http://familybusinessindex.com/.

3   The Model

We present a discrete-time, trinomial model of the evolution of the distribution of wealth. Time is denoted by $t = 0, 1, 2, \ldots$, and the length of a time period in calendar time measured in fractions of a year is denoted by Δ_t.

The economy is populated by a continuum of infinitely lived dynasties. The gross assets of an individual dynasty are the product of two components, $W_t = W_{c,t}W_{i,t}$, where $W_{c,t}$ is a component of assets that is common to all dynasties and $W_{i,t}$ is a component of assets that is idiosyncratic to each individual dynasty. We assume that the support of the idiosyncratic component of assets for a dynasty is on a grid with nodes indexed by $n = 0, 1, 2, \ldots$. The nodes of this grid are evenly spaced in logs so that $W_i(n) = \exp(n\Delta)$, where Δ is the step size of the grid.

The fraction of dynasties with idiosyncratic assets equal to $W_i(n)$ at time t is denoted by $g_t(n)$, and $G_t(n) \equiv \sum_{k \geq n} g_t(k)$ denotes the complementary cumulative distribution function (CCDF) of dynasties over nodes of the grid of the idiosyncratic component of assets at t. Aggregate assets at time t are equal to the common component of assets at that date $W_{c,t}$ times the aggregate of the idiosyncratic component of assets across dynasties at t given by $\sum_{n=0}^{\infty} W_i(n)g_t(n)$.

We assume that all dynasties hold a common and constant level of debt d relative to the common component of assets, so the net wealth of a dynasty at t is equal to their assets less their debt $W_{c,t}(W_{i,t} - d)$.

The share of aggregate net worth held by agents with net worth greater than or equal to a given percentile α of the cross-section distribution of net wealth at t is given by

$$S_t(\alpha) = \frac{\sum_{k \geq n(\alpha)}^{\infty} \exp(k\Delta)g_t(k) - G_t(n(\alpha))d}{\sum_{n \geq 0}^{\infty} \exp(n\Delta)g_t(n) - d}, \tag{1}$$

where $n(\alpha)$ is the smallest value of n such that $G_t(n) \leq \alpha$.

We refer to the tail coefficient of the distribution of assets as the negative of the

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slope of a graph with the logarithm of assets on the x-axis and the logarithm of the fraction of dynasties with assets at or above this level on the y-axis. This tail coefficient measured at node n of the grid of the idiosyncratic component of assets is denoted by

$$\zeta_t(n) = -\frac{\log G_t(n+1) - \log G_t(n)}{\Delta}.^9 \tag{2}$$

The evolution of assets for a given dynasty from period t to $t+1$ is as follows. A dynasty starts period t with assets $W_{c,t}W_{i,t}$ and is subject to shocks to both the common and idiosyncratic components of assets. The evolution of the common component of assets $W_{c,t+1}$ follows an exogenous stochastic process. While the details of this stochastic process are important for understanding the evolution of aggregate wealth, they are not important for understanding the evolution of its distribution as captured by wealth shares as defined in equation (1).

Idiosyncratic innovations to assets   The idiosyncratic component of assets evolves according to the following type-dependent trinomial process. We assume that at each date t, each dynasty has one of two types $j \in \{D, F\}$, where the type j indexes the distribution of changes in the idiosyncratic component of assets from t to $t+1$. For dynasties of type j at t with idiosyncratic component of assets $W_{i,t} = \exp(n\Delta)$ for nodes on the grid $n \geq 1$, the probability that the idiosyncratic components of assets move one step up on the grid $(W_{i,t+1} = \exp((n+1)\Delta))$ is denoted by $p_{u,j,t}$, the probability that this component of assets moves down one step down on the grid $(W_{i,t+1} = \exp((n-1)\Delta)$ is denoted by $p_{d,j,t}$, and the probability that this component of assets remains at the same node on the grid $(W_{i,t+1} = \exp(n\Delta))$ is given by $1 - p_{u,j,t} - p_{d,j,t}$.

The dynamics of the idiosyncratic component of assets are modified for dynasties of type j at t with idiosyncratic component of assets at the lowest node on the grid

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[9]For large values of net wealth, the tail coefficient of the distribution of net wealth across dynasties is equal to the tail coefficient of assets, since the level of debt d is fixed and small relative to large levels of assets.

$W_{i,t} = 1$. For such a dynasty, the probability that the idiosyncratic component of assets moves one step up on the grid to node $n = 1$ is denoted by $p_{u,j,t}$, and the probability that this component of assets remains at node $W_{i,t} = W_i(0)$ is given by $1 - p_{u,j,t}$. It is not possible for the idiosyncratic component of assets to fall below this lowest node on the grid of assets. This assumption is referred to as a reflecting barrier for the idiosyncratic component of assets at the lowest level of assets on the grid.

This lowest level of the idiosyncratic component of assets is positive and normalized to 1 (since $n = 0$ at this node). The net worth of dynasties at this lowest level of grid is given by $W_{c,t}(1 - d)$, which may be negative if $d > 1$.

Under these assumptions, the expected value at t of the innovations to the logarithm of the idiosyncratic component of assets for all dynasties of type j, except those at the lowest node on the grid, is given by

$$\mathbb{E}_t \left[\log W_{i,t+1} - \log W_{i,t} \right] = (p_{u,j,t} - p_{d,j,t}) \Delta. \tag{3}$$

The uncentered second moment of these innovations to the logarithm of the idiosyncratic component of assets is given by

$$\mathbb{E}_t \left[\log W_{i,t+1} - \log W_{i,t} \right]^2 = (p_{u,j,t} + p_{d,j,t}) \Delta^2. \tag{4}$$

Transitions over types   At the end of period t, after these realizations to the idiosyncratic component of assets have been realized, each dynasty of type j experiences a shock to its type. It remains of the same type j with probability ϕ_j and transitions to the opposite type with probability $1 - \phi_j$. These transitions of types are independent over time and of dynastic wealth. We assume that at each date t, the fraction of dynasties of type j is equal to the fraction ν_j corresponding to the stationary distribution induced by this Markov process over types.

The forward equation for the distribution of wealth The notation $g_{j,t}(n)$ denotes the fraction of dynasties of type j at time t with idiosyncratic component of assets equal to $W_i(n)$. The overall fraction of dynasties with this level of assets is given by $g_t(n) = \sum_{j=D,F} g_{j,t}\nu_j$. With our assumption that ν_j is given by the stationary distribution over types, the evolution through time of these conditional distributions of the idiosyncratic component of assets by type is given by the following second order difference equation for all $n \geq 1$:

$$g_{j,t+1}(n) = \phi_j \left[p_{u,j,t}g_{j,t}(n-1) + p_{d,j,t}g_{j,t}(n+1) + (1 - p_{u,j,t} - p_{d,j,t})g_{j,t}(n) \right] + \quad (5)$$

$$(1 - \phi_j) \left[p_{u,-j,t}g_{-j,t}(n-1) + p_{d,-j,t}g_{-j,t}(n+1) + (1 - p_{u,-j,t} - p_{d,-j,t})g_{-j,t}(n) \right],$$

where $-j$ denotes the type opposite to j. For $n = 0$, this evolution is given by

$$g_{j,t+1}(0) = \phi_j \left[p_{d,j,t}g_{j,t}(1) + (1 - p_{u,j,t})g_{j,t}(0) \right] + \quad (6)$$

$$(1 - \phi_j) \left[p_{d,-j,t}g_{-j,t}(1) + (1 - p_{u,-j,t})g_{-j,t}(0) \right].$$

We consider specifications of the model in which the type-specific moments of the innovations to the logarithm of the idiosyncratic component of assets for dynasties are constant over time. For these specifications of the model, we refer to the *steady-state distribution* of the idiosyncratic component of wealth across dynasties as the stationary solution to the transition equations (5) and (6).

To summarize, the full set of parameters of our model are the grid step size Δ, the probabilities for wealth innovations of both types $\{p_{u,j,t}, p_{d,j,t}\}_{t=0}^{T}$, the unconditional fractions of each type ν_j, the transition probabilities for types ϕ_j, the level of debt d, and the initial distributions $g_{j,0}(n)$. We compute the evolution of the distribution of assets across types using equations (5) and (6) and the implied evolution of top wealth shares for T periods corresponding to 100 years using equation (1). We use the notation $\eta_{j,t} \equiv p_{u,j,t}/p_{d,j,t}$.

Parameterization for comparison with continuous time results   To compare results in our discrete time model with closely related results in continuous time versions of the model as presented in Luttmer (2016), Gabaix et al. (2016), and elsewhere, we use the following procedure to adjust the parameters of our model as we change the length of the time period Δ_t. This is done to consider the limiting implications of our model as the time period gets short. We set $p_{d,j,t}$ and $\eta_{j,t}$ to match annualized date- and time-specific means $\mu_{j,t}$ and variances $\sigma_{j,t}^2$ of innovations to the logarithm of the idiosyncratic component of assets. Specifically, we set the grid step size Δ as a function of the length of a time period Δ_t as

$$\Delta = \sigma_{max}\sqrt{2\Delta_t},$$

where σ_{max} is the largest annualized standard deviation of innovations to the logarithm of assets that we consider. We then choose the parameters $p_{d,j,t}$ and $\eta_{j,t}$ so that the expression in equation (3) is equal to the implied per period mean $\Delta_t\mu_{j,t}$, and the expression in equation (4) is equal to the implied per period uncentered second moment $\Delta_t\sigma_{j,t}^2 + \Delta_t^2\mu_{j,t}^2$. We set the transition probabilities over types as $1 - \phi_j = \kappa_j\Delta_t$ for fixed values of κ_j.

In the transition experiments that we conduct, we specify the initial distributions of the idiosyncratic component of assets by type $g_{j,0}(n)$, the parameters κ_j governing the Markov process over types for dynasties, and sequences of type-specific moments $\{\mu_{j,t}, \sigma_{j,t}\}_{t=0}^T$ governing the innovations to the logarithm of the idiosyncratic component of assets for dynasties. Using the equations above, we compute the corresponding per-period switching probabilities ϕ_j and the per period transition probabilities $\{p_{u,j,t}, p_{d,j,t}\}_{t=0}^T$.

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4 Steady-State, Wealth Mobility, and Transitions in the One-Type Model

The steady-state and transitions in the version of the model in which all dynasties are of a single type j can be solved analytically. These analytical formulas extend results in Luttmer (2016), Gabaix et al. (2016), and Gomez (2019). We present these analytical formulas to offer intuition for the mechanics of our model with two types.

We show that our model implies a tight link between the speed of convergence of the distribution of assets to steady-state and the degree of wealth mobility in the steady-state for dynasties starting out with the lowest level of assets on our grid. We show in turn that this speed of wealth mobility starting from the bottom is also intimately linked to the speed with which dynasties starting out very high up in the distribution of assets dissipate their fortunes.

In the one-type version of our model, we assume that $j = F$ for all dynasties (so $\nu_F = 1$ and $\phi_F = 1$). To simplify the notation, we suppress reference to types j for the remainder of this section on the one-type model.

Steady-State In the one-type model, there is a steady-state distribution of the idiosyncratic component of assets across dynasties that solves equations (5) and (6) if $p_u < p_d$. This steady-state distribution of dynastic assets over asset nodes is given by

$$g_{ss}(n) = (1 - \lambda_{ss})\lambda_{ss}^n, \tag{7}$$

where $\lambda_{ss} = p_u/p_d$. The CCDF of this distribution is given by $G_{ss}(n) = \lambda_{ss}^n$.

Aggregate assets are defined in steady-state if $\exp(\Delta)\lambda_{ss} < 1$. In this case, the steady-state tail coefficient of the distribution of assets across dynasties as defined in equation (2) is given by

$$\zeta_{ss} = -\frac{1}{\Delta}\log(\lambda_{ss}).$$

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Note that holding the grid step size Δ fixed, all versions of the one-type model that have the same steady-state tail coefficient ζ_{ss} have the same ratio of probabilities $\lambda_{ss} = p_u/p_d$ and hence, from equations (3) and (4), the same ratio of first and second moments of innovations to the logarithm of the idiosyncratic component of assets. Different specifications of the one-type model with the same steady-state tail coefficient vary in the magnitude of these moments, as indexed by the level of the probability p_d.[10]

We use the procedure described above to set parameters of the model as a function of the length of a time period Δ_t to consider the implications of our model as the time period becomes short. With this procedure for setting parameters, it is straightforward to show that

$$\lim_{\Delta_t \to 0} \zeta_{ss} = -\frac{2\mu}{\sigma^2}, \tag{8}$$

which is the standard formula in continuous time for the tail coefficient of the steady-state distribution of assets in this model.

Thus, as we said before, there is a family of specifications of the one-type model that all share the same ratio of moments of innovations to the idiosyncratic component of assets $2\mu/\sigma^2$. Thus, they all share the same steady-state tail coefficient of assets. Members of this family of model specifications differ in the magnitude of these moments.

We now show that these different specifications of the one-type model differ in the speed with which the distribution of assets transitions to steady-state from a given initial distribution of assets different from steady-state. The larger the magnitude of the moments μ and σ^2 is, the faster the transition.

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[10]Note that this result does not hold in the binomial specification of innovations to the logarithm of the idiosyncratic component of assets. In the binomial specification of this model, the steady-state distribution is given by equation (7). But in the binomial model, the probabilities $p_{u,t}$ and $p_{d,t}$ are restricted to satisfy $p_{u,t} + p_{d,t} = 1$ since assets must move either up or down one step on the grid. Thus, in the binomial specification of this model, there is a unique choice of p_d consistent with a given tail coefficient ζ_{ss}. In the trinomial specification of the model we consider, the probabilities need only satisfy the restriction $p_{u,t} + p_{d,t} \leq 1$.

Transitions We now consider the speed with which the distribution of assets transitions to steady-state from a given initial distribution of assets different from steady-state in the one-type model. Specifically, we study the transition experiment considered in Gabaix et al. (2016). We assume that the initial distribution of the idiosyncratic component of assets across dynasties is given by

$$g_0(n) = (1 - \lambda_0)\lambda_0^n,$$

for some $\lambda_0 \neq \lambda_{ss}$. We provide an analytical formula for the transition of the distribution $\{g_t(n)\}_{t=0}^\infty$ in the one-type model implied by the transition law given in equations (5) and (6) given this initial distribution.

To develop this analytical formula, we use the following notation. Let $\mathbb{T}$ be the operator mapping distributions over nodes n of our grid to new distributions defined by equations (5) and (6) for the one-type model. Let Λ_0 be a vector corresponding to the initial distribution $g_0(n) = (1 - \lambda_0)\lambda_0^n$. Let Λ_{ss} be the distribution to which the economy converges, $g_{ss}(n) = (1 - \lambda_{ss})\lambda_{ss}^n$. Let $\mathbf{1}$ denote a distribution that places weight 1 on the node $n = 0$ and weight 0 on every node $n \geq 1$. That is, $\mathbf{1}$ corresponds to the distribution of assets for a cohort of dynasties all starting with the minimum level of assets. With this notation, we have the following result.

Proposition 1. *Assume that the initial distribution at $t = 0$ of the idiosyncratic component of assets across dynasties is given by Λ_0 and that the transition probabilities in equations (5) and (6) are constant at p_d and $p_u = \lambda_{ss}p_d$ so that the stationary distribution of the idiosyncratic component of assets across dynasties is given by Λ_{ss}. Then the distribution at date t implied by equations (5) and (6) is given recursively by*

$$(g_{t+1} - \Lambda_{ss}) = A(g_t - \Lambda_{ss}) + (1 - A)\left(\mathbb{T}^t(\mathbf{1}) - \Lambda_{ss}\right), \tag{9}$$

where A is a scalar given by

$$A \equiv \left(p_d(1 - \lambda_0)(\frac{\lambda_{ss}}{\lambda_0} - 1) + 1\right),$$

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Proof. Direct calculation gives that

$$\mathbb{T}(\Lambda_0) = A\Lambda_0 + (1 - A)\mathbf{1}.$$

The operator $\mathbb{T}$ is linear, and $\mathbb{T}(\Lambda_{ss}) = \Lambda_{ss}$. Repeated application of this operator to $g_{t+1} = \mathbb{T}(g_t)$ starting from $g_0 = \Lambda_0$ then gives the result (9). $\square$

We provide an extension of this result for the two-type version of our model in the appendix in section 7.

Note that since $\lambda_0 < 1$, $A \neq 1$ when $\lambda_{ss} \neq \lambda_0$. We have $A < 1$ when $\lambda_{ss} < \lambda_0$, that is, when the the distribution of the idiosyncratic component of assets to which the economy is converging is more equal than the initial distribution, and we have $A > 1$ when the opposite is true.

Consider now the implications of this result for the connections between the transition dynamics of the distribution of wealth and wealth mobility. Note that the term $\mathbb{T}^t(\mathbf{1})$ in equation (9) corresponds to the distribution of the idiosyncratic component of assets in period t of a cohort of dynasties that all started with idiosyncratic assets at the bottom of our grid at date 0. With this observation, we see that equation (9) implies an intimate link between the speed of convergence of the distribution of the idiosyncratic component of assets in our transition experiment and the extent of wealth mobility starting from the bottom in the new economy that we are considering. That is, at any node n, the gap between $g_t(n)$ and $g_{ss}(n)$ stops changing when the conditional distribution of assets for a cohort of dynasties starting at the bottom at that node n converges to the unconditional distribution of assets at that node in the new steady-state.

Which parameters govern the degree of wealth mobility from the bottom implied by our model? Given that we have fixed λ_{ss}, only the single parameter p_d remains to influence the speed with which the distribution of the idiosyncratic component of assets for a cohort of agents starting at the bottom of the support of our grid converges to the steady-state distribution $\mathbb{T}^t(\mathbf{1}) \to \Lambda_{ss}$. The speed of this convergence

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is our measure of the degree of wealth mobility from the bottom. From equations (3) and (4), we see that the larger p_d is the larger the mean and variance of innovations to the logarithm of the idiosyncratic component of assets. Below,, we show numerically that the larger these moments are, the faster this measure of wealth mobility from the bottom is.

Wealth mobility from the bottom We now consider our model's quantitative implications for the extent of wealth mobility from the bottom of the distribution of assets, depending on the magnitude of the first and second moments of innovations to the logarithm of the idiosyncratic component of assets. We proceed as follows.

We hold fixed the tail coefficient ζ_{ss} of the steady-state distribution of assets. In the experiments below, we set this tail coefficient to $\zeta_{ss} = 1.5$, which is a value consistent with current estimates of that tail coefficient in the United States. We set the time period Δ_t to a small value and choose the grid step size Δ accordingly. This procedure thus pins down λ_{ss}, leaving only the single parameter p_d to be chosen. We consider two values for p_d. The first corresponds to a standard deviation of innovations to the logarithm of the idiosyncratic component of assets of $\sigma = 0.5$. The second corresponds to the smaller value of $\sigma = 0.25$ of this standard deviation.

To illustrate the speed of wealth mobility from the bottom for these two specifications of our model, we compute time paths for $G_t(n(\alpha))/\alpha$, where G_t is the CCDF of the distribution of assets of the distribution $\mathbb{T}^t(\mathbf{1})$ evaluated at the node $n(\alpha)$ on the grid of the idiosyncratic component of assets consistent with the percentile α of the steady-state distribution of assets. This statistic converges to one as the distribution of assets for a cohort of dynasties starting at the bottom converges to the unconditional distribution of assets. We are particularly interested in the convergence of this statistic for top percentiles of the distribution of wealth. In our plots, we show the convergence of this statistic for percentiles $\alpha = 1\%; 0.1\%; 0.01\%; 0.001\%;$ and $\alpha = 0.0003\%$, which Gomez (2019) selects as the percentile corresponding to the Forbes 400. We compute this process of convergence over 100 years.

In figure 3a, we show this measure of wealth mobility from the bottom in our economy with the standard deviation of the logarithm of the idiosyncratic component of assets set to $\sigma = 0.5$. In this figure, we see that it takes roughly 50-60 years for the upper percentiles of the distribution of assets for a cohort starting from the bottom of the support of assets to converge to levels close to those in the steady-state distribution. This convergence is complete, even for the highest wealth levels in 100 years.

In figure 3b, we show this measure of wealth mobility from the bottom in our economy in which we set $\sigma = 0.25$ the standard deviation of the logarithm of the idiosyncratic component of assets. In this figure, we see that wealth mobility from the bottom is much slower when σ is lower. In particular, the convergence of this distribution for the highest wealth levels has barely begun, even after 100 years.
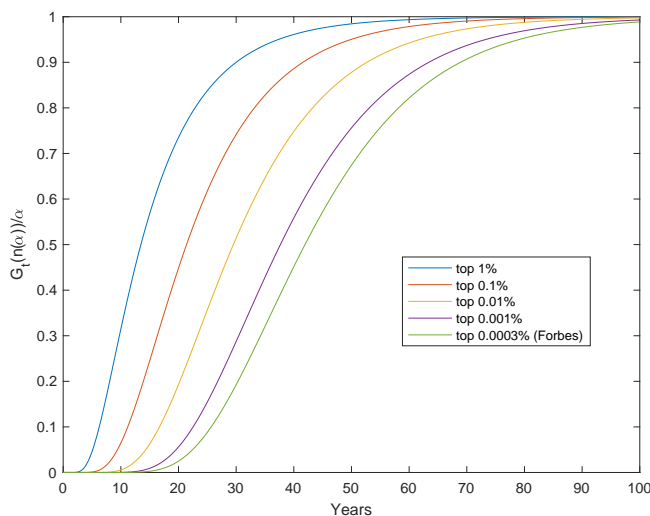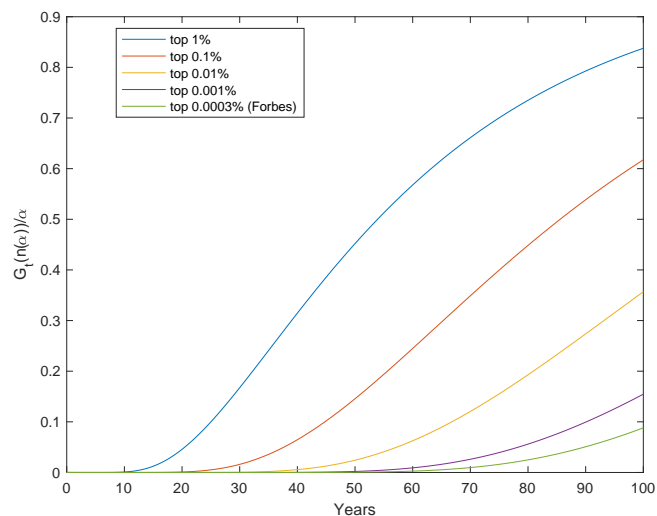
(a) High $\sigma = 0.5$ (b) Low $\sigma = 0.25$

Figure 3: Time paths for $G_t(n(\alpha))/\alpha$ at top wealth percentiles α, for a cohort starting at the bottom of the asset distribution. $\alpha = 1\%, 0.1\%, 0.01\%, 0.001\%, 0.0003\%$. We see that the speed of the transition is much slower when σ is low.

This computational experiment indicates that the extent of wealth mobility from

the bottom is dramatically affected by the magnitude of the volatility of innovations to the idiosyncratic component of wealth.

The transition of the tail coefficient   We now illustrate quantitatively the connection between the extent of wealth mobility from the bottom and the speed of transition of the distribution of assets to steady-state. To do so, we revisit the transition experiment considered in Gabaix et al. (2016) and described in proposition 1. Note that our results from proposition 1 and in figure 3 indicate that the transition of the distribution of assets in this experiment is slowest at top wealth percentiles, as the time it takes for $\mathbb{T}^k(\mathbf{1})$ to converge to steady-state is longest at these top wealth percentiles. As a result, we focus on the transition of the tail coefficient of the distribution of assets at top wealth percentiles.

Specifically, in our transition experiment, we start the economy with an initial distribution of assets Λ_0 with an initial tail coefficient of $\zeta = 2$, and we compute the transition of this distribution to the new steady-state distribution Λ_{ss} in which the steady-state tail coefficient is $\zeta = 1.5$. We display the evolution of the tail coefficient of wealth at top wealth percentiles over a 100 year period of this transition. When the distribution of assets has an exact Pareto tail, the slope of this graph is constant. However, during the transition, the asset distribution will not be exactly Pareto, and hence the slope will depend on the wealth node at which it is computed. We therefore compute this wealth-level dependent slope, $\zeta_t(n)$, in accordance with equation 2 for nodes $n(\alpha)$ corresponding to the top wealth percentiles $\alpha = 1\%, 0.1\%, 0.01\%, 0.001\%$, and 0.0003%.

We plot the results of this computation in figure 4.

(a) High $\sigma = 0.5$

(b) Low $\sigma = 0.25$
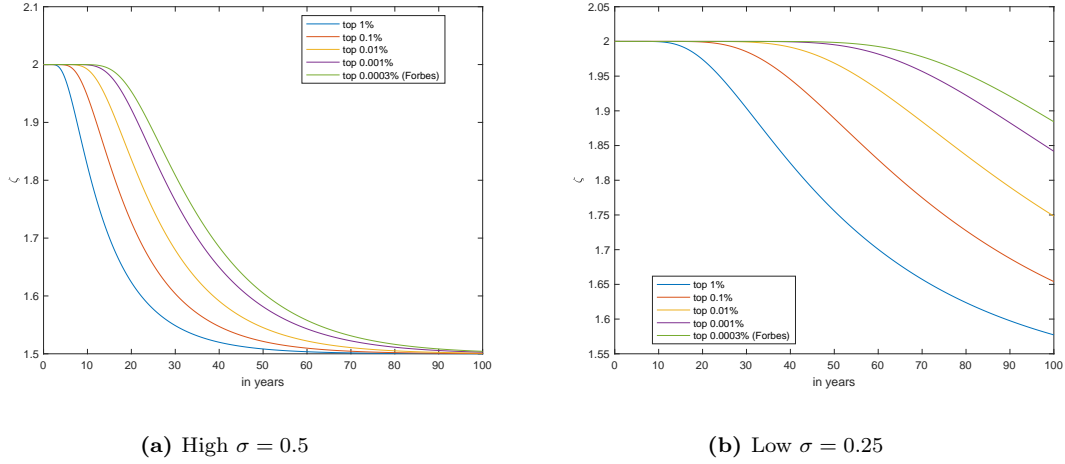
Figure 4: Time paths for the tail coefficient of the asset distribution, $\zeta_t(n(\alpha))$. The tail coefficient is computed at α corresponding to various top wealth percentiles.

We see in figure 4a that when the standard deviation of innovations to the log of the idiosyncratic component of assets is set to $\sigma = 0.5$, the transition of the tail coefficient for top wealth percentiles is fairly rapid. It takes roughly 50-60 years to transition from a tail coefficient of 2 to a coefficient of 1.5. The transition is slower the further up in the distribution we look when computing the tail coefficient. In contrast, we see in figure 4b that when the standard deviation is set to a smaller level, $\sigma = 0.25$, after a 100 years, the evolution of the tail coefficient has barely begun.

The dissipation of great wealth   The computational experiments on wealth mobility from the bottom that we have considered also have direct implications for the speed with which dynasties that start with great wealth dissipate their fortunes. Consider in particular the implications of equation (8). This relates the tail coefficient of the steady-state distribution of wealth to the annualized moments of innovations to the idiosyncratic component of assets. As the time period in our model becomes small, the distribution of changes in the logarithm of the idiosyncratic component of assets over a year for wealthy dynasties becomes Gaussian. Thus, the expected

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change in the level of the idiosyncratic component of assets for wealthy dynasties over the course of a year is given by $\exp(g_W)$, where $g_W = \mu + \sigma^2/2$. Equation (8) implies that

$$g_W = \frac{\sigma^2}{2}(-\zeta + 1).$$

Thus, with a tail coefficient of the distribution of assets in the steady-state of $\zeta = 1.5$ and $\sigma = 0.5$, we have that $g_W = -0.0625$. That is, with these parameters, wealthy dynasties decrease assets in relative terms at over 6% per year. In contrast, when $\sigma = 0.25$, we have $g_W = -0.015625$, so this dissipation of the relative wealth of great dynasties is much slower.

This dissipation of great wealth is illustrated in figure 5. For different values of σ, this figure depicts the evolution over time of the ratio of assets held by a cohort of dynasties all starting with idiosyncratic component of assets at the node on our grid corresponding to various top percentiles compared to average wealth. The speed with which this ratio converges to one indicates the rate at which dynasties starting out at top percentiles of the distribution of assets dissipate their wealth in relative terms. We see in figure 5a that top wealth is dissipated relatively rapidly when $\sigma = 0.5$, compared with what we find in Figure 5b when $\sigma = 0.25$.

(a) High $\sigma = 0.5$ (b) Low $\sigma = 0.25$
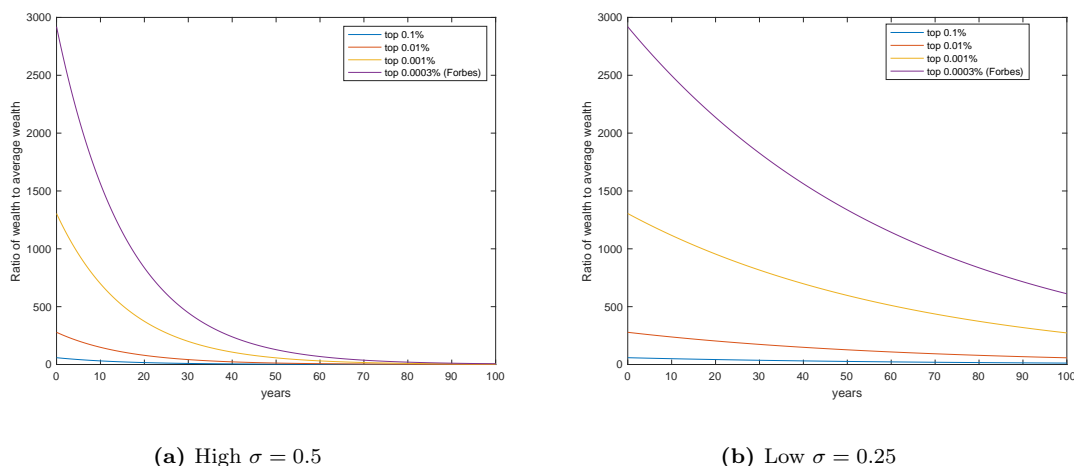
Figure 5: Time paths for the ratio of wealth held by cohorts starting at top percentiles relative to average wealth.

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These results for the one-type version of our model establish that if the standard deviation of innovations to the log of the idiosyncratic component of wealth is high (on the order of $\sigma = 0.5$), then our model can produce rapid transitions of top wealth shares. This specification of our model necessarily also predicts a high degree of wealth mobility from the bottom and relatively rapid dissipation of the relative fortunes of cohorts of families starting out at top wealth percentiles.[11]

This one-type model fails on two important grounds, however, as a full quantitative explanation of the evolution of top wealth shares. First, it is obviously not the case that every family runs its own business. Second, the evidence available in Gomez (2019) on the standard deviation of log changes in wealth for the Forbes 400 indicates that it is not the case that the standard deviation of innovations to the idiosyncratic component of assets in recent years is equal to 50%, even for the very wealthy.

To address these shortcomings of our one-type model, we now turn to an analysis of a two-type model in which only a small minority of dynasties derive their wealth from concentrated ownership of a single firm. We use this two-type model to reproduce the transition of top wealth shares in the United States over the past 100 years.

5   Steady-State, Wealth Mobility and Transitions in The Two Type Model

We now consider the quantitative implications of our model with two types of dynasties, $j \in \{D, F\}$. We use the index $j = F$ to denote a dynasty that currently has its assets concentrated in the equity of one firm, which we term the *family firm*. We assume that for this type of dynasty, the standard deviation of innovations to the logarithm of the idiosyncratic component of assets, denoted by $\sigma_{F,t}$, is large. The index $j = D$ denotes a dynasty that currently has its assets diversified across a

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[11]Gabaix et al. (2016) considered the implications of this model for transitions for much lower values of σ than 0.5. This accounts for their conclusions that this model fails to account for the transitions of top income shares.

broader range of investments. We assume that for this type of dynasty, the standard deviation $\sigma_{D,t}$ is small. We choose parameters so that the fraction of dynasties with a concentrated investment in a single firm is only a small minority of the overall population.

We now discuss the quantitative implications of this two-type model for the distribution of assets in the steady-state, the volatility of innovations to log wealth, wealth mobility from the bottom, the dissipation rate of great fortunes, and the evolution of top wealth shares over the past 100 years in response to a change in the volatility $\sigma_{F,t}$ consistent with the data on idiosyncratic equity volatility in Herskovic et al. (2016).

Steady-State In the two-type model, the steady-state distributions of the idiosyncratic component of assets by type are given by

$$g_j(n) = a_j(1 - \lambda_a)\lambda_a^n + b_j(1 - \lambda_b)\lambda_b^n$$

for $j \in \{D, F\}$, where a_j and b_j are non-negative constants and λ_a and λ_b are the two stable eigenvalues of the difference equation (5) stacked as a first order system and imposing steady-state. By convention, we assume that λ_a is the larger of the two stable eigenvalues. Under this assumption, the tail coefficient for the distribution of assets for high levels of assets converges to

$$\zeta_{ss} = -\frac{1}{\Delta} \log(\lambda_a).$$

Consider the fraction of dynasties of each type at the top end of the distribution of wealth. The fraction of the population of each type at node n of the grid is denoted by $\nu_j(h)$. At very high levels of wealth, these fractions of the population of each type converge to

$$\nu_j(n) \to \frac{\nu_j a_j}{\nu_F a_F + \nu_D a_D} \tag{10}$$

as n grows large.

There is a sense in which there are dynasties of type D with low volatility at the top of the wealth distribution only because these dynasties were of type F with high volatility in the recent past. To illustrate this point, consider the implications of the model as the rate at which type F dynasties switch type gets small ($\kappa_F \to 0$). We refer to this limiting case as the *non-communicating types* model. In this limit, the two stable eigenvalues of the model are given, as in the one-type model, by $\lambda_j = p_{u,j}/p_{d,j}$, and the weights a_j and b_j put weight on only one of the two eigenvalues for each type. Since $a_D = 0$ in this limiting case, we have that $\nu_D(n)$ goes to zero as n gets large.

Parameter choices: Initial steady-state We choose the parameters of the initial steady-state of our two-type model (corresponding to 1920) as follows. In the Appendix, we discuss how we map these calibration moments to the parameters of the model.

We set the time period $\Delta_t = 1/1000$, and choose the grid step size Δ consistent with the choice of $\sigma_{max} = 0.5$.

We set the overall fractions of dynasties of each type in the population to $\nu_F = 0.05$ and $\nu_D = 0.95$.[12] We set the annualized rate at which dynasties switch from type F to type D to $\kappa_F = 1/15$, or 6.66% per year.

We set the tail coefficient of the distribution of assets at high levels of wealth to $\zeta_0 = 1.5$.

We set parameters so that 53% of the population of the Forbes 400 in our model are of type F and 47% are of type D.

We set the initial values of the uncentered second moment of innovations to the log of the idiosyncratic component of assets by type to 0.45^2 for type F and 0.15^2

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[12]Cagetti and De Nardi (2009) find that the fraction of entrepreneurs in the U.S. population is 7.6%. Hurst and Pugsley (2009) argue that many of these entrepreneurs do not intend to grow their businesses. We choose to set $\nu_F = 5\%$ as a balance between these two papers. Most relevant for our results are the implications of our model for the fractions of each type at top wealth levels.

for type D.

To match the level of shares of net wealth held by top wealth percentiles, we choose the debt parameter $d = 1.1$. Note that our minimum level of assets is 1, so the net worth of those at the lowest level of assets is -0.1.

With these parameters, the model implies that in the initial steady-state of the model (which we use for the distribution of wealth in 1920), the share of net worth of the top 1% is 44%, while the shares of the top 0.1% and of the Forbes 400 (the top 0.0003%) are 21% and 2.8%, respectively. At the bottom end of the distribution of net worth, we have that 37% of the population has negative net worth.

Wealth volatility for the very wealthy The distribution of changes in the logarithm of the idiosyncratic component of assets for dynasties at node n of our grid of assets over a one year period is well approximated by a mixture of two normal distributions with moments μ_j, σ_j^2 and weights $\nu_j(n)$. For dynasties at the cutoff for the Forbes 400 in our model, the standard deviation of changes in the the log of the idiosyncratic component of assets is 34%, and the excess kurtosis of this distribution is 4.65. These numbers are roughly equal to the data shown in Gomez (2019) for the standard deviation and excess kurtosis of log wealth for the Forbes 400.

Wealth mobility Before conducting our analysis of the transition dynamics of top wealth shares implied by the model, we consider the implications of our two-type model for wealth mobility from the bottom and from the top in the initial steady-state. As we did with the one-type model, we compute time paths for $G_t(n(\alpha))/\alpha$, where G_t is the CCDF of the distribution of assets of the distribution $\mathbb{T}^t(\mathbf{1})$ evaluated at the node $n(\alpha)$ on the grid of the idiosyncratic component of assets consistent with percentile α of the steady-state distribution of assets. In figure 6, we show this measure of wealth mobility from the bottom over a period of 100 years. As with the one-type model, in roughly 50-70 years, the upper part of the CCDF of assets for a cohort of dynasties starting from the bottom converges to the overall CCDF in the

steady-state.
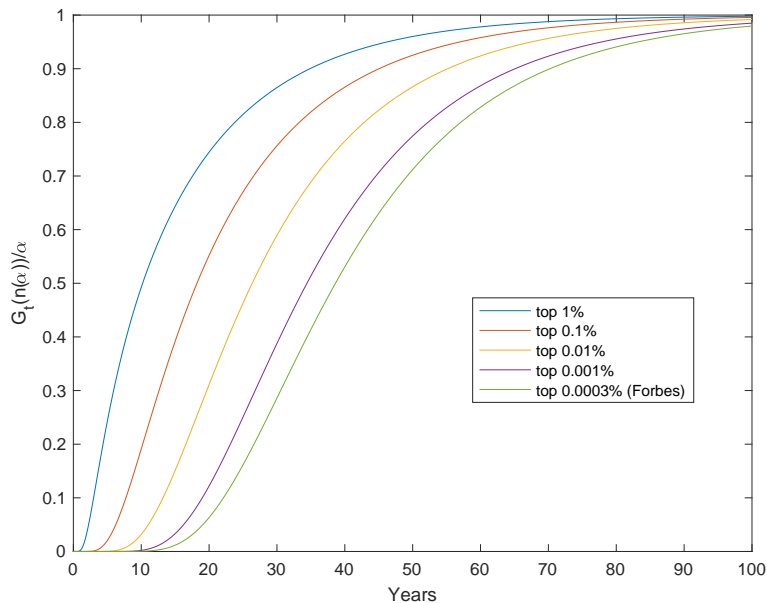
Figure 6: Time paths for $G_t(n(\alpha))/\alpha$ at top wealth percentiles α, for a cohort starting at the bottom of the asset distribution.

In figure 7, we plot the path of the ratio of total wealth of a cohort of dynasties starting at the cutoff level of wealth for various top wealth percentiles relative to aggregate wealth over a 100 year period. We see that a great portion of that wealth is expected to dissipate in relative terms over this time horizon.
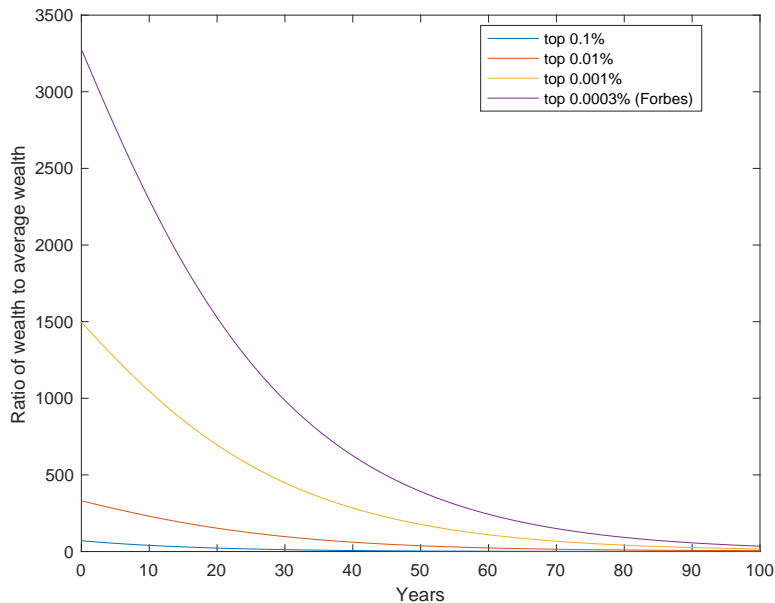
Figure 7: Transition of the ratio of top wealth shares to aggregate wealth.

100 years of top wealth shares with time-varying idiosyncratic volatility
We now turn to our two type model's the implications for the evolution of top wealth shares over the past 100 years in the United States when the volatility of innovations to the value of family firms is assumed to follow the path estimated by Herskovic et al. (2016) shown in Figure 2.

To do so, we assume that the initial distribution of wealth corresponding to 1920 is given by the initial steady-state considered above. We assume that the parameters governing the time interval (Δ_t), the grid step size (Δ), the debt level (d), the fractions of dynasties of each type (ν_j), and the rate at which dynasties switch types (ϕ_j) remain constant over time and that the probabilities $p_{u,D,t}$ and $p_{d,D,t}$ governing the innovations to the idiosyncratic component of wealth for dynasties of type D also remain constant throughout the 100 year transition considered.

We set the probabilities $p_{u,F,t}$ and $p_{d,F,t}$, which govern the innovations to the idiosyncratic component of wealth for dynasties with concentrated investments in family firms during the 100 year transition, as follows. We assume that the standard deviation of these innovations to the value of family firms, $\sigma_{F,t}$, declines linearly from 0.45 to a low of 0.25 over the course of 30 years (from 1920 to 1950), then rises linearly from 0.25 back up to 0.5 over the course of the next 50 years (from 1950 to 2000), and then remains constant at 0.5 over the final 20 years (from 2000 to 2020). We choose the path of $\mu_{F,t}$ so that the expected growth rate of the level of the idiosyncratic component of assets $g_{W,F,t} = \mu_{F,t} + \sigma_{F,t}^2/2$ remains constant at its initial value. We then use equations (3) and (4) to compute the corresponding probabilities $\{p_{u,F,t}, p_{d,F,t}\}$.

In figures 8 and 9, we show the transition path for top wealth shares from this experiment.[13] The transitions for the wealth shares of the top 1% and 0.1% are shown in Figure 8 along with the estimates of these wealth shares from Saez and Zucman (2016). The transition for wealth shares for the Forbes 400 (the top 0.0003%) is shown in Figure 9 along with the estimates of these wealth shares from Gomez (2019) normalized so that the model and data coincide in 1982. For 2017 and 2018, we complement the Forbes data from Gomez (2019) data with data from Zheng (2019).

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[13]Note that we compute net worth shares at t using equation (1) at or above fixed nodes $n(\alpha)$ on our grid, which corespond to wealth cutoffs for wealth percentiles α in the initial steady-state distribution.
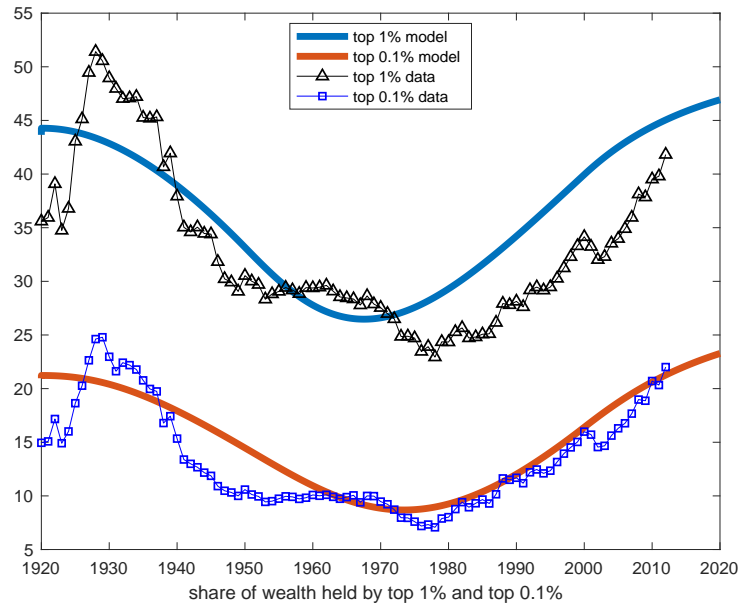
Figure 8: 100 year transition of top 1% and top 0.1% share of wealth (in %). Model vs. data.
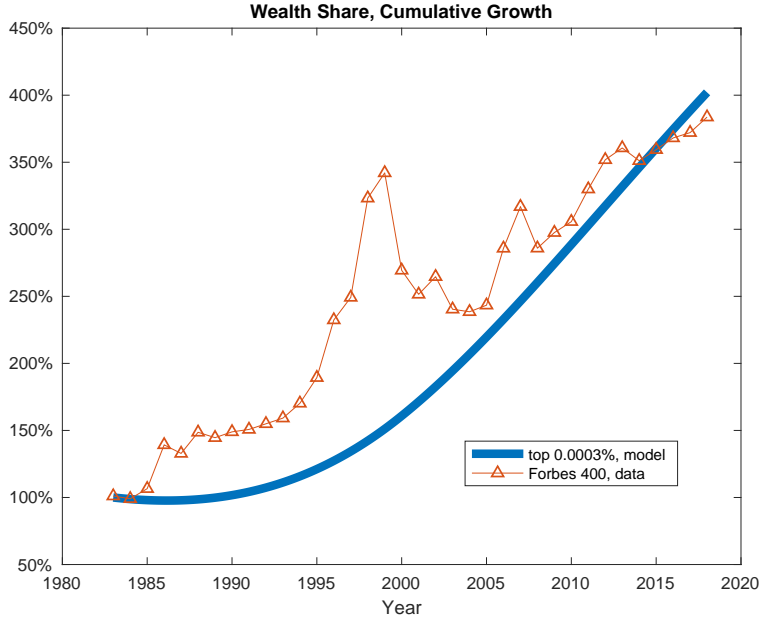
Figure 9: Cumulative growth of Forbes 400 wealth. Model vs. data.

The model captures the U-shape of the transitions of top wealth shares well. The fact that the distribution of assets changes, with a delay, from the bottom up implies that the time-variation in idiosyncratic volatility is reflected in top wealth shares. In particular, recall that the time-varying idiosyncratic volatility reaches its minimum around 1950 whereas top wealth shares reach their minimum in the 1970s or later. The bottom for wealth shares occurs later the higher the percentile of wealth that is being considered. This delay in the dynamics of top wealth shares emerges endogenously from the model.

Turnover in the Forbes 400 Gomez (2019) studies the turnover rate for the households on the Forbes 400 list. He documents that less than 10% percent of the households that were on list in 1983 were still on the list in 2017. We compare our model with the data in this dimension as follows. We consider the time paths

of wealth for a cohort of dynasties starting with a wealth level in the top 0.0003% (which Gomez (2019) takes as the cutoff for the Forbes 400) in the year 1983. We subject this cohort of dynasties to the transition dynamics of the years 1983-2017 taken from the 100-year experiment above. Finally, we compute how many of them are still above the top 0.0003% cutoff in 2017, and we find that 9.66% are. Hence, the model is fairly successful in accounting for the mobility out the Forbes 400, at least according to this rough measure of turnover.

The economics of family firms In our interpretation of our model and this transition experiment, the phenomenon of long-lived and highly volatile family firms is central not only to the highly skewed distribution of wealth in the United States and many other countries, but also to the rapid evolution of top wealth shares over time. In our model, family firms facing a large amount of idiosyncratic risk are also central to high degrees of wealth mobility from the bottom of the wealth distribution. Moreover, the observation that the wealth distribution is not more skewed suggests that if family firms are an important part of the wealth distribution, then it must be the case that great fortunes are also relatively rapidly dissipated.

These features of our model suggest that decisions by owners of family firms to sell out and diversify their holdings may play a central role in the evolution of the wealth distribution. To illustrate this point, we use our model to consider the counterfactual distribution of wealth that would occur if family firm owners were to choose to more rapidly diversify their portfolios. In the context of the two type model presented above, changes in the propensity to diversify can be modeled as changes in the type switching rate κ_F.

Consider, for instance, the effect of an increase in the rate at which entrepreneurs diversify their wealth on the steady-state wealth distribution. In particular, take as a starting point the initial steady-state wealth distribution implied by the dynamics from the previous transition experiment. How would that distribution change if the switching rate were increased from our benchmark value of $\kappa_F = 1/15$ to $\kappa_F = 1/5$, while keeping constant the other parameters governing the evolution of assets for

both family firms and diversified dynasties? In table 3, we compare the steady-state wealth shares implied by having dynasties diversify at different rates.

	Steady State Wealth Shares		
	top 1%	top 0.1%	top 0.0003% (Forbes 400)
$\kappa_F = 1/15$	44%	21%	2.8%
$\kappa_F = 1/5$	35%	10%	0.9%

Table 3: Model implied steady-state wealth shares for different values of the switching rate κ_F

Increasing the rate at which dynasties diversify dramatically reduces steady state top wealth shares and does so in a progressive fashion: the fall in the share of wealth held by the Forbes 400 is more dramatic than the fall for the top 0.1%, which in turn is more pronounced than the fall for the top 1%.

We interpret this experiment as indicating that the economics of family firms and the reasons for which wealthy families do or do not more rapidly diversify their holdings are fruitful for understanding extremes of wealth inequality in the United States.[14]

6   Conclusion

In this paper, we have shown that the evolution of top wealth shares in the United States over the past 100 years can be understood as being driven by the evolution of the idiosyncratic volatility of firm values as long as one allows "family firms" to play an important role in the model.

Throughout our paper, we take the moments of innovations to the value of family firms as a primitive. As discussed in Piketty (2014) and Saez and Zucman (2019), the moments of after-tax returns for families owning family firms are also affected

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[14]See Peter (2019) for a discussion of differences in the extent of diversification of firm ownership across European countries and top wealth shares in those countries.

by tax policies. See also Aoki and Nirei (2017) and Hubmer et al. (Forthcoming). We have made no effort to disentangle to what extent this evolution of top wealth shares was driven by changes in the volatility of firm values versus tax rates or some other aspect of the economy. We leave this for future research.

We do conjecture, however, that it may be fruitful going forward to emphasize the economics of family firms in the analysis, in order to deepen our understanding of the drivers of wealth inequality in the United States and likely many other countries. We have shown in our model that wealth is much more equal if entrepreneurs and their heirs sell out and diversify shortly after founding their firms. Why do families maintain their concentrated stakes in family firms over several generations? Why do these families resolve the tension between control of their firms and access to broad sources of capital available in public markets in favor of concentrated ownership? Could one implement policies that would tilt this choice in favor of rapid diversification by founding families? Would such a policy intervention result in a more equal distribution of wealth? We suggest that it would be worthwhile to think harder about the economics of family firms in considering policies to reduce wealth inequality in the United States.

7 Appendix

In this appendix, we present an extension Proposition 1 to the two-type version of our model. We show in particular that the degree of wealth mobility from the bottom of the distribution is critical in determining the speed of transition implied by our model. We then discuss how we map our calibration moments to the parameters of the two-type model.

As discussed in Section 5 the distribution of assets by type in the steady-state of our two type model has the form

$$g_j(n) = a_j(1 - \lambda_a)\lambda_a^n + b_j(1 - \lambda_b)\lambda_b^n$$

for $j = F, D$. Using the notation of Proposition 1, we write the distributions $\Lambda_i(n) = (1 - \lambda_i)\lambda_i^n$ for $i = a, b$ and $n \geq 0$, and we use **1** to denote a distribution that puts weight one on the node $n = 0$ and zero on every other node.

In the two-type model, the operator $\mathbb{T}$ defined by equations (5) and (6) maps a pair of distributions by type at t, $[g_{F,t}, g_{D,t}]'$ to a pair of distributions by type at $t + 1$, $[g_{F,t}, g_{D,t}]'$. Define $\mathbb{T}_j$ to be the operator which maps pairs of distributions at t, $[g_{F,t}, g_{D,t}]'$ to the distribution for type j at $t + 1$. With these definitions

$$\mathbb{T}[g_{F,t}, g_{D,t}]' = \left[\mathbb{T}_F[g_{F,t}, g_{D,t}]', \mathbb{T}_D[g_{F,t}, g_{D,t}]' \right]'$$

The transition experiment we consider is as follows. Fix the parameters of the operator $\mathbb{T}$ given by $\{p_{u,j}, p_{d,j}, \phi_j\}$. Let the initial distribution of assets by type be given by

$$g_{j,0} = a_{j,0}\Lambda_a + b_{j,0}\Lambda_b$$

with $a_{j,0} + b_{j,0} = 1$ for arbitrary nonnegative weights $a_{j,0}, b_{j,0}$ and arbitrary Λ_a, Λ_b defined by $\lambda_a, \lambda_b \in [0, 1)$. The distributions of assets by type converges over time to

$$g_{j,ss} = a_{j,ss}\Lambda_{a,ss} + b_{j,ss}\Lambda_{b,ss}$$

with new weights $a_{j,ss}$ and $b_{j,ss}$ on new vectors $\Lambda_{a,ss} \neq \Lambda_a$ and $\Lambda_{b,ss} \neq \Lambda_b$.

Extension of Proposition 1 We then have the following extension of our transition result in Proposition 1. Specifically, we have that, in this transition experiment in the two-type model, the distributions of assets by type at date t are given by

$$\begin{bmatrix} g_{F,t} \\ g_{D,t} \end{bmatrix} = \begin{bmatrix} a_{F,t}\Lambda_a \\ a_{D,t}\Lambda_a \end{bmatrix} + \begin{bmatrix} b_{F,t}\Lambda_b \\ b_{D,t}\Lambda_b \end{bmatrix} + \sum_{k=0}^{t-1} \mathbb{T}^k \begin{bmatrix} c_{F,t-k}\mathbf{1} \\ c_{D,t-k}\mathbf{1} \end{bmatrix}. \tag{11}$$

where $a_{j,0}, b_{j,0}$ are give by the initial distributions at $t = 0$,

$$\begin{bmatrix} a_{F,t+1} \\ a_{D,t+1} \end{bmatrix} = \begin{bmatrix} \phi_F A_F & (1 - \phi_F)A_D \\ (1 - \phi_D)A_F & \phi_D A_D \end{bmatrix} \begin{bmatrix} a_{F,t} \\ a_{D,t} \end{bmatrix} \tag{12}$$

and

$$\begin{bmatrix} b_{F,t+1} \\ b_{D,t+1} \end{bmatrix} = \begin{bmatrix} \phi_F B_F & (1 - \phi_F)B_D \\ (1 - \phi_D)B_F & \phi_D B_D \end{bmatrix} \begin{bmatrix} b_{F,t} \\ b_{D,t} \end{bmatrix} \tag{13}$$

where

$$A_j = \left[1 + p_{u,j}\frac{1 - \lambda_a}{\lambda_a} - p_{d,j}(1 - \lambda_a) \right] \tag{14}$$

$$B_j = \left[1 + p_{u,j}\frac{1 - \lambda_b}{\lambda_b} - p_{d,j}(1 - \lambda_b) \right] \tag{15}$$

and $c_{F,0} = c_{D,0} = 0$ and

$$c_{F,t+1} = \phi_F(a_{F,t} + b_{F,t}) + (1 - \phi_F)(a_{D,t} + b_{D,t}) - (a_{F,t+1} + b_{F,t+1})$$

$$c_{D,t+1} = \phi_D(a_{D,t} + b_{D,t}) + (1 - \phi_D)(a_{F,t} + b_{F,t}) - (a_{D,t+1} + b_{D,t+1})$$

In comparing this result to that in Proposition 1, we see that the transition dynamics of the distribution of assets in the two-type model is similar to those in the one-type model in that the distribution of assets in the transition at time t is a weighted average of the initial distributions Λ_a and Λ_b and distributions of assets built up by repeated application of the operator $\mathbb{T}$ to cohorts of agents all starting

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at the bottom of the distribution of assets (with initial distribution $\mathbf{1}$). Thus, the speed with which the tail coefficient of the distribution of assets depends on how fast distributions of assets built up from cohorts of agents starting at the bottom converge to the steady-state distributions based on $\Lambda_{a,ss}$ and $\Lambda_{b,ss}$.

We prove this result regarding the distribution of assets in the transition as follows.

Note that the operator $\mathbb{T}$ is linear in acting on pairs of distributions. Direct calculation gives that

$$\mathbb{T}_F \begin{bmatrix} a_{F,t}\Lambda_a \\ a_{D,t}\Lambda_a \end{bmatrix} = \left[\phi_F A_F a_{F,t} + (1 - \phi_F) A_D a_{D,t} \right] \Lambda_a +$$

$$\left[\phi_F (1 - A_F) a_{F,t} + (1 - \phi_F)(1 - A_D) a_{D,t} \right] \mathbf{1} =$$

$$a_{F,t+1}\Lambda_a + \left[\phi_F a_{F,t} + (1 - \phi_F) a_{D,t} - a_{F,t+1} \right] \mathbf{1}$$

$$\mathbb{T}_F \begin{bmatrix} b_{F,t}\Lambda_b \\ b_{D,t}\Lambda_b \end{bmatrix} = b_{F,t+1}\Lambda_b + \left[\phi_F b_{F,t} + (1 - \phi_F) b_{D,t} - b_{F,t+1} \right] \mathbf{1}$$

$$\mathbb{T}_D \begin{bmatrix} a_{F,t}\Lambda_a \\ a_{D,t}\Lambda_a \end{bmatrix} = a_{D,t+1}\Lambda_a + \left[\phi_D a_{D,t} + (1 - \phi_D) a_{F,t} - a_{D,t+1} \right] \mathbf{1}$$

$$\mathbb{T}_D \begin{bmatrix} b_{F,t}\Lambda_b \\ b_{D,t}\Lambda_b \end{bmatrix} = b_{D,t+1}\Lambda_b + \left[\phi_D b_{D,t} + (1 - \phi_D) b_{F,t} - b_{D,t+1} \right] \mathbf{1}$$

These results imply that when the operator $\mathbb{T}$ is applied to the initial distribution at $t = 0$, the pair of distributions that results at $t = 1$ is given by

$$\begin{bmatrix} g_{F,1} \\ g_{D,1} \end{bmatrix} = \begin{bmatrix} a_{F,1}\Lambda_a \\ a_{D,1}\Lambda_a \end{bmatrix} + \begin{bmatrix} b_{F,1}\Lambda_b \\ b_{D,1}\Lambda_b \end{bmatrix} + \begin{bmatrix} c_{F,1}\mathbf{1} \\ c_{D,1}\mathbf{1} \end{bmatrix}$$

Now consider applying the operator $\mathbb{T}$ to a pair of distributions at t of the form in

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equation (11). We get

$$
\begin{bmatrix} g_{F,t+1} \\ g_{D,t+1} \end{bmatrix} = \mathbb{T} \begin{bmatrix} g_{F,t} \\ g_{D,t} \end{bmatrix} = \begin{bmatrix} a_{F,t+1}\Lambda_a \\ a_{D,t+1}\Lambda_a \end{bmatrix} + \begin{bmatrix} b_{F,t+1}\Lambda_b \\ b_{D,t+1}\Lambda_b \end{bmatrix} +
$$

$$
\begin{bmatrix} c_{F,t+1}\mathbf{1} \\ c_{D,t+1}\mathbf{1} \end{bmatrix} + \sum_{k=0}^{t-1} \mathbb{T}^{k+1} \begin{bmatrix} c_{F,t-k}\mathbf{1} \\ c_{D,t-k}\mathbf{1} \end{bmatrix} =
$$

$$
\begin{bmatrix} a_{F,t+1}\Lambda_a \\ a_{D,t+1}\Lambda_a \end{bmatrix} + \begin{bmatrix} b_{F,t+1}\Lambda_b \\ b_{D,t+1}\Lambda_b \end{bmatrix} + \sum_{k=0}^{t} \mathbb{T}^k \begin{bmatrix} c_{F,t+1-k}\mathbf{1} \\ c_{D,t+1-k}\mathbf{1} \end{bmatrix}
$$

This proves the result.

Characterizing the Steady-State Distribution We take as given the parameters of the two-type model $\phi_F, \phi_D, p_{u,F}, p_{d,F}, p_{u,D}, p_{d,D}$. The steady state distribution is given by six parameters $\lambda_a, \lambda_b \in (0,1)$ and $a_F, a_D, b_F, b_D \in [0,1]$. These six parameters have to satisfy the following conditions. The weights a_F, a_D, b_F, b_D have to satisfy

$$
a_F + b_F = 1
$$

$$
a_D + b_D = 1
$$

and be a stationary solution to equations (12) and (13) with the coefficients A_j and B_j given by equations (14) and (15). These equations imply that

$$
\frac{a_F}{a_D} = \frac{(1 - \phi_F)A_D}{(1 - \phi_F A_F)} = \frac{(1 - \phi_D A_D)}{(1 - \phi_D)A_F} \tag{16}
$$

The second of these equations implies

$$
0 = 1 - (1 - \phi_F - \phi_D)A_F A_D - \phi_D A_D - \phi_F A_F \tag{17}
$$

Since A_F and A_D are both quadratic in λ (powers -1, 0, and 1), equation (17) is a fourth order polynomial when $(1 - \phi_F - \phi_D) \neq 0$. To have a unique stationary distribution, one must check that only two of the roots of this polynomial lie in the

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interval $(0, 1)$. By convention, λ_a is the largest root of this polynomial that lies in the interval $(0, 1)$ and λ_b is the smaller of the two roots in this interval. We have that b_F and b_D solve the analogous equation to (16) with λ_b being the smaller root in $(0, 1)$ of the analog to equation (17) defined by B_F and B_D in place of A_F and A_D.

Calibrating the Two-Type Model We calibrate the parameters of the two-type model as follows.

We fix the time interval $\Delta_t = 1/1000$ representing $1/1000$ of a year and $\sigma_{max} = 0.5$. The step size of the grid of assets is given by

$$\Delta = \sigma_{max}\sqrt{2\Delta_t}$$

We have the following calibration moments for the initial steady-state distribution of assets.

1. The unconditional fractions of each type in the population are given by ν_j for $j = F, D$. This sets these parameters directly at $\nu_F = 0.05$ and $\nu_D = 0.95$.

2. The switching rate per unit time from F to D given by $\kappa_F = 1/15$ or 6.66% per year. This condition and the unconditional fractions of each type in the population give us

$$\phi_F = 1 - \kappa_F\Delta_t$$

$$\phi_D = 1 - \frac{\nu_F}{\nu_D}(1 - \phi_F)$$

3. Let ν_j^{400} denote the fractions of each type in the limit as wealth gets large. The superscript 400 here refers to the Forbes 400. We set $\nu_F^{400} = 53\%$ and $\nu_D^{400} = 47\%$. To match these fractions, from equation (10) the ratio of the parameters a_F/a_D must satisfy

$$a_D/a_F = \frac{\nu_F - \nu_F\nu_F^{400}}{\nu_D\nu_F^{400}}$$

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Note that with this solution for a_F/a_D, the equations (16) can then be solved for A_F and A_D.

4. The limiting tail coefficient for the Pareto distribution of assets is denoted by ζ_a. We set $\zeta_a = 1.5$. To match this limiting tail coefficient, we must have

$$\lambda_a = \exp(-\Delta\zeta_a)$$

5. We set the two uncentered second moments of innovations to the log of assets by type to

$$(p_{u,F} + p_{d,F})\Delta^2 = 0.45^2\Delta_t$$

and

$$(p_{u,D} + p_{d,D})\Delta^2 = 0.15^2\Delta_t$$

Given our solutions for A_j in step 3 together with our calibrated value of λ_a, these equations together with the equations (14) imply solutions for these probabilities $p_{u,j}, p_{d,j}$ for $j = F, D$.

We finish solving for the parameters of the model as follows.

The second stable eigenvalue λ_b has to be a root of

$$0 = 1 - (1 - \phi_F - \phi_D)B_F B_D - \phi_D B_D - \phi_F B_F$$

with B_j given from equations (15) using the probabilities solved for in step 5. Once we find this second stable eigenvalue, we have that b_F/b_D must satisfy the analog to equation (16), which, together with the restrictions that $a_j + b_j = 1$ for $j = F, D$, gives us the four coefficients a_j, b_j.

Finally, we choose the level of debt $d = 1.1$ to match the share of net worth held by the top 0.1% using equation (1).

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