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# Market Structure and Monetary Non-neutrality*

*Simon Mongey*‡

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**Abstract**

I propose an equilibrium menu cost model with a continuum of sectors, each consisting of strategically engaged firms. Compared to a model with monopolistically competitive sectors that is calibrated to the same data on good-level price flexibility, the dynamic duopoly model features a smaller inflation response to monetary shocks and output responses that are more than twice as large. The model also implies (i) four times larger welfare losses from nominal rigidities, (ii) smaller menu costs and idiosyncratic shocks are needed to match the data, (iii) a $U$-shaped relationship between market concentration and price flexibility, for which I find empirical support.

**Keywords:** Oligopoly, menu costs, monetary policy, firm dynamics.

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# 1 Introduction

A standard assumption made for tractability in macroeconomic models is that firms behave non-strategically in the markets in which they sell their goods. This paper relaxes this assumption in a monetary business cycle model with nominal rigidity, exploring an oligopolistic market structure.

Motivation for investigating the macroeconomic implications of oligopolistic markets is straightforward: product markets are highly concentrated. Figure 1 documents this fact for a broad range of narrowly defined markets: a product category (e.g., ketchup) within a state in a particular month.[1] The median effective number of firms—a measure of market concentration given by the inverse Herfindahl index—is only 3.7, and the median revenue share of the two largest firms is over two-thirds.[2]

In this paper, I propose an equilibrium menu cost model of price adjustment that accommodates a duopoly within each sector. Firms face persistent, idiosyncratic shocks, must pay a cost to change their price, and compete strategically under a Markov perfect equilibrium (MPE) concept. Aggregating a continuum of oligopolistic sectors reveals how the strategic behavior of firms affects the equilibrium response of output to monetary shocks.

I compare the dynamic oligopoly model to a benchmark model with a monopolistically competitive market structure in which each sector is populated with a continuum of non-strategic firms. Both models are calibrated to the same features of good-level price change data and the same average markup. Since prices change frequently and by large amounts on average, matching these facts strongly curtails output fluctuations due to monetary shocks in a monopolistically competitive model (Golosov and Lucas, 2007). My main finding is that—in these two models of market structure that are equivalent in terms of idiosyncratic price flexibility—the aggregate price is less flexible under oligopoly, leading to output fluctuations in response to monetary shocks that are around two and half times as large.[3]

Understanding this main result requires understanding the particular way complementar-

[1]IRI data are used to construct measures of firm-level revenue, which are then used to construct measures of concentration. The IRI data are weekly good-level data for the universe of goods in a panel of over 5,000 supermarkets in the US from 2001 to 2011. For a detailed description of how these measures are constructed see Appendix A.

[2]The inverse Herfindahl index (IHI) admits an interpretation of "effective number of firms" as follows. The IHI of a sector with $n$ equally sized firms is $n$. Therefore, if a sector has an IHI of 2.4, then it has a Herfindahl index between that of a market with 2 and 3 equally sized firms. For more on this interpretation, see Adelman (1969). For a recent paper that uses this measure of market concentration, see Edmond, Midrigan, and Xu (2015).

[3]Real effects of monetary shocks are measured as the time series standard deviation of output in an economy with only monetary shocks. Under both market structures I assume that menu costs are random, which generates more monetary non-neutrality than a fixed menu cost model. Therefore, output fluctuations in the duopoly model are two and half times larger than a baseline that already features significant monetary non-neutrality.

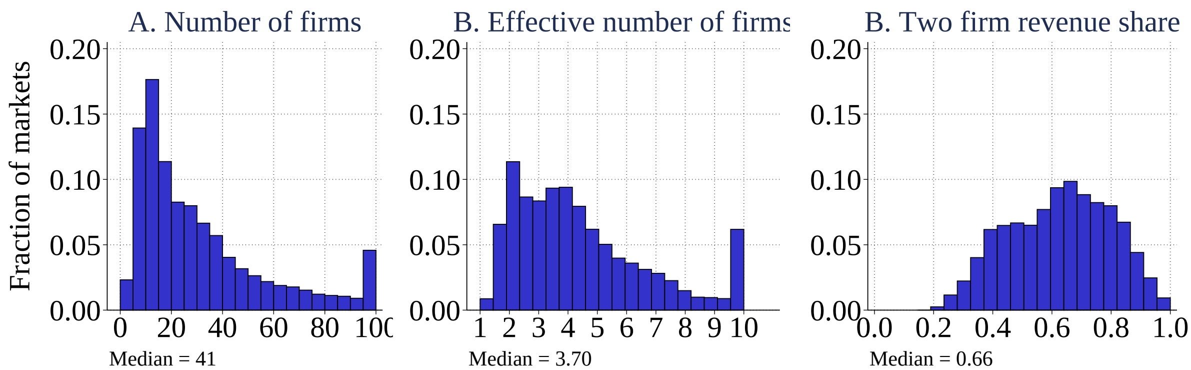


Figure 1: Market concentration in the IRI supermarket data

Notes: A market is defined as an IRI product category $p$ within state $s$ in month $t$ giving 191,833 observations. A firm $i$ is defined within a $pst$ market by the first 6 digits of a product's bar code. Revenue $r_{ipst}$ is the sum over the revenue from all products of firm $i$ in market $pst$. See Appendix A for more details on the data. Medians reported in the figure are revenue weighted. Unweighted medians are A. 21, B. 3.86, C. 0.64. Panel A: Number of firms is the total number of firms with positive sales in market $pst$. Panel B: Effective number of firms is given by the inverse Herfindahl index $h_{pst}^{-1}$, where the Herfindahl index is the revenue share weighted average revenue share of all firms in the market, $h_{pst} = \sum_{i \in \{pst\}} (r_{ipst}/r_{pst})^2$. Panel C: Two-firm revenue share is the share of total revenue in market $pst$ accruing to the two firms with the highest revenue.

ity in prices arises in the model, and how this dampens the inflation response to a monetary shock. Throughout I make a distinction between *static complementarity*, and *dynamic complementarity*, which I explain in turn.[4] When firms are strategic—and so understand how their price affects household demand across sectors—prices are *static complements* within sectors. That is, with respect to profits, a firm's optimal price is increasing in the price of its direct competitor. Absent adjustment frictions the Nash equilibrium price, $p^*$, would obtain, which is a a constant markup over nominal cost. In equilibrium, a monetary expansion increases nominal costs, causing all prices increase one for one.

Neutrality is broken by the interaction of static complementarity with nominal rigidity. In a dynamic environment with menu costs, prices are *dynamic complements*, as in the following example.[5] Suppose two competitors—Firm $A$ and Firm $B$—begin the period with prices $p_A \gg p_B > p^*$. Consider these actions: Firm $A$ keeps its price fixed, Firm $B$ pays the menu cost and increases its price to $p'_B \in (p_B, p_A)$. Given Firm $A$'s action, complementarity in pricing means that a price just undercutting $p_A$ is Firm $B$'s best response, and profitable net of the menu cost. Given Firm $B$'s action, menu costs mean that inaction is Firm $A$'s best response. Prices are dynamic, or intertemporal, complements in that a higher (lower) $p_A$ yields a higher (lower) $p'_B$ and a higher (lower)

[4] Throughout the paper I drop the term *strategic* when discussing *strategic complementarity*, and reserve the term to distinguish between the two models: under oligopoly firms behave strategically, and under monopolistic competition firms behave competitively, or non-strategically. This avoids confusion when making comparisons to models of strategic complementarity under a monopolistically competitive market structure in which, despite the terminology, behavior is non-strategic.

[5] I take this terminology—static vs. dynamic complementarity—from Jun and Vives (2004), who study a dynamic Bertrand game with two firms and convex costs of price adjustment.

probability of a price increase on the equilibrium path.

How does this dynamic complementarity dampen the response of inflation to a positive monetary shock? First note that complementarity will be in the relative prices that determine demand. Since households use nominal wage payments to buy goods, complementarity will be in prices relative to the wage: $\hat{p}_A = p_A/W$ and $\hat{p}_B = p_B/W$. In equilibrium, a monetary expansion increases nominal wages $W$, reducing $\hat{p}_A$ and $\hat{p}_B$. This selects more firms like Firm $B$, with an initially low price, to increase its price, and increase it by more to compensate for the increase in cost. Respectively, these *extensive* and *intensive* margin responses are large when—as they are in the data—the average size and frequency of price adjustment are large.[6] Under monopolistic competition, Firm $B$ contributes substantially to both margins, driving the response of inflation.

Dynamic complementarity dampens the response of Firm $B$ to a monetary shock on both margins. The increase in the wage brings Firm $A$'s high price into line with its costs, reducing its probability of a price cut. The falling relative price of its competitor $\hat{p}_A$, dampens Firm $B$'s impulse toward a price increase. Its optimal price increase is dampened, weakening intensive margin adjustment. Its value of a price change is dampened, weakening extensive margin adjustment. A statistical decomposition of movements in inflation into intensive and extensive margin components reveals that—relative to the competitive model—both are weakened equally in the duopoly model.

Dynamic complementarity in the strategic menu cost model yields a number of other quantitative results. First, output losses due to nominal rigidity are four times larger under duopoly relative to the competitive model. Pricing frictions enable strategic firms to achieve higher markups in equilibrium, reducing output. These output losses are first order and large relative to the second order losses from price dispersion, which are roughly equal in both models. Market structure therefore has implications both for the dynamics of output and its level. And, since the amount of dynamic complementarity that arises in equilibrium depends not on a single parameter but on all model features, invites future research that may consider how policies designed for smaller fluctuations in output may affect average output and vice versa.

Second, the value of the firm is non-monotonic in the menu cost. Small menu costs increase dynamic complementarity, thereby increasing markups and increasing value. Large menu costs

[6] The decomposition into extensive and intensive margin components in the spirit of Caballero and Engel (2007) has provided an accounting tool for this class of models and has been used by Midrigan (2011), Alvarez and Lippi (2014), and others. Figure E1 in Appendix E provides a diagrammatic representation of these margins of adjustment for a monopolistically competitive fixed menu cost model, and may be used as a reference throughout.

render firms unresponsive to the large idiosyncratic shocks they face, reducing value. From the firms' perspective, a value-maximizing, positive menu cost exists. The model therefore provides a novel rationale for actions that increase the cost of price adjustment, such as prices widely advertised as fixed for some period.

Third, when comparing market structures under the same parameters, prices are half as flexible under duopoly. That is, the strategic behavior of firms in the presence of menu costs generates some endogenous stickiness in prices. Low-priced firms are reluctant to adjust, since market share will fall in the short run. High-priced firms are reluctant to adjust, since doing so reduces the incentives of their competitor to choose a high price when they adjust. Accordingly, the oligopoly model requires 25 percent smaller menu costs, and slightly smaller idiosyncratic shocks, in order to match the same data on price adjustment.

I document empirical support for this prediction using variation across markets that plausibly have similar primitives. Defining a market by a product-state-month, I exploit variation in market concentration and price flexibility that exists across states, within product-months, controlling for market size. The empirical correlation is consistent with the causal implications of the model. Prices are less flexible in markets dominated by a small handful of firms, than those dominated by one very large firms, or many similarly sized firms. There is a robust $U$-shape (inverted $U$-shape) relationship between market concentration and the frequency (average size) of price adjustment.[7]

Fourth, that smaller menu costs and idiosyncratic shocks are required, indicates that oligopoly avoids issues that have led complementarity to be abandoned as a source of amplification. Within the competitive model, papers have tested whether the result of Golosov and Lucas (2007) survives the introduction of complementarity. Klenow and Willis (2016) introduce non-CES preferences.[8] Burstein and Hellwig (2007) introduce decreasing returns to scale in production.[9] Their findings are that such complementarities cannot be a source of propagation.[10] The reason: complementarity has the unwanted by-product of increasing price flexibility following idiosyncratic

[7]It is beyond the scope of this paper to pursue causal relationships between market concentration and price flexibility. I do not aim to address the endogeneity of market structure in this paper.

[8]A literature in international economics has employed the same Kimball (1995) demand specification to study the pass-through of exchange rate and foreign productivity shocks to domestic prices. See Gopinath and Itskhoki (2011) and Berger and Vavra (2013). This is also used to replicate the empirical slope of the Phillips curve (i.e. shallow) under the empirical frequency of price adjustment (i.e. high) in large scale New Keynesian models (for example, see Smets and Wouters (2007)).

[9]Woodford (2003, chap. 4) quantitatively compares the treatments of these types of complementarity in the earlier New Keynesian literature.

[10]Nakamura and Steinsson (2010) and Gopinath and Itskhoki (2011) provide elegant summaries of these conclusions.

shocks. Since idiosyncratic shocks determine most price changes, implausibly large menu costs and idiosyncratic shocks are required to match price adjustment data. The result that smaller menu costs and shocks are required under oligopoly, but amplification still occurs through firm-level complementarity is, therefore, significant. Section 5.3 details how the result is due to complementarity existing between two firms' prices, rather than between a firm's price and the aggregate price—as it is in the papers described above.

More generally, this paper demonstrates that the strategic interaction of firms can be quantitatively important for the cyclicality of macroeconomic aggregates. This may be of particular interest given rising concentration in many sectors of the US economy, which recent empirical work has linked to numerous macroeconomic trends.[11]

**Related Literature** The model is situated in two distinct literatures: (i) papers following Golosov and Lucas (2007) that have studied whether menu cost models of price adjustment can explain monetary non-neutrality, and (ii) dynamic games of price setting with adjustment frictions. I also contribute new facts regarding cross-sectional heterogeneity in price flexibility.

Golosov and Lucas (2007) show that in an equilibrium menu cost model of price adjustment that matches the large size and frequency of price change in good-level data, monetary shocks cause negligible output fluctuations. Extensions of the Golosov and Lucas (2007) model have been shown to mitigate this approximate neutrality. Midrigan (2011) and Alvarez and Lippi (2014) show that once the model accounts for small price changes, it can generate output responses similar to a Calvo model of price adjustment calibrated to the same moments.[12] These do not apply to models with complementarity and maintain the standard assumption: firms behave atomistically.

Nakamura and Steinsson (2010) contribute in two ways. First, they note that the size of output fluctuations is convex in the degree of price flexibility. Second, if firms purchase inputs from sectors with sticky prices, then aggregate nominal cost will respond slowly to a monetary shock. For both reasons, a multisector model that replicates the empirical heterogeneity in price

[11] Autor, Dorn, Katz, Patterson, and Reenen (2017) show that across sectors, declines in the labor share are correlated with increases in concentration. Gutierrez and Philippon (2016) show that the decline in the predictive power of Tobin's Q for aggregate investment is due to sectors that have experienced large increases in concentration. de Loecker and Eeckhout (2017) provide evidence for increasing average markups, which may also be linked to increasing concentration. In all cases, measures of concentration are computed nationally. Section 7 of this paper shows that there is significant regional heterogeneity in product market concentration even with very narrowly defined sectors.

[12] Both Midrigan (2011) and Alvarez and Lippi (2014) achieve this through multiproduct firms with economies of scope in price changes. Midrigan (2011) shows that the precise way that one accounts for small price changes is inconsequential: a single-product model with random menu costs that matches the distribution of price changes can also deliver large output responses.

flexibility across sectors generates significant non-neutrality. Like Klenow and Willis (2016) and Burstein and Hellwig (2007), the authors conclude that *macro* complementarities that slow the response of aggregate nominal cost are the most likely candidate for monetary non-neutrality.[13] The dynamic complementarity arising here is different and derives from explicitly strategic behavior under nominal rigidity. Moreover, the amount of equilibrium complementarity in prices in my model is endogenous, removing a free parameter that controls its strength, and leaving it open to changes in policy.[14] Section 5 carefully differentiates the model from those cited here.

The industrial organization literature established that nominal rigidities induce dynamic complementarity in prices when markets are oligopolistic. Maskin and Tirole (1988b) first make this point. In a stylized environment with exogenous short-run commitment to prices, MPE strategies may accommodate prices above the frictionless equilibrium. Jun and Vives (2004) extend this result in a differential game with convex costs of adjustment. Both also establish that, in response to small, unforeseen, perturbations in cost, prices may be stickier. In the data, however, idiosyncratic shocks are large, leaving open the questions as to whether such additional stickiness survives in a quantitative framework. Note also that a lower frequency of adjustment due to oligopoly is insufficient for the quantitative exercise in this paper. Comparing models of market structure requires that both models match the same data on price adjustment, which my calibration strategy ensures.

Nakamura and Zerom (2010) and Neiman (2011) study a single oligopolistic sector with menu costs of price adjustment. The former study three firms subject to a sectoral shock to the cost of inputs. Consistent with the monetary literature, I assume that firms face both idiosyncratic and aggregate shocks. The latter studies two firms subject to idiosyncratic shocks, but does not bring the model to the data on the size and frequency of price adjustment nor compare implications to a monopolistically competitive benchmark. Neither discusses the effect of nominal rigidity on the level of markups and firm value nor studies the model in general equilibrium.

I also contribute two new facts to a literature that has documented persistent heterogeneity in price flexibility across sectors (Bils and Klenow, 2004). First, I show that within a narrow product category, the average variation in price flexibility observed across geographic markets is two-thirds as large as the variation across all product categories. Price flexibility is as much market

[13] Nakamura and Steinsson (2010) follow the formulation of the *roundabout production structure* of Basu (1995). A similar structure is used in Weber, Pasten, and Schoenle (2017), in which sectoral heterogeneity in price flexibility is also taken as a primitive.

[14] For example, fixing the shape of the demand system faced by firms—which fixes the amount of *static complementarity*—there is more *dynamic complementarity* when firms are able to time their price changes. Therefore, when comparing Calvo and menu cost versions of the strategic model, there is not the same large difference that is found when comparing Calvo and menu cost versions of the competitive model.

specific as it is good specific. Second, I establish that a component of this variation across markets is systematic and relates to market concentration.[15]

**Outline** Section 2 presents the model. Section 3 describes the main mechanism using simulations of the model. Section 4 presents the calibration. Section 5 presents the main results, decomposition exercise, robustness, and distinguishes the results from the papers discussed above. Section 6 describes the additional results. Section 7 provides the empirical analysis. Section 8 concludes. An Appendix contains—among other details—further discussions of modelling assumptions, and theoretical results for a one-period price-setting game under menu costs and static complementarity.

## 2 Model

Time is discrete. There are two types of agents: households and firms. Households are identical, consume goods, supply labor, and buy shares in a portfolio of all firms in the economy. Firms are organized in a continuum of sectors indexed $j \in [0,1]$. Each sector contains two firms indexed $i \in \{1,2\}$. Goods are differentiated first across, then within sectors. Good $ij$ is produced by a single firm operating a technology with constant returns to scale in labor. Aggregate uncertainty arises from shocks to the growth rate $g_t$ of the money supply $M_t$, and idiosyncratic uncertainty arises from shocks to preferences for each good $z_{ijt}$. Each period every firm draws a menu cost $\xi_{ijt} \sim H(\xi)$ and may change their price $p_{ijt}$ conditional on paying $\xi_{ijt}$.

I write agents' problems recursively, such that the time subscript $t$ is redundant. The aggregate state is denoted $\mathbf{S} \in \mathcal{S}$. The sectoral state is denoted $s \in S$. The measure of sectors with state $s$ is given by $\lambda(s, \mathbf{S})$. When integrating over sectors, I integrate $s$ over $\lambda(s, \mathbf{S})$ rather than $j$ over $U[0,1]$.

[15] Existing models incorporating the empirical cross-sectional heterogeneity in price flexibility assume it is caused by sectoral heterogeneity in nominal rigidity. Nakamura and Steinsson (2010) incorporate heterogeneity in menu costs. Weber (2016) and Gorodnichenko and Weber (2016) incorporate heterogeneity in the Calvo parameter. For identical exogenous menu costs, I find that prices endogenously change less frequently under duopoly. In a related result, and in the context of an international menu cost model, Berger and Vavra (2013) reject substantial heterogeneity in menu costs on the basis of its poor performance in accounting for the positive cross-sector covariance of average size of adjustment and pass-through of exchange rate shocks. If sectors had a low average size of adjustment *because* of low menu costs, then they would have, counterfactually, higher pass-through. In an estimation exercise that allows for cross-sectional heterogeneity in a number of parameters, the authors find that heterogeneity in the curvature of demand best explains the data. Heterogeneity in market structure could be one way of accounting for this variation.

## 2.1 Household

Given prices for all goods in all sectors $p_i(s, \mathbf{S})$, wage $W(\mathbf{S})$, price of shares $\Omega(\mathbf{S})$, aggregate dividends $\Pi(\mathbf{S})$, the distribution of sectors $\lambda(s, \mathbf{S})$, and law of motion for the aggregate state $\mathbf{S}' \sim \Gamma(\mathbf{S}'|\mathbf{S})$, households' policies for consumption demand for each good in each sector $c_i(s, \mathbf{S})$, labor supply $N(\mathbf{S})$, and share demand $X'(\mathbf{S})$, solve

$$\begin{aligned}
\mathbf{W}(\mathbf{S}, X) &= \max_{c_i(s),N,X'} \log C - N + \beta \mathbb{E}\Big[\mathbf{W}(\mathbf{S}', X')\Big], \\
\text{where} \quad C &= \left[\int_S \mathbf{c}(s)^{\frac{\theta-1}{\theta}} d\lambda(s, \mathbf{S})\right]^{\frac{\theta}{\theta-1}}, \\
\mathbf{c}(s) &= \left[\Big(z_1(s)c_1(s)\Big)^{\frac{\eta-1}{\eta}} + \Big(z_2(s)c_2(s)\Big)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta-1}{\eta}},
\end{aligned}$$

subject to the nominal budget constraint

$$\int_S \Big[p_1(s, \mathbf{S})c_1(s) + p_2(s, \mathbf{S})c_2(s)\Big] d\lambda(s, \mathbf{S}) + \Omega(\mathbf{S})X' \;\leq\; W(\mathbf{S})N + \Big(\Omega(\mathbf{S}) + \Pi(\mathbf{S})\Big)X.$$

Households discount the future at rate $\beta$, have time-separable utility, and derive period utility from consumption adjusted for the disutility of work, which is linear in labor.[16] Utility from consumption is logarithmic in a CES aggregator of consumption utility from the continuum of sectors. The cross-sector elasticity of demand is denoted $\theta > 1$. Utility from sector $j$ goods is given by a CES utility function over the two firms' goods. The within-sector elasticity of demand is denoted $\eta > 1$. These elasticities are ranked $\eta > \theta$ indicating that the household is more willing to substitute goods within a sector (Pepsi vs. Coke) than across sectors (soda vs. laundry detergent). Finally, household preference for each good is subject to a shifter $z_i(s)$ that evolves according to a random walk,

$$\log z_i'(s') = \log z_i(s) + \sigma_z \varepsilon_i', \quad \varepsilon_i \sim \mathcal{N}(0, 1). \tag{1}$$

The shock $\varepsilon_i'$ is independent over firms, sectors, and time.

The solution to the household problem consists of demand functions for each firm's output $c_i(s, \mathbf{S})$, a labor supply condition $N(\mathbf{S})$, and an equilibrium share price $\Omega(\mathbf{S})$ which will be used to price firm payoffs. Demand functions are given by

[16] A parameter controlling the utility cost of labor can be normalized to one, so is not included.

$$
\begin{aligned}
c_i(s,\mathbf{S}) &= z_i(s)^{\eta-1}\left(\frac{p_i(s,\mathbf{S})}{\mathbf{p}(s,\mathbf{S})}\right)^{-\eta}\left(\frac{\mathbf{p}(s,\mathbf{S})}{P(\mathbf{S})}\right)^{-\theta} C(\mathbf{S}), \qquad (2)\\
\text{where} \quad \mathbf{p}(s,\mathbf{S}) &= \left[\left(\frac{p_1(s,\mathbf{S})}{z_1(s)}\right)^{1-\eta}+\left(\frac{p_2(s,\mathbf{S})}{z_2(s)}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}},\\
P(\mathbf{S}) &= \left[\int_S \mathbf{p}(s,\mathbf{S})^{1-\theta} d\lambda(s,\mathbf{S})\right]^{\frac{1}{1-\theta}}.
\end{aligned}
$$

Aggregate real consumption is $C(\mathbf{S})$. The allocation of $C(\mathbf{S})$ to sector $s$ depends on the level of the sectoral price $\mathbf{p}(s,\mathbf{S})$ relative to the aggregate price $P(\mathbf{S})$. The allocation of expenditure to firm $i$ is then determined by $z_i(s)$ and the level of firm $i$'s price relative to $\mathbf{p}(s,\mathbf{S})$.

The aggregate price index satisfies $P(\mathbf{S})C(\mathbf{S}) = \int_S \left[p_1(s,\mathbf{S})c_1(s,\mathbf{S}) + p_2(s,\mathbf{S})c_2(s,\mathbf{S})\right] d\lambda(s,\mathbf{S})$, such that $P(\mathbf{S})C(\mathbf{S})$ is equal to aggregate nominal consumption. I assume that aggregate nominal consumption must be paid for using money $M(\mathbf{S})$ such that $M(\mathbf{S}) = P(\mathbf{S})C(\mathbf{S})$ in equilibrium.[17] Nominal money supply is exogenous. Its growth rate $g' = M'/M$ evolves as follows:

$$
\log g'(\mathbf{S}') = (1-\rho_g)\log \bar{g} + \rho_g \log g(\mathbf{S}') + \sigma_g \varepsilon'_g, \quad \varepsilon'_g \sim \mathcal{N}(0,1). \qquad (3)
$$

Hence, the nominal economy is trend stationary around $\bar{g}$. An intratemporal condition determines labor supply and Euler equation prices shares under the nominal discount factor $Q(\mathbf{S},\mathbf{S}')$:

$$
\begin{aligned}
W(\mathbf{S}) &= P(\mathbf{S})C(\mathbf{S}), \qquad (4)\\
\Omega(\mathbf{S}) &= \mathbb{E}\left[Q(\mathbf{S},\mathbf{S}')\left(\Omega(\mathbf{S}')+\Pi(\mathbf{S}')\right)|\mathbf{S}\right], \quad Q(\mathbf{S},\mathbf{S}') = \beta\frac{P(\mathbf{S})C(\mathbf{S})}{P(\mathbf{S}')C(\mathbf{S}')}. \qquad (5)
\end{aligned}
$$

### 2.2 Firms

I consider the problem for firm $i$, denoting its direct competitor $-i$. The sectoral state vector $s$ consists of previous prices $p_i$, $p_{-i}$ and current preferences $z_i$, $z_{-i}$. After these states are revealed, both firms, independently, draw a menu cost for the period $\xi_{ij}$ from the known distribution $H(\xi)$. I make the additional assumption, discussed below, that these draws are private information. Simultaneously with its competitor, firm $i$ then chooses whether to adjust its price, $\phi_i \in \{0,1\}$ and price conditional on adjustment, $p_i^*$. Prices are revealed, firms produce the quantity demanded by households, and preference shocks evolve $(z_i, z_{-i})$ to $(z'_i, z'_{-i})$.

When determining its actions, firm $i$ takes as given the policies of its direct competitor: $\phi_{-i}(s,\mathbf{S},\xi_{-i})$, and $p^*_{-i}(s,\mathbf{S})$. Since menu costs are sunk, $p^*_{-i}(s,\mathbf{S})$ is independent of $\xi_{-i}$. This

[17] An alternative assumption is that money enters the utility function as in Golosov and Lucas (2007). As noted in that paper, if utility is separable, the disutility of labor is linear, and the utility of money is logarithmic, one obtains the same equilibrium conditions studied here.

description of the environment explicitly restricts firm policies to depend only on payoff relevant information $(s, \mathbf{S})$, that is, they are *Markov strategies*. A richer dependency of policies on the history of firm behavior is beyond the scope of this paper.[18]

Let $V_i(s, \mathbf{S}, \xi_i)$ denote the present discounted expected value of nominal profits of firm $i$ after the realization of the sectoral and aggregate states $(s, \mathbf{S})$ and its menu cost $\xi_i$. Then $V_i(s, \mathbf{S}, \xi_i)$ satisfies the following recursion:

$$
\begin{aligned}
V_i(s, \mathbf{S}, \xi_i) &= \max_{\phi_i \in \{0,1\}} \phi_i \left[V_i^{adj}(s, \mathbf{S}) - W(\mathbf{S})\xi_i\right] + (1-\phi_i)V_i^{stay}(s, \mathbf{S}), \qquad (6)\\
V_i^{adj}(s, \mathbf{S}) &= \max_{p_i^*} \int \Bigg[ \phi_{-i}(s, \mathbf{S}, \xi_{-i})\left\{\pi_i\left(p_i^*, p_{-i}^*(s, \mathbf{S}), s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}')V_i\left(s'_{adj}, \mathbf{S}', \xi_i'\right)\right]\right\} \\
&\quad + \left(1 - \phi_{-i}(s, \mathbf{S}, \xi_{-i})\right)\left\{\pi_i\left(p_i^*, p_{-i}, s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}')V_i\left(s'_{adj}, \mathbf{S}', \xi_i'\right)\right]\right\}\Bigg] dH(\xi_{-i}),\\
\pi_i\left(p_i, p_{-i}, s, S\right) &= d_i\left(p_i, p_{-i}, s, \mathbf{S}\right)\left(p_i - z_i(s)W(\mathbf{S})\right),\\
s'_{adj} &= \phi_{-i}(s, \mathbf{S}, \xi_{-i}) \times (p_i^*, p_{-i}^*(s, \mathbf{S}), z_i', z_{-i}') + \left(1 - \phi_{-i}(s, \mathbf{S}, \xi_{-i})\right) \times (p_i^*, p_{-i}, z_i', z_{-i}')\\
S' &\sim \Gamma\left(\mathbf{S}'|\mathbf{S}\right).
\end{aligned}
$$

The first line states the extensive margin problem, where adjustment requires a payment of menu cost $\xi_i$ in units of labor. The value of adjustment is independent of the menu cost and requires choosing a new price $p_i^*$. The firm integrates out the unobserved state of its competitor—the menu cost $\xi_{-i}$—and takes as given the effect of its competitor's pricing decisions on current payoffs and future states. The term in braces on the second (third) line gives the flow nominal profits plus continuation value of the firm if its competitor does (does not) adjust its price. Non-adjustment value $V_i^{stay}(s, \mathbf{S})$ and state $s'_{stay}$ are identical, up to $p_i^* = p_i$.

The flow payoff introduces a role for $z_i(s)$ in costs. As in Midrigan (2011), I assume that $z_i(s)$—which increases demand for the good with an elasticity of $(\eta - 1)$ — also increases total costs with a unit elasticity. This technical assumption, discussed below, will reduce the state space of the firm's problem, a crucial step to maintain computational tractability of the model.

The household's nominal discount factor $Q(\mathbf{S}, \mathbf{S}')$ is used to discount future nominal profits, and expectations are taken with respect to both the equilibrium transition density $\Gamma(\mathbf{S}'|\mathbf{S})$ and

[18] In the words off Maskin and Tirole (1988a), "Markov strategies...depend on as little as possible, while still being consistent with rationality." Rotemberg and Woodford (1992) study an oligopoly with arbitrary history dependence of policies but no nominal rigidity or idiosyncratic shocks. Implicit collusion leads to countercyclical markups: the value of deviating from collusion increases when demand is high, reducing the level of the markup that the trigger strategies can sustain.

firm-level shocks. Through the household's demand functions $d_i(p_i, p_{-i}, s, \mathbf{S})$, nominal profit depends on aggregate consumption $C(\mathbf{S})$, the aggregate price index $P(\mathbf{S})$, which the firm takes as given.

That menu costs are sunk and *iid* allows for a number of simplifications. Since $p^*_{-i}$ is independent of $\xi_{-i}$, firm $i$ need only know the probability that its competitor changes its price: $\gamma_{-i}(s,\mathbf{S}) = \int \phi_{-i}(s,\mathbf{S},\xi_{-i})dH(\xi_{-i})$. Since $\xi_i$ is *iid*, it can be integrated out of firm $i$'s Bellman equation. These observations reduce the state space:

$$\begin{aligned} V_i\left(s,\mathbf{S}\right) &= \int \max\left\{V_i^{adj}\left(s,\mathbf{S}\right) - W(\mathbf{S})\xi_i,\ V_i^{stay}\left(s,\mathbf{S}\right)\right\}dH(\xi_i), \qquad (7)\\ V_i^{adj}\left(s,\mathbf{S}\right) &= \max_{p_i^*}\ \gamma_{-i}(s,\mathbf{S})\left\{\pi_i\left(p_i^*, p^*_{-i}(s,\mathbf{S}), s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S},\mathbf{S}')V_i\left(s',\mathbf{S}'\right)\right]\right\}\\ &\quad + \left(1-\gamma_{-i}(s,\mathbf{S})\right)\left\{\pi_i\left(p_i^*, p_{-i}, s, \mathbf{S}\right) + \mathbb{E}\left[Q(\mathbf{S},\mathbf{S}')V_i\left(s',\mathbf{S}'\right)\right]\right\}. \end{aligned}$$

Given $p^*_{-i}(s,\mathbf{S})$ and $\gamma_{-i}(s,\mathbf{S})$, the solution to this problem delivers firm $i$'s optimal price adjustment $p_i^*(s,\mathbf{S})$ and probability of price adjustment $\gamma_i(s,\mathbf{S}) = H[(V^i_{adj}(s,\mathbf{S}) - V^i_{stay}(s,\mathbf{S}))/W(\mathbf{S})]$.

### 2.3 Equilibrium

Given the above, the aggregate state vector $\mathbf{S}$ must contain the level of nominal demand $M$, its growth rate $g$, and distribution of sectors over sectoral state variables $\lambda$. A *recursive equilibrium* is

(i) Household demand functions $d_i(p_i, p_{-i}, s, \mathbf{S})$

(ii) Functions of the aggregate state: $W(\mathbf{S})$, $N(\mathbf{S})$, $P(\mathbf{S})$, $C(\mathbf{S})$, $Q(\mathbf{S},\mathbf{S}')$

(iii) Law of motion $\Gamma(\mathbf{S},\mathbf{S}')$ for the aggregate state $\mathbf{S} = (g, M, \lambda)$

(iv) Firm value functions $V_i(s,\mathbf{S})$ and policies $p_i^*(s,\mathbf{S})$, $\gamma_i(s,\mathbf{S})$

such that

(a) Demand functions in (i) are consistent with household optimality conditions (2).

(b) The functions in (ii) are consistent with household optimality conditions (4).

(c) Given functions (i), (ii), (iv), and competitor policies; $p_i^*$, $\gamma_i$, and $V_i$ are consistent with firm $i$ optimization and Bellman equation (7).

(d) Aggregate price $P(\mathbf{S})$ equals the household price index under $\lambda(s,\mathbf{S})$, $p_i^*(s,\mathbf{S})$ and $\gamma_i(s,\mathbf{S})$.

(e) Nominal aggregate demand satisfies $P(\mathbf{S})C(\mathbf{S}) = M(\mathbf{S})$.

(f) The household holds all shares $(X(\mathbf{S}) = 1)$ and the price of shares is consistent with (4).

(g) The law of motion for $g$ and path for $M$ are determined by (3).

(h) The law of motion for $\lambda$ is consistent with firm policies and (1). Let $X = P_1 \times P_2 \times Z \times Z \in \mathbb{R}^4_+$, and the corresponding set of Borel sigma algebras on $X$ be given by $\mathcal{X} = \mathcal{P}_1 \times \mathcal{P}_2 \times \mathcal{Z}_1 \times \mathcal{Z}_2$. Then $\lambda : \mathcal{X} \to [0,1]$ and obeys the following law of motion for all subsets of $\mathcal{X}$:[19]

$$\lambda'(\mathcal{X}) = \int_X \mathbb{E}_{\gamma_1(s,\mathbf{S}),\gamma_2(s,\mathbf{S})} \mathbf{1}\left\{(p_1^*(s,\mathbf{S}), p_2^*(s,\mathbf{S})) \in \mathcal{P}_1 \times \mathcal{P}_2\right\} \mathbb{P}\left[z_1' \in \mathcal{Z}_1 | z_1\right] \mathbb{P}\left[z_2' \in \mathcal{Z}_2 | z_2\right] d\lambda(s,\mathbf{S}).$$

This extends of the standard definition of a recursive competitive equilibrium by assuming that firms are competitive with respect to firms in other sectors of the economy, but strategic with respect to firms in their own sector. Condition (c) requires that these strategies constitute an MPE.

## 2.4 Monopolistic competition and monopoly

The monopolistically competitive model is identical to the above, but where firm $i$ belongs to a continuum of firms $i \in [0,1]$ in sector $j$. The demand system is identical to (2), but where $\mathbf{p}_j(\mathbf{S}) = [\int (p(s,\mathbf{S})/z(s))^{1-\eta} d\lambda_j(s,\mathbf{S})]^{1/(1-\eta)}$. Since firms are competitive, they take $\mathbf{p}_j(\mathbf{S})$ as given, so the state of the firm is limited to its own $z_i$ and past price $p_i$. Moreover, since sectors are homogeneous in parameters, and the law of large numbers applies for each sector, then the distribution of firms $\lambda_j$ is the same in all sectors. Therefore $\mathbf{p}_j(\mathbf{S}) = \mathbf{p}_k(\mathbf{S})$ for all $j$ and $k$, and $P(\mathbf{S}) = \mathbf{p}_j(\mathbf{S})$. The cross-sector elasticity of demand $\theta$ is absent from the firm problem and all equilibrium conditions.

Note, therefore, the connection between monopolistic competition and another market structure: sectoral monopoly. Under monopoly, the sectoral price index is the monopolist's price, and the within-sector elasticity of demand $\eta$ is redundant. Sectoral monopolistic competition under $(\theta, \eta) = (\theta_{mc}, \eta_0)$ will therefore be identical in firm and aggregate dynamics to sectoral monopoly with $(\theta, \eta) = (\eta_0, \eta_m)$ for any values of $\theta_{mc}$ and $\eta_m$. I return to this point when discussing the model's implications for the empirical relationship between concentration and price flexibility.

## 2.5 Markups

A sectoral MPE, nested in a macroeconomic equilibrium, is computationally infeasible with four continuous state variables. However, it may be restated in terms of markups, which are the ratio of nominal price to nominal marginal cost: $\mu_{ij} = p_{ij}/(z_{ij}W)$. Similarly, I define the sectoral markup $\mu_j = \mathbf{p}_j/W$ and aggregate markup $\mu = P/W$. Along with (2), these definitions imply $\mu_j = [\mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta}]^{1/(1-\eta)}$, and $\mu = [\int_0^1 \mu_j^{1-\theta} dj]^{1/(1-\theta)}$.

Expressed in markups and normalized by the wage, the profit of the firm is

[19] In this definition, $\mathbb{E}_{\gamma_1(s,\mathbf{S}),\gamma_2(s,\mathbf{S})}[f(s,\mathbf{S})]$ is the expectation of $f$ under the sector $s$ probabilities of price adjustment.

$$\pi_i\Big(\mu_i,\mu_{-i},\mathbf{S}\Big)/W(\mathbf{S}) \;=\; \tilde{\pi}_i\Big(\mu_i,\mu_{-i}\Big)\mu(\mathbf{S})^{\theta-1}, \quad \tilde{\pi}_i\Big(\mu_i,\mu_{-i}\Big) = \mu_i^{-\eta}\mu_j\left(\mu_i,\mu_{-i}\right)^{\eta-\theta}\left(\mu_i-1\right), \tag{8}$$

which implies that complementarity in prices carries over to complementarity in markups.[20] Value functions can also be normalized $v(s,\mathbf{S}) = V(s,\mathbf{S})/W(\mathbf{S})$:

$$\begin{aligned} v_i\left(\mu_i,\mu_{-i},\mathbf{S}\right) &= \int \max\left\{v_i^{adj}\left(\mu_i,\mu_{-i},\mathbf{S}\right) - \xi_i,\; v_i^{stay}\left(s,\mathbf{S}\right)\right\} dH(\xi_i), \qquad (9)\\ v_i^{adj}\left(\mu_i,\mu_{-i},\mathbf{S}\right) &= \max_{\mu_i^*} \gamma_{-i}(\mu_i,\mu_{-i},\mathbf{S})\left\{\tilde{\pi}_i\Big(\mu_i^*,\mu_{-i}^*(\mu_i,\mu_{-i},\mathbf{S})\Big)\mu\left(\mathbf{S}\right)^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}},\frac{\mu_{-i}^*(\mu_i,\mu_{-i},\mathbf{S})}{g'e^{\varepsilon_{-i}'}},\mathbf{S}'\right)\right]\right\}\\ &\quad + \Big(1-\gamma_{-i}(\mu_i,\mu_{-i},\mathbf{S})\Big)\left\{\tilde{\pi}_i\Big(\mu_i^*,\mu_{-i}\Big)\mu\left(\mathbf{S}\right)^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}},\frac{\mu_{-i}}{g'e^{\varepsilon_{-i}'}},\mathbf{S}'\right)\right]\right\}. \end{aligned}$$

This renders the firm problem stationary and clarifies the mechanics of the shocks. A random walk idiosyncratic shock $\varepsilon_i'$ is a permanent *iid* shock to the markup of firm $i$ should the firm not adjust its price. A single positive innovation to money growth causes equilibrium nominal marginal cost to increase, which reduces both firms' markups. As money growth returns to $\bar{g}$ at rate $\rho_g$, the markup continues to decline. Firm $i$ pays a real cost $\xi_i$ to adjust its markup.

In this way, all equilibrium conditions can be stated in markups. Note that aggregate consumption is $C(\mathbf{S}) = 1/\mu(\mathbf{S})$. An increase in the money supply causes an equilibrium increase in wages, reducing all firms' markups. If all prices do not increase one for one with wages, the real wage increases, labor supply increases, and output increases.

A solution for the equilibrium involves the function $\mu(\mathbf{S})$, requiring the infinite dimensional distribution $\lambda\left(\mu_i,\mu_{-i}\right)$ as a state variable. To make the problem tractable, I follow the lead of Krusell and Smith (1998). Since I already need to specify a price function for $\mu$, a convenient choice of moment to characterize $\lambda$ is last period's aggregate markup, $\mu_{-1}$. The following then serves as both pricing function and law of motion for the approximate aggregate state:

$$\mu\left(\mu_{-1},g\right) = \exp\left(\bar{\mu} + \beta_1(\log\mu_{-1} - \log\bar{\mu}) + \beta_2(\log g - \log\bar{g})\right).$$

Applying this to (9) verifies that the approximate aggregate state consists of $\mathbf{S} = (\mu_{-1},g)$. Appendix B provides more details on the solution of the firm problem and equilibrium.

Appendix D discusses a number of modeling assumptions: CES preferences, structure of idiosyncratic shocks, and random menu costs and their information structure. Following the insight of Doraszelski and Satterthwaite (2010), this last assumption is made to accommodate a solution in pure strategies. A model with fixed costs would yield mixed strategy equilibria, becoming

[20] When $\mu_{-i}$ is large, the effect of a change in $\mu_i$ on $\mu_j(\mu_i,\mu_{-i})$ is larger: $\partial\mu_j/\partial\mu_i = (\mu_j/\mu_i)^{\eta}$. Since $\eta > \theta$, then $\tilde{\pi}_i$ is increasing in $\mu_j$. Combined, these imply that the cross-partial derivative of $\tilde{\pi}_i$ is positive.

computationally infeasible. In Appendix C, I prove a number of results for a one-period game of price adjustment with a fixed menu cost, equal initial prices, and a general profit function with complementarity. For any menu cost, even in this simple setting, there always exist a range of initial prices such that multiple equilibria may arise (see Figure C1).

## 3 Illustrating the mechanism

To understand the dynamics of markups in the two models of market structure, I consider an exercise that corresponds to the central experiment in Golosov and Lucas (2007). Inflation and aggregate shocks are zero, and I study the response to a one-time unforeseen increase in money in period $t$ ($g_t > 0$, $\rho_g = 0$). Firms assume that the aggregate markup remains at its steady-state level.[21] Both models are solved and simulated under the parameters estimated in Section 4.

### 3.1 Monopolistic competition

Figure 2 describes the behavior of firms in the monopolistically competitive model. Black (grey) lines describe a firm that, from period five onward, has received a string of positive (negative) idiosyncratic shocks. For $t < 5$, firms draw zero menu costs, and for $t \geq 5$, both firms draw large menu costs such that their prices do not adjust. Thin solid lines in panel A plot the evolution of each firm's markup absent the increase in money supply. Dashed lines in panel A describe the optimal reset markup of each firm $\mu^*_{it}$. Since $\mu_{it}$ is payoff irrelevant once the firm decides to change its price, the reset markup is constant and the same for both firms. Thin lines in panel B plot the firm's probability of adjustment $\gamma_{it} = \gamma(\mu_{it})$.

The thick lines in Figure 2 describe the response to a permanent increase in the money supply in period 40 which, absent adjustment, reduces both firms' markups. The low-markup firm's probability of adjustment increases as its markup moves away from its reset value. The size of its optimal adjustment increases by $\Delta M$, accommodating the entire increase in aggregate nominal cost. The high-markup firm moves closer to its reset value, its probability of adjustment falls, and its size of adjustment falls by $\Delta M$. The firms' optimal markups are unaffected by the shock.[22]

[21] This turns out to be a good approximation for three reasons. First, the aggregate markup $\mu(\mathbf{S})$ has only a second order effect on the policies of the firm (see (8)). Second, aggregate shocks are small so $\mu(\mathbf{S})$ fluctuates very little. Third, since $\theta$ is close to one, then movements in $\mu(\mathbf{S})$ change firm profits by little. In the monopolistically competitive model, this intuition is formalized in Proposition 7 of Alvarez and Lippi (2014). For further discussion, see Appendix B.

[22] Since the shock to money growth is not persistent, the optimal markup of the firm does not change. If $\rho_g > 0$, then the optimal markup would itself increase. The firm increases its markup by more in period 40, knowing that higher than steady-state money growth will wear down its markup in consecutive periods.

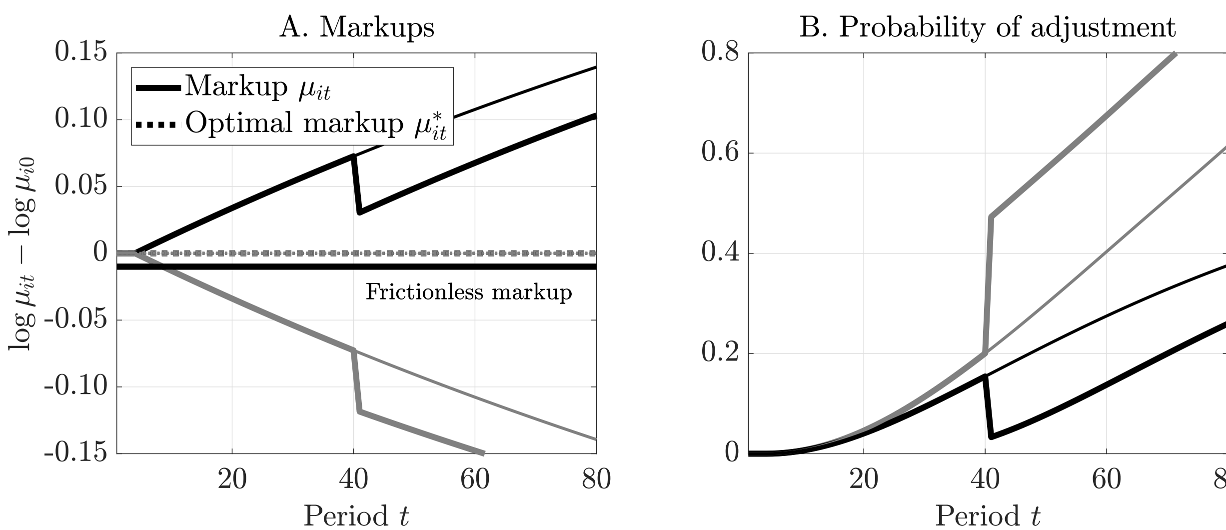


Figure 2: Example - Positive monetary shock in monopolistically competitive model

Notes: Thin solid lines give exogenous evolution of markups for two firms *within the same sector* absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment, where $\mu_1^* = \mu_2^*$. Thick solid lines include a monetary shock in period 40 which decreases both firms' markups. Thick dashed lines—which lie on top of the thin dashed lines before period 40—give the corresponding optimal markups. The model is solved in steady state, and the monetary shock is a one-time unforeseen level increase in money. The parameters of the model are as in Table 1. The $y$-axis in panel A describes the log deviation of markups from the value chosen when realizations of shocks and menu costs are zero, $\bar{\mu} = 1.30$, which is equal to the average markup.

As detailed by Golosov and Lucas (2007), this behavior sharply curtails the real effects of the monetary expansion. The distribution of adjusting firms shifts toward those with already low prices. These are firms that are increasing their prices and now by larger amounts. Monetary neutrality owes to the behavior of these firms with low markups and a high probability of adjustment that are *marginal* with respect to the shock.

### 3.2 Duopoly

I now repeat this exercise in the duopoly model for two firms in the same sector. The firms differ both in their policies absent the shock and in their response to the shock. These differences are due to the interaction of menu costs and complementarity in prices that arise in the duopoly model.

**Static complementarity** Prices are *static complements* when the cross-partial derivative of a firm's profit function ($\tilde{\pi}_{12} > 0$) is positive. Economically, this is the case for two reasons: (i) firms are strategic, so they understand how their price affects the sectoral price, and (ii) the household has a lower ability to substitute across sectors than within sectors. As $\mu_2$ increases, firm 1 sells to more of the market. Because of (i), firm 1 understands how this changes its demand elasticity. Because of (ii), the elasticity it faces falls, encouraging a higher markup. Figure 3A plots the static best

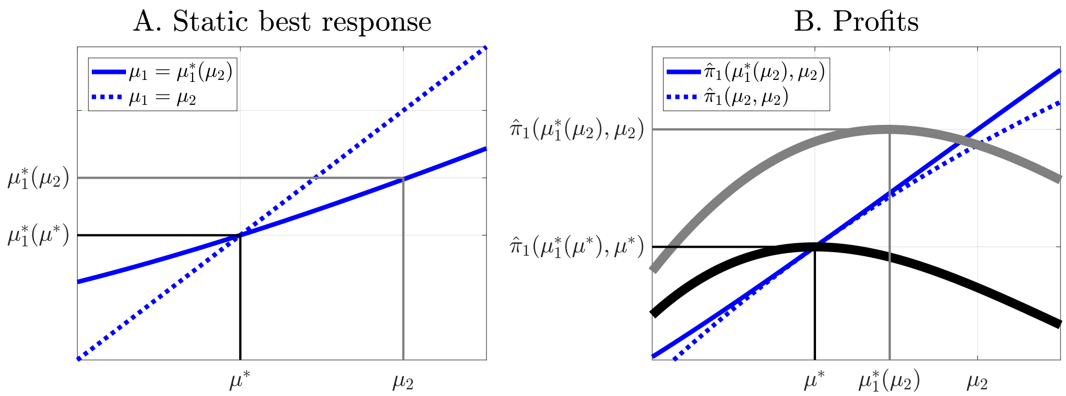


Figure 3: Static complementarity

Notes: Thick curves in panel B plot the component of firm 1's profit function due to the two firms' markups: $\hat{\pi}_1(\mu_1, \mu_2)$ from the normalized profit function in equation (8). The only parameters that enter this function are $\eta$ and $\theta$, which are set to their calibrated values of 10.5 and 1.5 (see Table 1). The upper (grey) curve describes firm 1 profits when $\mu_2 = 1.30$, which equals the average markup under the baseline calibration. The lower (black) curve describes firm 1 profits when $\mu_2 = 1.20$, which equals the frictionless Nash equilibrium markup under the baseline calibration. Given a value of $\mu_2$ on the $x$-axis, the solid thin line describes $\hat{\pi}_1(\mu_1^*(\mu_2), \mu_2)$, under firm 1's static best response. The static best response $\mu_1^*(\mu_2)$, is plotted in panel A. Given a value of $\mu_2$ on the $x$-axis, the dotted thin line describes $\hat{\pi}_1(\mu_2, \mu_2)$, under firm 1 setting its markup equal to firm 2's.

response function of firm 1: $\mu_1^*(\mu_2)$.[23]

**Dynamic complementarity** In an MPE with zero menu costs, static complementarity does not lead to monetary non-neutrality. The unique equilibrium actions consist of both firms choosing the static Nash equilibrium markup in all periods. In other words, the MPE policy function, $\mu_i^*(\mu_i, \mu_{-i}) = \mu^*$, is independent of $\mu_i$ and $\mu_{-i}$. An increase in money supply which reduces both firms' markups at the start of the period is immediately passed through to prices.

In the presence of menu costs, however, this static complementarity is reflected in the MPE, and $\mu_i^*$ and $\gamma_i$ will depend on initial markups. Menu costs make future price reductions costly. So in equilibrium, a high $\mu_2$ at the start of the period illicits a high equilibrium response of firm 1 within the period: a low-priced firm adjusts to a price that is below but close to its high-priced competitor. Prices are *dynamic complements* in that increases in the pre-determined state-variable of firm 1 illicits an increasing response of firm 2.[24]

Figure 3B provides an intuition for how such strategies may be accommodated. While the static best response $\mu_1^*(\mu_2)$ is to undercut $\mu_2$, it does not substantially increase firm 1's profit above what is obtained under $\mu_1 = \mu_2$. Small values of menu costs can lend credibility to following a

[23]In Appendix C I show that the best response function in a static, frictionless model under CES preferences with $\eta > \theta$ is upward sloping with a slope less than one. This implies that if $\mu_{-i}$ is greater than the frictionless Nash equilibrium markup $\mu^*$, then the static best response of firm $i$ is to undercut: $\mu_i^*(\mu_{-i}) \in (\mu^*, \mu_j)$. Figure C2 provides—around the calibrated values of $\theta$ and $\eta$—comparative statics with respect to $\eta$ of the best response function and other features of the profit function.

[24]I take this language from Jun and Vives (2004), who differentiate between static and dynamic complementarity in the MPE of dynamic oligopoly models of Cournot and Bertrand competition with convex costs of adjustment.

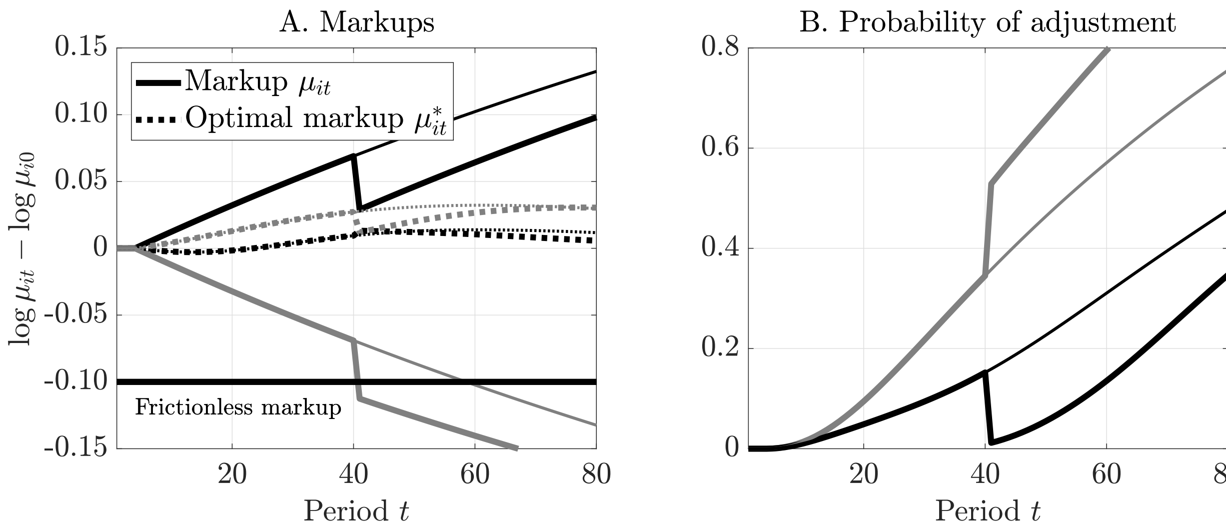


Figure 4: Example - Positive monetary shock in duopoly model

Notes: Thin solid lines give exogenous evolution of markups for two firms *within the same sector* absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment $\mu_1^*(\mu_1, \mu_2)$ and $\mu_2^*(\mu_1, \mu_2)$. Thick solid lines include a monetary shock in period 40 which decreases both firms' markups. Thick dashed lines—which lie on top of the thin dashed lines before period 40—give the corresponding optimal markups. The model is solved in steady state, and the monetary shock is a one-time unforeseen level increase in money. The parameters of the model are as in Table 1. The $y$-axis in panel A describes the log deviation of markups from the value chosen when realizations of shocks and menu costs are zero, $\bar{\mu} = 1.30$, which is equal to the average markup.

competitor's high price, and allow firms to sustain markups and profits significantly higher than those that occur at the frictionless Nash equilibrium $\mu^*$. Figure 4 shows how the MPE policy functions of firms reflect this dynamic complementarity.

**Steady-state policies** As opposed to the monopolistically competitive policies, optimal markups $\mu_i^*(\mu_i, \mu_j)$ are no longer equal, and the low-markup (grey) firm sets $\mu_{it}^*$ to below, but near, that of its competitor. Choosing a high optimal markup and high probability of adjustment discourages undercutting by the high markup (black) firm. This maintain's the low-markup firm's market share in the short run while also supporting a high sectoral price in the long run. The menu costs faced by the high-markup firm makes its low probability of a price cut a credible response to the low-markup firm's policy.

In this way, the non-cooperative MPE of the model sustains markups substantially above the frictionless Bertrand-Nash equilibrium, even in the presence of large idiosyncratic shocks. Note, however, that the size of this wedge is limited by the size of the menu cost. Figure 3B shows that higher initial markups increasingly invite undercutting: $\tilde{\pi}_1(\mu_1^*(\mu_2), \mu_2)) - \tilde{\pi}_1(\mu_2, \mu_2)$ increases as $\mu_2$ exceeds $\mu^*$. In Figure 4A, this is reflected in the flattening out of the grey firm's optimal markup. If the grey firm adjusted to an even higher markup, the menu cost would be insufficient to commit the black firm to not undercut its price. While static complementarity depends only on $\theta$ and $\eta$, the amount of dynamic complementarity in the MPE depends on the price change technology and all other features of the economic environment. As I show below, the MPE of a Calvo model features less dynamic complementarity: initial prices are less influential when adjustment is random.

**Response to monetary shock** Dynamic complementarity leads the duopoly model to respond differently to the monopolistically competitive model following a monetary shock. The desired price increase at the low-markup firm still jumps to cover the increase in aggregate nominal cost, but this is tempered by the decline in its competitor's markup. The equilibrium best response of the marginal firm is increasing in the initial markup of the inframarginal firm, so with a lower markup at the inframarginal firm, the optimal markup of the marginal firm falls. With a lower markup at its competitor, the increase in the value of a price change is also dampened since any price increase will be met with lower, more elastic demand.[25] In the example of Figure 4, the probability and size of price adjustment at the marginal firm increase by half as much as they do in Figure 2.[26]

Monetary non-neutrality occurs because price adjustment at marginal firms is weakened by the falling relative price at inframarginal firms. Figure 4 provides a stark example, considering firms with markups below and above their reset markups. Figure E4 repeats the experiment for two low-priced firms. In such sectors, the desired markup of both firms increases. With both firms' probability of adjustment increasing, the firms choose as high a markup as is sustainable given menu costs. The decomposition below reveals that sectors representative of Figure 4 dominate in shaping the aggregate price response.

I now return to the full model with stochastic, persistent money growth shocks for a quantitative comparison of monetary non-neutrality under both market structures.

[25] For completeness, consider the symmetric case of a negative money supply shock. The nominal wage falls and—conditional on non-adjustment—markups increase. The marginal firm now has the high markup and considers decreasing its markup, while the shock has increased the markup of its competitor. The increasing markup at its competitor shifts the marginal firm's demand curve out and lowers its elasticity, reducing the value of a price decrease and its optimal size.

[26] Note the small increase in $\mu^*_{it}$ at the high markup firm. Increasing $\mu^*_{it}$ encourages its competitor to choose a high markup conditional on adjustment, which is now a more likely event.

# 4 Calibration

Both models are calibrated at a monthly frequency with $\beta = 0.95^{1/12}$. I follow the same procedure as Midrigan (2011) for calibrating the persistence and size of shocks to the growth rate of money: $\rho_g = 0.61$, $\sigma_g = 0.0019$.[27] I set $\bar{g}$ to replicate 2.5 percent average inflation in the US from 1985 to 2016. The final parameter set externally is the cross-sector elasticity $\theta$ which I set to 1.5, consistent with Nechio and Hobijn (2017), one of the few studies to provide empirical estimates of upper-level demand elasticities.[28]

The same set of parameters remain in both models: (i) within-sector elasticity of substitution $\eta$, (ii) size of idiosyncratic shocks $\sigma_z$, (iii) distribution of menu costs. I assume menu costs are uniformly distributed $\xi_{ijt} \sim U\left[0, \bar{\xi}\right]$ and refer to $\bar{\xi}$ as the menu cost. These parameters are chosen to match the average absolute size and frequency of price change in the IRI data, as well as a measure of the average markup.[29]

As shown by Golosov and Lucas (2007), matching these first two moments severely constrains the ability of the monopolistically competitive menu cost model to generate sizeable output fluctuations. A large average size of price change implies that the additional low-markup firms adjusting after a monetary shock will have large positive price changes. If prices change frequently, then the increase in nominal cost is quickly incorporated into the aggregate price index. The average absolute log size of price change is 0.10, and the average frequency of price change is 0.13. Appendix A details the construction of these measures, noting here that I exclude sales and small price changes that may be deemed measurement error.

The third moment, the average markup, is motivated two ways. First, note that the duopolist

[27] Specifically, I take monthly time series for $M1$ and regress $\Delta \log M1_t$ on current and 24 lagged values of the monetary shock series constructed by Romer and Romer (2004). I then estimate an AR(1) process on the predicted values. The coefficient on lagged money growth is $\rho_g = 0.608$, with standard error 0.045. The standard deviation of residuals gives $\sigma_g$.

[28] Edmond, Midrigan, and Xu (2015) estimate $\theta = 1.24$ and $\eta = 10.5$ in a static oligopoly model with trade. In their quantitative application Atkeson and Burstein (2008) choose $\theta$ "close to one" and $\eta = 10$. When estimating within-sector elasticities of substitution, it is common practice in industrial organization to assume that $\theta = 1$ such that preferences are Cobb-Douglas across sectors (for an example, see Hottman, Redding, and Weinstein (2014)).

[29] The argument for identification is as follows. The parameter $\eta$ has an overwhelming effect on the average markup. Given a value of $\eta$, one can match the size and frequency of price change by changing $\bar{\xi}$ and $\sigma_z$. Let $x_{it} = \left|\log\left(\mu^*_{it}/\mu_{it}\right)\right|$. Increasing $\bar{\xi}$ lowers adjustment probabilities for any $x_{it}$, lowering frequency of price change. The average size of price change increases since $x_{it}$ will on average be larger by the time the firm adjusts. Increasing $\sigma_z$ increases frequency of price change since any large value of $x_{it}$ now occurs more often, and increases average size of price change since more frequent adjustment leads the firm to wait until $x_{it}$ is larger before adjusting. This leads to an indirect effect that pushes the frequency of adjustment down. Theoretically, this argument leads to exact identification in a continuous time, fixed menu cost model (Barro, 1972). However, the widening of adjustment boundaries due to higher $\sigma_z$ leads to an indirect effect that pushes frequency of adjustment down. Similarly to Vavra (2014), Berger and Vavra (2013), and others, I find that quantitatively the indirect effect is dominated by the direct effect, allowing for identification.

| | | Duopoly | Monopolistic competition | | | |
|---|---|---|---|---|---|---|
| | | | Base | Alt. I | Alt. II | Alt. III |
| A. Parameter | | | | | | |
| Within-sector elasticity of demand | $\eta$ | 10.5 | 4.5 | 10.5 | 6 | 10.5 |
| Upper bound of menu cost distribution | $\xi \sim U[0, \bar{\xi}]$ | 0.17 | 0.21 | 0.17 | 0.29 | 0.42 |
| Size of shocks (percent) | $\sigma_z$ | 3.8 | 4.0 | 3.8 | 4.0 | 4.0 |
| B. Moments | | | | | | |
| Markup | $\mathbb{E}\left[\mu_{it}\right]$ | 1.30 | 1.30 | 1.12 | 1.22 | 1.13 |
| Frequency of price change | $\mathbb{E}\left[\mathbf{1}\{p_{it} \neq p_{it-1}\}\right]$ | 0.13 | 0.13 | 0.19 | 0.13 | 0.13 |
| Log absolute price change, cond. on price change | $\mathbb{E}\left[\lvert \log(p_{it}/p_{it-1}) \rvert\right]$ | 0.10 | 0.10 | 0.05 | 0.10 | 0.10 |
| C. Results | | | | | | |
| Std. deviation consumption (percent) | $\sigma\left(\log C_t\right)$ | 0.31 | 0.13 | 0.06 | 0.13 | 0.13 |
| Average markup minus frictionless markup | $\mathbb{E}\left[\mu_{it}\right] - \mu^*$ | 0.10 | 0.02 | 0.01 | 0.02 | 0.02 |

Table 1: Parameters in the duopoly and monopolistically competitive models

Notes: The table presents three alternative calibrations of the monopolistically competitive model. *Alt. I* has the same parameters as the baseline duopoly calibration. *Alt. II* has a value of $\eta$ chosen such that it delivers the same frictionless markup as the baseline duopoly calibration. *Alt. III* has a value of $\eta$ equal to the baseline duopoly calibration. The value of $\bar{\xi}$ in *Alt. II* and *Alt. III* is chosen to match the frequency and size of adjustment. Given that $\log z_{ij}$ follows a random walk, $\sigma_z$ measures percentage innovations to $z_{ij}$.

faces an overall elasticity of demand between $\theta$ and $\eta$, since it does not take the sectoral markup as given. Therefore, if $\eta$ and $\theta$ were the same in both models, then the lower demand elasticity facing the duopolist would be a force toward less frequent price adjustment, requiring a significantly lower menu cost to match the data. Calibrating to the same average markup means the elasticity of demand faced by firms in both models is approximately the same.

Second, equating average markups equates average profits. A ranking of calibrated menu costs is therefore preserved when transformed into the ratio of menu costs to profits, which is an economically more meaningful measure. I can therefore make statements regarding the price stickiness endogenously generated by each model by simply comparing the calibrated menu costs. Note that by calibrating both models to match the same frequency of price change, there is no role for any such endogenous price stickiness in the comparison of aggregate dynamics. The spirit of the experiment is to control for price flexibility with respect to idiosyncratic shocks and examine the differential response to aggregate shocks.

I target an average markup of 1.30, which forms the consensus of a range of studies using various techniques. In their estimation of markups across 50 sectors ,Christopoulou and Vermeulen (2008) find an average markup in the US of 1.32. For the US auto industry, Berry, Levinsohn, and Pakes (1995) estimate an average markup of 1.31. For retail goods, Hottman (2016) estimates an average markup between 1.29 and 1.33. For Compustat firms, de Loecker and Eeckhout (2017) estimate an average markup between 1.20 and 1.30 for the pre-1990 period. Macroeconomic models with monopolistic competition commonly calibrate to a lower average markup around 1.20. This would require a higher elasticity of demand in the duopoly model, implying greater complementarity in prices and larger output fluctuations.

Table 1 provides calibrated parameter values. The baseline calibration of the monopolistically competitive model appears in the column *Base*. The remaining columns provide alternative calibrations of the monopolistically competitive model, which I will refer to below. Since idiosyncratic markup shocks ($\sigma_z \approx 0.04$) are more than twenty times larger than aggregate markup shocks ($\sigma_g = 0.0019$), aggregate shocks could, in practice, be shut off and the moments of the model be unaffected. Hence, the calibration delivers models that have the same good-level price flexibility following good-level shocks.

To demonstrate the importance of equating these good-level price dynamics before comparing aggregate dynamics, compare the *Base* and *Alt I* parameterizations of the monopolistically competitive model. Under *Alt I* the model is evaluated at the calibrated duopoly parameters. With a higher $\eta$, lower $\bar{\xi}$ and smaller $\sigma_z$, *Alt I* features more frequent and smaller price adjustments. With more flexible firm-level prices, output fluctuations—as measured by the standard deviation of log aggregate consumption $\sigma(\log C_t)$—are half as large (0.06 vs. 0.13).[30] The calibration strategy therefore works toward comparatively less, rather than more, amplification in the duopoly model.

## 5 Aggregate dynamics

Table 1 delivers the main result of the paper, which is that fluctuations in output are around 2.4 times larger in the duopoly model (0.31 vs. 0.13).[31] Figure 5A plots the impulse response of aggregate consumption to a one standard deviation shock to money growth, computed via local projection.[32] Panel B shows that the cumulative response is more than twice as large in the

[30]The standard deviation of log consumption is a common summary statistic for the output effects of monetary shocks in the menu cost models cited in Section 1. Specifically, $\sigma(\log C_t)$ is equal to the standard deviation of HP-filtered deviations of log of consumption from its value in an economy in which $g_t = \bar{g}$.

[31]Random menu costs imply that the monopolistically competitive model generates larger output fluctuations than under a fixed menu cost, calibrated to the same data (e.g. Golosov and Lucas (2007)). In such a model I find that $\sigma(\log C_t) = 0.06$. This difference is for the reason discussed extensively in Midrigan (2011): random menu costs generate some small price changes, dampening the extensive margin response of inflation—or selection effect—following a monetary shock. For a model based on this mechanism, see Dotsey, King, and Wolman (1999).

[32]Impulse response functions in this section are computed as follows, an approach that is econometrically equivalent to the approach used by Jorda (2005). The economy is simulated for 5,000 periods with aggregate and idiosyncratic shocks. Given the known time series of aggregate shocks to money growth $\varepsilon_t^g$, the horizon $\tau$ IRF is $IRF_\tau = \sum_{s=0}^{\tau} \hat{\beta}_\tau$, where $\hat{\beta}_\tau$ is computed using estimated values of $\beta_\tau$ from $\Delta \log C_t = \alpha + \beta_\tau \varepsilon_{t-\tau}^g + \eta_t$ The benefits of computing the IRF in this manner are (i) it is exactly what one would compute in the data if the realized path of monetary shocks was known, which is consistent with the approach that uses identified monetary shocks from either a narrative or high-

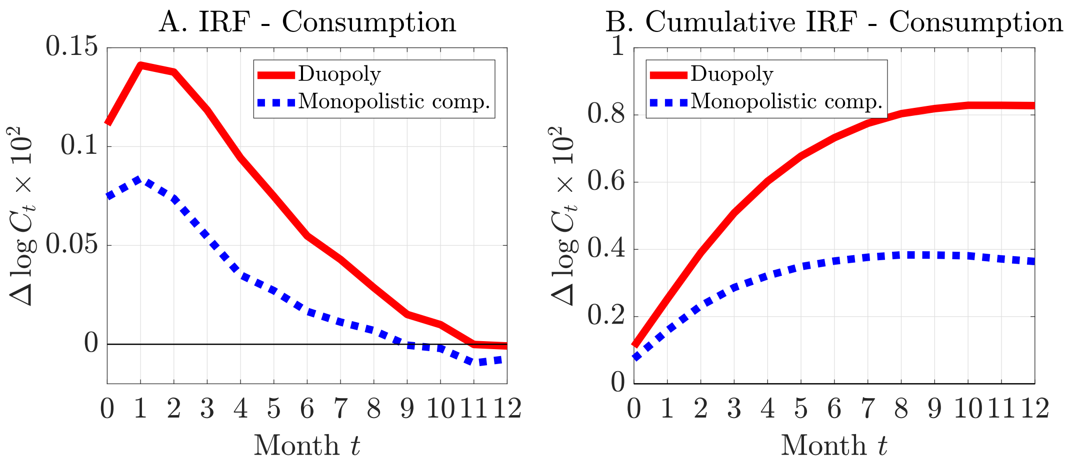


Figure 5: Monetary non-neutrality in the duopoly and monopolistically competitive models

Notes: Parameters for both models are as in Table 1, with the monopolistically competitive model under *Base*. Impulse response function computed by local projection; see footnote 32. The response function plotted $IRF_\tau$ for $\Delta \log C_t$ is multiplied by the standard deviation of innovations to money growth $\sigma_g = 0.0019$. This is then multiplied by 100, such that units are log points.

duopoly model (0.83 vs. 0.36).

These results can be compared with other papers that study the neutrality of money in extensions of the Golosov and Lucas (2007) model. Output fluctuations are slightly larger than in the multiproduct model of Midrigan (2011) ($\sigma(\log C_t) = 0.29$). The ratio of $\sigma(\log C_t)$ under duopoly to monopolistic competition is also larger than what Nakamura and Steinsson (2010) find when comparing single and multisector menu cost models (a ratio of 1.82 compared to 2.38 here).[33] My paper therefore adds realism—markets are concentrated—and moves the model toward the large real effects of monetary shocks found in the data.[34]

### 5.1 Verifying the mechanism I: Impulse responses

To check whether the intuition from Section 3 holds in the full model, I study the response of the size and frequency of price change for low- and high-markup firms following a positive monetary shock. Figure 6 shows that the broad dynamics of both models are the same. Low-markup firms

frequency approach (Gertler and Karadi, 2015); (ii) it avoids the time-consuming approach of simulating the model many times, as is usually done in heterogeneous agents models with aggregate shocks; and (iii) it averages over any state dependence which might bias the results if computing an IRF from a specific state, as well as any non-linearity in the size of the response following positive or negative and small or large shocks. Berger, Caballero, and Engel (2017) extensively assess the benefits of this approach in accurately capturing the persistence of aggregate dynamics in lumpy adjustment models.

[33]See their Table VI (first row, first two columns). This ratio is 1.63 when comparing single and multisector menu cost models of their Calvo+ model (a menu cost model where $\xi > 0$ with probability $\alpha$ and $\xi = 0$ with probability $(1 - \alpha)$.

[34]In a VAR study in which nominal shocks are identified by long-run restrictions on the effects of nominal and real shocks Shapiro and Watson (1988) attribute 28 percent of the variation in output to nominal shocks. The standard deviation of HP-filtered log consumption in the United States (1947-2006) is $1.28 \times 10^{-2}$, and in the model this is $0.31 \times 10^{-2}$. The model, therefore, generates fluctuations in output around 24 percent of what appears in the data.

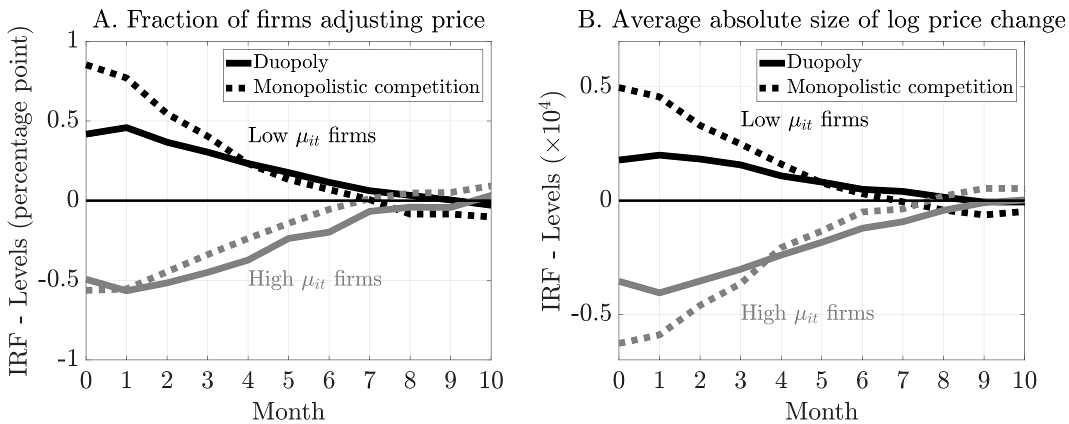


Figure 6: Impulse responses of frequency and size of adjustment to a positive monetary shock

Notes: Impulse response functions are computed by local projection (see footnote 32). For panel A, the dependent variable is the change in the fraction of firms adjusting price. For panel B, the dependent variable is the change in the average absolute size of log price changes. To isolate the effect of a positive monetary shock, only positive innovations to money growth $\varepsilon_t^g > 0$ are included in the regressions. Black (grey) lines correspond to low (high) markup firms. In the duopoly model, firms are assigned to the low-markup group if, within their sector, they have the lowest markup. In the monopolistically competitive model, pairs of firms are drawn at random and assigned to the low-markup group if their markup is the lowest in the pair.

adjust more (panel A), and the size of their price change increases (panel B). High-markup firms adjust less, and the size of their price change falls. However, both the frequency and size of price change of low-markup firms respond by less in the duopoly model. The falling markup of their competitor, on average, reduces the value of a price increase and the optimal price conditional on adjustment.

Observe that the average size of price changes at high-markup firms falls by less in the duopoly model. High markup firms' optimal price decrease is reduced now that their competitors have a higher probability of increasing their price. This is a force toward a larger increase in inflation response in the duopoly model. However, the falling probability of adjustment for high markup firms implies that this differential response is rarely incorporated into the aggregate price index.

## 5.2 Verifying the mechanism II: Decomposing inflation

The response of inflation can be more formally decomposed into an extensive and intensive margin response, and these margins compared across sectors of the economy. I follow in the spirit of the theoretical decomposition of Caballero and Engel (2007) which can be applied to a wide class of lumpy adjustment models.[35]

Consider two simulations of the model, where the model has been solved in the presence of aggregate shocks. In one simulation, aggregate shocks are set to zero such that there is only trend

[35] See Figure E1 for a diagrammatic representation of this decomposition in a monopolistically competitive model with fixed menu costs.

| | | 1. Intensive | 2. Extensive | 3. Covariance |
|---|---|---|---|---|
| A. Fraction of inflation accounted for by each margin | | | | |
| Monopolistic competition | $\pi_t^{mc}$ | 0.40 | 0.55 | 0.05 |
| Duopoly | $\pi_t^{d}$ | 0.41 | 0.58 | 0.01 |
| B. Fraction of the difference in inflation accounted for by each margin | | | | |
| Monopolistic competition minus duopoly | $\left(\pi_t^{mc} - \pi_t^{d}\right)$ | 0.36 | 0.45 | 0.19 |
| C. Fraction of the difference in each margin accounted for by regions of the distribution of markups | | | | |
| Both markups below the median | $\left(\mu_i^L, \mu_j^L\right)$ | -0.90 | -0.73 | -0.50 |
| One below, one above the median | $\left(\mu_i^L, \mu_j^H\right)$ | 1.81 | 1.65 | 1.05 |
| Both markups above the median | $\left(\mu_i^H, \mu_j^H\right)$ | 0.09 | 0.08 | 0.45 |

Table 2: Market structure and the composition of monetary non-neutrality

inflation. A second simulation features identical draws of idiosyncratic shocks, but includes a single shock to the money growth at date $t$. Denote by $\Delta \bar{p}_t$ the log change in the aggregate price index in the first simulation and by $\Delta \hat{p}_t$ the same statistic in the simulation with the shock. Inflation generated by the shock is $\pi_t = \Delta \hat{p}_t - \Delta \bar{p}_t$. Let $x_{it} = \log p_{it}^* - \log p_{it-1}$ denote the desired log price change of firm $i$, and $\gamma_{it}$ denote the probability of price change. Then $\Delta p_t \approx N^{-1} \sum_{i=1}^N \gamma_{it} x_{it}$. This implies the following decomposition of inflation:

$$\pi_t \approx N^{-1} \sum_{i=1}^{N} \underbrace{\bar{\gamma}_{it}\left(\hat{x}_{it} - \bar{x}_{it}\right)}_{\text{1. Intensive}} + \underbrace{\bar{x}_{it}\left(\hat{\gamma}_{it} - \bar{\gamma}_{it}\right)}_{\text{2. Extensive}} + \underbrace{\left(\hat{\gamma}_{it} - \bar{\gamma}_{it}\right)\left(\hat{x}_{it} - \bar{x}_{it}\right)}_{\text{3. Covariance}}. \tag{10}$$

Panel A of Table 2 provides this decomposition for each of the two models. The first two lines show that in both models, inflation is generated roughly equally by adjustment on the intensive and extensive margins. The main result from the previous section was that inflation responds by less in the duopoly model, generating larger output effects. Panel B shows that the difference in inflation is roughly equally accounted for by decreases in all margins of adjustment.

Panel C accounts for these differences across the distribution of sectors. For example, the bottom left entry states that 9 percent of the difference in the intensive margin of adjustment can be accounted for by sectors in which both firms have markups above the median markup.[36] Panel C supports the earlier statement that sectors with dispersed markups account for the difference between the two models. This is despite the fact that sectors with low markups contribute sub-

[36] In these experiments, the realizations of random numbers used to generate the simulations are the same across models. Two firms in one sector in the duopoly model therefore have two corresponding, but unrelated, firms in the monopolistically competitive model. The different parameters of each model map random numbers into different idiosyncratic shocks and menu costs, but the underlying random numbers are the same for each of these pairs. In each model, these pairs of firms are then assigned to quadrants of the distribution of markups according to their markups relative to the median markup.

stantially toward greater aggregate price flexibility. Quantitatively, the dispersed markup sectors shape the aggregate inflation response for two reasons. First, there are simply twice as many sectors with low-high markups than low-low. Second, there is little difference in the behavior of sectors with two initially high markups.

### 5.3 Robustness

**State-dependent price setting** A motivation for studying state-dependent menu cost models of price adjustment is that they realistically allow firms to choose when to change their prices, as opposed to time-dependent Calvo models of price adjustment which assume that adjusting firms are randomly chosen. Comparing the monopolistically competitive and duopoly models under Calvo pricing—where the exogenous frequency of price adjustment $\alpha$ and size of the shocks are again recalibrated to match the data—I find that output fluctuations are only 10 percent larger in the duopoly model (0.41 vs. 0.38; Figure 7 plots comparative statics with respect to $\alpha$). Compare this to the main result in the state-dependent model: output fluctuations were nearly 250 percent larger in the duopoly model. Put differently, a monopolistically competitive model exhibits far greater neutrality under menu costs than Calvo (0.13 vs. 0.38), which is the central result of Golosov and Lucas (2007). However, the same is not true for the duopoly model (0.31 vs. 0.41).[37]

The real effects of monetary shocks are less dependent on market structure when adjustment is random for two reasons. First, returning to the decomposition in equation (10), note that under Calvo, both the extensive margin and covariance terms are zero.[38] From Table 2B, the majority of the difference in the inflation responses of the monopolistically competitive and duopoly menu cost models was due to these margins. Under Calvo, the value of a price change still falls for a low-markup firm facing a high-markup competitor, but by assumption, this does not affect its probability of a price increase.

Second, dynamic complementarity is weaker under Calvo. With random adjustment, a high-priced firm cannot choose when to lower its price. This reduces the incentive of a low-priced firm to reprice close to its competitor, which reduces the dampening of the intensive margin response.

[37] These results imply that the duopoly model accounts for around three-quarters of the difference between monopolistically competitive Calvo and menu cost models. This comparison may seem unwarranted. However, a feature of the literature has been to ask whether state-dependent models can deliver output fluctuations as large as time-dependent models. For example, in Midrigan (2011), a Golosov-Lucas model delivers $\sigma(C_t) = 0.07$, a Calvo model $\sigma(C_t) = 0.35$, and the author's benchmark multiproduct model $\sigma(C_t) = 0.29$. The main result is that the multiproduct model generates real effects of monetary shocks that are 78 percent as large as a Calvo model. In my case this number is around 71 percent, but note that random menu costs lead to less neutrality in the monopolistically competitive model.

[38] Formally, the decomposition (10) is limited to only the first, intensive margin component, since $\hat{\gamma}_{it} = \bar{\gamma}_{it} = \alpha$.

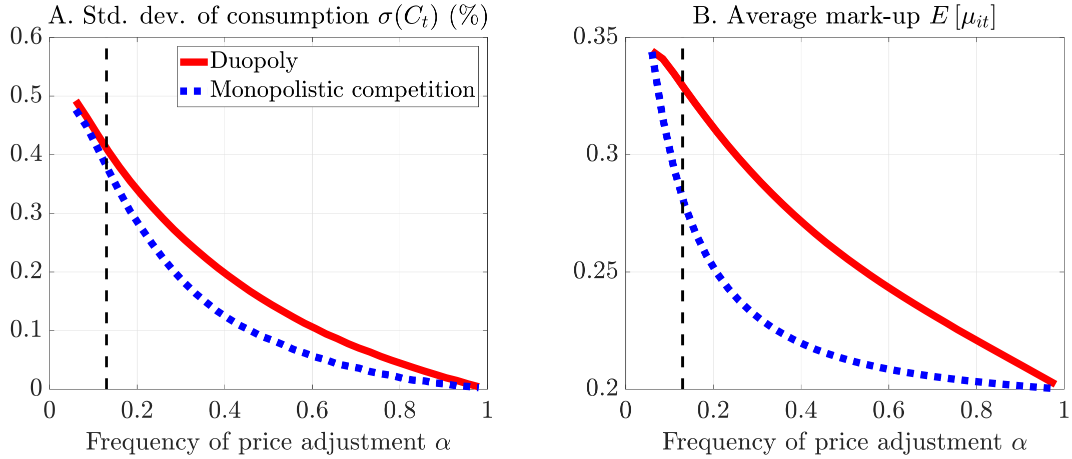


Figure 7: Market structure and monetary non-neutrality in a Calvo model of price rigidty

Notes: Vertical dashed lines mark the empirical frequency of price adjustment $\alpha = 0.13$. In both models, $\theta = 1.5$ and the elasticity of demand is chosen to obtain a frictionless markup of 1.20: $\eta_d = 10.5$, $\eta_m = 6$. This can be seen in panel B, in which the average markup is equal when prices are perfectly flexible ($\alpha = 1$). In both duopoly and monopolistically competitive models, the size of shocks $\sigma_z$ is set to 0.05, which matches the average size of price changes at $\alpha = 0.13$.

Although the degree of static complementarity remains unchanged, its affect on MPE strategies—the degree of dynamic complementarity—is weakened.

**Demand elasticity** An alternative strategy for calibrating the elasticity of demand would have been to choose $\eta$ such that markups in a frictionless economy coincided exactly.[39] In Appendix C I derive the familiar frictionless markups in each model:

$$\mu_d^* = \frac{\frac{1}{2}(\eta_d + \theta)}{\frac{1}{2}(\eta_d + \theta) - 1}, \quad \mu_{mc}^* = \frac{\eta_{mc}}{\eta_{mc} - 1}.$$

In the baseline calibration $\eta_d = 10.5$, which implies $\mu_d^* = 1.20$. This is substantially less than the observed average markup, a point I return to below. Setting $\mu_{mc}^* = 1.20$ therefore requires $\eta_{mc} = 6$. Calibration *Alt II* in Table 1 uses this value of $\eta_{mc}$ and a higher value of the menu cost in order to match the same moments. Business cycles are of the same magnitude as *Base*. Calibration *Alt III* shows that even if $\eta_{mc} = \eta_d = 10.5$, then, again recalibrating the menu cost, $\sigma(\log C_t)$ is again unaffected.[40]

[39]Such an approach is appealing, since it is better situated to ask "How do the affects of nominal rigidity depend on market structure?" This is more in the spirit of Maskin and Tirole (1988b), who ask how *introducing* exogenous price stickiness may affect the pricing of oligopolists.

[40]Note that a higher value of $\eta$ will, however, imply a lower *level* of output. Since *Base*, *Alt II* and *Alt III* have the same average size of price changes and the same size of idiosyncratic shocks, this implies they have roughly the same price dispersion. But since the demand elasticity increases across these calibrations, firms with suboptimally low (high) prices relative to their productivity will produce even more (less), reducing total output. The baseline calibration keeps the overall elasticity of demand roughly the same across the duopoly and monopolistically competitive models, such that the output losses due to price dispersion are approximately equal. I return to this in Section 6.

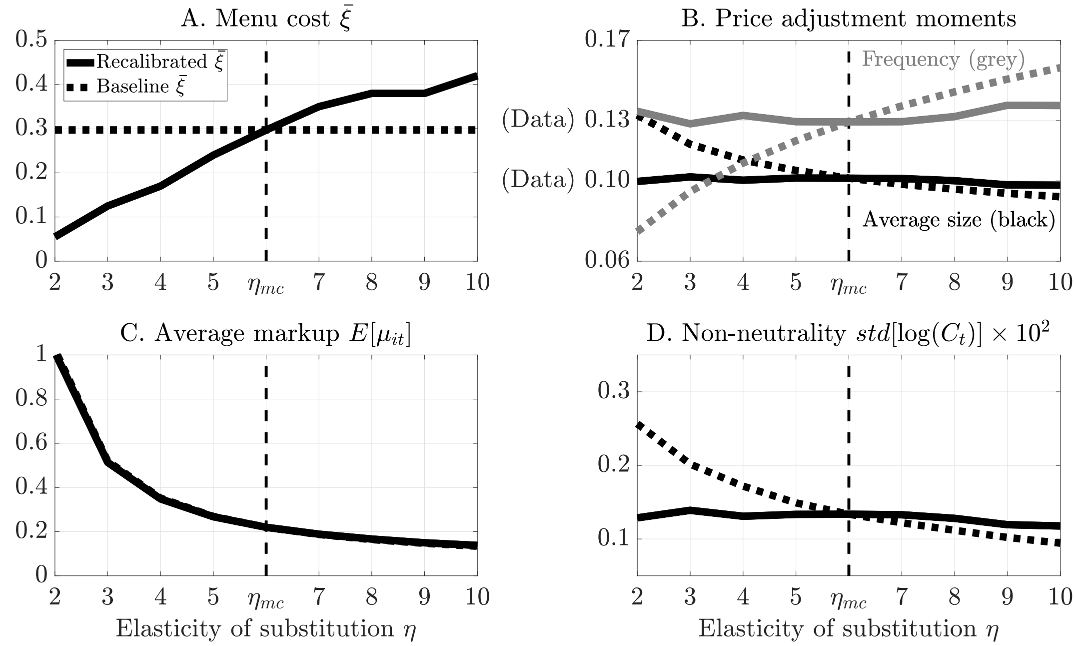


Figure 8: Elasticity of substitution comparative statics and monopolistic competition

Notes: Solid lines denote values for the monopolistically competitive model under $\sigma_z = 0.04$ and the recalibrated values of $\bar{\xi}$ given by the solid line in panel A. These values of $\bar{\xi}$ are chosen to match the same data on frequency and size of price change (panel B). Dashed lines denote values for the monopolistically competitive model under $\sigma_z = 0.04$, with $\bar{\xi}$ fixed at its value from calibration *Alt III* of Table 1. The vertical black lines mark the value of $\eta_{mc} = 6$ under this calibration. In panel C, the dashed line lies slightly above (below) the solid line to the left (right) of $\eta_{mc} = 6$. For low values of $\eta$, and fixed $\bar{\xi}$, frequency of price change is lower (panel B), leading firms to choose higher markups for precautionary reasons. These effects on the average markup are, however, very small.

Figure 8 shows that this holds across all values of $\eta_{mc} \in [2, 10]$, or equivalently, $\mu^*_{mc} \in [1.11, 2.00]$. Solid lines describe the monopolistically competitive model under different values of $\eta_{mc}$, each time recalibrating the menu cost (panel A) to match the data (panel B). Dashed lines describe the same economies but with the menu cost fixed at 0.29 from *Alt II*. In all cases, $\sigma(\log C_t) \approx 0.13$. It does not matter which monopolistically competitive economy—indexed by $\eta_{mc}$—I compare the duopoly model to, so long as it is calibrated to match the same moments. Put differently, larger output fluctuations are not obtained by simply giving more market power to monopolistically competitive firms.[41]

The irrelevance of $\eta_{mc}$ for the aggregate dynamics of the monopolistically competitive model does not, however, carry over to the duopoly model. Decreasing $\eta_d$ weakens complementarity. In the limit, $\eta_d = \theta$, and firms behave monopolistically competitively.[42] As per Figure 8D and

[41] Alvarez, LeBehin, and Lippi (2016) prove that to a second order approximation, the real effects of small monetary shocks in monopolistically competitive menu cost models will be equal provided they match the same frequency, average absolute size, and kurtosis of price changes. Changing the elasticity of demand while recalibrating the model ensures that these statistics are the same. One can therefore interpret Figure 8 as demonstrating that their theorems hold in a model without any such approximations, and under the empirical size of monetary shocks.

[42] This is verified by noting that the sectoral price index—which contains a firm's direct competitor's price—drops out of the firm's demand function when $\eta = \theta$ (see equation (2)).

the above discussion, such a model will, once recalibrated, imply $\sigma(\log C_t) \approx 0.13$. Increasing $\eta_d$ strengthens complementarity. This increases output fluctuations as inframarginal firms' initial prices have a larger impact on marginal firm adjustment. With respect to the calibration of the model, increasing (decreasing) $\eta_d$ monotonically increases (decreases) the average markup. Therefore, if one believes markups to be lower than 30 percent—as tends to be the case in the calibration of most macroeconomic models—then output fluctuations will be even larger in a recalibrated duopoly model (see Figure E2).

### 5.4 Alternative extensions of Golosov and Lucas (2007)

Previous extensions of Golosov and Lucas (2007) lead to monetary non-neutrality through (i) increasing the kurtosis in the distribution of desired price changes, and (ii) introducing complementarities through preferences or technology. I contrast the mechanism in the duopoly model to these alterations of the microeconomic environment.[43]

**Kurtosis** Holding the average size of price changes fixed, the size of the extensive margin response is determined by the increase in the mass of firms increasing their prices following a positive monetary shock. This, in turn, is determined by the gradient of the distribution of firms near the adjustment thresholds. In a model with Gaussian shocks, this gradient is steep (see Figure E1). More kurtosis reduces this gradient.

In Midrigan (2011) and further work by Alvarez and Lippi (2014), additional kurtosis stems from multiproduct firms with economy of scope in price changes. When the markup of one good hits an adjustment threshold, the firm reprices all of its goods, despite its other goods' markups being close to their optimum. In Gertler and Leahy (2008), large infrequent shocks throw the firm's markup conditional on non-adjustment beyond the adjustment threshold, forcing the firm to adjust while its previous markup has not moved far from its reset value. Alvarez, LeBehin, and Lippi (2016) (hereafter, ALB) formalize these types of results by showing that—within this class of models—the frequency and kurtosis of price changes are sufficient statistics for the real effects of small monetary shocks.

Figure 9 verifies that changing market structure—while keeping the size and frequency of price change the same—does not change the kurtosis of the distribution of desired price changes.

[43] Since the macroeconomic environment of the duopoly and monopolistically competitive model are the same, I do not compare the model to those that alter the macroeconomics of the model in order to slow the pass-through of the monetary shock to movements in nominal cost (for example, Nakamura and Steinsson (2010)).

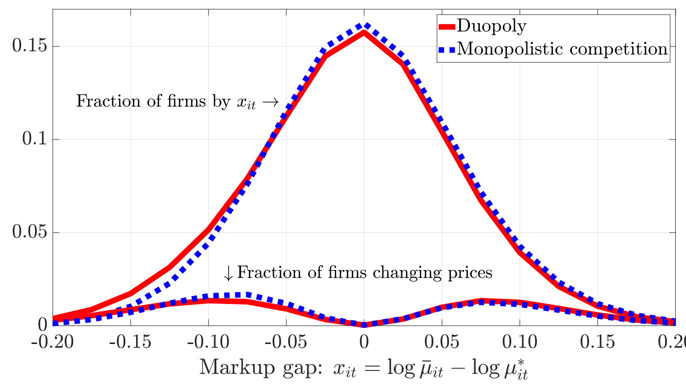


Figure 9: Distributions of markup gaps and changes (dashed)

Notes: The markup gap $x_{it} = \log \mu_{it} - \log \mu^*_{it}$ is defined with respect to the markup at the beginning of the period following the realization of shocks $\mu_{it}$ and the coincident optimal markup $\mu^*_{it}$. Firms are binned in 0.025 intervals of $x_{it}$. The top set of lines plot the fraction of firms in each bin. The bottom set of lines plot the fraction of firms in each bin multiplied by the fraction of firms in that bin changing prices. First, note that in a Calvo model of price adjustment, the dashed lines would be a multiple $\alpha$ of the solid lines, where $\alpha$ is the exogenous probability of price adjustment. Second, summing points on the lower set of lines obtains the total fraction of firms changing prices, and is equal to 0.13 in both models due to the calibration.

Some additional left skewness arises under duopoly due to the lower frequency of price change at low-markup firms. That the duopoly model generates larger output effects confirms that it does not belong to the class of models for which these sufficient statistics apply.

The duopoly model—and those discussed next—are outside the class of models studied by ALB due to complementarities in price setting. The results of ALB require that—to a first order—a firm's optimal markup is independent of all other prices. In the duopoly model a competitor's price enters the first order conditions of the firm, breaking the application of these sufficient statistics.

**Complementarity** As summarized by Nakamura and Steinsson (2010), "monetary economists have long relied heavily on complementarity in price setting to amplify monetary non-neutrality generated by nominal rigidities." Under monopolistic competition, complementarity may be introduced between the firm's price and the aggregate price level through alternative preferences or technology.[44]

What are these features? First, Kimball (1995) preferences, as studied by Klenow and Willis (2016) and Beck and Lein (2015), imply variable marginal revenue. When the quantity a firm sells decreases, its elasticity of demand increases, as approximated by a demand function of the form

$$\frac{y_i}{Y} = \left(\frac{\mu_i}{\mu}\right)^{-\eta \exp\left(\Delta \log\left(\frac{\mu_i}{\mu}\right)\right)}, \quad \Delta \geq 0. \tag{11}$$

[44]The sufficient statistics of ALB also do not apply to these models, since, due to complementarity, the aggregate price has a first order effect on firm profits.

Second, a decreasing returns to scale technology (DRS), as studied by Burstein and Hellwig (2007), implies variable marginal cost. When the quantity a firm sells decreases, its marginal cost decreases:[45]

$$mc_i = \Omega \frac{y_i^{\Delta}}{z_i W}, \quad \Delta \geq 0. \tag{12}$$

In both cases, $\Delta$ controls the degree of complementarity.

These features amplify a positive monetary shock as follows. Consider a marginal firm with a relatively low markup of $\mu_{it}$ and an optimal markup of $\mu_{it}^*$. As the money supply increases, aggregate marginal cost increases and since some firms do not increase their price, the aggregate markup $\mu_t$ falls to $\mu_t' < \mu_t$. Is $\mu_{it}^*$ still the firm's optimal markup? With $(\mu_{it}^*/\mu_t') > (\mu_{it}^*/\mu_t)$, the firm would now sell a lower quantity at $\mu_{it}^*$. In the Kimball (DRS) model, this increases the elasticity of demand (decreases marginal cost) at $\mu_{it}^*$, implying a lower optimal markup $\mu_{it}^{*\prime} < \mu_{it}^*$. So as $\Delta$ is increased, low-priced firms reduce their desired markup following a monetary expansion, slowing inflation.

Yet these features also affect firm responses to idiosyncratic shocks. Consider the same firm's response to a decrease in $z_{it}$ to $z_{it}' < z_{it}$, reducing $(\mu_{it}/\mu_t)$, and increasing output. In the Kimball (DRS) model, this decreases the elasticity of demand (increases marginal cost) at $\mu_{it}$, increasing the value of a price increase. So as $\Delta$ is increased, low-priced firms become more responsive to negative idiosyncratic shocks.

Quantitatively, most price changes are due to idiosyncratic shocks, which are large, not aggregate shocks, which are small. Because of this, Klenow and Willis (2016) and Burstein and Hellwig (2007) find that values of $\Delta$ that reduce monetary neutrality, require large menu costs and idiosyncratic shocks in order to match the same data on good-level price adjustment.[46]

In the duopoly model amplification occurs due to complementarity, but at even lower $\bar{\xi}$ and $\sigma_z$ than the monopolistically competitive model. Why does the model avoid the issues in the existing literature? Under DRS or Kimball, it is almost guaranteed that a shock that reduces $\mu_{it}$ also causes $(\mu_{it}/\mu_t)$ to fall sharply. Under duopoly, the firm cares about $(\mu_{it}/\mu_{-it})$, which may increase or decrease depending on shocks to $\mu_{-it}$. In some sectors, $\mu_{-it}$ also falls, strongly reducing the marginal firm's incentive to increase its price. In some sectors, $\mu_{-it}$ increases, strongly increasing

[45] If $y_i = z_i^{\alpha} n_i^{\alpha}$, then $\Delta = (1-\alpha)/\alpha$, and $\Omega = 1/\alpha$.

[46] Klenow and Willis (2016) find that the standard deviation of shocks at a monthly frequency would need to be 28 percent to accommodate $\Delta = 10$, which delivers amplification similar to my main result. In an exhaustive study of the model under Kimball preferences, Beck and Lein (2015) reach the same conclusion. As Nakamura and Steinsson (2010) conclude, "introduction of such strategic complementarities render the models unable to match the average size of price changes for plausible parameter values...requir[ing] massive idiosyncratic shocks and large menu costs...cast[ing] doubt on strategic complementarity as a source of amplification in menu cost models with idiosyncratic shocks."

the marginal firm's incentive to increase its price. Following an aggregate shock, however, $\mu_{-it}$ decreases on average.

With respect to the Kimball model, another way of stating this is as follows. If a duopolist is draw at random from an interval of $\mu_{it}$'s, then the average elasticity of demand faced by the firm would increase slightly as $\mu_{it}$ increases. In the Kimball model, since $\mu_t$ barely moves, the elasticity of demand increases sharply. Figure E5 highlights this, plotting the Kimball profit function under $\varepsilon = 10$ as in Klenow and Willis (2016). The variable demand elasticity results in a substantially more concave profit function compared to the duopoly profit function.[47]

The duopoly model succeeds in decoupling the response of firms to idiosyncratic and aggregate shocks while still presenting a mechanism based on complementarity in price setting. The amount of this complementarity is also an endogenous feature of the environment, which unlike a parameter $\Delta$, is responds to shocks and policy. This is potentially interesting given recent evidence that the responsiveness of firms to shocks is counter-cyclical (Berger and Vavra, 2013) and has decreased over time (Decker, Haltiwanger, Jarmin, and Miranda, 2017).

# 6 Welfare implications of nominal rigidity

In the presence of menu costs, strategic firms are able to sustain markups that are higher than the frictionless markup. Higher markups reduce the real wage, reducing output. Table 3 shows that the output lost due to nominal rigidity is four times larger under duopoly. Moreover, more than three quarters of this difference is due to the difference in the level, rather than dispersion of markups.[48]

Figure 10 quantifies a related result: the value of the firm may be increasing in the degree of exogenous pricing frictions. Larger frictions lead to greater dynamic complementarity, accommodating higher markups and increasing firm value. But larger frictions also reduce price flexibility, reducing firm value. The resulting non-monotonic relationship is clear in both the menu cost and

[47] Compared to the monopolistically competitive profit function under *Alt III*—under which the duopoly and monopolistically competitive demand functions have the same elasticity at the frictionless markup—Figure E5 shows that the duopoly profit function does exhibit slightly more curvature as a low (high) priced firm sells to more (less) of the market and so faces a lower (higher) elasticity of demand. This additional curvature is small and roughly equivalent to that which occurs under Kimball with $\Delta \approx 0.7$. Beck and Lein (2015) estimate $\Delta \approx 1$ using European retail goods, and Gopinath and Itskhoki (2011) estimate $\Delta \approx 1.5$ using evidence on pass-through of exchange rate shocks. The varying overall demand elasticity that occurs naturally under oligopoly with nested CES preferences and reasonable $(\theta, \eta)$ is, therefore, consistent with empirical evidence on the curvature of demand functions.

[48] Total menu costs are smaller in the duopoly model since prices are endogenously stickier. However, menu costs are such a small fraction of output that they do not affect Table 3 at two decimal places.

| | | | Mon. Comp. | Duopoly |
|---|---|---|---|---|
| (1) | Output | | 0.76 | 0.75 |
| (2) | ... under no dispersion | $\mu_{it} = \mathbb{E}[\mu_{it}]$ | 0.77 | 0.77 |
| (3) | ... under no menu costs | $\mu_{it} = \mu^*$ | 0.78 | 0.83 |
| (3)-(1) | Output loss due to nominal rigidity | | 2.6% | 9.6% |
| (2)-(1) | ... fraction due to dispersion in markups | | 0.51 | 0.23 |
| (3)-(2) | ... fraction due to level of markups | | 0.49 | 0.77 |

Table 3: Market structure and output losses due to nominal rigidity

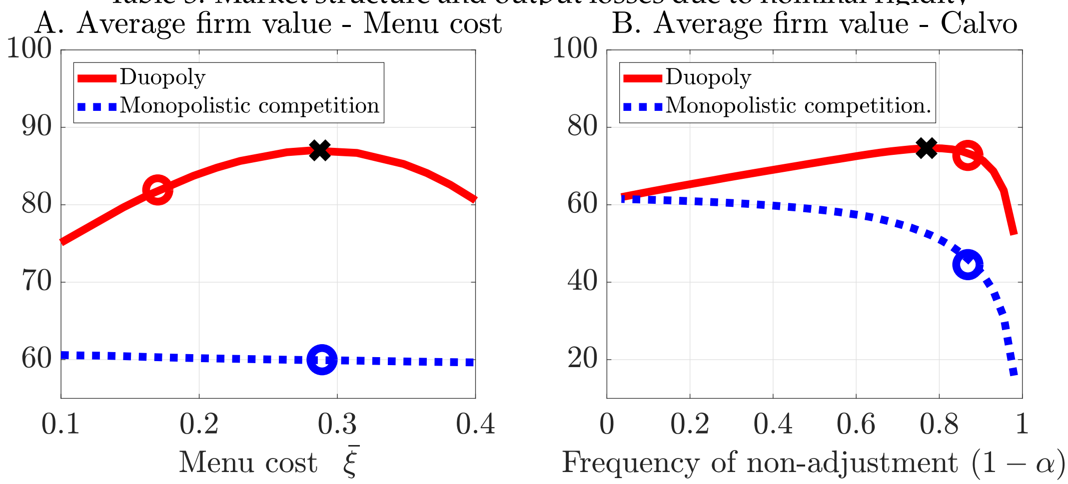


Figure 10: Comparative statics: Markups and firm value

Notes: Figures plot the comparative statics of the average firm value given by Bellman equation (9), with respect to the size of nominal rigidity in the menu cost (panel A) and Calvo model (panel B). Firm value is computed from a simulation of 20,000 firms $v_{i\tau} = \sum_{t=\tau}^{T} \beta^{t-\tau} (\pi_{it} - \xi_{it})$, where initial states in period $\tau$ are due to a burn-in simulation. This is the baseline calibration of the duopoly model and *Alt III* calibration of the monopolistically competitive model (see Table 1). This implies that both models have the same frictionless markup of 1.20, such that firm values are equal in both models when frictions are zero. The circles mark the size of the friction under these calibrations. The cross mark gives the size of the friction that maximizes firm value. Panel A is truncated on the $x$-axis due to computational issues associated with approximating policy functions—which are required to solve the duopoly model—under very small and very large menu costs. The scale of the $y$-axis differs because menu cost and Calvo models are not comparable in terms of firm value: value is larger in the menu-cost model since the value gained by being able to time price changes more than offsets the value lost in menu costs. The Calvo model has a baseline frequency of price change equal to the data $\alpha = 0.13$, as given by the circle marks, and is calibrated such that at this frequency of price change, the average size of price change matches the data.

Calvo models. While monopolistically competitive firms always prefer smaller frictions and more adjustment, for duopolists, there is an optimal, positive degree of friction. Compared to the baseline menu cost model ($\bar{\xi}_d = 0.17$), at $\bar{\xi}_d^* = 0.29$, the frequency of price change is 3 percentage points lower, and the real value of the firm 9 percent larger. In the Calvo model, smaller frictions are optimal from the firms' perspective. At the calibrated values, the weaker dynamic complementarity in the Calvo model implies that the second force dominates.

Four potentially interesting paths for future research arise. First, the fact that firms desire some, but not too much, nominal rigidity may rationalize why firms engage in investments that increase the cost of price changes.[49] Second, if policies such as higher trend inflation weaken the ability of firms to commit to higher markups, reducing dynamic complementarity, then such policies can have first order output effects.[50] Third, these results imply a systematic downward bias in markup estimates from static models of competition. Conditional on unbiased estimates of preference parameters, one would infer $\mu_d^*$, which is substantially less than $\mathbb{E}[\mu_{it}]$. Finally, these results distort the usual welfare implications of frictions in macroeconomics. The standard intuition holds in the monopolistically competitive model: firms and households both dislike frictions. In an oligopoly, there is a range over which higher frictions cause profits to increase but consumption to fall.

## 7 Endogenous price stickiness and market concentration

Table 1 reveals that a duopoly requires a smaller menu cost to match the data on price adjustment. In a duopoly, price decreases are less valuable due to a long-run incentive to maintain a high sectoral price, and price increases are less valuable due to a short-run incentive to maintain market share. Nominal prices therefore change less often for any $\bar{\xi}$. Calibration *Alt I* highlights this. Evaluating the monopolistically competitive model at the same parameters as the duopoly model implies a much larger frequency of price change (0.19 vs. 0.13) and smaller average size (0.05 vs. 0.10). Recall that a monopolistically competitive market structure is mathematically identical to a model with a monopolist in each sector. Prices are therefore more flexible in the competitive limiting cases, and less so under duopoly.[51]

**What to test?** Suppose firms in all markets faced an economic environment determined by the same parameters. What should we expect as we compare markets with one and two firms? First, competing with an additional firm, each firm's revenue share is lower, so its elasticity of demand is higher, making deviations from its optimal markup more costly. This *elasticity effect* leads to more price flexibility. Second, the strategic forces documented in this paper lead to less flexible

[49] For example, firms print brochures with prices fixed for some period of time.

[50] In the limit, high trend inflation would cause firms to reset their prices every period and the frictionless Nash equilibrium markup would be obtained, eliminating the first order welfare losses of nominal rigidity but also eliminating any possibility of counter cyclical monetary policy.

[51] The case of three and four firms, and so on, I leave to future work. I note briefly that the computational complexity of solving the model with more firms comes not with (i) integrating over more firms' actions when computing payoffs or (ii) adding state variables, which increases the dimensionality of the value function problem. These can be handled computationally. The additional complexity derives from converging on the MPE policy functions which are problematic to approximate in higher dimensions.

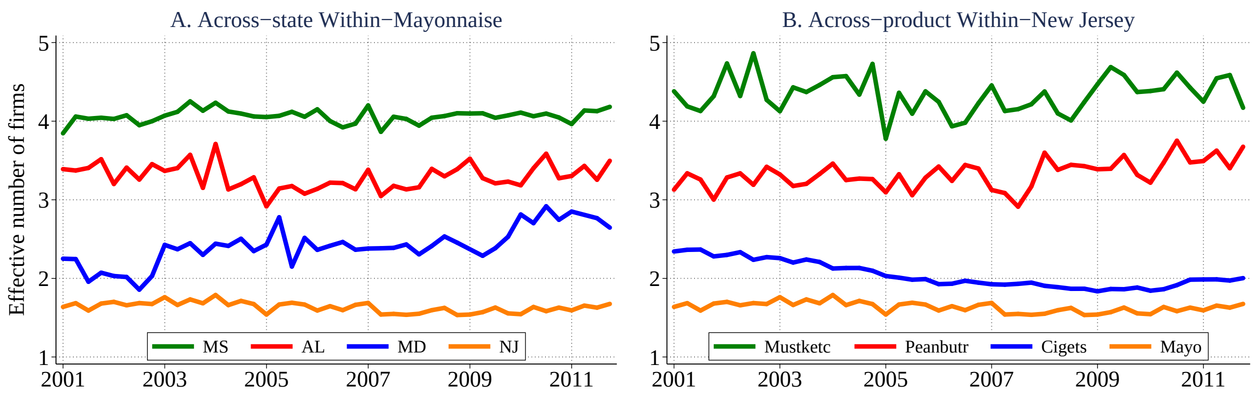


Figure 11: Empirical variation in market concentration

**Notes** For construction of the *Effective number of firms* measure see the notes to Figure 1. Each series gives the quarterly average of effective number of firms, where effective number of firms is computed in each product-state-month market.

prices, an *oligopoly effect*. Considering markets with more firms, the oligopoly effect dissipates as firms behave less strategically, and the elasticity effect dominates.

This leads me to test for a $U$-shaped (hump-shaped) relationship between frequency (size) of price change and market concentration. Note that an increase in flexibility as firms are added does not suggest that the oligopoly effect is not present, only that it is weaker than the counterpoised elasticity effect. In this sense, the right tail of a $U$-shape is confounded. However, decreasing flexibility under more firms indicates that the oligopoly effect is present and large enough to offset the elasticity effect.

**Variation in concentration** To carry out these tests, I return to the IRI data and exploit two separate sources of variation in the concentration of markets. The first uses variation *across states, within products*. The second uses variation *across products, within states*. Figure 11 provides examples. Panel A describes the time series of the effective number of firms in the market for mayonnaise in four different states. Clearly, there is very little variation in the time dimension, whereas variation across states is large. Panel B describes the same time series but for different products within the state of New Jersey. Here most of the variation is across products.

A useful feature of this persistent variation in the data is that cases arise where the market for product $p$ may be very concentrated in state $s$ and less in state $s'$, while the market for product $p'$ is more concentrated in $s'$ than $s$. Market concentration is location-good-specific and a highly persistent feature of markets.[52] This implies that any explanation for heterogeneity in price flexibility

[52] This variation in market concentration has been studied using the same data by Bronnenberg, Dhar, and Dubé

across markets cannot rely on only across-good or across-region heterogeneity in menu costs or the stochastic processes facing the firm. But is there much variation in price flexibility along these two dimensions?

**Variation in flexibility** Bils and Klenow (2004) document heterogeneity in price flexibility across goods categories in the CPI microdata. A number of papers address this in structural models by introducing sectoral heterogeneity in the severity of adjustment frictions.[53] However, the IRI data reveal that even within a narrow product group, there is substantial variation across markets. The average within-product across-state standard deviation of log frequency (size) of price change is 0.20 (0.13).[54] Nationally, the across-product standard deviation of log frequency (size) of price change is 0.28 (0.20).[55,56] Therefore around two-thirds as much variation exists *within* products, *across* states, as does across products, suggesting that modeling price stickiness as good specific may miss some factors that are market-specific. I now quantify the extent to which this variation can be explained by differences in market concentration.

**Estimating equations** Let $y_{pst}$ be a measure of price flexibility for product $p$, in state $s$, month $t$. Let $x_{pst}$ be a measure of concentration and $X_{pst}$ be other data at the market level. The across-product, within-state-month specification is

$$\left(y_{pst} - \bar{y}_{st}\right) = \alpha + \beta\left(x_{pst} - \bar{x}_{st}\right) + \delta\left(x_{pst} - \bar{x}_{st}\right)^2 + \gamma X_{pst} + \varepsilon_{pst}, \tag{13}$$

(2009) and Bronnenberg, Dube, and Gentzkow (2012). The latter points to the migration of individuals—who carry with them brand preferences—as a major determinant of market shares. Exploiting this variation innovates on Bils and Klenow (2004), who also study the relationship between concentration and price flexibility. However, since they use CPI microdata—which takes small samples of goods from stores—they cannot compute concentration measures locally, so cannot examine within-product variation in concentration. They instead regress national price flexibility for a good, on national market concentration. The latter can be a misleading measure of product market *competition* if, for example, there are 50 different monopolists operating in 50 states. They find no significant relationship.

[53] For examples, see Nakamura and Steinsson (2010), Weber (2016), and Weber, Pasten, and Schoenle (2017).

[54] These statistics are computed as follows. Let $f_{pst}$ denote the frequency of price change in market $pst$. Unweighted within product-state across-time averages are first computed so as to focus on permanent differences: $f_{ps} = T_{-1}\sum_{t=1}^{T} f_{pst}$. The average within-product across-state standard deviation of log frequency of price change is then $P^{-1}\sum_{p=1}^{P} std[\log(f_{ps})|p]$, where the within-product across-state standard deviation is computed using weights $w_{ps} = r_{ps} / \sum_{s=1}^{S} r_{ps}$, and $var[\log(f_{ps})|p] = \sum_{s=1}^{S} w_{ps}(\log f_{ps} - \sum_{s=1}^{S} w_{ps}\log f_{ps})^2$. Figure E6 plots distributions of these objects.

[55] When computed within states, across products, the average standard deviation of log frequency (size) of price change is 0.32 (0.22), which is only a little larger than national across-product variation.

[56] Data from Table A1 of Bils and Klenow (2004) describe frequency of price change across a wide array of product categories. My computations using their data yield a standard deviation of log frequency of 0.79. Specifically, this is computed using $var[\log f_p] = \sum_{p=1}^{P} w_p(\log f_p - \sum_{p=1}^{P} w_p \log f_p)^2$, where $w_p$ are given by 1995 CPI expenditure shares, and $P = 350$ categories determined by ELI numbers. Therefore, across products nationally, the IRI data capture around 35 percent of the dispersion found in the broader CPI basket.

| | Across-product w/in state | | Across-state w/in product | |
|---|---|---|---|---|
| | Size (%) | Frequency | Size (%) | Frequency |
| Eff. number of firms | 0.450 | -0.968 | 0.325 | -0.753 |
| | (0.062) | (0.155) | (0.073) | (0.181) |
| Eff. number of firms$^2$ | -0.077 | 0.156 | -0.044 | 0.168 |
| | (0.015) | (0.038) | (0.019) | (0.078) |
| Observations | 133,340 | 133,340 | 133,340 | 133,340 |
| R-squared | 0.072 | 0.074 | 0.021 | 0.012 |
| $Rev_{pst}$ control | ✓ | ✓ | ✓ | ✓ |

Table 4: Regression results - Cross-product regression

Notes: Results for the estimation of equation (13) (first two columns) and symmetric across product within state-month specification (last two columns). Data points in the regression consist of product-month-state-level observations. *Size* of price change is the product-month-state average of monthly log absolute price changes for all products conditional on price change. *Freq* is frequency of price change computed as the fraction of goods changing price. Effective number of firms is given by the inverse Herfindahl index $h_{pst}^{-1}$ for market $pst$, where the Herfindahl index is the revenue share weighted average revenue share of all firms in the market, $h_{pst} = \sum_{i \in \{pst\}} (rev_{ipst}/rev_{pst})^2$. Errors are clustered at the $ps$ level.

where $\bar{y}_{st}$ is the across-product mean for state $s$ in month $t$. The across-state, within-product-month specification is symmetric.

The effective number of firms is used as a measure of concentration, and frequency and average size of price change as measures of flexibility. I include an additional control for revenue in the market $pst$.[57] Errors are clustered at the product-state level. Results are described in Table 4.

**Results** Consistent with the more price stickiness in oligopolist markets, the quadratic terms are negative (positive) in the case of size (frequency) of price changes. Coefficient estimates are similar across both specifications, despite each using very different sources of variation in the data. Figure 12 displays these results graphically, plotting the average fitted values of frequency and size of price change from (13). Dashed lines denote lower and upper quantiles. The model's interpretation is that oligopolistic forces are strong, counteracting the elasticity-effect, but weaken at around five equally sized firms. Price flexibility is therefore similar in markets with very low and very high levels of concentration in which firm behavior may be approximated as competitive. Figure E7 shows that when $x_{pst}$ is the raw number of firms in a market, the hump shapes disappear and quadratic terms are exactly zero (Table E2). This reinforces the importance of the competitive structure of the market for price flexibility. Future studies of models with more than two firms per sector can be used to understand when and how these oligopoly forces peak.

[57]This controls for the fact that if there is economy of scope in the cost of price change then flexibility will be higher when revenues are higher.

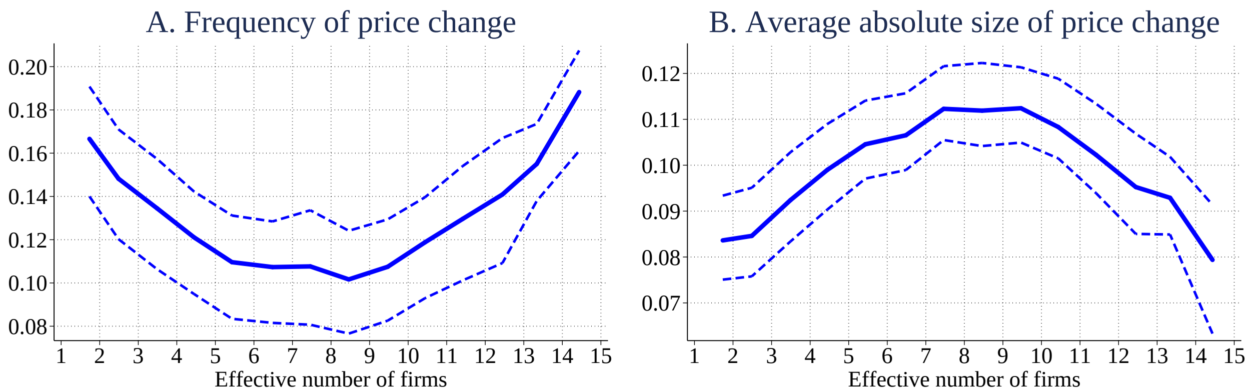


Figure 12: Market concentration and price flexibility

Notes: Solid (dashed) lines are medians (25th/75th percentiles) of fitted values from regression (13), where averages for both effective number of firms and the dependent variable are taken within bins of effective number of firms of width one.

## 8 Conclusion

This paper establishes that the competitive structure of markets can be quantitatively important for the transmission of macroeconomic shocks. In particular, in a menu cost model of firm-level price setting—which aggregates to a monetary business cycle model—I show that a monopolistically competitive market structure and a duopoly market structure can generate different levels of monetary non-neutrality. Even when calibrated to match the same salient features of price flexibility in the data, the duopoly model generates larger output responses. Following a monetary expansion, the incentive for low-priced firms to respond to the shock increases less sharply as a lower sectoral price reduces the incentive to adjust.

More broadly, this paper aims to bridge an inconsistency between data and macroeconomic models that aggregate idiosyncratic firm behavior. Recently, macroeconomic models with heterogeneous firms have been used to answer questions of the following type: micro-data suggest a certain friction at the firm level; does incorporating this friction affect the aggregate dynamics of the economy with respect to aggregate shocks? Examples of such frictions include fixed costs of investment, equity issuance costs, collateral constraints on borrowing, and—as studied here—menu costs of price adjustment. The models are used to interpret datasets that have a key feature: the size distribution is fat tailed. Yet in these models, firms are assumed to behave competitively regardless of their size. This paper expands the structure of models used to answer these questions to allow for non-competitive behavior and finds—in the case of nominal rigidities and monetary shocks—that this can be important for aggregate dynamics.

The structure of the model studied in this paper also allows one to study a larger set of microeconomic behavior and its implications for macroeconomic outcomes. One could draw motivation from simple, well-studied models of strategic interaction that, when aggregated, may either amplify or attenuate macroeconomic shocks. Do firms accumulate excess capacity as a threat against the expansion of competitors, and if so, does this have implications for the business cycle properties of investment? Can oligopsony in labor markets help explain why wages do not fall sharply in a recession? Did recent changes in market concentration contribute to the low-inflation recovery from the Great Recession? These may be answered with modifications of the existing model.

## References


ADELMAN, M. (1969): "Comment on the 'H' Concentration Measure as a Numbers-Equivalent," *Review of Economics and Statistics*, 51(1), 99–101.

ALVAREZ, F., H. LEBEHIN, AND F. LIPPI (2016): "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach," *Econometrica*, 106(10), 1–37.

ALVAREZ, F., AND F. LIPPI (2014): "Price Setting With Menu Cost for Multiproduct Firms," *Econometrica*, 82(1), 89–135.

ANDERSON, S. P., A. DE PALMA, AND J.-F. THISSE (1992): *Discrete choice theory of product differentiation*. Cambridge, MA: MIT Press.

ATKESON, A., AND A. BURSTEIN (2008): "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, 98(5), 1998–2031.

AUTOR, D., D. DORN, L. F. KATZ, C. PATTERSON, AND J. V. REENEN (2017): "Concentrating on the Fall of the Labor Share," NBER Working Paper 23108, National Bureau of Economic Research.

BARRO, R. J. (1972): "A Theory of Monopolistic Price Adjustment," *Review of Economic Studies*, 39(1), 17–26.

BASU, S. (1995): "Intermediate Goods and Business Cycles: Implications for Productivity and Welfare," *American Economic Review*, 85(3), 512–31.

BECK, G. W., AND S. M. LEIN (2015): "Microeconometric Evidence on Demand-side Real Rigidity and Implications for Monetary Non-neutrality," Working paper 2015/13, Faculty of Business and Economics - University of Basel.

BERGER, D., R. J. CABALLERO, AND E. ENGEL (2017): "Missing Aggregate Dynamics: On the Slow Convergence of Lumpy Adjustment Models," NBER Working Paper 9898, National Bureau of Economic Research.

BERGER, D., AND J. S. VAVRA (2013): "Volatility and Pass-through," NBER Working Paper 19651, National Bureau of Economic Research.

BERRY, S., J. LEVINSOHN, AND A. PAKES (1995): "Automobile Prices in Market Equilibrium," *Econometrica*, 63(4), 841–890.

BILS, M., AND P. J. KLENOW (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947–985.

BRONNENBERG, B. J., S. K. DHAR, AND J.-P. H. DUBÉ (2009): "Brand History, Geography, and the Persistence of Brand Shares," *Journal of Political Economy*, 117(1), 87–115.

BRONNENBERG, B. J., J.-P. H. DUBE, AND M. GENTZKOW (2012): "The Evolution of Brand Preferences: Evidence from Consumer Migration," *American Economic Review*, 102(6), 2472–2508.

BRONNENBERG, B. J., M. W. KRUGER, AND C. F. MELA (2008): "The IRI Marketing Data Set - Database Paper," *Marketing Science*, 27(4), 745–748.

BURSTEIN, A., AND C. HELLWIG (2007): "Prices and Market Shares in a Menu Cost Model," NBER Working Paper 13455, National Bureau of Economic Research.

CABALLERO, R. J., AND E. M. ENGEL (2007): "Price Stickiness in Ss Models: New Interpretations of Old Results," *Journal of Monetary Economics*, 54(Supplement), 100–121.

CHRISTOPOULOU, R., AND P. VERMEULEN (2008): "Markups in the Euro Area and the US Over the Period 1981-2004 - A Comparison of 50 Sectors," *ECB - Working Paper Series*, (856).

COIBION, O., Y. GORODNICHENKO, AND G. H. HONG (2015): "The Cyclicality of Sales, Regular and Effective Prices: Business Cycle and Policy Implications," *American Economic Review*, 105(3), 993–1029.

DE LOECKER, J., AND J. EECKHOUT (2017): "The Rise of Market Power and the Macroeconomic Implications," http://www.janeeckhout.com.

DECKER, R. A., J. HALTIWANGER, R. S. JARMIN, AND J. MIRANDA (2017): "Changing Business Dynamism and Productivity: Shocks vs. Responsiveness," Mimeo, University of Maryland.

DORASZELSKI, U., AND M. SATTERTHWAITE (2010): "Computable Markov-Perfect Industry Dynamics," *RAND Journal of Economics*, 41(2), 215–243.

DOSSCHE, M., F. HEYLEN, AND D. V. DEN POEL (2010): "The Kinked Demand Curve and Price Rigidity: Evidence from Scanner Data," *Scandinavian Journal of Economics*, 112(4), 723–752.

DOTSEY, M., R. G. KING, AND A. L. WOLMAN (1999): "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics*, 114(2), 655–90.

EDMOND, C., V. MIDRIGAN, AND D. Y. XU (2015): "Competition, Markups, and the Gains from International Trade," *American Economic Review*, 105(10), 3183–3221.

ERICSON, R., AND A. PAKES (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies*, 62(1), 53–82.

GERTLER, M., AND P. KARADI (2015): "Monetary Policy Surprises, Credit Costs, and Economic Activity," *American Economic Journal: Macroeconomics*, 7(1), 44–76.

GERTLER, M., AND J. LEAHY (2008): "A Phillips Curve with an Ss Foundation," *Journal of Political Economy*, 116(3), 533–572.

GOLOSOV, M., AND R. E. LUCAS (2007): "Menu Costs and Phillips Curves," *Journal of Political Economy*, 115, 171–199.

GOPINATH, G., AND O. ITSKHOKI (2011): "In Search of Real Rigidities," in *NBER Macroeconomics Annual*, ed. by D. Acemoglu, and M. Woodford, vol. 25, pp. 261–309. Chicago, IL: University of Chicago Press.

GORODNICHENKO, Y., AND M. WEBER (2016): "Are Sticky Prices Costly? Evidence from the Stock Market," *American Economic Review*, 106(1), 165–99.

GUTIERREZ, G., AND T. PHILIPPON (2016): "Investment-less Growth: An Empirical Investigation," NBER Working Paper 22897, National Bureau of Economic Research.

HOTTMAN, C. (2016): "Retail Markups, Misallocation, and Store Variety in the US," *Working paper*.

HOTTMAN, C. J., S. J. REDDING, AND D. E. WEINSTEIN (2014): "Quantifying the Sources of Firm Heterogeneity," *Quarterly Journal of Economics*, 131(3), 1291–364.

JORDA, O. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95(1), 161–182.

JUN, B., AND X. VIVES (2004): "Strategic Incentives in Dynamic Duopoly," *Journal of Economic Theory*, 116(2), 249–281.

KIMBALL, M. S. (1995): "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 27(4), 1241–77.

KLENOW, P. J., AND J. L. WILLIS (2016): "Real Rigidities and Nominal Price Changes," *Economica*, 83, 443–472.

KRUSELL, P., AND A. A. SMITH (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

MASKIN, E., AND J. TIROLE (1988a): "A Theory of Dynamic Oligopoly, I: Overview and Quantity Competition with Large Fixed Costs," *Econometrica*, 56(3), 549–69.

——— (1988b): "A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles," *Econometrica*, 56(3), 571–99.

MIDRIGAN, V. (2011): "Menu Costs, Multiproduct Firms, and Aggregate Fluctuations," *Econometrica*, 79(4), 1139–80.

MIRANDA, M. J., AND P. L. FACKLER (2002): *Applied Computational Economics and Finance*. Cambridge, MA: MIT Press.

NAKAMURA, E., AND J. STEINSSON (2010): "Monetary Non-neutrality in a Multisector Menu Cost Model," *Quarterly Journal of Economics*, 125(3), 961–1013.

NAKAMURA, E., AND D. ZEROM (2010): "Accounting for Incomplete Pass-through," *Review of Economic Studies*, 77, 1192–1230.

NECHIO, F., AND B. HOBIJN (2017): "Sticker Shocks: Using VAT Changes to Estimate Upper-Level Elasticities of Substitution," Working Paper Series 2015-17, Federal Reserve Bank of San Francisco.

NEIMAN, B. (2011): "A State-Dependent Model of Intermediate Goods Pricing," *Journal of International Economics*, 85(1), 1–13.

ROMER, C. D., AND D. H. ROMER (2004): "A New Measure of Monetary Shocks: Derivation and Implications," *American Economic Review*, 94(4), 1055–1084.

ROTEMBERG, J. J., AND M. WOODFORD (1992): "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity," *Journal of Political Economy*, 100(6), 1153–1207.

SHAPIRO, M., AND M. WATSON (1988): "Sources of Business Cycle Fluctuations," in *NBER Macroeconomics Annual*, ed. by S. Fischer, vol. 3. Cambridge, MA: MIT Press.

SMETS, F., AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

STROEBEL, J., AND J. VAVRA (2014): "House Prices, Local Demand, and Retail Prices," NBER Working Paper 20710, National Bureau of Economic Research.

VAVRA, J. (2014): "Inflation Dynamics and Time-Varying Volatility: New Evidence and an Ss Interpretation," *Quarterly Journal of Economics*, 129(1), 215–258.

WEBER, M. (2016): "Nominal Rigidities and Asset Pricing," Mimeo, University of Chicago Booth Business School.

WEBER, M., E. PASTEN, AND R. SCHOENLE (2017): "Price Rigidities and the Granular Origins of Aggregate Fluctuations," Mimeo, University of Chicago Booth Business School.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ: Princeton University Press.

ZBARACKI, M. J., M. RITSON, D. LEVY, S. DUTTA, AND M. BERGEN (2004): "Managerial and Customer Costs of Price Adjustment: Direct Evidence from Industrial Markets," *Review of Economics and Statistics*, 86(2), 514–533.

## Appendix For Online Publication

This Appendix is organized as follows. Section A describes the IRI data and their treatment in the paper. Section B describes the computational methods used to solve the model in Section 2. Section C proves the results for a static game with menu costs and exogenously specified initial markups. I also derive properties of the firm's frictionless best response function and profit functions under general complementarity in pricing and for CES preferences. Section D discusses some of the assumptions of the model. Section E includes additional figures and tables referenced in the text.

# A Data description

The data used throughout this paper come from the IRI Symphony data. Details can be found in the summary paper by Bronnenberg, Kruger, and Mela (2008).[58] The data are at a weekly frequency from 2001 to 2011 and contain revenue and quantity data at the good level, where a good is defined by a unique bar code number (Universal Product Code—UPC). Data are collected in over 5,000 stores covering 50 metropolitan areas.[59] For each store, data are recorded for all UPCs within each of 31 different product categories. Product categories—for example toothpaste—are determined by IRI and were designed such that the vendor could sell data, by product category, to interested firms.[60] This therefore provides an economically meaningful way to separate goods categories, since firms presumedly would be interested in purchasing data relevant to their product market. The measures that I construct from these data and use in the paper relate to (i) market concentration, and (ii) price changes. In both cases I define a market by product category $p$, state $s$ and month $t$.

Constructing measures of market concentration requires market-level sales for each firm. To identify a firm, I use the first five digits of a good's UPC. This uniquely identifies a company. For example, the five digits 00012 in the bar code 00012100064595 identify Kraft within a market for mayonnaise; 48001 would identify Hellman's. As my measures are constructed within a market

[58] Other recent papers to use these data include Stroebel and Vavra (2014) and Coibion, Gorodnichenko, and Hong (2015). See http://www.iriworldwide.com/en-US/solutions/Academic-Data-Set.

[59] Details on the identification of stores are removed from the data and replaced with a unique identifying number. Walmart is not included in the data.

[60] For completeness, the categories are: beer, razor blades, carbonated beverages, cigarettes, coffee, cold foods, deodorant, diapers, facial tissues, frozen dinner entrees, frozen pizza, household cleaning goods, hot dogs, laundry detergent, margarine and butter, mayonnaise, milk, mustard and ketchup, paper towels, peanut butter, photo products, razors, salted snacks, shampoo, soup, pasta sauces, sugar and substitutes, toilet tissue, toothbrushes, toothpaste, and yogurt.

$pst$, I consider Kraft within the mayonnaise market in Ohio as a different firm from Kraft within the margarine market in Ohio. Revenue $r_{fpst}$ for each firm $f$ in market $pst$ is the sum of weekly revenue from all UPCs at all stores within $pst$. The preferred concentration measure in the paper is the effective number of firms, as measured by the inverse Herfindahl index, which is $h_{pst} = \sum_{f \in pst}(r_{fpst}/r_{pst})^2$.

Computing measures of price changes first requires a measure of price. To obtain weekly prices for each good, I simply divide revenue by quantity. I compute price change statistics monthly and measure prices in the third week of each month. I focus only on regular price changes and deem a price to have been changed between month $t-1$ and $t$ if it (i) changes by more than 0.1 percent, considering price changes smaller than this to be due to rounding error from the construction of the price, and (ii) was on promotion neither in month $t-1$, nor in month $t$. The IRI data include indicators for whether a good is on promotion, and so I use this information directly rather than using a sales filter. This second requirement means that I exclude both goods that go on promotion and come off promotion. The frequency of price change in market $pst$ is the fraction of goods that change price in market $pst$ between $t-1$ and $t$. The size of price change in market $pst$ is the average absolute log change in prices for all price changes in market $pst$ between $t-1$ and $t$.

When computing moments for use in the calibration of the model, I first take a simple average over $s$ and $t$ for each product $p$. I then take a revenue-weighted average across products, where revenue weights are computed using average national revenue for product $p$: $r_p = T^{-1} \sum_{t=1}^{T} \left( \sum_{s=1}^{S} r_{pst} \right)$.

# B Computation

First I show that Bellman equation (7) corresponds to the Bellman equation in markups under the equilibrium conditions of the model (9), as the latter is used in computation. Second, I describe the numerical methods used in computing the equilibrium of the model.

**Price indices** Denote the first firm's markup $\mu_{ij} = p_{ij}/z_{ij}W$. Using this, the sectoral price index $\mathbf{p}_j$ can be written

$$\mathbf{p}_j = \left[ \left( \frac{p_{1j}}{z_{1j}} \right)^{1-\eta} + \left( \frac{p_{2j}}{z_{2j}} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} . = W \left[ {\mu_{1j}}^{1-\eta} + {\mu_{2j}}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

Define the sectoral markup $\mu_j = \mathbf{p}_j/W$, which implies that $\mu_j = \left[\mu_{1j}^{1-\eta} + \mu_{2j}^{1-\eta}\right]^{1/(1-\eta)}$. Using the sectoral markup, the aggregate price index $P$ can be written

$$P = \left[\int_0^1 \mathbf{p}_j^{1-\theta} dj\right]^{\frac{1}{1-\theta}} = \left[\int_0^1 \mu_j^{1-\theta} dj\right]^{\frac{1}{1-\theta}} W.$$

Define the aggregate markup $\mu = P/W$, which implies that $\mu = \left[\int_0^1 \mu_j^{1-\theta} dj\right]^{1/(1-\theta)}$.

**Profits** The expressions for markups can be used to rewrite the firm's profit function. Start with the baseline case

$$\pi_{ij} = z_{ij}^{\eta-1}\left(\frac{p_{ij}}{\mathbf{p}_j}\right)^{-\eta}\left(\frac{\mathbf{p}_j}{P}\right)^{-\theta}\left(p_{ij} - z_{ij}W\right)C.$$

The equilibrium household labor supply condition requires $PC = W$. The the definition of the aggregate markup, therefore implies that $C = 1/\mu$. This, along with $p_{ij} = \mu_{ij} z_{ij} W$, $p_j = \mu_j W$, and $P = \mu W$, gives

$$\pi_{ij} \quad = \quad \left(\frac{\mu_{ij}}{\mu_j}\right)^{-\eta}\left(\frac{\mu_j}{\mu}\right)^{-\theta}\left(\mu_{ij} - 1\right)\frac{W}{\mu} = \tilde{\pi}\left(\mu_{ij}, \mu_{-ij}\right)\mu^{\theta-1}W.$$

The function $\tilde{\pi}$ depends on the aggregate state only indirectly through the policies of each firm within the sector. This makes clear the use of the technical assumption that the demand shifter $z_{ij}$ also increases average cost, allowing profits to be expressed only in markups.

**Markup dynamics** Suppose that a firm sells at a markup of $\mu_{ij}$ this month. The relevant state next month is the markup that it will sell at if it does not change its price $\mu'_{ij} = p_{ij}/z'_{ij}W'$. Replacing $p_{ij}$ with $\mu_{ij}$, we can write $\mu'_{ij}$ in terms of this month's markup, the equilibrium growth of the nominal wage, and the growth rate of idiosyncratic demand:

$$\mu'_{ij} = \mu_{ij}\frac{z_{ij}}{z'_{ij}}\frac{W}{W'} = \mu_{ij}\frac{1}{g' e^{\varepsilon'_{ij}}}.$$

The random walk assumption for $z_{ij}$ implies that $z'_{ij}/z_{ij} = \exp(\varepsilon'_{ij})$. The equilibrium condition on nominal expenditure $PC = M$, combined with the equilibrium household labor supply condition $PC = W$, implies that in equilibrium $W = M$. The stochastic process for money growth then implies that $W'/W = g'$.

**Bellman equation** Using these results in the firm's Bellman equation reduces the value of adjustment from (7) to the following (here for clarity I assume that the competitor's markup $\mu_{-i}$ is fixed):

$$V_i^{adj}\left(\mu_i, \mu_{-i}, \mathbf{S}\right) = \max_{\mu_i^*} \tilde{\pi}\left(\mu_i^*, \mu_{-i}\right)\mu(\mathbf{S})^{\theta-1}W(\mathbf{S}) + \mathbb{E}\left[Q(\mathbf{S}, \mathbf{S}')V_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}}{g'e^{\varepsilon_{-i}'}}, \mathbf{S}'\right)\right].$$

The equilibrium discount factor is $Q(\mathbf{S}, \mathbf{S}') = \beta W(\mathbf{S})/W(\mathbf{S}')$. This implies that all values can be normalized by the wage, where $v_i = V_i/W$:

$$v_i^{adj}\left(\mu_i, \mu_{-i}, \mathbf{S}\right) = \max_{\mu_i^*} \tilde{\pi}\left(\mu_i^*, \mu_{-i}\right)\mu(\mathbf{S})^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}}{g'e^{\varepsilon_{-i}'}}, \mathbf{S}'\right)\right].$$

Replacing the aggregate state $\mathbf{S} = (g, \lambda)$ with that used in the approximation $\mathbf{S} = (g, \mu_{-1})$, we have the following:

$$v_i^{adj}\left(\mu_i, \mu_{-i}, g, \mu_{-1}\right) = \max_{\mu_i^*} \tilde{\pi}\left(\mu_i^*, \mu_{-i}\right)\hat{\mu}\left(g, \mu_{-1}\right)^{\theta-1} + \beta\mathbb{E}\left[v_i\left(\frac{\mu_i^*}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}}{g'e^{\varepsilon_{-i}'}}, g', \hat{\mu}\left(g, \mu_{-1}\right)\right)\right],$$

where $\hat{\mu}$ is given by the assumed log-linear function: $\log\hat{\mu} = \alpha_0 + \alpha_1 g + \alpha_2 \log\mu_{-1}$.

The equilibrium condition requiring the price index be consistent with firm prices has also been restated in terms of markups, which implies the entire equilibrium is now restated in terms of markups. To simulate changes in prices, it is sufficient to know a path for markups $\mu_{ijt}$, innovations $\varepsilon_{ijt}$, and money growth $g_t$. To determine quantities I need to also simulate paths for $M_t$ and $z_{ijt}$.

**Solution of the MPE** First, for simplicity, suppose that $\theta = 1$ such that no function of the aggregate state enters the firm's problem. Suppose also that shocks to the growth rate of money supply are entirely transitory ($\rho_g = 0$). In this case, the state variables of the firm's problem are only $\mu_i$ and $\mu_{-i}$. Since the parameters associated with each firm in each sector are symmetric, I only consider solutions in symmetric policies $\mu(\mu_i, \mu_{-i})$ and $\gamma(\mu_i, \mu_{-i})$. Suppose that these functions are known; then solving the firm's problem amounts to solving a simple Bellman equation. Define the firm's expected value function $v_i^e(\mu_i', \mu_j') = \mathbb{E}\left[v_i\left(\frac{\mu_i'}{g'e^{\varepsilon_i'}}, \frac{\mu_{-i}'}{g'e^{\varepsilon_{-i}'}}\right)\right]$. I can approximate $v_i^e$ with a cubic spline and, given a starting guess, use standard collocation tools to solve the firm's Bellman equation. This requires specifying a grid of collocation nodes for $\mu_i$ and $\mu_{-i}$, and then solving for splines with as many coefficients as collocation nodes. Given an approximation of $v_i^e$, the choices of a firm on these nodes can be solved for, and the values on these nodes used to update the approximation using Newton's method (see Miranda and Fackler (2002)). An alternative approach is to iterate on the Bellman equation.

When solving the MPE, the competitor policies are not initially known. In solving the model, I take a number of approaches, each of which yields the same equilibrium policies. In all cases, I approximate the optimal markup and probability of adjustment policies using cubic splines. The first approach is to consider some large $T$ and assume that from this period onward, prices are perfectly flexible such that the unique frictionless Nash equilibrium is obtained. This determines a starting guess for the policies and value function. Random menu costs imply that each stage game has a unique equilibrium for each point in the state space, which implies that this long subgame perfect Nash equilibrium is unique. One can then iterate backward to $t = 0$, or truncate iterations once the policy functions and values of the firm converge. The second approach is to fix a competitor's policies, solve a firm's Bellman equation, use this to compute new policies, and then continue to iterate in this manner until all objects converge. In practice, both approaches were found to lead to the same policy and value functions. The second approach is faster, since collocation methods can be used to quickly solve the Bellman equation, keeping the competitor policies fixed.

Under $\theta > 1$ and persistent shocks to money growth, then the approximate aggregate state $(g, \mu_{-1})$ also enters the firm's state vector. The solution algorithms for the MPE, however, do not change. I approximate the firm's policies using linear splines in each of these additional dimensions. Policy and value functions are approximated using 25 evenly spaced nodes, and the aggregate states are approximated using 7 evenly spaced nodes.[61] Approximating the expected value function implies that expectations are only taken once in each iterative step while solving the value function, rather than on every step of the solver for the optimal $\mu_i^*$. This, along with the use of a continuous approximation to the value function, allows for a high degree of precision in updating the expected value function. Given an expected value function, an optimal policy can be computed, delivering a new value function, which is then integrated over 100 points in both $\varepsilon_i'$ and $\varepsilon_{-i}'$ in order to compute a new expected value function.[62]

[61] Note that when solving the problem for a firm, a competitor's policy is never evaluated off the collocation nodes. The only computations that involve the splines are evaluating the expected value function for proposed $\mu_i^*$ values in the maximization step, and the simulation of sectors.

[62] "Quadrature" methods, by contrast, only use a small handful of points in the approximation of the integral. Working with continuous splines and iterating on the expected value function allow a much more precise computation of

**Issues for high and low menu costs** For a fixed set of collocation nodes, issues arise when trying to solve the model for very low or very high menu costs. For very low menu costs, the adjustment probabilities of the firm take on a steep $V$-shape, and small deviations in markups lead to a sharp increase in the probability of adjustment. Approximating such functions is difficult with a conservative number of nodes for the approximant of $\gamma(\mu_i, \mu_{-i})$. When menu costs are very large, the adjustment probabilities take on a very shallow $U$-shape, and markups deviate more widely. This also is hard to approximate with a conservative number of nodes for the approximants.

Figure 10 is symptomatic of this issue. Note that in the Calvo model of adjustment these issues do not arise, since I no longer have to approximate the probability of adjustment function. Therefore the Calvo model can be solved at a very high frequency of adjustment. Figure 10 verifies that as $\alpha$ tends towards one, the value of the firm in the duopoly model smoothly approaches the value of the firm in the monopolistically competitive model, since both models are calibrated to the same frictionless markup.

**Krusell-Smith algorithm** I first solve the economy under $\mu_t = \mu^*$, where $\mu^*$ is the frictionless Nash equilibrium markup. I then proceed with the Krusell-Smith algorithm, refining the firm's forecast. Solving the model under the initial forecasting rule, I can then simulate the economy. Since firm-level shocks are large, then even for large numbers of simulated sectors, there will be small fluctuations in aggregates. In implementing the Krusell-Smith algorithm I therefore proceed as follows. Let $\{E_t\}_{t=0}^{T}$ be a sequence of matrices of idiosyncratic shocks—to both productivity and menu costs—to all firms in all sectors, and consider some simulated path of money growth $\{\varepsilon_t^g\}_{t=0}^{T}$. I simulate two economies, both under $\{E_t\}_{t=0}^{T}$ and with the same initial distribution of markups, but one under $\{\varepsilon_t^g\}_{t=0}^{T}$ and the other under $g_t = \bar{g}$ for all $t$. From the second simulation, I then compute the sequence of aggregate markups and call this $\bar{\mu}_t$, with corresponding $\mu_t$ from the first simulation. I then run the following regression on simulated data from $\underline{T}$ to $T$:

$$(\log \mu_t - \log \bar{\mu}_t) = \alpha_1 (\log g_t - \log \bar{g}) + \alpha_2 (\log \mu_{t-1} - \log \bar{\mu}_{t-1}) + \eta_t.$$

I also compute the average aggregate markup $\bar{\mu} = 1/(T - \underline{T}) \sum_{t=\underline{T}}^{T} \mu_t$. When solving the model on the next iteration, I renormalize the aggregate state space to $S = (\log g - \log \bar{g}, \log \mu_{-1} - \log \bar{\mu})$

the integral.

and provide firms with the forecasting rule

$$\log \mu(S) = \log \bar{\mu} + \hat{\alpha}_1 S_1 + \hat{\alpha}_2 S_2.$$

In practice, I simulate 10,000 sectors, set $T = 2,000$, and $\underline{T} = 500$, and iterate to convergence on $\{\bar{\mu}, \alpha_1, \alpha_2\}$. In the monopolistically competitive model, I simulate a single sector with 20,000 firms. This approach controls for simulation error, and allows me to keep the nodes of the state space for $S_2$ the same across solutions of the model, while incorporating changes in the forecast of the average markup.

The algorithm converges quickly and the rule provides a high $R^2$ in simulation. This works especially well in the context of this model for a number of reasons, which all relate to the role of $\mu_t$ in the firm's problem. First, $\mu_t$ simply shifts the level of the firm's profit function, which implies that in a static model, it only affects the value of a price change, not the firm's optimal markup. Second, if $\theta$ is close to one, then this movement in the profit function is small for any given fluctuations in $\mu_t$. Third, these fluctuations in $\mu_t$ are in fact small, given the empirical magnitude of money growth shocks. From a robustness perspective, this is reassuring: if the rule used by firms was incorrect, then this misspecification would have little impact on the policies of the firm. In practice, this means that the coefficients for $\{\bar{\mu}, \alpha_1, \alpha_2\}$ from the first solution of the model under the rule $\mu_t = \mu^*$, are very close to the final coefficients.

**Computing aggregate fluctuations** I also correct the computation of other moments for simulation error which might otherwise bias one toward finding larger time-series fluctuations. For example, the key statistic of $\sigma(C_t)$ is computed using $std\left[\log C_t - \log \bar{C}_t\right]$, where $\bar{C}_t$ is aggregate consumption computed under the simulation with aggregate money growth equal to $\bar{g}$ in all periods. In this "steady-state" economy, there are still fluctuations in aggregate consumption, but these are due only to large shocks to firms not washing out in a simulation of finitely many firms. The same approach is taken when computing impulse response functions for moments such as the frequency of price adjustment of low-markup firms in Figure 6.

# C Static game

In this appendix, I study a two-player price-setting game in which the profit function of the firm displays complementarities in prices, firms face a fixed cost of changing prices, and initial prices are above the frictionless Nash equilibrium price. I establish that (i) the frictionless best response function of the firm has a positive gradient bounded between zero and one, (ii) menu costs can sustain higher prices than obtain in a frictionless setting, (iii) the only pure strategy equilibria that exist are ones in which both firms change their price or both keep them fixed, (iv) for any given menu cost, there is always a range of initial prices for which both equilibria exist. I then show that the profit functions—derived from nested CES preferences—in the body of the paper satisfy the necessary assumptions for these results.

**Environment** Consider two firms with symmetric profit functions $\pi^1(p_1, p_2) = \pi^2(p_2, p_1)$. In what follows, I drop the superscripts on the profit function and prices, with the second argument always referring to the competitor's price. Assume that $\pi$ is twice continuously differentiable and that the derivatives of $\pi$ have the following properties: $\pi_{11} < 0$, $\pi_{12} > 0$. The second assumption is the definition of complementarity in prices.

There is one period. Firms begin the period with initial price $\bar{p}$, which is larger than the frictionless Nash equilibrium price $p^*$. To deviate from this price, a firm must pay a cost $\xi$. The objective function of firm $i$ is therefore $v(p_i, p_j) = \pi(p_i, p_j) - \mathbf{1}[p_i \neq \bar{p}]\,\xi$.

**Static best response function** The *frictionless best response function* $p^*(p)$ is the best response of a firm to its competitor's price $p$ when $\xi = 0$. The key property of the static best response which is discussed in the text is that it has a positive gradient between zero and one. To prove this, take the firm's first order condition: $\pi_1(p^*(p), p) = 0$. By the implicit function theorem, the derivative of $p^*(p)$ can be obtained by rearranging the total derivative of the first order condition:

$$\frac{\partial p^*(p)}{\partial p} = -\frac{\pi_{12}(p^*(p), p)}{\pi_{11}(p^*(p), p)}.$$

The frictionless Nash equilibrium price $p^* = p^*(p^*)$ solves both firms' first order conditions simultaneously. The second order conditions must hold at $(p^*, p^*)$, requiring that the principal minors of the Hessian—evaluated at $p^* = p^*(p^*)$—alternate in sign:

$$\pi_{11}(p^*, p^*) < 0, \quad \text{and} \quad \pi_{12}(p^*, p^*)^2 < \pi_{11}(p^*, p^*)^2.$$

The first condition holds by assumption. The second condition, jointly with the assumption of complementarity ($\pi_{12} > 0$), gives the result that any Nash equilibrium

$$\left.\frac{\partial p^*(p)}{\partial p}\right|_{p=p^*} = -\frac{\pi_{12}(p^*, p^*)}{\pi_{11}(p^*, p^*)} \in (0, 1).$$

Multiple equilibria would require $p^{*\prime}(p^*)$ have a slope greater than one at some other equilibria, so clearly the equilibrium is also unique. Note that $p^*(p) \in (p^*, p)$ for $p > p^*$, that is, the best response function exhibits "undercutting."

**Equilibria of the menu cost game** Categorize possible pure strategy equilibria into three types : (I) neither firm changes its price, (II) both firms change their price, (III) one firm changes its price.

A necessary and sufficient condition for a Type-I equilibrium is

$$\pi(\bar{p}, \bar{p}) \geq \max_{p} \pi(p, \bar{p}) - \xi, \tag{C1}$$

or equivalently

$$\xi \geq \Delta_I(\bar{p}) = \pi\left(p^*(\bar{p}), \bar{p}\right) - \pi\left(\bar{p}, \bar{p}\right). \tag{C2}$$

This condition for a Type-I equilibrium holds when (i) $\xi$ is very large or (ii) $\bar{p}$ is small. To show that $\Delta_I(\bar{p})$ is increasing in $\bar{p}$, it is useful to represent $\Delta_I(\bar{p})$ as an integral. The derivative is then

$$\frac{\partial \Delta_I(\bar{p})}{\partial \bar{p}} = \frac{\partial}{\partial \bar{p}}\left[-\int_{p^*(\bar{p})}^{\bar{p}} \pi_1(u, \bar{p}) du\right] = \int_{p^*(\bar{p})}^{\bar{p}} \pi_{12}(u, \bar{p}) du + \pi_1(p^*(\bar{p}), \bar{p}) - \pi_1(\bar{p}, \bar{p}) > 0. \tag{C3}$$

This is positive due to complementarity ($\pi_{12} > 0$), the definition of $p^*(\bar{p})$ implies the second term is zero, and $\pi_1(\bar{p}, \bar{p}) < 0$ since $\bar{p} > p^*(\bar{p})$. The change in value that accompanies the optimal deviation from $p^*(\bar{p})$ increases in $\bar{p}$. Sustaining initial deviations from the frictionless Nash equilibrium requires the initial deviation to be not too large or menu costs to be not too small.

In a Type-II equilibrium, in which both firms change their price, it must be that the prices chosen are $(p^*, p^*)$. Given that both firms are changing their prices, then the price chosen by each firm must be a best response to its competitor. We then need to check that it is not optimal for a firm to leave its price at $\bar{p}$, which requires

$$\xi \leq \pi\left(p^*, p^*\right) - \pi\left(\bar{p}, p^*\right) \quad \left(= \Delta_{II}(\bar{p})\right). \tag{C4}$$

This condition for a Type-II equilibrium holds when (i) $\xi$ is small or (ii) $\bar{p}$ is large. To see that $\Delta_{II}(\bar{p})$ is increasing in $\bar{p}$, note that $\pi(\bar{p}, p^*)$ is decreasing in $\bar{p}$ for all $\bar{p} > p^*$. The frictionless equilibrium will still obtain when $\bar{p}$ is large relative to the menu cost.

Type-III equilibria do not exist. Observe that a Type-III equilibrium requires that the firm that changes its price, changes it to $p^*(\bar{p})$. There are therefore two necessary conditions for a Type-III equilibrium. First, firm 2 must find it profitable to change its price given that firm 1's price remains at $\bar{p}$:

$$\pi\left(p^*(\bar{p}),\bar{p}\right)-\xi\geq\pi\left(\bar{p},\bar{p}\right). \tag{C5}$$

This holds when (i) $\xi$ is small or (ii) $\bar{p}$ is large. Second, the frictionless best response of firm 1 to firm 2's price must not be a best response under a positive menu cost. Letting $p^{**}(\bar{p})$ denote the frictionless best response to $p^*(\bar{p})$, we then require

$$\pi\left(p^{**}(\bar{p}),p^*(\bar{p})\right)-\xi\leq\pi\left(\bar{p},p^*(\bar{p})\right). \tag{C6}$$

This holds when (i) $\xi$ is large or (ii) $\bar{p}$ is small. Intuitively, it seems that these conditions should not simultaneously hold. If one firm finds it valuable to undercut its competitor, then its competitor should find it valuable to respond. Indeed, this can be proven, with the proof found at the end of this appendix.

Having asserted that the only pure strategy equilbria are of Type-I and Type-II, we can also show that for any value of $\xi$, there exist an interval of $\bar{p}$ for which both Type-I equilibria and Type-II equilibria may exist. First note that $\Delta_I(p^*)=\Delta_{II}(p^*)=0$. That is, both equilibria trivially exist for zero menu costs at $\bar{p}=p^*$. A sufficient condition for both equilibria to exist for any value of $\xi$ is to show that $\Delta_{II}(\bar{p})>\Delta_I(\bar{p})$:

$$\pi(p^*,p^*)-\pi(\bar{p},p^*)>\pi(p^*(\bar{p}),\bar{p})-\pi(\bar{p},\bar{p}). \tag{C7}$$

Since $p^*$ is the best response to $p^*$ then $\pi(p^*,p^*)>\pi(p^*(\bar{p}),p^*)$, so showing the following is sufficient:

$$\pi(p^*(\bar{p}),p^*)-\pi(\bar{p},p^*)>\pi(p^*(\bar{p}),\bar{p})-\pi(\bar{p},\bar{p}). \tag{C8}$$

If $\pi$ displays complementarity, then this holds.[63]

These results characterize equilibria in $(\bar{p},\xi)$-space as follows. Consider fixing $\bar{p}$ and starting at a high value of $\xi$. In this region, only the Type-I equilibrium exists. Menu costs are sufficiently high that the best response of each firm to the initially high price of its competitor is to keep a high price. As $\xi$ decreases, we reach a point at which Type-II equilibria are also feasible. In this region, if firm 2 changes its price, then the best response of firm 1 is to also change its price (Type-II), but

[63] To see this, rearrange the condition and then express both sides as integrals:

$$\begin{aligned}\pi(\bar{p},p^*)-\pi(p^*(\bar{p}),p^*) &< \pi(\bar{p},\bar{p})-\pi(p^*(\bar{p}),\bar{p}),\\ \int_{p^*(\bar{p})}^{\bar{p}}\pi_1(u,p^*)du &< \int_{p^*(\bar{p})}^{\bar{p}}\pi_1(u,\bar{p})du.\end{aligned}$$

Due to complementarity, $\bar{p}>p^*$ implies $\pi_1(u,\bar{p})>\pi_1(u,p^*)$. Since both integrals are over the same support, then the inequality must always hold.

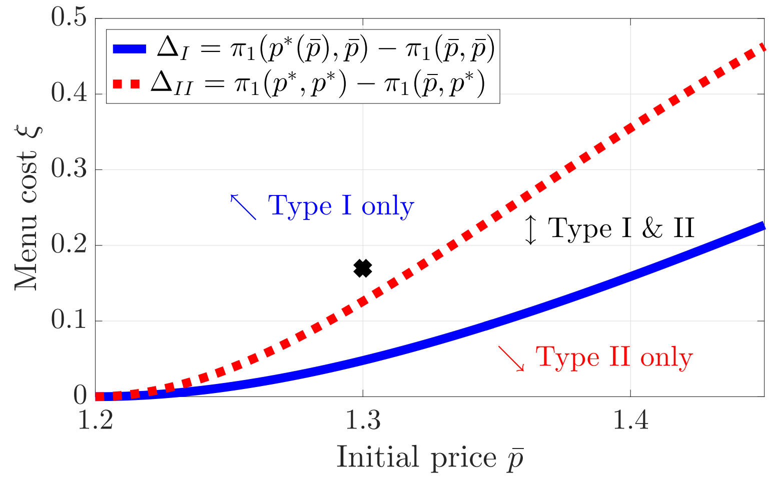


Figure C1: Regions of equilibria in a static price setting game

if firm 2 leaves its price fixed, then the best response of firm 1 is to also leave its price fixed (Type-I). As $\xi$ decreases further, the Type-I equilibrium can no longer be sustained as the menu cost is insufficient to commit firms not to respond to a price decrease at their competitor. Alternatively, fixing $\xi$ and increasing $\bar{p}$, first only the Type-I equilibrium exists, then both, then as the value of a price decrease becomes large, only the Type-II equilibrium exists. Figure C1 plots these regions for a profit function discussed below.

In the case of the existence of multiple equilibria, the equilibria are ranked as we would expect: the fixed price Type-I equilibrium is preferred. This requires that $\pi(\bar{p}, \bar{p}) > \pi(p^*, p^*) - \xi$. Since the Type-I equilibrium exists, then $\xi \geq \pi(p^*(\bar{p}), \bar{p}) - \pi(\bar{p}, \bar{p})$, and therefore this ranking holds if $\pi(p^*(\bar{p}), \bar{p}) > \pi(p^*, p^*)$. Since prices are complements, this is true: the best response to a high price yields a larger profit than the best response to a low price.

From this static game we learn that for a given menu cost $\xi$, high prices $\bar{p}$ can be sustained so long as they are not too far from the frictionless Nash equilibrium. If the initial price is too high, one firm has a profitable deviation even it pays the menu cost. If the value of one firm's deviation exceeds the menu cost, then the value of an iterative undercutting strategy from its competitor must also exceed the menu cost. Both firms change their prices, and only the frictionless Nash equilibrium price is attainable. If initial prices are not too high, then the menu cost is enough to negate the small value of the optimal frictionless downward deviation in price, making the high-priced strategy credible. We also learn that the equilibrium is not unique for certain combinations of $\xi$ and $\bar{p}$, while these equilibria are clearly Pareto ranked: if firms could coordinate on an equilibrium, they would choose not to change their prices.

**Getting to $\bar{p}$** Consider the game when the firms prices are initially at $p^*$. Regardless of the size of $\xi$, the only equilibrium that can exist is $(p^*, p^*)$. One firm increasing its price is not an equilibrium, since $p^*$ is already the best response to $p^*$. Both firms raising prices to the same price $\bar{p}$ is not an equilibrium since conditional on changing price $\bar{p}(p^*) \in (p^*, \bar{p})$ is the best response. In a dynamic game, firm 2 may "take the high road" by posting $\bar{p}_2$ today. Its competitor may choose $p_1' \in (p^*(\bar{p}_2), \bar{p}_2)$ next period, knowing that at $(p_1', \bar{p}_2)$, the menu cost faced by firm 2 will make a downward response unprofitable. In this way, firms can constructively distribute gains and losses from policies across periods and achieve prices above $p^*$.

**Numerical example** In the main text, the profit function of the firm is

$$
\begin{aligned}
\pi_1(p_1, p_2) &= \left(\frac{p_1}{p(p_1, p_2)}\right)^{-\eta} \left(\frac{p(p_1, p_2)}{P}\right)^{-\theta} (p_1 - 1)C, \\
p(p_1, p_2) &= \left[p_1^{1-\eta} + p_2^{1-\eta}\right]^{1/1-\eta}.
\end{aligned}
$$

To be consistent with notation in this appendix, I have replaced markups with prices and a unit marginal cost. From this profit function we can solve in closed form for the Nash equilibrium price as follows.

The first order condition of the firm's problem is

$$
\left[p_1^{-\eta} - \eta p_1^{-\eta-1}(p_1 - 1)\right] p^{\eta-\theta} + (\eta - \theta) p_1^{-\eta} p^{\eta-\theta-1}(p_1 - 1)\frac{\partial p}{\partial p_1} = 0,
$$

where the term in square brackets gives the first order condition of a monopolistically competitive firm facing elasticity of demand $\eta$. The second term gives the marginal profit due to the firm increasing the sectoral price. Since $\eta > \theta$, this second term is positive, implying that the term in brackets is negative, and so the equilibrium price must be larger than the monopolistically competitive price under $\eta$.

Two additional results for a CES demand system allow us to solve the first order condition in closed form. First,

$$
\frac{\partial p}{\partial p_1} = \left[p_1^{1-\eta} + p_2^{1-\eta}\right]^{\frac{1}{1-\eta}-1} p_1^{-\eta} = \left(\frac{p_1}{p}\right)^{-\eta}.
$$

Second, the revenue of the firm is $r_1 = p_1(p_1/p)^{-\eta}(p/P)^{-\theta}C$, which gives the following revenue share:

$$
s_1 = \frac{r_1}{r_1 + r_2} = \frac{p_1^{1-\eta}}{p_1^{1-\eta} + p_2^{1-\eta}} = \left(\frac{p_1}{p}\right)^{-\eta} \frac{p_1}{p} = \frac{\partial p}{\partial p_1}\frac{p_1}{p}.
$$

Using these results in the first order condition we obtain

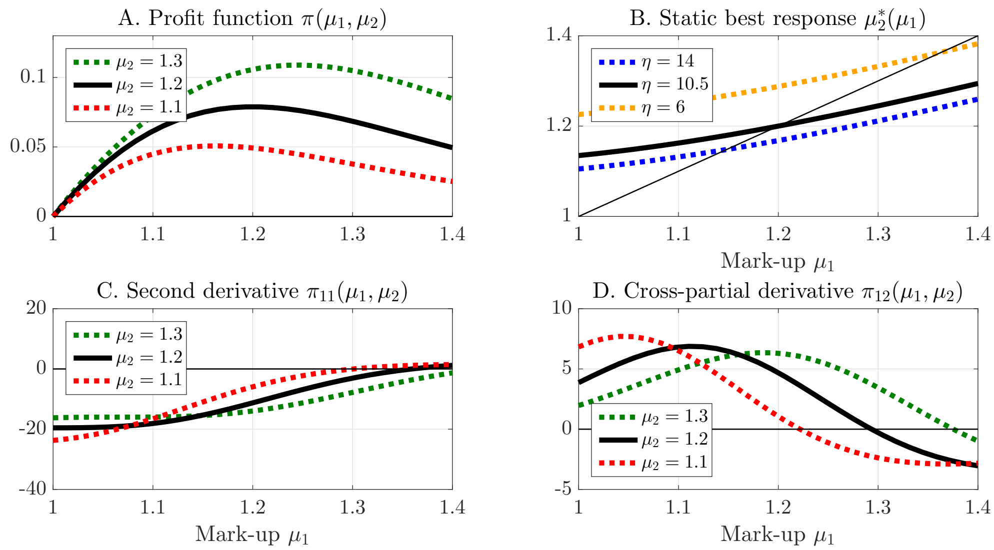


Figure C2: Properties of firm profit functions

Notes: Panels A, C, and D display features of the duopoly profit functions under $\theta = 1.5$, $\eta = 10.5$ as in Table 1. Given these parameters, the frictionless Nash-Bertrand markup is 1.20 due to an effective elasticity of demand of $\varepsilon = (1/2)(\theta + \eta)$ and a symmetric equilibrium. Panel B plots the static best response function $\mu_i^*(\mu_j)$ under $\theta = 1.5$ and different values of $\eta$. Higher values of $\eta$ reduce the Nash equilibrium markup—given by the intersection of the best response with the 45-degree line—and increase the slope of the best response function.

$$p_1 - \eta(p_1 - 1) + (\eta - \theta)(p_1 - 1)s_1 = 0.$$

Since firms are symmetric, the equilibrium will yield equal revenue shares $s_1 = 0.5$, and $p^* = \varepsilon/(\varepsilon - 1)$, where $\varepsilon$ is an average of the within- and across-sector demand elasticities $\varepsilon = 0.5 \times (\eta + \theta)$. The form of the solution implies that markups are consistent with those chosen by a monopolistically competitive firm facing an elasticity of demand equal to $\varepsilon$. Note that since $P$ and $C$ are first order terms in the firm's profit function, they do not affect the Nash equilibrium markup.

**Calibration** The calibration of the dynamic duopoly model yielded $\theta = 1.5$ and $\eta = 10.5$ (see Table 1). For these values, $\varepsilon = 6$ and $p^* = 1.2$.[64] I apply these values to the equilibrium profit function from the text (8), in which $P^{\theta-1}$ would multiply the profit function instead of $PC^{-\theta}$. Setting $P$ to the average markup 1.30, Figure C1 shows how $(\xi, \bar{p})$-space separates across different equilibria. It is entirely consistent with the theoretical results. Recall that the model was calibrated to the average size and frequency of price change, so the menu cost was not chosen with a particular equilibrium in mind. Note that the average markup in the model is $\bar{p} = 1.3$, and the upper bound on the menu cost is $\bar{\xi} = 0.17$ (marked with an $x$ in the figure). Zbaracki, Ritson, Levy, Dutta, and Bergen

[64] Recall that the *Alt III* calibration of the monopolistically competitive model set $\eta = 6$ to deliver this as a frictionless markup.

(2004) find that price adjustment costs make up 1.2 percent of firm revenue. As a benchmark, $\Delta_{II}(\bar{p})/rev(\bar{p},\bar{p}) = 0.012$ at $\bar{p} = 1.27$, so a menu cost around empirical estimates as a share of revenue would, in this static game under the calibrated parameters of the model, guarantee a Type-I equilibrium.

Figure C2 plots various features of this profit function, varying $p_2$. Under the profit function derived from CES preferences, it is not true that $\pi_{12} > 0$ everywhere, but this is true at $(p_1, p_2) = (1.3, 1.3)$, so around the average markup in the calibrated model.[65]

**Summary** From this exercise, the following is a heuristic understanding of the dynamic model. Nominal rigidity allows firms to fluctuate around a markup which is larger than the frictionless Nash equilibrium. However, this is constrained by the size of the menu cost, which is pinned down by the average frequency of price change. Given a menu cost $\xi$, firms choose reset prices around a real price $\bar{p}$ that supports a Type-I equilibrium, but not so high as to risk a Type-II equilibrium. Idiosyncratic shocks force the firms' real prices apart, but the firms keep on adjusting their prices so as to not let them get too far away from $\bar{p}$. Prices that are too high invite undercutting, and prices that are too low reduce profitability. Menu costs in the range of empirical estimates can sustain markups in the range of empirical estimates. Finally, getting to these high prices requires firms to reduce profit in the short run in order to lay the incentives for their competitor to choose a price that maintains high long run profits for the sector.

**Calvo model** Finally, consider a Calvo version of the static model, where each firm changes its price with probability $\alpha$. Let $\tilde{p}$ be the optimal reset price of the firm. Then a Nash equilibrium requires that each firm's first order condition be satisfied at $\tilde{p}$:

$$\alpha\pi_1(\tilde{p},\tilde{p}) + (1-\alpha)\pi_1(\tilde{p},\bar{p}) = 0.$$

It is straightforward to show that $\tilde{p} < p^*(\bar{p})$. A sufficient condition is that $\pi_1(\tilde{p},\bar{p}) < 0$, since $\pi_1(p^*(\bar{p}),\bar{p}) = 0$. The first order condition implies that this is true if $\pi_1(\tilde{p},\bar{p}) > \pi_1(\tilde{p},\tilde{p})$, which is true due to complementarity and $\bar{p} > \tilde{p}$. Note that as $\alpha \to 1$, then $\tilde{p} \to p^*$.

[65] An unusual property of the CES profit function is that profits are always positive for $p > 1$, regardless of price. This implies, as shown in Panel C, that the second derivative must, for high prices, become positive.

**Proof** For the Type-III equilibrium to exist, conditions (C5) and (C6) must hold simultaneously, requiring that

$$\pi(p^{**}(\bar{p}), p^*(\bar{p})) - \pi(\bar{p}, p^*(\bar{p})) \leq \pi(p^*(\bar{p}), \bar{p}) - \pi(\bar{p}, \bar{p}).$$

I prove that the negation of this inequality always holds. Note that the expression on the left hand side can be decomposed as follows:

$$\begin{aligned} \pi(p^{**}(\bar{p}), p^*(\bar{p})) - \pi(\bar{p}, p^*(\bar{p})) \quad = \quad & [\pi(p^{**}(\bar{p}), p^*(\bar{p})) - \pi(p^*(\bar{p}), p^*(\bar{p}))] \\ & + [\pi(p^*(\bar{p}), p^*(\bar{p})) - \pi(\bar{p}, p^*(\bar{p}))] \,. \end{aligned}$$

Since the best response function is upward sloping, then $p^{**}(\bar{p}) < p^*(\bar{p}) < \bar{p}$, and the profit function $\pi(p, p^*(p))$ is downward sloping for $p > p^{**}(\bar{p})$. This implies that each of the two terms on the right-hand side is positive. A sufficient condition for the non-existence of a Type-III equilibrium is therefore

$$\pi(p^*(\bar{p}), p^*(\bar{p})) - \pi(\bar{p}, p^*(\bar{p})) \geq \pi(p^*(\bar{p}), \bar{p}) - \pi(\bar{p}, \bar{p}).$$

Noting that $p^*(\bar{p}) < \bar{p}$, then the fundamental theorem of calculus can be used to express this condition as

$$\int_{p^*(\bar{p})}^{\bar{p}} \pi_1(u, p^*(\bar{p}))\, du \leq \int_{p^*(\bar{p})}^{\bar{p}} \pi_1(u, \bar{p})\, du.$$

Since $p^*(\bar{p}) < \bar{p}$ and the firms' prices are complements, then $\pi_1(u, p^*(\bar{p})) \leq \pi_1(u, \bar{p})$ for all $u \in [p^*(\bar{p}), \bar{p}]$, so this condition holds.

# D Discussion of model assumptions

**1. CES demand structure** An alternative formulation of the demand system could have been chosen. A pertinent example is a nested logit system commonly used in structural estimation of demand systems. However, as shown by Anderson, de Palma, and Thisse (1992), the nested CES structure is isomorphic to a nested logit with a population of consumers that each choose a single option at each stage.[66] That is, consumers may have identical preferences for Kraft and Hellman's mayonnaise, up to an *iid* taste shock that shifts each consumer's tastes toward one or the other each period. A CES structure with equal weights will deliver the same market demand functions

[66] I thank Colin Hottman for making this point and take its presentation from Hottman (2016).

under an elasticity of substitution that reflects the distribution of taste shocks and reduced form elasticity of indirect utility to price.[67]

**2. Random menu costs** Random menu costs serve two purposes in the model. First, they generate some small price changes. Some firms, having recently changed their price and accumulating little change in sectoral productivity, draw a small menu cost and again adjust their price. Figure 9 shows that a monopolistically competitive model with random menu costs gives a distribution of price changes that appear as smoothed versions of the bimodal spikes of Golosov and Lucas (2007). Midrigan (2011) explicitly models multiproduct firms and shows that the implications for aggregate price and quantity dynamics are—when calibrated to the same price-change data—the same as in a model with random menu costs. What is important for these dynamics is that the model generates small price changes—which dampen the extensive margin effect—leading to the statement that the conclusions drawn are not sensitive to the exact mechanism used to generate small price changes. In this sense, one can think of the random menu costs in my model as standing in for an unmodeled multiproduct pricing problem.

Second, and most important, random menu costs that are private information allow me to avoid solving for mixed-strategy equilibria. This technique I borrow from Doraszelski and Satterthwaite (2010), who use it to address the computational infeasibility of solving the model of Ericson and Pakes (1995), which has potential equilibria in mixed strategies as well as issues with existence of any kind of equilibrium.[68] One could imagine solving the model under mixed strategies with fixed menu costs. Given the values of adjustment and non-adjustment and a fixed menu cost $\xi$, the firm may choose its probability of adjustment

$$\gamma_i(s,\mathbf{S}) = \arg\max_{\gamma_i\in[0,1]} \gamma_i \left[v_i^{adj}(s,\mathbf{S}) - \xi\right] + (1-\gamma_i)\, v_i^{stay}(s,\mathbf{S}).$$

If firm $-i$ follows a mixed strategy such that $v_i^{adj}(s,\mathbf{S}) - \xi = v_i^{stay}(s,\mathbf{S})$, then a mixed strategy is a best-response of firm $i$. If one believes that menu costs are fixed, then this provides an alternative rationale for small price changes. Some firms may not wish to adjust prices this period, yet their mixed strategy over adjustment leads them to change prices nonetheless. However, the solution

[67]For estimation of alternative static demand systems using scanner data similar to that used in this paper see Beck and Lein (2015) (nested logit), Dossche, Heylen, and den Poel (2010) (AIDS), and Hottman, Redding, and Weinstein (2014) (nested CES). Only the latter studies an equilibrium, imperfectly competitive model.

[68]This technique is also used by Nakamura and Zerom (2010) and Neiman (2011) in menu cost models.

of this model would be vastly more complicated and at this stage infeasible. Appendix C proves that even in a simple static game of price adjustment with menu costs, such multiple equilibria may arise.

**3. Information** I assume that the evolution of product demand within the sector $(z_{1j}, z_{2j})$ is known by both firms at the beginning of the period and only menu costs are private information. An alternative case is that menu costs are fixed, but firms know only their own productivity and the past prices of both firms. This would add significant complexity to the problem. First, if productivity is persistent, then firms would face a filtering problem and a state vector that includes a prior over their competitor's productivity. Second, computation is still complicated even if productivity is *iid*. From firm 1's perspective $z_{2j}$ would be given by a known distribution, which firm 1 must integrate over when computing expected payoffs. Integrating over firm 2's policy functions—which depend on $z_{2j}$—would be computationally costly. Since the menu cost is sunk, I avoid these issues.

**4. Idiosyncratic shocks** Three key assumptions are made regarding idiosyncratic shocks, they (i) follow a random walk, (ii) move both marginal revenue and marginal productivity schedules of the firm, and (iii) are idiosyncratic rather than sectoral. These are made for tractability but are not unrealistic.

The first is plausible given that the model is solved monthly. It achieves tractability in that future states depend on growth rates of $z_{ij}$, which are *iid*. An alternative assumption deployed in similar studies is a random walk in money growth and AR(1) in firm-level shocks, which reduces the total state variables of a monopolistically competitive model in the same way.[69] In the duopoly model, this would leave the total sectoral state vector with four elements, rendering the sectoral problem infeasible. Moreover, at a monthly frequency the estimated persistence of money growth is significantly less than one (see Section 4).

The second seems acceptable if one does not hold a strong view on whether demand or productivity shocks drive firm price changes, a reasonable stance given that most commonly only revenue productivity is observed in the data. Midrigan (2011) interprets $\varepsilon_{ij}$'s as shocks to "quality": the good has higher demand but is more costly to produce. This assumption is necessary—along with random walk shocks—to express the sectoral state vector in two rather than four states.

[69] Specifically, such an assumption would allow the aggregate state—following the Krusell-Smith approximation—to be captured by only the aggregate markup.

The third assumption is not for tractability of the duopoly model but the monopolistically competitive model. The latter with sectoral shocks would introduce two additional state variables to the firm's problem: the sectoral markup and sectoral shock. Firms would require forecasting rules for these on top of forecasting rules for the aggregate markup. This would render the problem infeasible. In addition, the existing literature does not take this approach.

Empirically, I offer some new evidence from a decomposition of firm revenue in the IRI data that suggests this may not be a bad approximation. Changes in firm $f$ revenue $r_{fpst}$ can expressed as:

$$\Delta \log r_{fpst} = \Delta \log \left( \frac{r_{fpst}}{r_{pst}} \right) + \Delta \log \left( \frac{r_{pst}}{r_{st}} \right) + \Delta \log r_{st}. \tag{D1}$$

The first component is the change in expenditure on firm $f$ relative to the market, the second component is the change in expenditure on product $p$ relative to total expenditure in the region, and the final term is due to changes in total expenditure in the region. Taking the time series variance of this equation admits the following identity for each pair of product $p$ and state $s$:

$$1 = \underbrace{\frac{var\left(\Delta \log \left(\frac{r_{fpst}}{r_{pst}}\right)\right)}{var\left(\Delta \log r_{fpst}\right)}}_{\text{(1) Firm share in market}} + \underbrace{\frac{var\left(\Delta \log \left(\frac{r_{pst}}{r_{st}}\right)\right)}{var\left(\Delta \log r_{fpst}\right)}}_{\text{(2) Market share in state}} + \underbrace{\frac{var\left(\Delta \log r_{st}\right)}{var\left(\Delta \log r_{fpst}\right)}}_{\text{(3) State expenditure}} + \underbrace{\frac{\text{Cov. terms}}{var\left(\Delta \log r_{fpst}\right)}}_{\text{(4) Covariance terms}}. \tag{D2}$$

I compute this decomposition for the largest firm in each market. Figure D1 plots the first three elements of equation (D2) against each other. Table D1 provides the average for each of these elements. The first column is a simple average across all pairs $ps$, and the second is weighted by average revenue $\bar{r}_{pr}$. Both point to fluctuations in the revenue share of the firm within the market as the most important in accounting for fluctuations in firm-level revenues, followed by fluctuations in the revenue share of the product within the state and finally fluctuations in total state expenditure.

The majority of fluctuations in the revenue of large firms are due to changes in the firm's share of expenditures within their product-state market, and not changes in the product's share of state expenditure or changes in the state's share of national expenditure. This suggests that as an initial approximation, firm rather than sectoral shocks are the most relevant.

| | Unweighted | Revenue weighted |
|---|---|---|
| (1) Firm share in market | 1.27 | 1.07 |
| (2) Market share in state | 0.72 | 0.40 |
| (3) State expenditure | 0.20 | 0.13 |
| (4) Covariance terms | -1.20 | -0.61 |

Table D1: Decomposing changes in firm revenue

Notes: Table gives the averages of the elements of equation (D2), computed for each product $p$, region $r$, where the firm $f$ has the largest revenue in market $pst$. There are 1,333 observations (31 products and 43 regions). Since these are averages, each column does not necessarily sum to one.

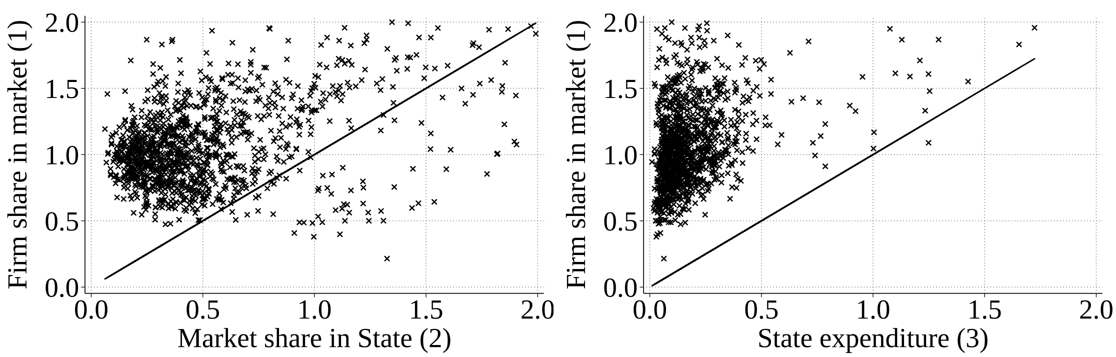


Figure D1: Decomposition of the variance of largest firm revenue changes

Notes: Figures plot the elements of equation D2, computed for each product $p$, region $r$, where the firm $f$ has the largest revenue in market $pst$. There are 1,333 observations (31 products and 43 regions).

# E Additional figures and tables

## E.1 Figures

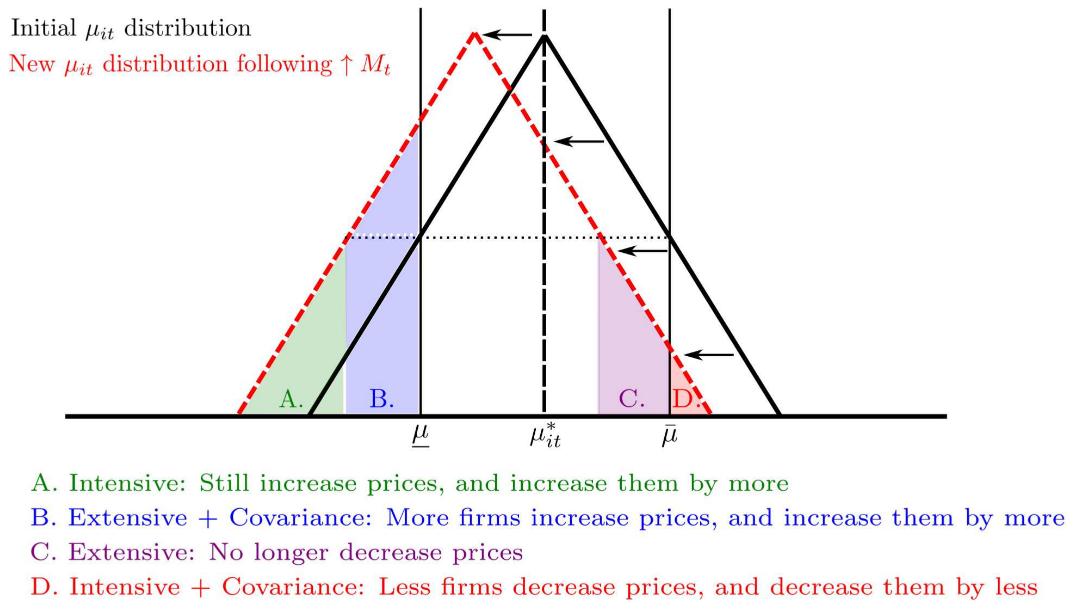


Figure E1: Decomposing markup adjustment in a monopolistically competitive, fixed menu cost model

Notes: Vertical solid lines give the thresholds for adjustment $\underline{\mu} < \bar{\mu}$. Following an increase in the money supply, all markups decrease by the same amount, as given by the leftward shift in the distribution. For a permanent one-time increase in the money supply, the optimal markup $\mu_{it}$ and thresholds for adjustment are not affected by the shock.

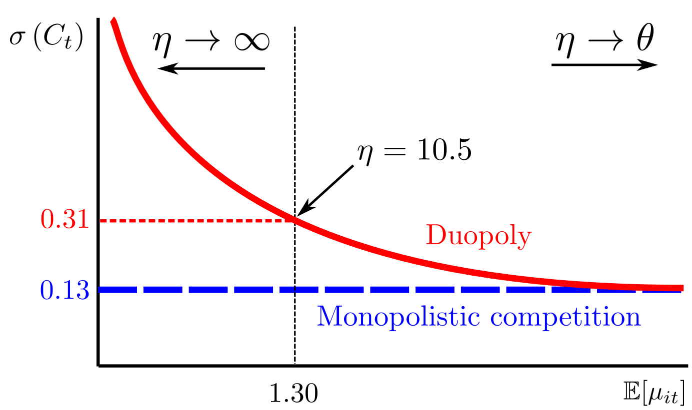


Figure E2: Monetary non-neutrality and the targeted value of the average markup

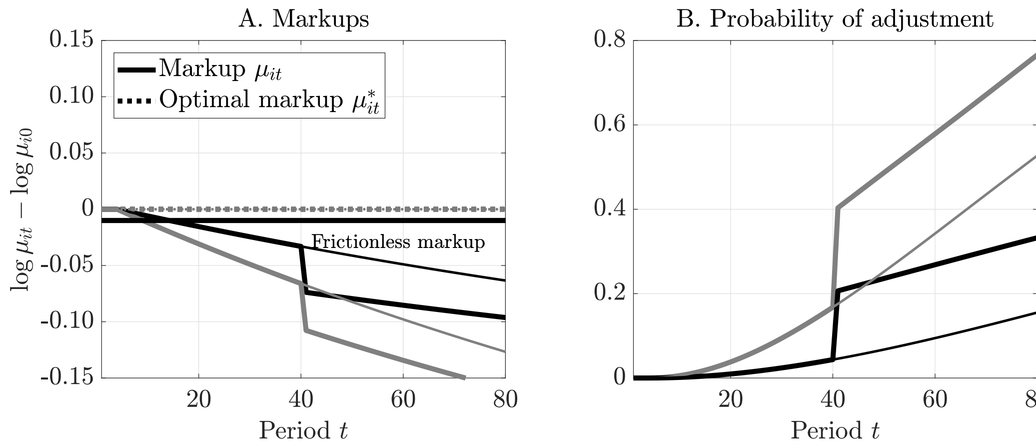


Figure E3: Positive monetary shock in monopolistically competitive model - Low markup firms

Notes: Thin solid lines give exogenous evolution of markups for two firms absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment $\mu_1' = \mu^*$ and $\mu_2' = \mu^*$. Thick solid lines include a monetary shock in period 40, which decreases both firms' markups. Thick dashed lines (which lie on top of the thin dashed lines) give the corresponding optimal markups. The model is solved in steady state and the monetary shock is a one-time unforeseen level increase in money. The $y$-axis in panel A describes the log deviation of markups from the value chosen when shocks and menu costs are zero, $\bar{\mu} = 1.30$, which is equal to the average markup.

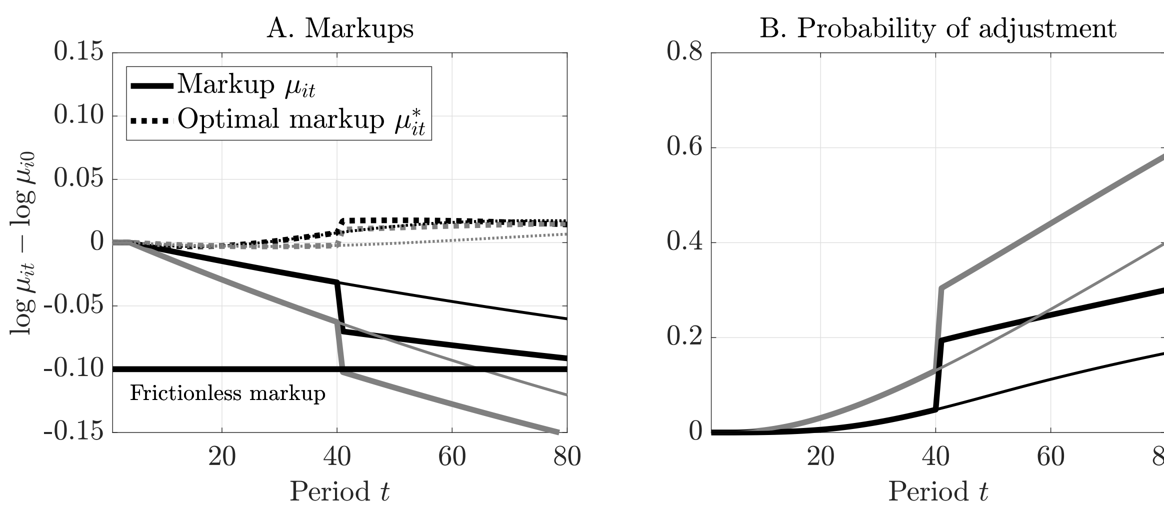


Figure E4: Positive monetary shock in duopoly model - Low markup firms

Notes: Thin solid lines give exogenous evolution of markups for two firms *within the same sector* absent a monetary shock. Thin dashed lines give corresponding optimal markups conditional on adjustment $\mu_1'(\mu_1, \mu_2)$ and $\mu_2'(\mu_1, \mu_2)$. Thick solid lines include a monetary shock in period 40, which decreases both firms' markups. Thick dashed lines (which lie on top of the thin dashed lines) give the corresponding optimal markups. The model solution is solved in steady state and the monetary shock is a one-time unforeseen level increase in money. The $y$-axis in panel A describes the log deviation of markups from the value chosen when shocks and menu costs are zero, $\bar{\mu} = 1.30$, which is equal to the average markup.

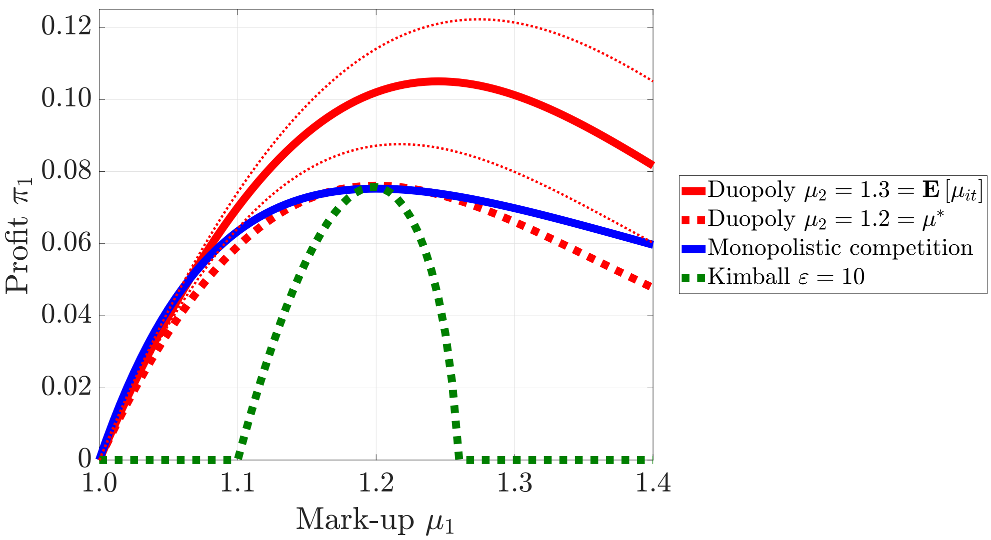


Figure E5: Market structure and the profit function of a firm

Notes: In all three models, the frictionless optimal markup is $\mu^* = 1.20$, with $\eta_d = 10.5$ (*Baseline*) and $\eta_m = 6$ (*Alt III*). The solid upper (red) line describes $\pi_1(\mu_1, \mathbb{E}[\mu_{it}])$, the flow profit of firm 1 when the markup of firm 2 is equal to the average markup in the duopoly model, which is 1.30. Its maximum $\mu_1^*(\mu_2) = \arg\max_{\mu_1} \pi_1(\mu_1, \mu_2)$ is obtained at 1.24. The thin dashed lines describe the same profit function when $\mu_2$ is one standard deviation above and below $\mathbb{E}[\mu_{it}]$. The thick dashed red line describes $\pi_1(\mu_1, \mu^*)$, the flow profit of firm 1 when the markup of firm 2 is equal to the frictionless optimal markup $\mu^*$, which is 1.20.

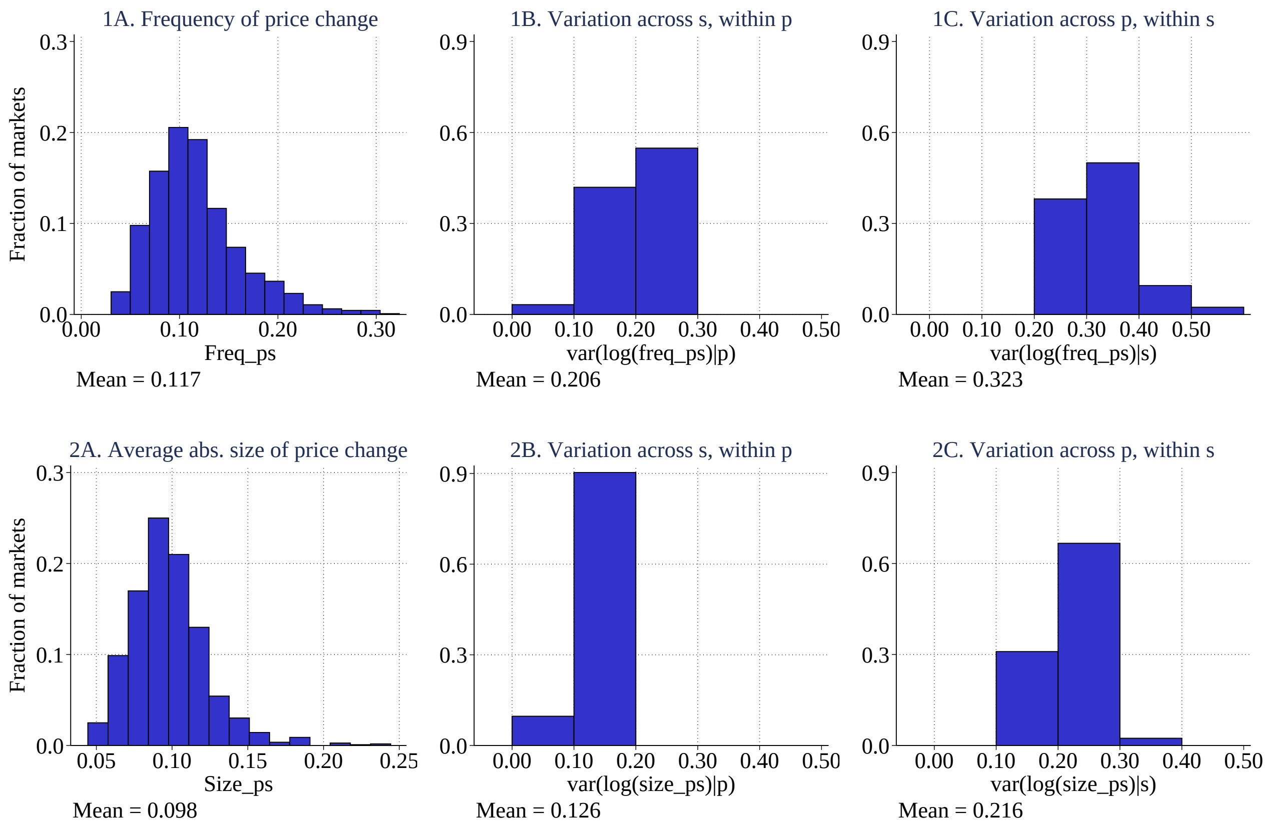


Figure E6: Empirical variation in (1) frequency and (2) absolute log size of price change

Notes: The first (second) row of figures refers to the average monthly frequency of price change (log absolute size of price change). Let $y_{pst}$ refer to a market $pst$ observation of this moment. In each row the histograms are as follows. Panel A: Histogram of the market average of $y_{pst}$: $y_{ps} = T^{-1}\sum_{t=1}^{T} y_{pst}$. Panel B: Histogram of the revenue-weighted across $s$, within $p$ coefficient of variation of $y_{ps}$: $CV_p = \sum_{s=1}^{S} w_{ps}(y_{ps} - \bar{y}_p)^2/\bar{y}_p$, where $\bar{y}_p = \sum_{s=1}^{S} w_{ps} y_{ps}$, and weights are $w_{ps} = r_{ps}/\sum_{s=1}^{S} r_{ps}$ and $r_{ps} = T^{-1}\sum_{t=1}^{T} r_{pst}$. Panel C: Histogram of the revenue-weighted across $p$, within $s$ coefficient of variation of $y_{ps}$: $CV_s = \sum_{p=1}^{P} w_{ps}(y_{ps} - \bar{y}_s)^2/\bar{y}_s$, where $\bar{y}_s = \sum_{p=1}^{P} w_{ps} y_{ps}$, and weights are $w_{ps} = r_{ps}/\sum_{p=1}^{P} r_{ps}$ and $r_{ps} = T^{-1}\sum_{t=1}^{T} r_{pst}$. In both cases, time variation is removed by first averaging so as to be comparable with Bils and Klenow (2004).

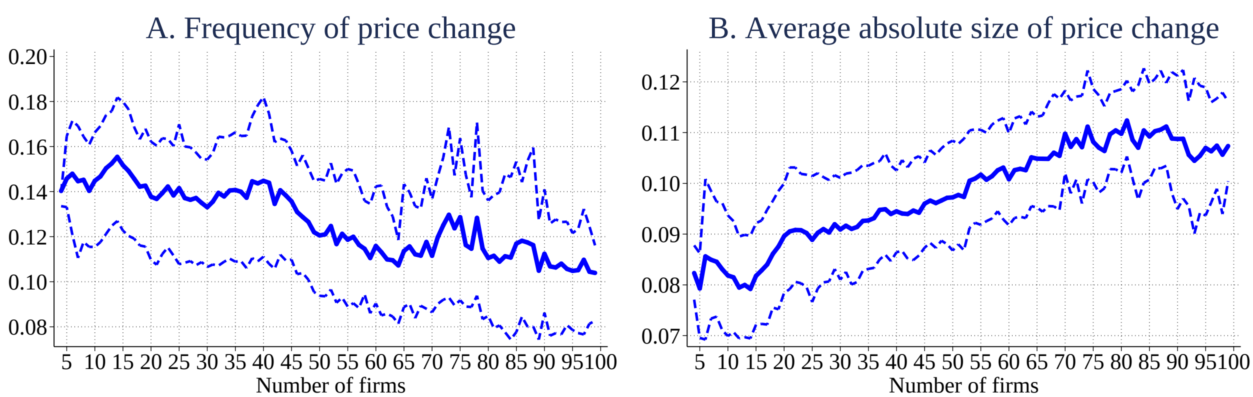


Figure E7: Number of firms and price flexibility

Notes: Solid (dashed) lines are medians (25th/75th percentiles) of fitted values from regression (13), where averages for both number of firms and the dependent variable are taken within bins of number of firms of width one.

## E.2 Tables

| | Across-product w/in state | | Across-state w/in product | |
|---|---|---|---|---|
| | Size (%) | Frequency | Size (%) | Frequency |
| Eff. number of firms | 0.454 | -1.002 | -0.360 | -0.749 |
| | (0.068) | (0.176) | (0.083) | (0.192) |
| Eff. number of firms$^2$ | -0.078 | 0.165 | -0.049 | 0.168 |
| | (0.016) | (0.044) | (0.019) | (0.078) |
| Observations | 133,340 | 133,340 | 133,340 | 133,340 |
| R-squared | 0.071 | 0.065 | 0.014 | 0.012 |
| $Rev_{pst}$ control | ✗ | ✗ | ✗ | ✗ |

Table E1: Regression results - No control for revenue

Notes: See notes for Table 4. This table provides results for the same regressions except where no additional controls are used.

| | Across-product w/in state | | Across-state w/in product | |
|---|---|---|---|---|
| | Size (%) | Frequency | Size (%) | Frequency |
| Number of firms | 0.051 | -0.089 | -0.001 | -0.011 |
| | (0.004) | (0.009) | (0.005) | (0.014) |
| Number of firms$^2$ | -0.001 | 0.001 | 0.000 | 0.000 |
| | (0.000) | (0.000) | (0.000) | (0.000) |
| Observations | 133,340 | 133,340 | 133,340 | 133,340 |
| R-squared | 0.107 | 0.083 | 0.010 | 0.001 |
| $Rev_{pst}$ control | ✓ | ✓ | ✓ | ✓ |

Table E2: Regression results - Alternative concentration measure - Number of firms

Notes: See notes for Table 4. This table provides results for the same regressions except where the number of firms in the market is used as the control variable.