Monetary Policy in the Presence
of a Stochastic Deficit

John Bryant
and
Neil Wallace

April 1979

Staff Report #: 42

Federal Reserve Bank of Minneapolis
and
University of Minnesota

This paper presents a welfare analysis of monetary policy rules that differ as regards the extent to which monetary policy accommodates an exogenous, stochastic deficit. Examples show that a nonaccommodating rule, one involving a higher ratio of bonds to currency the higher the deficit, is not necessarily better than rules that accommodate: either a rule involving a constant ratio of bonds to currency or one involving a lower ratio of bonds to currency the higher the deficit. Moreover, the nonaccommodating rule can imply more variation in the price level than the accommodating rules.

Table 1: Stationary Monetary Equilibrium for Four Monetary Policies

Endogenous Variable	Monetary Policy					
	1	2	3		4	
			i=1	i=2	i=1	i=2
	$\beta=0$	$\beta=.3$	$\beta=.3326$	$\beta=.2658$	$\beta=.2658$	$\beta=.3326$
$P(t+1,1)/P(t)$	.9091	.9091	.9091	.9063	.9091	.9119
$P(t+1,2)/P(t)$	1.1000	1.1000	1.1030	1.1000	1.0970	1.1000
$k(i)$	.0396	.0373	.0359	.0370	.0386	.0376
$z(i)$	.9586	1.0000	1.0000	1.0000	1.0000	1.0000
$S(i)$	1.0000	.9933	.9925	.9943	.9942	.9924
$s(i,1)$	.7688	.7574	.7541	.7624	.7608	.7525
$s(i,2)$	.3495	.3523	.3532	.3511	.3515	.3534
$d(P)$	.4594	.4540	.4543	.4557	.4538	.4524
$EU(i)$	6.1950	2.9325	2.8674	2.6288	3.0015	3.2438

In our earlier paper [1979b], we described a model that accounts for positive interest on default-free government bonds. That model allows us to analyze the consequences for economic welfare of alternative mixes of total government debt, mixes that differ with regard to how much of the debt is interest-bearing and how much is currency (high-powered or base money). The earlier paper considered only alternative fixed proportions of bonds to currency taking as given a fixed real deficit, indeed, a zero deficit. In this paper, we use the model to study more complicated fiscal and monetary policies. In particular, we study several monetary policy rules taking as given a stochastic deficit.

Roughly speaking, we study monetary policy rules that differ with regard to the degree to which monetary policy accommodates the financing of deficits and surpluses. Accommodation in this sense is one of the issues that divides monetarists from Keynesians. It is also the issue that separates the Milton Friedman of 1959 and thereafter, the monetarist Friedman, from the Friedman of 1948. In 1948 Friedman advocated that the budget be balanced on average and that all deficits be financed by printing currency and that all surpluses be financed by retiring or destroying currency. In contrast, the 1959 Friedman proposal and the present monetarist proposal is that there be no variation in currency to accommodate deficits and surpluses; deficits should be financed by borrowing and surpluses by debt retirement.

Despite the sharp disagreement about the Federal Reserve's role in accommodating deficits and surpluses, there is, so far as we know, not a single rigorous analysis of the consequences of alternative accommodation rules. Simulation studies using existing macroeconometric models do not qualify because the environments assumed to generate the behavioral relationships of these

models do not correspond to the environments implied by the models themselves.[1/]
One among many aspects of this inconsistency concerns expectation formation. Any sensible model of individual behavior would make current portfolio decisions depend on views about the future and, in particular, on views about future fiscal and monetary policy. No such dependence appears in existing macroeconometric models.

In a way, it is not surprising that there do not exist rigorous analyses of alternative monetary policy rules for accommodating stochastic deficits. In a dynamic equilibrium model, one in which the environment that confronts individuals is the same as that implied by the model, it is difficult to display the equilibrium for any but the simplest policy rules. In fact, one of our purposes is to describe a class of easily analyzable accommodation rules, rules that are stationary in the presence of stochastic deficits.

Generally speaking, the stationary rules are ones that specify how the ratio of bonds to currency varies with the state of the system. It turns out that in the face of stochastic deficits of the kind we analyze, the monetarist proposal is not in this class; it is not a stationary policy. This, by itself, is a serious indictment of it, since the proposal is often defended on the ground that it is simple. It turns out not to be at all simple.

Our earlier paper showed that once-for-all, open-market purchases of bonds need not be inflationary. Here we show, again through examples, that this result can carry over to various accommodation rules in the face of a stochastic deficit. In particular, the inflationary and deflationary effects of stochastic deficits are not offset by a monetary policy that leans toward monetarism by imposing a higher ratio of bonds to currency when there is a deficit than when there is a surplus.

_____

[1/]For a detailed statement of this criticism, see Lucas [1976].

More important, we show that such a monetarist-like rule is noncomparable in the Pareto sense to two other rules: one that holds the ratio of bonds to currency fixed and one that imposes a lower ratio of bonds to currency when there is a deficit than when there is a surplus.

Although the model we use is a special case of that presented in Bryant-Wallace [1979b], the exposition of the model in Sections I and II is self-contained. In Section III we describe the class of policy rules to be studied and in Section IV the class of stationary equilibria consistent with those rules. Section V contains the examples, while Section VI contains concluding remarks.

I.   The Physical Environment

Our model is a complicated version of Samuelson's [1958] pure consumption loans model. Time is discrete and there is a single good. At any date t, a new generation of N 2-period lived individuals (generation t) appears. Thus, at any date t the population consists of 2N people, N members of generation t-1 (the old or age two people), and N members of generation t (the young or age one people). Each member of generation t acts to maximize the expected value of $u(e_1(t), e_2(t))$ where $e_j(t)$ is age j consumption of the single good by a member of generation t. We assume that u is twice differentiable, strictly concave, that the arguments of u are normal goods, and that $u_1(e_1, e_2)/u_2(e_1, e_2)$ approaches infinity as e_1/e_2 approaches zero and approaches zero as e_1/e_2 approaches infinity.

At each date t there is a new aggregate endowment of Y units of the consumption good. This good may be consumed or stored. If K(t) is the amount placed into storage at t, there is a time t proportional storage cost, gK(t), where $0 \leq g \leq 1$. The aggregate payoff from the storage is time t+1 consumption good in the amount x(t+1)K(t), where x(t+1), which has the units time t+1 consumption good per unit of time t consumption good, is a random variable drawn

independently from period to period from a discrete probability distribution: $x(t+1) = x_i$ with probability $\pi_i > 0$ where x_i is an element of $\{x_1, x_2, \ldots, x_I\}$ and $0 < x_i \leq x_{i+1}$.[2/] The value of $x(t+1)$ is observed only after time t portfolios and consumption are determined. It is, however, known before generation $t+1$ appears so that intergeneration risk sharing via markets is precluded.

In addition to storage of the consumption good, there are two government-created assets in the model, currency or base money and one-period government bonds. Each government bond when issued is a default-free promise to one unit of currency at the next date. The total supply of such bonds at time t at face value in units of time $t+1$ currency is denoted $B(t)$. The total supply of currency at time t is denoted $H(t)$. We assume that there are no direct resource costs to the government of issuing either currency or bonds.

There are two storage technologies for $H(t)$. One is costless and individually risky. This risk is specified so as to maintain symmetry among all the young and so as to preserve the amount of currency in the system. Thus, let the N members of generation t be comprised of two groups: $N_1 = N_2 = N/2$. Subsequent to the determination of time t portfolios but prior to the appearance of generation $t+1$, a fair coin is tossed. If "heads," then each member of N_1 loses a fraction δ of his or her currency holding. The loss shows up as a lump-sum transfer to each member of N_2. If "tails," then vice versa. The crucial restriction is that bets cannot be placed on the outcome of the coin tossing. Such bets or contingent contracts would prevent the coin tossing from producing uncertainty.

The second storage technology for currency is physically safe but costly in terms of resources. If $P(t)$ is the value of a unit of currency at

---

[2/]Nothing of substance depends upon the discreteness of the $x(t+1)$ distribution.

time t in units of the consumption good, then physically safe storage of r units of currency from t to t+1 requires that $gP(t)r$ units of the consumption goods be surrendered at time t.

For government bonds, we assume a single, physically safe and costly storage technology. If $S(t)$ is the time t price of a bond in units of time t currency (so that $1/S(t)$ is one plus the nominal interest rate on bonds at time t), then physically safe storage of b bonds from t to t+1 requires that $gP(t)S(t)b$ units of the consumption good be surrendered at t.

These assumptions on storage technologies for currency and government bonds are simple ways of allowing for (a) the possibility that default-free government bonds will bear interest ($S(t)<1$) and (b) the possibility that individuals will diversify between, on the one hand, individually held currency via the risky storage technology and, on the other hand, safe assets stored by the costly technology.[3]

II. The Market Scheme

As an individual endowment scheme, we assume that each member of generation t comes into the world with Y/N units of time t consumption. In addition, the young person is subject to taxes. These taxes (which may be negative in which case there is a transfer) are of the lump-sum variety in the sense that the individual does not view them as depending on his or her saving or portfolio choices.

We find it convenient to describe the market scheme in terms of markets for contingent claims on second-period consumption where the contingency is the realization of the return on storage of the consumption good. If individuals were not identical, it would be necessary to proceed in this way.

---

[3] See our earlier papers, (1979a, 1979b), for a more detailed discussion of the rationale for such storage technologies.

As a matter of notation, let i denote the state at time t, the state that determines x(t), and i' denote the state at time t+1, the state that determines x(t+1).

Our first task is to describe competitive demands on the part of the young of generation t for currency stored via the individually risky storage technology and for claims on second-period consumption in state i' for i' = 1,2,...,I. We will then describe supplies of such contingent claims, all of which will be by price-taking firms using the costly storage technology.

Demand

Since the young of generation t make portfolio choices knowing x(t) or i, we adopt the following notation:

$e_1(t)$ — First-period consumption of a member of generation t.

$e_2(t,i',\delta)$ — Second-period consumption of a member of generation t in state (i', "loss") where "loss" means that this individual is in the half of the population to suffer a proportional loss on currency stored via the individually risky storage technology.

$e_2(t,i',0)$ — Second-period consumption of a member of generation t in state (i', "gain") where "gain" means that this individual is in the half of the population that does not suffer a loss and, indeed, that finds the currency that the other group lost.

Moreover, since the probabilities of "loss" and "gain" are equal and independent of the state i', we can write expected utility for any young person as

(1) $\qquad EU = .5 \sum_{i'} \pi_{i'} \{ u[e_1(t), e_2(t,i',\delta)] + u[e_1(t), e_2(t,i',0)] \}.$

The e_2's are defined in terms of portfolio choices and taxes by

(2) $\quad e_2(t,i',\delta) = q(t,i') + (1-\delta)P(t+1,i')c(t) - T_2(t,i')$

(3) $\quad e_2(t,i',0) = q(t,i') + P(t+1,i')c(t) + \delta P(t+1,i')c'(t) - T_2(t,i')$

where

$q(t,i')$ — Purchased claims on time t+1 consumption good in state i', (that is, in state (i', "loss") and in state (i', "gain")).

$c(t)$ — Currency held via the individually risky storage technology.

$c'(t)$ — Currency held by each member of the other group via the individually risky storage technology.

$P(t+1,i')$ — Value in terms of time t+1 consumption good of a unit of currency at time t+1.

$T_2(t,i')$ — The tax payable in state i' in the second period of life.

Upon substituting (2) and (3) into (1), we may state the individual's choice problem as one of maximizing EU by choice of $e_1(t) \geq 0$, $c(t) \geq 0$, and $q(t,i')$, i' = 1,2,...,I, subject to

(4) $\quad e_1(t) + P(t)c(t) + \sum_{i'} s(t,i')q(t,i') + T_1(t) - y \leq 0$

where

$P(t)$ — The price of currency at time t in units of time t consumption.

$s(t,i')$ — The time t price of a claim on second-period consumption in state i' in units of time t consumption.

$T_1(t)$ — The tax payable in the first period of life.

The individual treats all the prices and taxes and c'(t) parametrically.

For prices that imply a bounded and nonempty budget set, the necessary and sufficient conditions for a maximum are (4) at equality and

(5) $\sum_{i'} \pi_{i'} \left\{ u_1[e_1(t), e_2(t, i', \delta)] + u_1[e_1(t), e_2(t, i', 0)] \right\} - \lambda(t) = 0$

(6) $(1-\delta) \sum_{i'} \pi_{i'} P(t+1, i') \left\{ u_2[e_1(t), e_2(t, i', \delta)] + \right.$

   $\left. u_2[e_1(t), e_2(t, i', 0)] \right\} - \lambda(t)P(t) \leq 0$

(7) $\pi_{i'} \left\{ u_2[e_1(t), e_2(t, i', \delta)] + u_2[e_1(t), e_2(t, i', 0)] \right\} -$

   $\lambda(t)s(t, i') = 0; \quad i' = 1, 2, \ldots, I$

where $\lambda(t)$ is the nonnegative multiplier associated with (4) and where (6) holds with equality if c(t) is positive. Since all individuals are identical, we will work directly with (4)–(7) rather than with the implied demand functions.

Supplies

There are three possible lines of business in this economy, all of which involve supplying contingent claims on second-period consumption while taking prices as given.

One of the lines of business is storing the consumption good. Profits, revenues minus costs, in terms of time t consumption are given by

$$\sum_{i'} s(t, i') x_{i'} k(t) - (1+g)k(t)$$

where $k(t) \geq 0$ is the amount of the consumption good stored. Being linear in k(t), this implies that prices in any competitive equilibrium must satisfy

(8) $\sum_{i'} s(t, i') x_{i'} - (1+g) \leq 0$

and with equality if k(t) > 0.

Another line of business is storing currency safely using the costly storage technology. Profits from storing $r(t)$ units of currency are

$$\sum_{i'} s(t,i')P(t+1,i')r(t) - (1+g)P(t)r(t).$$

Being linear in $r(t)$, this implies that prices in any competitive equilibrium with $H(t) > 0$ must satisfy

$$(9) \qquad \sum_{i'} s(t,i')P(t+1,i') - (1+g)P(t) \leq 0$$

and with equality if $r(t) > 0$.

The third possible line of business involves storing government bonds. Profits from storing $b(t)$ bonds are

$$\sum_{i'} s(t,i')P(t+1,i')b(t) - (1+g)P(t)S(t)b(t).$$

Since this is the only way to store bonds, we have that prices in any competitive equilibrium with $B(t) > 0$ must satisfy

$$(10) \qquad \sum_{i'} s(t,i')P(t+1,i') - (1+g)P(t)S(t) = 0.$$

From (9) and (10) we have: (i) if $H(t) > 0$ and $B(t) > 0$, then $S(t) \leq 1$; and (ii) if $S(t) < 1$, then $r(t) = 0$.

The first of these propositions says that bonds never sell at more than par. To prove it, note that if $B(t) > 0$, then (10) holds with equality. But, then, if $S(t) > 1$, (9) is violated. The second statement, the proof of which we leave to the reader, says that if bonds bear interest (sell at a discount), then no currency is stored via the costly storage technology.

Equilibrium Conditions

For equilibrium we equate demands and supplies. Thus, if we let $R(t)$ be the total amount of currency stored via the costly storage technology, we must

have

(11) $Nc(t) = Nc'(t) = H(t) - R(t)$

(12) $Nq(t,i') = x_{i'}K(t) + P(t+1,i')[R(t)+B(t)]; \quad i' = 1,2,\ldots,I.$

The first equality of (11) is implied by the symmetry among all the young. Thus, roughly speaking, an equilibrium consists of prices and quantities that satisfy (4) at equality and (5)-(12). In order to go beyond this very rough statement, we must describe the modeling of $P(t+1,i')$, the time $t+1$ prices.

The model we have set up has the feature that its only dynamics come from the presence of $P(t+1,i')$ in the demand and supply functions. One way to ignore the dynamics is to assume some given subjective distribution of $P(t+1,i')$ for members of generation t. For example, one could make this distribution an exogenous function of $P(t-j)$ and/or of $x(t-j)$, $j = 0,1,\ldots,J$. This is the route of "temporary equilibrium" theory and of old-style macroeconomics. If one takes this route, then given H, B, and the taxes at t, one can solve for the endogenous variables at t: $P(t)$ and so on.

But to proceed in this fashion is unsatisfactory because matters are left very open. For each different subjective distribution of $P(t+1,i')$, one gets a different solution. This leads one to try to choose "reasonable" subjective distributions for $P(t+1,i')$.

Any given subjective distribution of $P(t+1,i')$ will imply a particular kind of dependence between $P(t)$, on the one hand, and H, B, and the taxes at t, on the other hand. But, then, it would seem that a reasonable subjective distribution for $P(t+1,i')$ would be consistent with some sort of dependence between $P(t+1,i')$ and H, B, and taxes at $t+1$. That, though, implies a dependence of $P(t)$ on H, B, and taxes at $t+1$. Nor do matters end there. This line of reasoning implies a dependence of $P(t)$ and the other endogenous variables at t on

the entire future paths of H, B, and taxes. This implication is what drives one to an analysis of rules that describe the future paths of H, B, and taxes.

Once we think in terms of dependencies between subjective distributions of future prices and policy rules, the obvious sort of dependence to impose is the true dependence; namely, that implied by the model. This, of course, is the same as requiring that the $P(t+1,i')$ that appears in the demand and supply functions of members of generation t be equal to the equilibrium price next period if state i' is realized. Before we can say any more about the properties of possible equilibria of this kind, we have to describe the policy rules.

III. Policy Rules

Fiscal policy in this model involves a specification of $T_1(t)$ and $T_2(t,i')$ for all t. The specification we adopt is one that gives rise to a stochastic deficit, meaning that the change in total nominal indebtedness, $H(t)+B(t) - [H(t-1)+B(t-1)]$, is random. Monetary policy involves a specification of the proportion of the total of nominal indebtedness that is in the form of bonds, a specification of the ratio of $B(t)$ to $H(t)$.

Fiscal Policy

We assume that

$$(13) \qquad T_1(t) = P(t)[1-S(t)]B(t)/N$$

$$(14) \qquad T_2(t,i') = P(t+1,i')[1-\alpha(i')][H(t)+B(t)]/N$$

where

$$\alpha(1) \geq \alpha(2) \geq \ldots \geq \alpha(I) > 0 \text{ and } \sum_{i'} \pi_i \ln\alpha(i') = 0.$$

The specification of $T_1(t)$ makes the young at t liable for the interest implied by the time t supply of bonds, $B(t)$.[4]

The specification of $T_2(t,i')$ allows for a stochastic nominal deficit. The assumed ordering of the α's makes the tax a monotone increasing function of the return on storage of the consumption good. While this is meant to mimic the idea of tax collections minus transfers being higher when things turn out well (large $x(t)$) than when they turn out poorly (small $x(t)$), it is important to note that the scheme is not an income tax; the individual perceives no connection between his or her portfolio choices and his or her tax.

We chose this specification because it is a stochastic version of the kind of fiscal policy long examined in the money and growth literature. It is no accident that it turns out to be easily analyzable. The condition on the expected value of $\ln\alpha(i')$ insures budget balance on average in a sense we will describe below.

The tax scheme implies the following cash outlay statement for a consolidated government-central bank at time t in state i:

Cash Outlays	Cash Receipts
$B(t-1)$	$[1-\alpha(i)][H(t-1)+B(t-1)]$
	$S(t)B(t)$
	$[1-S(t)]B(t)$

The item on the LHS is the outlay implied by the maturing bonds. The first item on the RHS is the state i tax on the old at t, the members of generation t-1. The

---

[4] The assumption that the tax liability for bond interest is payable when young tends to make the value of money a declining function of the ratio of bonds to currency. It does this for standard reasons; the tax reduces disposable income and, hence, saving. If the tax were, instead, payable when old, this effect would be weakened and, perhaps, reversed.

second item is the time t receipt from new bond sales, while the third item is the receipt from the taxes imposed on the young at t.

It follows that cash outlays minus cash receipts, which is the same as the change in the supply of currency, satisfies

$$H(t) - H(t-1) = -[1-\alpha(i)]H(t-1) + \alpha(i)B(t-1) - B(t)$$

which may be rewritten

$$(15) \qquad H(t) + B(t) = \alpha(i)[H(t-1)+B(t-1)].$$

Since the expected value of $\ln(\alpha(i))$ is zero, (15) implies average budget balance in the sense that the expected value of $\ln[H(t)+B(t)]$ is equal to the expected value of $\ln[H(t-1)+B(t-1)]$.

Monetary Policy

We study monetary policies in the following class

$$(16) \qquad B(t) = \beta(i)\alpha(i)[H(t-1)+B(t-1)]$$

where $0 \le \beta(i) < 1$ for all i. It follows, by (15), that

$$\beta(i) = B(t)/[H(t)+B(t)].$$

Thus, we are making the composition of total indebtedness in the sense of the ratio of bonds to money dependent at most on the deficit at t or, more directly, at most on x(t) or i. In the examples, we study three different kinds of monetary policies: $\beta(i) = \beta$ for all i; (ii) $\beta(i)$ monotone increasing in $\alpha(i)$; and (iii) $\beta(i)$ monotone decreasing in $\alpha(i)$. The second of these leans toward monetarism by making the ratio of bonds to currency greater, the greater the deficit.

As we will indicate in a moment, these rules are consistent with a monetary equilibrium (one in which currency has value) that is stationary in the sense that all real variables at t depend only on i or, equivalently, on x(t). For the fiscal policy we have assumed, (16) is the only class of monetary policies consistent with such stationarity.

While we cannot stay within this class of rules and impose monetarism, meaning a fixed value of H(t), we can, as noted, lean toward monetarism by a rule of the form (ii). Indeed, for this rule, we choose the dependence of $\beta(i)$ on $\alpha(i)$ so as to minimize the variance of $\ln[H(t+1)/H(t)]$ subject to keeping $\sum_i \pi_i \ln(1-\beta(i)) = \ln(1-\beta)$, a constant.

It is worth noting what monetarism in the sense of a fixed H(t) would imply given our fiscal policy. By (15) and the constraint on the α's, $\ln(H(t)+B(t))$ follows a random walk with the property that the expected value of $\ln(H(t+j)+B(t+j))$ conditional on x(t) is equal to $\ln(H(t)+B(t))$ for all positive j. It follows that holding H(t) fixed makes $\ln[H(t)/(H(t)+B(t))]$ obey a random walk; the ratio of bonds to money can take on an infinite number of different values, the particular value taken on at any date being dependent on the entire history of the x(t) series. Since our model is one in which real variables depend on the ratio of bonds to money, it follows that holding H(t) constant is inconsistent with _any_ kind of stationarity. In other words, a constant H(t) is inconsistent with any equilibrium in which all real variables at t depend only on a finite history of the x's.

Nor is our model the only one in which the ratio of bonds to money has real effects. We cannot imagine a sensible model in which this would fail to be the case. Recall that even in so-called macroeconomic models with money and government bonds, neutrality holds only for proportional changes in both and not for different values of the ratio of bonds to money (see Patinkin (1961)). Thus,

a constant H(t) seems to make sense only if fiscal policy is specified in such a way as to keep the sum H(t)+B(t) stationary. More generally, advocacy of a given monetary rule makes sense only in conjunction with advocacy of a particular fiscal policy. For example, advocacy of money growth at the rate ρ requires that $H(t) + B(t) = D(t)e^{\rho t}$ where D(t) is a stationary process.

IV. Stationary Monetary Equilibrium

Under the policy rules described above, it makes sense to look for a monetary equilibrium in which all real variables at t depend at most on x(t) or i. To be precise, our surmise is that there are equilibria that for all t satisfy: $P(t)[H(t)+B(t)]/N = d(i)$, $\lambda(t) = \lambda(i)$, $K(t)/N = k(i)$, $Nc(t)/H(t) = z(i)$, $S(t) = S(i)$, $s(t,i') = s(i,i')$, $q(t,i') = q(i,i')$. Our procedure is to impose this guess and then to show for several numerical examples that there are indeed monetary equilibria of this kind.

Our guess is motivated by the following considerations. Suppose $x(t) = x(t+j) = x_i$. Then, initial conditions at t and t+j differ in only two respects. First, K(t-1) may differ from K(t+j-1). While this may imply different consumption for the current old at each of these dates, it affects neither asset demands at the different dates nor asset supplies. Second, the sum, H + B, may be different at the two dates. Our guess is that the only effect of this difference is on the value of money. This is plausible since $x(t) = x(t+j)$ implies $H(t+j)/H(t) = B(t+j)/B(t)$.

To proceed, we first note what is implied by our guess for several of the endogenous variables. Thus,

$$(17) \qquad P(t+1,i')/P(t) = \{d(i')N/[H(t+1)+B(t+1)]\}/\{d(i)N/[H(t)+B(t)]\} =$$

$$d(i')/\alpha(i')d(i).$$

This, in turn, implies that

(18) $T_2(t,i') = [1-\alpha(i')]d(i')/\alpha(i')$

(19) $T_1(t) = [1-S(i)]\beta(i)d(i)$.

Thus, in an equilibrium of this kind with $c(t) = c'(t) = z(i)H(t)/N$, (2) and (3) become, respectively,

(20) $e_2(t,i',\delta) = q(i,i') + (1-\delta)z(i)[1-\beta(i)]d(i')/\alpha(i') -$

$[1-\alpha(i')]d(i')/\alpha(i')$

(21) $e_2(t,i',0) = q(i,i') + (1+\delta)z(i)[1-\beta(i')]d(i')/\alpha(i') -$

$[1-\alpha(i')]d(i')/\alpha(i')$

while (4) at equality may be written as

(22) $e_1(t) = y - [1-S(i)]\beta(i)d(i) - z(i)[1-\beta(i)]d(i) - \sum_{i'}s(i,i')q(i,i')$.

Using (17)-(22), we can write (5)-(10) entirely in terms of $d(i)$, $\lambda(i)$, $z(i)$, $k(i)$, $S(i)$, $s(i,i')$, and $q(i,i')$. Moreover, upon substituting the second equality of (11) into (12), we get

$Nq(t,i') = x_i,K(t) + [H(t)+B(t)-Nc(t)]P(t)P(t+1,i')/P(t)$.

In terms of our surmise, this becomes

(23) $q(i,i') = x_i,k(i) + \{1+z(i)[1-\beta(i)]\}d(i')/\alpha(i')$; $i' = 1,2,\ldots,I$.

We are not going to attempt any general existence proof. First of all, monetary equilibria, equilibria with $d(i) > 0$, do not always exist. For example, it is easy enough to specify x_1, x_2, $\ldots$, x_I in such a way as to rule out such equilibria. Nonmonetary equilibria, those with $d(i) = 0$ for all i, do, in

general, exist. But since we are here not interested in nonmonetary equilibria, we will not attempt a general existence proof for this kind of equilibrium either. However, before turning to numerical examples, we can, at least, check that we have as many unknowns as we have equilibrium conditions. We have $5I+2I^2$ unknowns. Equations (7) and (23) are $2I^2$ equations, while (5), (6), (8), (9), (10) are $5I$ other equations.

V. Examples

This section describes the stationary monetary equilibrium for each of four monetary policies. Taken as given are a particular stochastic fiscal policy and a particular specification of the physical environment.

The economy is described by $u(e_1,e_2) = 60 - e_1^{-6} - e_2^{-6}$, $g = 1/12$, $(x_1,x_2) = (0.5,2.0)$ with $\pi_1 = \pi_2 = 0.5$, $\delta = 0.1$, and $Y/N = 1.0$. This is the same specification we used in our earlier paper [1979b]. It was chosen because it was the first specification we tried and because it is consistent with diversified portfolios under policy rules that imply neither too much inflation nor deflation.

The fiscal policy is $(\alpha(1),\alpha(2)) = (1.1,1/1.1)$; the old are subsidized by way of a deficit when they experience the bad outcome on storage $(x_1=0.5)$ and are taxed by way of a surplus when they experience the good outcome on storage $(x_2=2.0)$.

Each monetary policy is described by a vector $(\beta(1),\beta(2))$.

1. $\beta(1) = \beta(2) = 0$. There are no bonds outstanding; deficits and surpluses are financed by currency creation and destruction. Together with our fiscal policy, this is a version of the 1948 Friedman proposal.

2. $\beta(1) = \beta(2) = .3$. The ratio of bonds to currency is held constant (at 3/7); deficits and surpluses are financed by proportional increases and decreases in both bonds and currency.

3. $\beta(1) = .3326, \beta(2) = .2658$. These are the values (rounded) implied by choosing $\beta(i) \geq 0$ to minimize the variance of $\ln[H(t+1)/H(t)]$ subject to $\sum_i \pi_i \ln(1-\beta(i)) = \ln(1-\beta) = \ln(1-.3)$, our monetarist-like policy. If the constraint $\beta(i) \geq 0$ is not binding, the general (many state) solution is $\ln(1-\beta(i)) = \ln(1-\beta) - \ln(1-\alpha(i))/2$ which implies $H(t+1) = H(t)(\alpha(i)\alpha(i'))^{1/2}$. This implies that H and B increase and decrease proportionally whenever $i = i'$ and, in our two-state example, that $H(t+1) = H(t)$ otherwise. This policy is nonaccommodating in that when a deficit follows a surplus, the entire deficit is financed by debt creation, while when a surplus follows a deficit, the entire surplus is financed by debt retirement.

4. $\beta(1) = .2658$, $\beta(2) = .3326$. This policy is chosen because it is the reverse of policy 3. (It does not maximize the variance of $\ln[H(t+1)/H(t)]$.)

The stationary monetary equilibrium values of the endogenous variables are displayed in Table 1.

Before describing how these policies rank according to the Pareto criterion, we want to comment briefly on the behavior of the interest rate $(1/S(i)-1)$, investment $(k(i))$, and the rate-of-change of the value of money $(P(t+1,i')/P(t)=d(i')/\alpha(i')d(i))$ under policies 2, 3, and 4.

There is nothing surprising about the behavior of the interest rate. Under policy 3 it is higher when there is a deficit than when there is a surplus, while the reverse happens under policy 4. Nor is there anything surprising about the behavior of investment. Under each of policies 3 and 4, investment is high when the interest is low.

As regards the rate-of-change of the value of money, note first that the value of money is always changing from one period to the next. Under policy 2 (and policy 1), in each period there is either a 10 percent inflation (state 1) or a 10 percent deflation (state 2). Under policies 3 and 4, there are four

possible inflation rates, all of which are equally probable. The price level fluctuations are greatest under policy 3, the monetarist-like policy, and are least under policy 4, the strongly accommodating policy. The monetarist-like policy exacerbates price level fluctuations relative to what would occur under a policy of holding the ratio of bonds to currency fixed. This result is consistent with our earlier results [1979b] for the price level effects of alternative constant magnitudes of bonds and currency.

As regards welfare comparisons, one policy is better than another—Pareto superior to another—if adoption of the former at some date t makes everyone better off (in a weak sense) than would adoption of the latter at that same date. Everyone includes the current old (the members of generation t-1), the current young, and the members of all future generations. The well-being of the current and future young is given by the values of expected utility (EU(i)). The well-being of each of the current old is affected by the choice of policy only by way of its effect on the value of money or equivalently, d(i). The higher is d(i), the better off are members of generation t-1.

We will comment on two aspects of the examples: (i) policy 1 is Pareto superior to each of the other three policies; (ii) among policies 2, 3, and 4, any pair are noncomparable.

The first result, the Pareto superiority of policy 1, is implied by higher values of both expected utility and of d(i) uniformly over states for policy 1 than for the other policies. It reflects nothing more than the inefficiency of interest-bearing government bonds under our assumptions. As explained in our earlier paper [1979b], the presence of interest-bearing bonds amounts to a subsidy on the storage of nominal assets via the physically safe but costly storage technology, a subsidy financed by lump-sum taxes on the young, $T_1(t)$.

The second result, noncomparability among policies 2, 3, and 4, and the form that the noncomparability takes is not so easily "explained." Since the average amount of bonds is the same under all three policies, noncomparability should not be a surprise. But why it turns out that the current old rank policy 3 first, policy 2 next, and policy 4 worst while the young rank them in the reverse order is not obvious. We are somewhat surprised by the uniformity across states. In any case, given that noncomparability shows up in a simple model with identical individuals, there are grounds for thinking that it is a general feature for comparisons among policies like 2, 3, and 4.

VI. Concluding Remarks

Advocates of monetarism make two claims about a monetary policy that gives rise to a fixed and low rate of growth of some monetary aggregate like M_1: (i) it will prevent the occurence of major deflations and inflations; and (ii) it will prevent monetary policy itself from being a source of disturbances. We accept neither claim.

First of all, and most serious from our point of view, is the notion implicit in the first claim--and widely accepted not only by monetarists--that monetary policy (open market operations) stands on a par with fiscal policy as a macroeconomic tool. Put differently, it is widely believed that expansionary monetary policy can, at least in part, offset the effects of contractionary fiscal policy--for example, large surpluses--and that contractionary monetary policy can, at least in part, offset the expansionary effects of expansionary fiscal policy--for example, large deficits. This seems doubtful. Government deficits and surpluses involve changes in net government indebtedness. These may or may not have important effects. They do if the model has an equilibrium with voluntarily-held valued fiat money. Monetary policy, in contrast, involves only asset exchanges. These matter only if there are transaction costs and a private

sector-government asymmetry with regard to those costs. How monetary policy matters depends on those transaction costs and on the taxes levied to cover cost asymmetries.

As for the second claim, whether a policy is itself a source of disturbances cannot be determined outside the context of a model. And whether a particular monetary policy is disturbing depends upon what fiscal policy is in effect. Given the kind of fiscal policy we have assumed, monetarism _is_ a source of disturbances since under a monetarist regime, a stationary monetary equilibrium does not exist. Indeed it would seem that the second claim holds not for monetarism, but for a policy that holds constant the ratio of bonds to currency.

References

Baumol, W. J., 1952, "The Transactions Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics 66, 545-556.

Bryant, J., and N. Wallace, 1979a, "The Inefficiency of Interest-Bearing National Debt," Journal of Political Economy (forthcoming).

_____, 1979b, "Open Market Operations in a Model of Regulated, Insured Intermediaries," Journal of Political Economy (forthcoming).

Friedman, M., 1960, A Program for Monetary Stability (Fordham University Press, New York).

_____, 1948, "A Monetary and Fiscal Framework for Economic Stability," American Economic Review 38, 245-264.

Lucas, R. E., Jr., 1976, "Econometric Policy Evaluation: A Critique," in: K. Brunner and A. Meltzer, eds., The Phillips Curve and Labor Markets (North Holland, Amsterdam), 19-46.

Miller, M., and D. Orr, 1966, "A Model of the Demand for Money by Firms," Quarterly Journal of Economics 80, 413-435.

Patinkin, D., 1961, "Financial Intermediaries and the Logical Structure of Monetary Theory," American Economic Review 51, 95-116.

Samuelson, P. A., 1958, "An Exact Consumption-Loan Model of Interest With or Without the Social Contrivance of Money," Journal of Political Economy 66, 467-482.

Tobin, J., 1956, "The Interest Elasticity of Transactions Demand for Cash," Review of Economics and Statistics 38, 241-247.