Rational Expectations, Econometric
Exogeneity and Consumption

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Introduction

This paper computes estimates of the parameters of Friedman's [1956]
permanent income model of the consumption function. The estimates are
obtained from aggregate time series data for the U.S. by imposing the
restrictions across the consumption and income processes that are implied
by the hypothesis of rational expectations in the context of the permanent
income theory of consumption. That the hypothesis of rational expectations
is an important element of Friedman's application of his theory, especially
to the cross-section data, has recently been emphasized by Lucas [1976].

To implement the rational expectations theory in the present
context requires characterizing the stochastic structure of the consumption,
income process from the point of view of extracting optimal forecasts of
future income from records of past income and consumption. For us the
important aspects of this characterization happen to coincide with the
Wiener-Granger notion of "causality." According to Wiener and Granger's [1969]
definition, a process C is said to cause a process Y if given past values
                                                                    [1972]
of Y, past values of C help to predict future Y. A theorem of Sims/informs
us that if C fails to Granger-cause Y, then there exists a model consistent
with the data in which Y is strictly econometrically exogenous with respect
to C.   This is of interest here because, as is well known, in standard
Keynesian stochastic macroeconomic models, income Y is predicted not to
be exogenous with respect to consumption C. Despite this fact, this paper
devotes some space to describing an estimator that is appropriate
in the case in which Y is strictly econometrically exogenous with respect
to C. By way of indicating that the hypothesis of exogeneity of Y with

respect to C is not uniformly rejected by macroeconomic theory, section 1 describes a version of Tobin's Dynamic Aggregative Model [1955] which predicts that Y is exogenous with respect to C. In Tobin's model, movements in government purchases leave GNP unaltered in the short run, while movements in the money supply cause sympathetic movements in real GNP, in sharp contrast to the Keynesian model. The differing implications of Tobin's model and the Keynesian model for the Granger-causal structure of income and consumption indicate that the econometric exogeneity tests that are reported in section 6 are of independent macroeconomic interest, over and above their implications about appropriate procedures for estimating the consumption function under rational expectations. Section 2 digresses somewhat from the main theme of this paper by indicating how in Tobin's model there may emerge a spurious investment accelerator, this despite the fact that in Tobin's model firms have no investment demand schedules.

Sections 3 and 4 describe estimation procedures for the cases in which income is and is not exogenous, respectively, while section 5 takes a stand on the question of whether seasonally adjusted or unadjusted data are the appropriate ones to use. Exogeneity tests are reported in section 6 while the rational expectations restrictions are imposed and tested in section 7.

This paper stays with Friedman's [1956] original very simple specification of the consumption function, supplemented only by the imposition of Muth's hypothesis [1961] that expectations of future income are "rational." Proceeding in this way is subject to the valid criticism that it ignores the insights into the form of optimal consumption rules

provided by Merton [1970]. Further, from the viewpoint of equilibrium theories of the business cycle, which envision agents as confronting not incomes but rather sequences of wages and prices at which they can supply labor and consume (as in Lucas and Rapping [1968]), the consumption schedule estimated in this paper is misspecified.[1/] Both of these criticisms ought to be taken seriously.

Let me warn the reader in advance that the econometric findings are not encouraging from the viewpoint of the permanent income model of consumption. But finding confirmation of that theory ought not to be the only reason for reading this paper. The paper describes a useful technology for imposing rational expectations on time series models, a technology which is applicable to a number of dynamic econometric models of rational agents. The paper illustrates the feasibility of that technology, and also the strongly overidentifying nature of the restrictions embodied in rational expectations models.

The reader not interested in the macroeconomic issues associated with the question of the econometric exogeneity of consumption can without loss advance to Section 3 and find a discussion of the estimation technology.

1. Econometric Exogeneity of Income

in the Consumption Function

Most macroeconomists would find unacceptable the specification that income is strictly econometrically exogenous in the consumption function. Indeed, ever since Haavelmo's famous paper [1943], the failure of the consumption function to be a regression equation in the context of a Keynesian model has been a classic example of simultaneous equations bias. It perhaps bears pointing out, however, that there exist interesting macroeconomic models in which Y is econometrically exogenous in the consumption function. One such model is illustrated here, though it is certainly not the only possible example of such a model.[2]

Consider the following stochastic version of Tobin's "Dynamic Aggregative Model" [1955]:

(a) $\log Y_t = \sigma \log N_t + \varepsilon_{1t}$, $\quad \sigma \approx 1$ $\qquad$ (production function)

(b) $\log w_t - \log p_t = g \cdot (\log N_t - \log K_t) + \varepsilon_{2t}$, $\quad g<0$, (aggregate demand schedule for employment)

(c) $r_t = h(\log N_t - \log K_t) + \varepsilon_{3t}$, $\quad h>0$, (marginal productivity for capital)

(d) $\log M_t - \log p_t - \log Y_t = br_t + \varepsilon_{4t}$, $\quad b<0$, (portfolio balance schedule)

(e) $\log w_t = \sum_{i=1}^{n} v_i \log w_{t-i} + \varepsilon_{5t}$ (exogenous money wage process)

(f) $C_t = BY_{pt} + u_t$, $\quad B>0$, (consumption function)

(g) $Y_{pt} = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j E_t Y_{t+j}$, $\quad 0<\alpha<1$ (definition of permanent income)

(h) $C_t + I_t + G_t = Y_t$ (GNP identity)

(i) $K_{t+1} - K_t = I_t$

Here Y is GNP, N is employment, r_t the interest rate, G_t government

purchases, p_t the price level, w_t the money wage, M_t the money supply,

and I_t investment; ε_{1t}, ε_{2t}, ε_{3t}, ε_{4t}, ε_{5t}, and u_t are stationary

random processes with finite means and variances. I assume that the u

process is statistically independent of the ε_1, ε_2, ε_3, ε_4, and ε_5

processes, though these latter five processes can be statistically

dependent. The notation $E_t Y_{t+j}$ denotes the linear least squares fore-

cast of Y_{t+j} based on information available at time t.

Tobin's model is of substantial interest to macroeconomists

partly because it is a concrete example of a model in which "pure"

fiscal policy (e.g., a change in government expenditures or a lump sum

transfer) does not affect output in the short run, while monetary policy

does. The present version of the model is nine equations in the nine

endogenous stochastic processes Y_t, N_t, r_t, p_t, w_t, C_t, Y_{pt}, I_t, and K_t

with exogenous driving processes M_t, G_t, u_t, and the ε_{jt}'s. As it

happens, equations (a), (b), (c), (d), and (e) form a recursive block

that each period determines Y_t, r_t, p_t, w_t, and N_t given values of the

exogenous M_t and the predetermined K_t (predetermined in a fashion to be

explained shortly). Given Y_t so determined, equations (f) and (g)

determine C_t, while (h) determines I_t, which through equation (j)

determines K_{t+1}.

To indicate in more detail how (a)-(e) determine Y, r, p, w,

and N, substitute (a) and (b) into (d) to get the modified "LM" curve

(LM) $br_t = \log M_t - \log w_t + (g-\sigma) \log N_t - g \log K_t + \varepsilon_{2t} - \varepsilon_{4t}$,

which, given M_t, w_t, ε_{2t}, and ε_{4t}, depicts the (r_t, N_t) locus that assures

portfolio balance, assuming that (a) and (b) are satisfied. Equation (c)

is a second curve in the (r,N) plane, one along which the demand for the existing stock of capital equals the supply. Both (c) and the (LM) curve are upward sloping curves in the (r,N) plane. Stability[3] requires that the LM curve be the steeper, which is the condition that $\frac{g-\sigma}{b} > h$. Substituting (LM) into (c) and solving for log N_t gives the reduced form

$$b(h - \frac{g-\sigma}{b})\log N_t = \log M_t - \log w_t + (bh-g)\log K_t$$

$$+ (\varepsilon_{2t} - \varepsilon_{4t} - b\varepsilon_{3t}).$$

The assumption of stability guarantees that the coefficient on log N_t is positive, so that employment varies directly with M_t, inversely with w_t.[4] Because the sign of $(bh-g)$ is ambiguous, so is the effect of capital on employment and also, given (a), on output. Increases in log K_t have two contrary effects on employment. On the one hand, an increase in K_t, given N_t, calls for a higher real wage via (b) and, given the w_t fixed by (e), a lower p_t. This lowers the portfolio-balancing interest rate and is expansionary (see equation LM). On the other hand, an increase in K_t lowers the marginal product of capital given N_t, and therefore lowers the capital-market clearing interest rate, which is contractionary (see equation c). The two effects are opposing.[5]

In this model, Y_{pt} will be strictly econometrically exogenous in the consumption schedule under two conditions.[6] First, the u_t process must, as I have assumed, be statistically independent of the ε processes. Second, we must have the condition $(bh-g) = 0$, so that capital does not appear in the reduced form for employment, the two opposing effects of the preceding paragraph just offsetting one another. With these two conditions met, (a)-(j) is an example of a model in which Y_{pt} is strictly econometrically exogenous in the consumption function.

It is worth remarking that were we to add the side condition
that u_t be serially independent, then even without our second condition,
the consumption function (f) _is_ a regression equation; that is, it
satisfies the population least squares orthogonality condition $Eu_t Y_{ps} = 0$
for all $t \geq s$. The force of our second condition is to guarantee $Eu_t Y_{ps} = 0$
for $s > t$. Under the side condition / that u be serially uncorrelated, estimation of the consumption
function (f) by variants of least squares methods (methods modified to
extract proxies for the unobservable Y_{pt}) would yield consistent estimates
even without our second condition.

Obviously, _a priori_ setting down of models like the above one
(or like the standard Keynesian model) cannot settle the issue of whether
Y_{pt} is properly regarded as econometrically exogenous in the consumption
function. All that the model in this section is intended to suggest is
that testing for econometric exogeneity in the consumption-income process
is an interesting enterprise from the perspective of alternative macro-
economic models.

2. A Spurious Accelerator in Tobin's Dynamic Aggregative Model

The preceding model is one in which a spurious investment accelerator emerges in the data, despite the fact that the model is one in which firms have no investment demand schedules. To illustrate things simply, I assume the special version of the model in the preceding section,

(k) $\qquad C_t = \dfrac{B}{1-\lambda L} Y_t + u_t \qquad , \qquad 0 < \lambda < 1$

(m) $\qquad Y_t = C_t + I_t$

where $Eu_t \cdot Y_s = 0$ for all t and s, so that Y is strictly econometrically exogenous in (k) Assume that Y and u are covariance stationary stochastic processes. Now solve (k) and (m) for I_t to get

(n) $\qquad I_t = \dfrac{(1-B)-\lambda L}{1-\lambda L} Y_t - u_t$

Equation (n) expresses I as a one-sided distributed lag of Y with a disturbance $-u_t$ that is orthogonal to the entire Y process. That is, Y is strictly econometrically exogenous in (k). If $(1-B) \doteq \lambda$, then (n) approximately assumes the form of the distributed lag accelerator. To see this more generally, the model (m) implies that the spectrum of Y is related to the cross-spectra of Y with C and I, respectively, by

$$S_Y(w) = S_{CY}(w) + S_{IY}(w) \quad .$$

Dividing by the spectrum of Y gives

(o) $\qquad \dfrac{S_{IY}(w)}{S_Y(w)} = 1 - \dfrac{S_{CY}(w)}{S_Y(w)}$

If the gain $| S_{CY}(w) / S_Y(w) |$ of consumption on income has the characteristic permanent income consumption function form with gain generally falling with increases in angular frequency w, it follows from (o) that the gain of investment on income will display the characteristic (distributed lag) accelerator form with the gain of investment on income generally rising with increases in angular frequency. Indeed, as (o) shows, the gain of investment on income is in a sense the reflection of the gain of consumption on income.

It would obviously be a mistake to interpret (n) as a structural investment demand schedule reflecting firms' behavior. Rather, it is a saving schedule reflecting choices of consumers. The present illustration is in effect an extreme example of the fact that least squares estimates of investment schedules of the form (n) in the context of most macroeconomic models typically represent a confounding of firms' investment demand with households' saving behavior.

Of course, it is possible to set up examples of macroeconomic models in which the dual of the above error occurs; that is, models in which there is a true structural investment demand schedule, and in which the projection of consumption on income looks like a consumption function despite its truly being a mere reflection of the investment schedule..

3. Estimation of the Consumption Function under

Rational Expectations with Income Exogenous

Suppose the consumption function is

(1) $C_t = BY_{pt} + u_t$

(2) $Y_{pt} = (1-\alpha)[Y_t + \alpha E_t Y_{t+1} + \alpha^2 E_t Y_{t+2} + \ldots]$

$= (1-\alpha) \sum_{j=0}^{\infty} \alpha^j E_t Y_{t+j}$ $0 < \alpha < 1$

where α is a discount factor, $E_t Y_{t+j}$ is the linear least squares forecast

of Y_{t+j} conditional on information assumed to be known at time t, and u_t is

a stationary random process that obeys $Eu_t Y_s = 0$ for all t and s, which is

equivalent with the condition that Y_s is strictly econometrically exogenous

in (1). Subsequently, I will assume that information at time t consists of

$C_t, C_{t-1}, \ldots, Y_t, Y_{t-1}, \ldots$, so that $E_t x \equiv Ex | C_t, C_{t-1}, \ldots, Y_t, Y_{t-1}, \ldots$ where E

is the linear least squares projection operator.[7] The assumption that Y_t

is econometrically exogenous in (1) implies that,[8] given lagged values of Y_t,

lagged values of C do not help predict Y_t. Therefore, so far as concerns

the covariance structure of the (C,Y) process, it involves no further

restrictions to assume that $E_t Y_{t+j}$ is simply the regression (projection)

of Y_{t+j} on current and past Y_t's, since even if current and lagged C's

were included in the conditioning set, they would bear zero coefficients.[9]

By the linearity of the projection operator E_t, (2) can be

rearranged to read[10]

(3) $Y_{pt} = (1-\alpha) E_t \sum_{j=0}^{\infty} \alpha^j Y_{t+j}$.

Suppose that

Y_t is a linearly regular stationary stochastic process with covariance
generating function

(4) $\qquad R_y(z) = \sigma^2 b(z)b(z^{-1})$

where $b(z) = \sum_{j=0}^{\infty} b_j z^j$, $\sigma^2 > 0$. Any linearly regular (strictly linearly indeter-

ministic) stochastic process has a covariance-generating function that can

be represented in the form (4). By a famous formula of Wiener and

Kolmogorov,[11] the z-transform of the coefficients in the projection of

Y_{t+j} against Y_t, Y_{t-1}, ... is given by

$$\frac{1}{b(z)} \, [z^{-j} b(z)]_+$$

where $[\ ]_+$ means ignore all negative powers of z, i.e., $[\sum_{j=-\infty}^{\infty} h_j z^j]_+ \equiv \sum_{j=0}^{\infty} h_j z^j$.
Then using (3), Y_{pt} can be written

(5) $\qquad Y_{pt} = h(L)Y_t$

(6) $\qquad h(z) = \frac{(1-\alpha)}{b(z)} \, [\frac{1}{(1-\alpha z^{-1})} \, b(z)]_+$

In practice, $b(z)$ could be assumed to be a ratio of finite order polynomials

in z so that Y would be a moving average, autoregressive process. Under

such assumptions, by using the method of partial fractions, convenient

closed form expressions for $\frac{1-\alpha}{b(z)}$ and $[\frac{1}{(1-\alpha z^{-1})} \, b(z)]_+$ in terms of the

roots of $(b(z)=0)$ can be obtained. In this way, for fixed α and given

$b(z)$, a closed form expression for $h(z)$ can be obtained.

Rather than pursue those calculations here, I propose to exhibit

the following more compact calculations. I assume temporarily that $\{Y_t\}$

is a covariance stationary stochastic process, though the calculations below permit limited departures from stationarity. From Wold's decomposition theorem,[12] it follows that Y_t can be represented uniquely as the sum of two orthogonal stochastic processes

$$Y_t = Y_t^i + d_t$$

where $EY_t^i d_s = 0$ for all t and s. Here d_t is a linearly deterministic stochastic process, which can be predicted arbitrarily well, arbitrarily far into the future from knowledge of past d's or past Y_t's alone; Y_t^i is a linearly indeterministic process which generally can't be predicted perfectly from its own past. Wold's decomposition theorem states that Y_t^i always has a moving average representation $Y_t^i = \sum_{j=0}^{\infty} d_j a_{t-j}$ where $\sum_{j=0}^{\infty} d_j^2 < \infty$ and $a_t = Y_t - EY_t | Y_{t-1}, Y_{t-2}, \ldots$. A bivariate version of Wold's theorem asserts the existence of a similar orthogonal decomposition into deterministic and indeterministic processes for bivariate jointly covariance stationary stochastic processes.

The model formed by (1) and (2) imposes restrictions across both the deterministic and the indeterministic part of the C and Y processes. In the remainder of this section, I shall focus on characterizing the restrictions imposed on the indeterministic parts only. I shall assume that the strictly deterministic parts have been subtracted off; certain of the estimates reported in section 7 utilize as data residuals from regressions on a trend and seasonal dummies in an effort to approximate the purely indeterministic parts of the C and Y processes. I shall defer until section 4 the task of characterizing the restrictions that (1) and (2) impose across the deterministic parts of the (C_t, Y_t) process.

I now assume that Y_t is strictly linearly indeterministic (or that the deterministic parts have been subtracted). I further restrict things by supposing that Y_t is an nth order Markov process

$$(7) \quad Y_t = \delta_1 Y_{t-1} + \ldots + \delta_n Y_{t-n} + \epsilon_t$$

where ϵ_t is a covariance stationary stochastic process with $E\epsilon_t = 0$, $E\epsilon_t^2 = \sigma_\epsilon^2$ and where $E\epsilon_t Y_{t-s} = 0$ for all $s > 0$. Write (7) in matrix form as

$$(8) \quad x_t = A x_{t-1} + \eta_t$$

where

$$x_t = \begin{bmatrix} Y_t \\ Y_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ Y_{t-n+1} \end{bmatrix} \quad , \quad \eta_t = \begin{bmatrix} \epsilon_t \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \quad ,$$

and

$$A = \begin{bmatrix} \delta_1 & \delta_2 & \cdot & \cdot & \delta_n \\ 1 & 0 & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & 0 \\ \cdot & & & & \\ 0 & 0 & \cdot & 1 & 0 \end{bmatrix}$$

We can recover Y_t from x_t by

$$Y_t = d x_t$$

where $d = [1, 0, \ldots, 0]$, a $(1 \times n)$ row vector.

From (8) we have

$$x_{t+1} = A x_t + \eta_{t+1}$$

$$x_{t+2} = A^2 x_t + A \eta_{t+1} + \eta_{t+2}$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$x_{t+j} = A^j x_t + A^{j-1} \eta_{t+1} + \ldots + \eta_{t+j}$$

Since by assumption $E\eta_{t+j} \cdot x_t' = 0$ for all $j>0$ (because $E\varepsilon_{t+j}Y_t = 0$ for all $j>0$), we have

(9) $E_t x_{t+j} = A^j x_t$.

If the eigenvalues of A are distinct, A can be expressed as

(10) $A = P\Lambda P^{-1}$

where the columns of the nxn matrix P are the eigenvectors of A and Λ is a diagonal matrix of eigenvalues of A. Using (10), we can write

(11) $A^j = P\Lambda^j P^{-1}$

Thus we have

$$E_t x_{t+j} = P\Lambda^j P^{-1} x_t$$

and in particular

(12) $E_t Y_{t+j} = dE_t x_{t+j}$

$$= dP\Lambda^j P^{-1} x_t$$

To form permanent income we take the appropriate weighted sum of (12)

$$Y_{pt} = (1-\alpha) \sum_{j=0}^{\infty} \alpha^j E_t Y_{t+j}$$

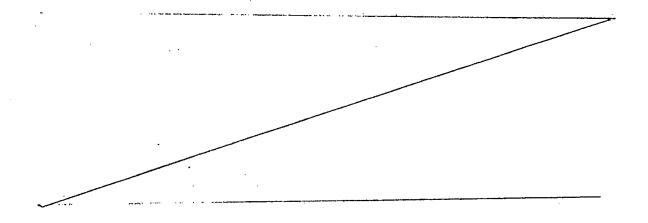$$= (1-\alpha)dP\left(\sum_{j=0}^{\infty} \alpha^j \Lambda^j\right) P^{-1} x_t$$

(13) $Y_{pt} = (1-\alpha)dPH_\alpha P^{-1} x_t$

where H_α is the diagonal matrix

$$H_\alpha = \begin{pmatrix} \dfrac{1}{1-\alpha\lambda_1} & 0 & . & . & . & 0 \\ & & & & & . \\ 0 & \dfrac{1}{1-\alpha\lambda_2} & . & & & . \\ & & . & & & . \\ 0 & . & . & . & . & \dfrac{1}{1-\alpha\lambda_n} \end{pmatrix}$$

where the λ_j's are the eigenvalues of A. I am assuming that $|\alpha\lambda_i|<1$ for all i, in order to guarantee that the infinite series $\sum\limits_{j=0}^{\infty} \alpha^j \lambda_i^j$ converges to $1/(1-\alpha\lambda_i)$. For $\alpha<1$, this condition is weaker than the condition $|\lambda_i|<1$ for all i, which is the condition required if $\{Y_t\}$ is to be a stationary stochastic process. It is possible for permanent income (2) to be a well-defined convergent series even if income is nonstationary in the sense that max $|\lambda_i|>1$. Provided $\alpha<1$, permanent income is thus potentially well defined by (2) even in the face of a stochastically upward-trending rate of income. Equation (13) is a convenient closed form for Y_{pt} in terms of the eigenvalues and eigenvectors of A (i.e. in terms of the roots of $1-\delta_1 z \ldots -\delta_n z^n = 0$), which can easily be implemented on a computer.

In summary, we have the structure

(1) $C_t = BY_{pt} + u_t$

(8) $x_t = Ax_{t-1} + \eta_t$

            $Y_t = dx_t$

(13) $Y_{pt} = (1-\alpha)dPH_{\alpha}P^{-1}x_t$

We thus have the observable consumption function,

(14) $C_t = B(1-\alpha)dPH_{\alpha}P^{-1}x_t + u_t$

The force of the rational expectations hypothesis and the restrictions

imposed on the error process u is to impose (highly nonlinear) restrictions

across the exogenous Y process (8) and the observable consumption function

(14). The most straightforward method of testing the model is to

estimate (8) and (14) simultaneously using the method of maximum likelihood,

estimating first subject to the cross-equation constraints imposed by (8)

and (14) and then with those constraints relaxed.   A likelihood

ratio test can be computed to evaluate the validity of the constraints.

I propose the following algorithm for estimating the vector

autogregression for the (Y,C) process under the restrictions implied by

rational expectations, an algorithm applicable when Y is strictly econo-

metrically exogenous with respect to C. Suppose that u_t follows the

second order Markov process

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \xi_t$$

where ξ_t is a serially uncorrelated process with mean zero. I assume

that ξ_t is orthogonal to the entire Y process. (It will be clear how to

proceed where u_t follows an nth order Markov process.) Quasi-differencing

(1) then gives

$$C_t - \rho_1 C_{t-1} - \rho_2 C_{t-2} = B[Y_{Pt} - \rho_1 Y_{Pt-1} - \rho_2 Y_{Pt-2}] + \xi_t$$

Since $E_{t-1}\xi_t = 0$, projecting both sides of the above equation on things

dated t-1 and earlier gives

$$(15) \qquad E_{t-1}C_t - \rho_1 C_{t-1} - \rho_2 C_{t-2} = B[E_{t-1}Y_{Pt} - \rho_1 Y_{Pt-1} - \rho_2 Y_{Pt-2}]$$

But we know that

$$E_{t-1}Y_{pt} = (1-\alpha)dP\{\frac{\lambda i}{1-\alpha\lambda_i}\}P^{-1}x_{t-1}$$

$$Y_{Pt-1} = (1-\alpha)dP \{\frac{1}{1-\alpha\lambda_i}\} P^{-1}x_{t-1}$$

$$Y_{Pt-2} = (1-\alpha)dP \{\frac{1}{1-\alpha\lambda_i}\} P^{-1}x_{t-2}$$

Substituting these into equation (15) gives

$$E_{t-1}C_t = \rho_1 C_{t-1} + \rho_2 C_{t-2} + B(1-\alpha)dP \left\{\frac{\lambda_i}{1-\alpha\lambda_i}\right\} P^{-1}x_{t-1}$$

$$-\rho_1 B(1-\alpha)dP \left\{\frac{1}{1-\alpha\lambda_i}\right\} P^{-1}x_{t-1}$$

$$-\rho_2 B(1-\alpha)dP \left\{\frac{1}{1-\alpha\lambda_i}\right\} P^{-1}x_{t-2}$$

(16)
$$E_{t-1}C_t = \rho_1 C_{t-1} + \rho_2 C_{t-2} + B(1-\alpha)dP \left\{\frac{\lambda_i-\rho_1}{1-\alpha\lambda_i}\right\} P^{-1}x_{t-1}$$

$$-\rho_2 B(1-\alpha)dP \left\{\frac{1}{1-\alpha\lambda_i}\right\} P^{-1}x_{t-2}$$

Equation (16) gives the vector autoregression for C in terms of the ρ's that characterize the Markov process for u and the δ's that characterize the autoregression for Y_t and that determine $A = P\Lambda P^{-1}$.

If we had assumed that u_t followed the sth-order Markov process

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} \cdots + \rho_s u_{t-s} + \xi_t$$

where $E_{t-1}\xi_t = 0$, then (16) would be

$$E_{t-1}C_t = \rho_1 C_{t-1} + \rho_2 C_{t-2} + \cdots + \rho_s C_{t-s}$$

$$+ B(1-\alpha)dP \left\{\frac{\lambda_i-\rho_1}{1-\alpha\lambda_i}\right\} P^{-1}x_{t-1}$$

$$- B(1-\alpha)dP \left\{\frac{1}{1-\alpha\lambda_i}\right\} P^{-1} (\rho_2 x_{t-2} + \rho_3 x_{t-3} + \cdots + \rho_s x_{t-s})$$

Proceeding with our second order example, we can now write the vector autoregression for (Y_t, C_t) as

$$Y_t = \delta_1 Y_{t-1} + \ldots + \delta_n Y_{t-n} + a_{yt}$$

$$C_t = \rho_1 C_{t-1} + \rho_2 C_{t-2} + B(1-\alpha)dP \left\{ \frac{\lambda_i - \rho_1}{1 - \alpha \lambda_i} \right\} P^{-1} x_{t-1}$$

$$-\rho_2 B(1-\alpha)dP \left\{ \frac{1}{1-\alpha \lambda_i} \right\} P^{-1} x_{t-2} + a_{ct}$$

and where $a_{yt} \equiv \varepsilon_t$. where the innovations (a_{yt}, a_{ct}) obey $E_{t-1} a_{yt} = E_{t-1} a_{ct} = 0$/ Assuming that (a_{yt}, a_{ct}) is jointly normally distributed, given a sample over $t=1,\ldots,T$, maximum likelihood estimates can be obtained by minimizing $|V|$ where

$$V = \begin{pmatrix} \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{yt}^2 & \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{yt} \hat{a}_{ct} \\[4mm] \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{yt} \hat{a}_{ct} & \frac{1}{T} \sum_{t=1}^{T} \hat{a}_{ct}^2 \end{pmatrix}$$

The minimization is carried out over the parameters $\delta_1, \ldots, \delta_n, \rho_1, \rho_2$, B, and α. The log likelihood function at the maximum likelihood parameter estimates can be shown to be (see Wilson [1973])

$$\log L = -\tfrac{1}{2}mT \log 2\pi - \tfrac{1}{2}T (\log |V| + m)$$

where m is the number of variables being modelled (in our case 2, C and Y).

To test the model, it is appropriate to estimate the vector autoregression twice: once subject to the constraints in (16); another time unconstrained. Let $\log L_u$ be the value of the log likelihood unconstrained, while $\log L_r$ is its value under the constraints. Then

$$-2 (\log L_r - \log L_u)$$

is asymptotically distributed as / chi-square with q degrees of freedom, where q is the number of restrictions

imposed. Equivalently, we can express the likelihood ratio as

$$T \{ \log |V_r| - \log |V_u| \}$$

where V_r and V_u are the values of V under the restriction and unrestricted, respectively.[13]

Notice that the parameters B and α are estimated only from the systematic part of the vector autoregression of the (C,Y) process. In particular, information about the contemporaneous covariance matrix of the innovations a_{ct}, a_{yt} is not used in estimating the parameters B and α. This estimator takes no stand on how the contemporaneous covariance between a_{ct} and a_{yt} is to be accounted for. In Granger's language, this estimator permits "instantaneous feedback" between C and Y in either direction, but ignores it in estimating B and α. Notice, however, that Sims' theorems assure us that if C fails to Granger-cause Y, then there exists a model of the following form that is consistent with the data:

$$(*) \qquad C_t = \sum_{i=0}^{\infty} h_i Y_{t-i} + \xi_t$$

$$Y_t = \sum_{i=1}^{\infty} \delta_i Y_{t-i} + \psi_t \quad ,$$

where
$/E\xi_t \psi_s = 0$ for all t and s. In this model, all contemporaneous covariance between the innovations a_{ct} and a_{yt} is swept into the first equation, in our case, the consumption function. A maximum likelihood estimator of B and α could be based on this model provided that the instantaneous feedback between Y and C is partitioned a priori, as it is, for example, by the model of section 1. In that model, all instantaneous feedback flows from Y to C through the consumption function, thereby in effect identifying

the first equation of (*) as the consumption function. A maximum likelihood

estimator along these lines uses more information than is incorporated

by the estimator described above.[14] However, at this point I prefer an

estimator that does not, for example, attribute all contemporaneous

covariance between a_{yt} and a_{ct} to the workings of the consumption function.

The estimator that I have recommended above does permit instantaneous

causality to flow from C to Y, a feature that would characterize a class

of macroeconomic models in which C still fails to Granger-cause Y.[15] In

summary, the maximum likelihood estimator that estimates B and α by

imposing the restrictions (16) requires weaker conditions on the covariance

between u_t and disposable income than are imposed, for example, by the

macroeconometric model of section 2.[16]

4. Estimation of the Consumption Function under

Rational Expectations with Income Endogenous

a. Purely Indeterministic Process

Now return to the structure (1) and (2), and drop the assumption that Y_t is orthogonal to u_s at all t and s. This change in assumption permits income to be econometrically endogenous and renders the preceding estimation procedures invalid, i.e. the estimators for B and α proposed above will not in general be statistically consistent when Y fails to be econometrically exogenous in (1). In this section I propose an estimator that, under special assumptions, is statistically consistent even where Y is not econometrically exogenous in (1). As will be shown, in obtaining consistent estimates, it is important to control for serial correlation in u_t so long as our information is confined to observations on the (C,Y) process alone. I begin by working things out for the case in which the (C_t, Y_t) process is purely linearly indeterministic. Later in the section, it is assumed that the (C_t, Y_t) process contains both deterministic and indeterministic parts.

Combining (1) with (2) gives

$$C_t = B(1-\alpha) \left[\sum_{j=0}^{\infty} \alpha^j F^j \right] E_t Y_t + u_t$$

or

(17) $\qquad C_t = B(1-\alpha) \dfrac{1}{1-\alpha F} E_t Y_t + u_t$

where F is the operator defined by $FE_t Y_{t+k} \equiv E_t Y_{t+k+1}$, so that application of F leaves the conditioning set unchanged while shifting the time index

on the random variable operated upon forward by one period. Operating

on both sides of (17) by the operator $(1-\alpha F)$ gives

(18) $\qquad C_t = \alpha E_t C_{t+1} + B(1-\alpha)Y_t + u_t - \alpha E_t u_{t+1}$.

Now suppose that u_t follows the first order Markov process

(19) $\qquad u_t = \rho u_{t-1} + w_t$

where w_t is serially uncorrelated and orthogonal to lagged Y's and C's.
Subtracting ρC_{t-1} from both sides of (18) and noting that $E_t u_{t+1} = \rho u_t$
gives

$$C_t - \rho C_{t-1} = \alpha(E_t C_{t+1} - \rho E_{t-1} C_t) + B(1-\alpha)(Y_t - \rho Y_{t-1})$$

$$+ (1-\alpha\rho)u_t - \rho(1-\alpha\rho)u_{t-1}$$

Projecting both sides of the above equation on information dated t-1 and

earlier gives

(20) $\qquad E_{t-1}(C_t - \rho C_{t-1}) = \alpha E_{t-1}(C_{t+1} - \rho C_t) + B(1-\alpha)E_{t-1}(Y_t - \rho Y_{t-1})$

Equation (20) is a restriction across the systematic part of the vector

autoregression of the $(C_t - \rho C_{t-1}, Y_t - \rho Y_{t-1})$ process. To characterize

the restriction, let the systematic part of the nth-order vector auto-

regression of $(C_t - \rho C_{t-1}, Y_t - \rho Y_{t-1}) \equiv (c_t, Y_t)$ be [17]

(21) $\qquad E_{t-1}c_t = c_1 Y_{t-1} + c_2 Y_{t-2} + \cdots + c_n Y_{t-n} + d_1 c_{t-1} + d_2 c_{t-2} + \cdots + d_n c_{t-n}$

$\qquad E_{t-1}Y_t = a_1 Y_{t-1} + a_2 Y_{t-2} + \cdots + a_n Y_{t-n} + b_1 c_{t-1} + b_2 c_{t-2} + \cdots + b_n c_{t-n}$

Application of the "chain rule of forecasting" gives [18]

$$(22) \quad E_{t-1}c_{t+1} = c_1 E_{t-1}y_t + c_2 y_{t-1} + \cdots + c_n y_{t-n+1}$$

$$+ d_1 E_{t-1}c_t + d_2 c_{t-1} + \cdots + d_n y_{t-n+1}$$

Substituting from (21) into (22) and rearranging gives

$$(23) \quad E_{t-1}c_{t+1} = (c_1 a_1 + c_2 + d_1 c_1)y_{t-1} + (d_1 a_2 + c_3 + d_1 c_2)y_{t-2} + \cdots$$

$$+ (c_1 a_{n-1} + c_n + d_1 c_{n-1})y_{t-n+1} + (c_1 a_n + d_1 c_n)y_{t-n}$$

$$+ (c_1 b_1 + d_1^2 + d_2)c_{t-1} + (c_1 b_2 + d_1 d_2 + d_3)c_{t-2}$$

$$+ (c_1 b_{n-1} + d_1 d_{n-1} + d_n)c_{t-n+1} + (c_1 b_n + d_1 d_n)c_{t-n} \quad .$$

Using (21) and (23), the restrictions implied by (20) become

$$(24) \quad c_1 = \beta(1-\alpha)a_1 + \alpha(d_1 a_1 + c_2 + d_1 c_1)$$

$$c_2 = \beta(1-\alpha)a_2 + \alpha(c_1 a_2 + c_3 + d_1 c_2)$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$c_{n-1} = \beta(1-\alpha)a_{n-1} + \alpha(c_1 a_{n-1} + c_n + d_1 c_{n-1})$$

$$c_n = \beta(1-\alpha)a_n + \alpha(c_1 a_n + d_1 c_n)$$

$$d_1 = \beta(1-\alpha)b_1 + \alpha(c_1 b_1 + d_1^2 + d_2)$$

$$d_2 = \beta(1-\alpha)b_2 + \alpha(c_1 b_2 + d_1 d_2 + d_3)$$

$$\cdot$$
$$\cdot$$
$$\cdot$$

$$d_{n-1} = \beta(1-\alpha)b_{n-1} + \alpha(c_1 b_{n-1} + d_1 d_{n-1} + d_n)$$

$$d_n = \beta(1-\alpha)b_n + \alpha(c_1 b_n + d_1 d_n)$$

These (2n) equations determine $a_1, \ldots, a_n, b_1, \ldots, b_n$ as functions of β, α, $c_1, \ldots, c_n, d_1, \ldots, d_n$. For example, notice that the first equation determines a_1 as a function of β, α, c_1, c_2, and d_1. The equations (24) embody the restrictions that equations (1), (2) and the assumption on the error process (19) impose on the systematic part of the nth order vector autoregression for (c, y).

Write the vector autoregression for (c_t, y_t) as

(25)

$$c_t = c_1 y_{t-1} + \ldots + c_n y_{t-n} + d_1 c_{t-1} + \ldots + d_n c_{t-n} + a_{ct}$$

$$y_t = a_1 y_{t-1} + \ldots + a_n y_{t-n} + b_1 c_{t-1} + \ldots + b_n c_{t-n} + a_{yt}$$

where (a_{ct}, a_{yt}) are disturbances orthogonal to $y_{t-1}, \ldots, y_{t-n}, c_{t-1}, \ldots, c_{t-n}$. Let $a_t = (a_{ct}, a_{yt})$. Given a sample extending over a period $t=1, \ldots, T$, maximum likelihood estimates of (25) under the restrictions (24) are obtained by minimizing

$$|V| = \left| \frac{1}{T} \sum_{t=1}^{T} a_t a_t' \right|$$

with respect to the free parameters $\beta, \alpha, c_1, \ldots, c_n, d_1, \ldots, d_n$. To test the model, (25) can be estimated unconstrained and then the appropriate likelihood ratio statistic can be constructed. Maximum likelihood estimation of (25) subject to no constraints amounts to least squares estimation of each equation separately.

b. Deterministic Components Present

So far, my calculations have assumed that consumption and income are purely linearly indeterministic processes, or else that the model (1)

and (2) applies only to the indeterministic parts of consumption and income. I now relax this assumption and assume instead that the (y_t, c_t) process is governed by

$$(26) \qquad c_t = c_1 y_{t-1} + \ldots + c_n y_{t-n} + d_1 c_{t-1} + \ldots + d_n c_{t-n} + \eta_t^c + a_{ct}$$

$$y_t = a_1 y_{t-1} + \ldots + a_n y_{t-n} + b_1 c_{t-1} + \ldots + b_n c_{t-n} + \eta_t^y + a_{yt}$$

where (a_{ct}, a_{yt}) are orthogonal to $y_{t-1}, c_{t-1}, \ldots, y_{t-n}, c_{t-n}, \eta_t^c$, and η_t^y, and where η_t^c and η_t^y are deterministic components that are given by

$$(27) \qquad \eta_t^c = \gamma^c t + \delta_1^c s_{1t} + \delta_2^c s_{2t} + \delta_3^c s_{3t} + \delta_4^c s_{4t}$$

$$\eta_t^y = \gamma^y t + \delta_1^y s_{1t} + \delta_2^y s_{2t} + \delta_3^y s_{3t} + \delta_4^y s_{yt} \qquad .$$

Here the seasonal dummy s_{1t} equals unity every fourth quarter and zero in the intervening three quarters, and $s_{2t+1} = s_{1t}$, $s_{3t+1} = s_{2t}$, $s_{4t+1} = s_{3t}$ and $s_{1t+1} = s_{4t}$. Notice that $s_{1t} + s_{2t} + s_{3t} + s_{4t} = 1$ for all t. I assume that agents are able to forecast η_t^y and η_t^c perfectly. Then from the chain rule of forecasting we have

$$E_{t-1} c_{t+1} = c_1 \{ a_1 y_{t-1} + \ldots + a_n y_{t-n} + b_1 c_{t-1} + \ldots + b_n c_{t-n} + \eta_t^y \}$$

$$+ c_2 y_{t-1} + \ldots + c_n y_{t-n+1}$$

$$+ d_1 \{ c_1 y_{t-1} + \ldots + c_n y_{t-n} + d_1 c_{t-1} + \ldots + d_n c_{t-n} + \eta_t^c \}$$

$$+ d_2 c_{t-1} + \ldots + d_n c_{t-n+1} + \eta_{t+1}^c$$

$$= \eta_{t+1}^c + d_1 \eta_t^c + c_1 \eta_t^y + \text{(terms in lagged c's and y's)}.$$

Equation (20) then places the following restriction across the η_t's:

$$\eta_t{}^C = \beta(1-\alpha)\eta_t{}^Y + \alpha[c_1\eta_t{}^Y + d_1\eta_t{}^C + \eta_{t+1}{}^C]$$

or

$$[1-\alpha d_1]\eta_t{}^C = [\beta(1-\alpha)+\alpha c_1]\eta_t{}^Y + \alpha\eta_{t+1}{}^C$$

Substituting from (27) gives

$$(1-\alpha d_1)[\gamma^C t + \delta_1{}^C s_{1t} + \delta_2{}^C s_{2t} + \delta_3{}^C s_{3t} + \delta_4{}^C s_{4t}] =$$

$$[\beta(1-\alpha) + \alpha c_1](\gamma^Y t + \delta_1{}^Y s_{1t} + \delta_2{}^Y s_{2t} + \delta_3{}^Y s_{3t} + \delta_4{}^Y s_{4t})$$

$$+ \alpha(\gamma^C t + \gamma^C(s_{1t}+s_{2t}+s_{3t}+s_{4t}) + \delta_1{}^C s_{1t+1} + \delta_2{}^C s_{2t+1}$$

$$+ \delta_3{}^C s_{3t+1} + \delta_4{}^C s_{4t+1}) \quad .$$

Noting that $s_{1t+1}=s_{4t}$, $s_{2t+1}=s_{1t}$, $s_{3t+1}=s_{2t}$, $s_{4t+1}=s_{3t}$, we deduce the following restrictions:

(28) $$(1-\alpha d_1)\gamma^C = [\beta(1-\alpha) + \alpha c_1]\gamma^Y + \alpha\gamma^C$$

$$(1-\alpha d_1)\delta_1{}^C = [\beta(1-\alpha) + \alpha c_1]\delta_1{}^Y + \alpha(\gamma^C + \delta_2{}^C)$$

$$(1-\alpha d_1)\delta_2{}^C = [\beta(1-\alpha) + \alpha c_1]\delta_2{}^Y + \alpha(\gamma^C + \delta_3{}^C)$$

$$(1-\alpha d_1)\delta_3{}^C = [\beta(1-\alpha) + \alpha c_1]\delta_3{}^Y + \alpha(\gamma^C + \delta_4{}^C)$$

$$(1-\alpha d_1)\delta_4{}^C = [\beta(1-\alpha) + \alpha c_1]\delta_4{}^Y + \alpha(\gamma^C + \delta_1{}^C)$$

These five equations determine γ^Y, $\delta_1{}^Y$, $\delta_2{}^Y$, $\delta_3{}^Y$, and $\delta_4{}^Y$ as functions of β, α, γ^C, $\delta_1{}^C$, $\delta_2{}^C$, $\delta_3{}^C$, $\delta_4{}^C$, c_1 and d_1. It is readily verified that the

restrictions (24) continue to hold across the a, b, c, and d's. Thus, equations (24) and (28) summarize the restrictions that the theory imposes on the parameters of the vector autoregression (26). Once again, the strategy is to estimate (26) by maximum likelihood, first subject to (24) and (28), then unconstrained, and to compute the pertinent likelihood ratio statistic.

5. Appropriate Data

The preceding sections have indicated the restrictions that the rational expectations hypothesis imposes across the autoregressive representation of the (C,Y) process. Evidently these restrictions are applicable to the unadulterated seasonally unadjusted data.[19] Even in the presence of a linearly indeterministic seasonal in Y, the model $(1)-(2)$ is supposed to hold. The restrictions in general are predicted not to hold for seasonally adjusted data.[20] Thus, let seasonally adjusted consumption and income, C_t^a and Y_t^a, respectively, be related to C_t and Y_t by

$$C_t^a = f(L)C_t$$

$$Y_t^a = g(L)Y_t$$

where $f(L)$ and $g(L)$ are two-sided seasonal adjustment filters. Wallis [1974] has argued that actual seasonal adjustment procedures are well approximated by the application of such linear filters. The cross-covariance generating matrix of the adjusted data is given by

$$\begin{bmatrix} S_c^a(z) & S_{cy}^a(z) \\ S_{yz}^a(z) & S_y^a(z) \end{bmatrix} = \begin{bmatrix} f(z)f(z^{-1})S_c(z) & f(z)g(z^{-1})S_{cy}(z) \\ g(z)f(z^{-1})S_{yc}(z) & g(z)g(z^{-1})S_y(z) \end{bmatrix}$$

where $S_c(z)$, $S_y(z)$, and $S_{cy}(z)$ are, respectively, the covariance generating functions of C and Y, and the cross-covariance generating function between C and Y. Restrictions that hold across elements of the cross-covariance

generating function of the unadjusted data will not in general hold across elements of the cross-covariance generating function for the left-hand side. To take an example, with Y exogenous, for the unadjusted data the generating function for the coefficients in the projection of C_t on the Y process, $S_{cy}(z)/S_y(z)$ is one-sided and has common parameters with the covariance-generating function of Y, $S_y(z) = \sigma^2 b(z)b(z^{-1})$, as indicated in section 3. The projection of C_t^a on the Y_t^a process has coefficients with generating

function $\dfrac{f(z)}{g(z)} \dfrac{S_{cy}(z)}{S_y(z)}$ which in general is two-sided unless $f(z) = g(z)$,

and in any event does not obey the section 3 restrictions with the parameters of the covariance generating function of the adjusted Y, $S_y^a(z) = \sigma^2 g(z)g(z^{-1})b(z)b(z^{-1})$.

Below I analyze both seasonally adjusted and unadjusted data. This is done partly because other investigators have usually used seasonally adjusted data, and partly because many readers may find the arguments of Sims [1976] compelling.

6. Exogeneity Tests

For the post-war U.S. for both seasonally adjusted and unadjusted data
I have defined the following variables:

Y_1 = GNP − capital consumption − state and local tax receipts

− Federal tax receipts + transfers

C = consumption of durables + consumption of nondurables

+ consumption of services

I = residential construction + nonresidential fixed investment

+ change in business inventories

Y_2 = C + I

$\tilde{I}$ = I + consumption of durables

D_t = .975 (consumption of durables) + .95 D_{t-1}

$\tilde{C}_t$ = C_t − .9875 (consumption of durables) + .025 D_t

The data are quarterly and "real." The seasonally adjusted data are
deflated by implicit price deflators and are measured in 1972 dollars.
The seasonally unadjusted data were deflated by quarterly averages of
the consumer price index and are measured in 1967 dollars. The sources
of the data are given in the appendix.

The disposable income concept Y_2 differs from the usual dispo-
sable income concept Y_1 by in effect subtracting from Y_1 the rate of
real government deficit.[21] To the extent that the government deficit is
financed by issuing bonds and to the extent that agents fully discount
anticipated future taxes levied to service the debt, Y_2 is the appropriate
disposable income concept.[22]

Table 1 reports the results of testing for Granger causality
over the period 1948III-1972IV. The regressions computed were of the form

$$z_t = \sum_{i=1}^{m} \alpha_i z_{t-i} + \sum_{i=1}^{n} \beta_i x_{t-i} + \text{residual} \quad .$$

The null hypothesis that x fails to Granger-cause z is tested by testing the null hypothesis $\beta_0 = \beta_1 = \ldots = \beta_n = 0$. The table reports the marginal significance levels associated with the F-statistic pertinent for testing this null hypothesis. The regressions include a constant, linear trend, and, for the seasonally unadjusted data, three seasonal dummies.

For the seasonally unadjusted data, the hypothesis that Y_1 is not Granger-caused by C or $\tilde{C}$ is generally rejected even at low marginal significance levels. A similar conclusion holds for Y_2. These results thus point against acceptance of the version of Tobin's Dynamic Aggregative Model described in Section 1. They also indicate that we should use the Section 4 estimator, which does not require exogeneity of Y, instead of the Section 3 estimator.

The results are substantially different where the seasonally adjusted data are used.[23] Here Y_1 seems not to be Granger-caused by C or $\tilde{C}$; Y_2 seems not to be caused by $\tilde{C}$, while the F-statistics are marginal at the usual significance levels for testing whether Y_2 is Granger-caused by C. All in all, the results with the seasonally adjusted data are much more consistent with the predictions of the special version of Tobin's model described in Section 1. With the seasonally adjusted data, these exogeneity results provide some justification for applying the Section 3 estimator.[24]

Table 1a

Granger Causality Test Results
Not Seasonally Adjusted Data

			Marginal Significant Level			
Z_t	X_t	Lags: (m,n) Statistic:	8-4 $F(4,79)$	6-4 $F(4,83)$	8-6 $F(6,77)$	6-6 $F(6,81)$
C	Y_1		.2062	.2135	.2841	.3418
Y_1	C		.0064	.0200	.0245	.0645
I	Y_1		.0943	.0226	.0316	.0073
Y_1	I		.0222	.0283	.0138	.0516
C	Y_2		.0013	.0003	.0003	.0001
Y_2	C		.0250	.0082	.0181	.0063
I	Y_2		.0291	.0033	.0014	.0002
Y_2	I		.0250	.0082	.0181	.0063
I	C		.0291	.0033	.0014	.0002
C	I		.0013	.0003	.0003	.0001
$\tilde{C}$	$\tilde{Y}_1$		.1694	.1450	.3105	.2748
$\tilde{Y}_1$	$\tilde{C}$		.0023	.0073	.0106	.0308
$\tilde{I}$	$\tilde{Y}_1$		.0909	.0386	.0410	.0187
$\tilde{Y}_1$	$\tilde{I}$		.1013	.0980	.0571	.1333
$\tilde{C}$	$\tilde{Y}_2$		.0023	.0007	.0039	.0012
$\tilde{Y}_2$	$\tilde{C}$		.2108	.0835	.0181	.0064
$\tilde{I}$	$\tilde{Y}_2$		.0871	.0341	.0030	.0009
$\tilde{Y}_2$	$\tilde{I}$		.1945	.0774	.0292	.0103
$\tilde{I}$	$\tilde{C}$		.0881	.0347	.0021	.0006
$\tilde{C}$	$\tilde{I}$		.0023	.0008	.0039	.0013

Columns 1 and 3: dependent variable spans 1949I-1972IV.
Columns 2 and 4: dependent variable spans 1948III-1972IV.
All regressions include constant, linear trend, and three seasonal dummies. Regressions are of the form

$$z_t = \sum_{i=1}^{m} \alpha_i z_{t-i} + \sum_{i=1}^{n} \beta_i x_{t-i} + \text{residual}.$$

Table reports marginal significance level of F-statistic pertinent for testing null hypothesis $\beta_1 = \beta_2 = \ldots = \beta_n = 0$, which is the null hypothesis "x fails to Granger-cause z." Where f is the calculated value of the pertinent F-statistic, the marginal significance level is defined as prob $\{F>f\}$ under the null hypothesis.

Table 1b

Granger Causality Test Results
Seasonally Adjusted Data

Z_t	X_t	Lags: (m,n)	Marginal Significant Level			
			8-4 $F(4,82)$	6-4 $F(4,86)$	8-6 $F(6,80)$	6-6 $F(6,84)$
C	Y_1		.0144	.0197	.0141	.0134
Y_1	C		.3540	.6709	.1901	.4771
I	Y_1		.0061	.0073	.0190	.0200
Y_1	I		.0386	.0843	.1050	.1850
C	Y_2		.0131	.0105	.0168	.0142
Y_2	C		.0760	.1199	.0876	.1272
I	Y_2		.0012	.0006	.0004	.0003
Y_2	I		.0760	.1199	.0876	.1272
I	C		.0012	.0006	.0004	.0003
C	I		.0131	.0105	.0168	.0142
$\tilde{C}$	Y_1		.0029	.0036	.0050	.0059
Y_1	$\tilde{C}$		.3145	.5615	.4059	.7285
$\tilde{I}$	Y_1		.0205	.0227	.0260	.0410
Y_1	$\tilde{I}$		.2848	.3533	.1902	.2972
$\tilde{C}$	Y_2		.0127	.0117	.0241	.0197
Y_2	$\tilde{C}$		.0640	.1091	.1647	.2241
$\tilde{I}$	Y_2		.0018	.0031	.0063	.0085
Y_2	$\tilde{I}$		.0666	.1224	.1618	.2322
$\tilde{I}$	$\tilde{C}$		.0016	.0025	.0055	.0070
$\tilde{C}$	$\tilde{I}$		.0112	.0099	.0222	.0175

Columns 1 and 3: dependent variable spans 1949I-1972IV.
Columns 2 and 4: dependent variable spans 1948III-1972IV.
All regressions include constant and linear trend. Regressions are
of the form

$$Z_t = \sum_{i=1}^{m} \alpha_i Z_{t-i} + \sum_{i=1}^{n} \beta_i X_{t-i} + \text{residual.}$$

Table reports marginal significance level of F-statistic pertinent for
testing null hypothesis $\beta_1 = \beta_2 = \ldots = \beta_n = 0$, which is the null hypoth-
esis "x fails to Granger-cause z." Where f is the calculated value of
the pertinent F-statistic, the marginal significance level is defined as
prob $\{F>f\}$ under the null hypothesis.

7. Tests of Cross-Equation Restrictions

Table 2 reports the results of implementing various versions of the Section 4 estimator with the seasonally unadjusted data where n, the order of the vector autoregression, is taken as four. Observations on the dependent variables span 1948II-1972IV. The table reports the marginal confidence level for the likelihood ratio test, defined as follows: Let X be a chi-square random variable with q degrees of freedom and let x be the computed value of the test statistic. Then the marginal confidence level is Prob {X<x} under the null hypothesis. High values of the marginal confidence level indicate that the null hypothesis is doing badly. Note that the marginal confidence level is one minus the marginal significance level as defined earlier. The various versions of the procedure differ in regard to whether the constraints (28) on the seasonals and trend terms were imposed and whether α and B were estimated or were imposed at the "reasonable" values of α = .95 and B = .9. The table records "yes" if the constraint on the trend (the first line of (28)) is imposed, and "yes" if the contraints on the seasonals (the next four lines of (28)) are imposed. The table also reports the Kolmogorov-Smirnov statistic on the cumulated periodogram of the consumption and income residuals from the constrained autoregressions. Tables for the distribution of this statistic are reported by Durbin [1969]. Finally, we set ρ=1 in all of the Section 4 calculations.

Overall, the pattern of results in Table 2 calls for rejection of the model (1) and (2) as supplemented by the hypothesis (19) that the disturbance u_t follows a first-order Markov process with ρ=1. In most cases, the likelihood ratio test emphatically calls for rejection of the

model. Further, very unreasonable parameter estimates, especially of B are obtained. Where they are left unconstrained, the point estimates of α do not resemble quarterly discount factors. Those estimates that correspond to likelihood ratios that don't call for rejecting the model typically involve theoretically unacceptable point estimates. Notice in particular that the first line of each of Table 2a-2d, which imposes the rational expectations constraints across the trends and seasonals, does not yield estimates of the discount factor α that would cause one to embrace the permanent income theory over a theory in which consumption is related only to contemporaneous income.

Tables 3a-3d report some results for the seasonally adjusted data. Qualitatively, the results are much the same as for the unadjusted data and are discouraging from the point of view of the model.

Table 4 reports results obtained by using the estimator of
Section 3 where Y_t is assumed to be an exogenous fourth-order Markov
process and u_t a second-order Markov process. Once again, when plausible
values of B and/or α are imposed, the likelihood ratio statistics call
for rejection of the cross-equation restrictions imposed by the model.
In the one case shown, set 3, in which the likelihood ratio does not
very strongly call for rejection, the estimate of B is highly implausible.[25]

8. Conclusions

As Lucas has observed,[26] if one's criterion of success is a high R^2, then macroeconometric theories that link flows (like consumption) to flows (like income) are bound to dominate theories that link flows to relative prices. And on that criterion, the permanent income version of the consumption function has certainly been a success. How can we reconcile that success with the discouraging results in this paper? The answer lies partly in another observation of Lucas [1975], namely that econometric imposition of the hypothesis of rational expections eliminates the usual emptiness of distributed lag estimation and delivers strongly overidentified models. Another reason for our results is that, contrary to common practice, we have resisted the temptation to attribute to the workings of the consumption function the correlation between innovations in consumption and income. This, together with a serial correlation correction, was adopted by way of eliminating simultaneous equations bias.

While the results here are discouraging from the viewpoint of the simple permanent income model of aggregate consumption, they perhaps raise the possibility that an equilibrium model, which confronts consumers with relative prices rather than incomes, ought not to be dismissed as being so obviously econometrically dominated by the Keynesian consumption hypothesis.

Table 2a

(Seasonally Unadjusted)*

C on Y_1

α	B	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
-.327	.473	Yes	Yes	1.000	.0363	.0699
Converged to $\alpha \gg 1$		Yes	No			
.95	35.430	Yes	Yes	1.000	.0321	.0999
.95	8.71×10^5	Yes	No	1.000	.0291	.0745
.95	4.38×10^5	No	No	1.000	.0281	.0779

Observations on the dependent variables span 1948II-1972IV.

Let X be a chi-square random variable with q degrees of freedom. Let x be the value of the test statistic. Then the marginal confidence level is defined as Prob {X<x} under the null hypothesis.

Table 2b

(Seasonally Unadjusted)
C on Y_2

α	B	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
-.0426	.991	Yes	Yes	1.000	.0267	.0426
.439	1.344	Yes	No	.917	.0277	.0338
.95	42.323	Yes	Yes	1.000	.0418	.0484
.95	20.025	Yes	No	.911	.0357	.0396
.914	11.561	No	No	.876	.0328	.0377
.95	20.213	No	No	.812	.0330	.0380

Observations on the dependent variables span 1948II-1972IV.

Table 2c

(Seasonally Unadjusted)
$\tilde{C}$ on Y_1

α	B	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
-.234	.401	Yes	Yes	1.000		
α >> 1 estimated		Yes	No			
.95	24.889	Yes	Yes	1.000	.0313	.0966
.95	1.174	Yes	No	1.000	.0530	.0192
.95	5.501	No	No	1.000	.0492	.0494

Observations on the dependent variables span 1948II-1972IV.

Table 2d

(Seasonally Unadjusted)
$\tilde{C}$ on Y_2

α	B	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
.0094	.791	Yes	Yes	1.000	.0420	.0525
.794	2.007	Yes	No	.8874	.0319	.0308
.95	30.006	Yes	Yes	1.000	.0401	.0433
.95	8.813	Yes	No	.8423	.0308	.0321
.95	9.025	No	No	.7548	.0310	.0306

Observations on the dependent variables span 1948II-1972IV.

Table 3a

(Seasonally Adjusted)
C on Y_1

α	B	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
.95	4.320	Yes	No	.997	.0498	.0265
.95	6.06×10^{7}	No	No	.697	.0385	.0294
.95	.90	No	No	1.000	.0528	.0381

Observations on the dependent variables span 1948II–1972IV.

Table 3b

(Seasonally Adjusted)
C on Y_2

α	β	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
.95	3.471	Yes	No	1.000	.0538	.0233
.95	3.561	No	No	1.000	.0545	.0240
.95	.90	No	No	1.000	.0554	.0380

Observations on the dependent variables span 1948II-1972IV.

Table 3c

(Seasonally Adjusted)

$\hat{C}$ on Y_1

α	B	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
.95	1.22×10^7	Yes	No	.6463	.0278	.0316
.95	1.91×10^7	No	No	.6138	.0271	.0294
.95	.90	No	No	.979	.0335	.0350

Observations on the dependent variables span 1948II-1972IV.

Table 3d

(Seasonally Adjusted)
$\tilde{C}$ on Y_2

α	β	Constraints on Trend	Constraints on Seasonal	Marginal Confidence Level Likelihood Ratio	K-S Statistic for C Residual	K-S Statistic for Y Residual
.95	2.372	Yes	No	.980	.0405	.0240
.95	2.471	No	No	.983	.0419	.0248
.95	.90	No	No	.997	.0450	.0311

Observations on the dependent variables span 1948II–1972IV.

Table 4

The Estimator of Section Three Which
Assumes Income is Exogenous

Set 1

Maximum likelihood
under rational expec-
tations with B fixed
at .9 (seasonally
adjusted)

$$Y_{1t} = 1.2820Y_{1t-1} - .2353Y_{1t-2} + .0208Y_{1t-3} - .0606Y_{1t-4} + a_{2t}$$

$$\tilde{C}_t = .1357Y_{1t-1} + .1098Y_{1t-2} - .0387Y_{1t-3} - .0141Y_{1t-4}$$

$$- .0085Y_{1t-5} + 1.5908\tilde{C}_{t-1} - .5764\tilde{C}_{t-2} + a_{ct}$$

$$B = .9 \qquad \alpha = .9903$$

Marginal confidence of likelihood ratio test: 1.000

Set 2

Maximum likelihood
under rational expec-
tations with B fixed
at .9 (seasonally
adjusted)

$$Y_{2t} = 1.5355Y_{2t-1} - .4950Y_{2t-2} - .1197Y_{2t-3} + .0869Y_{2t-4} + a_{yt}$$

$$\tilde{C}_t = .0658Y_{2t-1} + .0350Y_{2t-2} - .0365Y_{2t-3} + .0037Y_{2t-4}$$

$$+ .0060Y_{2t-5} + 1.6057\tilde{C}_{t-1} - .5954\tilde{C}_{t-2} + a_{ct}$$

$$B = .9 \qquad \alpha = .9952$$

Marginal confidence of likelihood ratio test: 1.000

Set 3

Maximum likelihood
under rational expec-
tations with B and α
free (not seasonally
adjusted)

$$Y_{1t} = .4600Y_{1t-1} + .2529Y_{1t-2} - .0919Y_{1t-3} + .3231Y_{1t-4} + a_{yt}$$

$$\tilde{C}_t = .0274Y_{1t-1} - .1007Y_{1t-2} - .0414Y_{1t-4} - .0534Y_{1t-4}$$

$$- .0202Y_{1t-5} + .7575\tilde{C}_{t-1} + .1559\tilde{C}_{t-2} + a_{yt}$$

$$B = -2.32 \qquad \alpha = .9334$$

Marginal confidence of likelihood ratio test: .9572

Set 4

Maximum likelihood under rational expectations with B=.9, α=.95 (not seasonally adjusted)

$$Y_{1t} = .4051Y_{1t-1} + .2420Y_{1t-2} - .0878Y_{1t-3} + .3780Y_{1t-4} + a_{yt}$$

$$\tilde{C}_t = -.0179Y_{1t-1} + .0307Y_{1t-2} + .0133Y_{1t-3} + .0169Y_{1t-4}$$

$$+ .0086Y_{1t-5} + .7913\tilde{C}_{t-1} + .2122\tilde{C}_{t-2} + a_{ct}$$

Marginal confidence of likelihood ratio test: .9934

Set 5

Maximum likelihood under rational expectations with B=.9, α=.95 (not seasonally adjusted)

$$Y_{1t} = .3765Y_{1t-1} + .2887Y_{1t-2} - .1010Y_{1t-3} + .3843Y_{1t-4} + a_{yt}$$

$$C_t = .0200Y_{1t-1} + .0785Y_{1t-2} + .0336Y_{1t-3} + .0400Y_{1t-4}$$

$$+ .0221Y_{1t-5} + .4937C_{t-1} + .4948C_{t-2} + a_{ct}$$

Marginal confidence of likelihood ratio test: .9908

Observations on Dependent Variables Span 1948III-1972IV

Footnotes

1/ Similarly, Hall's [1977] proposition that consumption is an
uncaused (in Granger's sense [1969]) first-order Markov process will not
in general hold in Lucas and Rapping's framework. To see this in Hall's
context, make instantaneous utility depend on current consumption and
employment, confront the household with an exogenous real-wage process
a la Lucas and Rapping, and then calculate the household's Euler equations
for employment and consumption.

2/ Equation (e) could be replaced by a Phillips curve without
affecting the properties of interest here. I use the simple specification
(e) here in order to emphasize that the exogeneity of Y_t with respect to
C_t in this model is not a result of imposing classical labor market
assumptions.

3/ See Henderson and Sargent [1973] or Sargent [1975].

4/ That G_t does not appear in the reduced form for N_t establishes
the lack of a short-run effect of G_t on N_t or Y_t (for "short-run" read
"given K_t"). It is possible for the G process to affect subsequent Y_t's
through the effects of G_t on I_t and therefore subsequent values of K.
This is the usual "classical" route by which fiscal policy affects the
rate of growth of output, though not its level in the "short run."

5/ The reader may question the exclusion of capital from the
production function (a). However, see the time series regressions
reported by Bodkin and Klein [1967] and Lucas [1970]. The exclusion of
capital from (a) together with its inclusion in the aggregate demand
schedule for labor (b) is rationalized theoretically by Lucas [1970].

$\underline{6/}$The reader is invited to work things out where (c) is replaced
by

$$r_t = h(\log N_t - \log K_t) + E_t(\log p_{t+1} - \log p_t) + \varepsilon_{3t},$$

where $E_t(x)$ is again the linear least squares forecast of the random
variable x. With this modification the two conditions in the text
continue to imply that Y_{pt} is strictly exogenous in the consumption
function. One of the virtues of Tobin's model is that it readily delivers
procyclical movements of the interest rate even if variance in M_t is
the prime force behind the cycle and even if $E_t(p_{t+1} - p_t)$ is omitted
from the right side of (c). Compare this with the predictions of
standard Keynesian models with downward sloping "IS" curves.

$\underline{7/}$Actually, the results below are valid so long as the
information set that the public uses to forecast includes at least
$C_t, C_{t-1}, \ldots, Y_t, Y_{t-1}, \ldots$.

$\underline{8/}$This is an implication of the two theorems proved by Sims [1972].

$\underline{9/}$As will be noted in the text below, the estimator proposed
in this section is statistically consistent under weaker conditions than
those that have been imposed on the u_t's in this paragraph.

$\underline{10/}$Hall [1977] has recently suggested that the permanent
income hypothesis implies that aggregate consumption follows an uncaused
(in Granger's sense) first-order Markov process. Within the context of
the consumption schedule (1), sufficient conditions for Hall's proposition
are (i) that theoretically permanent income follows an uncaused first-
order Markov process and (ii) that consumption is an exact linear function
of permanent income alone, u_t being identically zero. In general, under
the definition (2) of permanent income in the text, permanent income is

not predicted to be an uncaused first-order Markov process. (A process X_t is said to be uncaused relative to Ω_t in Granger's sense if $EX_{t+1}|\Omega_t$ is only a function of X_t, X_{t-1}, ... where Ω_t is an information set including at least X_t, X_{t-1}, ...) To see this, notice that Y_{pt} given by (2) is a solution to the stochastic difference equation

$$Y_{pt} = (1-\alpha)Y_t + \alpha E_t Y_{pt+1} \qquad\qquad 0 < \alpha < 1.$$

Rearranging we have

$$(*) \qquad E_t Y_{pt+1} = \frac{1}{\alpha} Y_{pt} - (1-\frac{1}{\alpha})Y_t.$$

In general, permanent income Y_{pt} does not follow an uncaused first-order Markov process, i.e., $E_t Y_{pt+1}$ is not a function of Y_{pt} alone; but there are two possible ways of restricting things to get that special result. The first way involves driving the discount factor α to unity. Heuristically, (*) indicates that with $\alpha=1$, permanent income follows a random walk (i.e., an uncaused first-order Markov process with Markov parameter unity). For this argument to be rigorous, permanent income must be redefined as a Cesaro sum,

$$Y_{pt} = \lim_{n\to\infty} (\frac{1}{n} \sum_{j=0}^{\infty} E_t Y_{t+j}),$$

since Y_{pt} given in (2) is not well defined with $\alpha=1$. A second way of making $E_t Y_{pt+1} = Y_{pt}$ would be first to redefine permanent income as

$$(**) \qquad Y_{pt} = (1-\alpha) \sum_{j=0}^{\infty} \alpha^i E_t Y_{t+j+1}.$$

Then follow John F. Muth [1960] and require that Y_t follow the special process $Y_t = Y_{t-1} + a_t - \lambda a_{t-1}$, $|\lambda| < 1$, where a_t is the process of one-step-ahead errors in predicting Y_t from its own past. Muth showed that

with this process, definition (**) implies that permanent income follows
a random walk for any $|\alpha| < 1$.

[11]/See Whittle [1963, p. 32]. It is necessary to assume that
Y_t possesses an autoregressive represenation, i.e., that b(z) is invertible.

[12]/For a proof of Wold's theorem, see T. W. Anderson [1971].

[13]/Christopher Sims points out that in small samples using T
rather than (T-k), where k is the number of parameters on the right-hand
side of a typical equation under the unrestricted parameterization,
biases the test statistic against the null hypothesis. As it will
turn out, correcting for this bias will not reverse any of our conclusions.

[14]/The sacrifice of information occurs in the step leading to
equation (15). The estimator imposes only the restriction across the
systematic part of the vector autoregression that is contained in (15).

[15]/An example can easily be constructed along the lines of the
model described by Sargent [1976]. In particular, unexpected movements in
u_t are permitted to have effects on disposable income via the sort of
mechanism described by Sargent [1973].

[16]/The following two-step estimator is appropriate where Y is
econometrically exogenous in the consumption function, so that all contem-
poraneous correlation between a_{yt} and a_{ct} is appropriately attributed to
the workings of the consumption function. First estimate (8) by least

squares. Then obtain the eigenvalues and eigenvectors of A, and estimate

(14) by forming $(1-\alpha)dPH_\alpha P^{-1}x_t$ for various values of α and searching over

α for the value that delivers the minimum sum of squared residuals in (14).

The resulting estimates of B and α are consistent on the assumptions of

the model, though they aren't fully efficient. The model could be tested

by adding to the righthand side of (14) current, past, and future values

of Y and testing the null hypothesis that these intruding variables bear

zero coefficients. Finding non-zero coefficients on future Y_t's would

reflect negatively on the assumed exogeneity of Y in (1).

[17]/Notice that (c_y, y_t) depends on ρ.

[18]/The "chain rule of forecasting" is a simple implication of

the "law of iterated projections." That law states that where E is the

linear least squares projection operator, and y, x, z are random variables,

(†) $\qquad E(y|z) = E(E(y|x,z)|z)$.

This law is easily proved as an implication of the fact that linear least

squares projections are uniquely determined by the condition that their

forecast errors be orthogonal to the conditioning variables. To get the

chain rule of forecasting, shift (21) forward one period, then project

both sides of the result on information available at (t-1), applying (†).

An analogous "law of iterated expectations" and chain rule of forecasting

are also wellknown properties of conditional mathematical expectations.

[19]/Here I am assuming that agents care about and see their

seasonally unadjusted income flow.

The position taken in this section is not at all unanimously accepted by serious students of rational expectations models. Sims [1976] has made an ingenious argument supporting the opposite position, that even if agents are responding to the seasonally unadjusted data, it is advisable to use seasonally adjusted data to implement and test rational expectations cross-equation restrictions. Sims's argument has several elements, one of which is a desire to have procedures that are robust to departures from stationarity together with the suspicion that departures from stationarity are in some sense most likely at the seasonal frequencies. The argument advanced in the text is predicated on the assumption of stationarity in the sense that the δ's of section 3 and the a, b, c, and d's of section 4 are required to be constant over time. (As mentioned earlier, though, we <u>can</u> tolerate nonstationarity in the sense of maximum eigenvalues of the transition matrix A exceeding unity, so long as $|\alpha\lambda| < 1$.)

21/Let the national income identity be

$$C + I + G + \delta K = Y$$

where Y is real GNP, I is real investment, C is real consumption, G is real government purchases and δK is depreciation. Then we have

$$Y_1 = Y - T - \delta K$$
$$= C + I + G - T$$

where T is the flow of real tax collections net of transfers. Hence

$$Y_2 = C + I = Y_1 - (G - T) .$$

22/For discussions of discounting of taxes and appropriate definitions of wealth and disposable income see Bailey [1972] and Sargent [1975].

23/The reader of Wallis [1974] and especially Sims [1974] will

not be surprised that the seasonally adjusted and seasonally unadjusted

data give substantially different results with respect to Granger-causality.

24/Hall's [1977] hypothesis that consumption is not Granger-caused

by income is generally soundly rejected for the seasonally adjusted data

summarized in Table 1b; on the other hand, with respect to some but not

all information sets, Hall's hypothesis cannot be rejected on the basis

of the seasonally unadjusted data in Table 1a. But the regressions

underlying Table 1a rather sharply contradict Hall's hypothesis that

consumption is a first-order Markov process as opposed to a higher-order

process, which is no surprise with seasonally unadjusted data.

25/Once again we note that using the test statistic

$T\{\log |V_r| - \log |V_u|\}$ rather than $(T-k)\{\log |V_r| - \log |V_u|\}$ where k is the number of

parameters estimated in a typical equation under the unrestricted para-

meterization, biases the test against the null hypothesis. In our case,

using $(T-k)$ rather than T to define the test statistic would not materially

affect our conclusions.

26/In discussions.

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Appendix on Data

Construction of Not Seasonally Adjusted Data

The six series, Y_1, Y_2, C, I, $\tilde{I}$, and $\tilde{C}$, are generated from fourteen series of quarterly data, <u>1947I</u> through <u>1972IV</u>.

The first thirteen are from "<u>The National Income and Product Accounts of the United States, 1929-74 Statistical Tables</u>," <u>a supplement to the Survey of Current Business</u>, published by the Department of Commerce/ Bureau of Economic Analysis.

All of the series are quarterly totals, not seasonally adjusted, in current dollars, with the exception of Capital Consumption Allowance, for which we have quarterly totals at annual rates, seasonally adjusted.

The series are:

		<u>Table</u>	<u>Line #</u>	<u>Page</u>
X1.	GNP	1.22	1	62
X2.	Consumption Durables	2.50	2	86
X3.	Consumption Nondurables	2.50	6	86
X4.	Consumption Services	2.50	12	86
X5.	Investment Nonresidential	1.22	8	86
X6.	Investment Residential	1.22	11	62
X7.	Change in Business Inventories	1.22	15	62
X8.	Capital Consumption Allowances With Capital Consumption Adjustment	1.90	2	22
X9.	State and Local Government Receipts	3.50	1	116
X10.	Federal Government Receipts	3.30	1	104
X11.	Federal Government Transfers to Persons	3.30	11	104
X12.	State and Local Transfers to Persons	3.50	9	116
X13.	Federal Government Grants-In-Aid	3.30	13	104

X14. The last series was obtained from the National Bureau of Economic Research Data Bank. It consists of quarterly averages of the monthly Consumer Price Index, all items 1967=100. Source is the U.S. Department of Labor.

Construction of the Seasonally Adjusted Data

The series Y_1, Y_2, C, I, $\tilde{I}$, and $\tilde{C}$ are generated from fourteen series of quarterly data, 1947I through 1972IV. They are all located in the statistical tables supplement of the National Income and Product Accounts. All are seasonally adjusted in current or 1972 dollars. Those in current dollars are deflated by the GNP implicit price deflator.

The series are:

		Table	Line #	Page
1.	GNP	1.5	1	14
2.	Consumption Durables	2.4	2	82
3.	Consumption Nondurables	2.4	6	82
4.	Consumption Services	2.4	12	82
5.	Investment Nonresidential	1.1	8	2
6.	Investment Residential	1.1	11	2
7.	Change in Business Inventories	1.1	15	2
8.	Capital Consumption Allowance	1.9	2	22
9.	State and Local Government Receipts	3.4	1	110
10.	Federal Government Receipts	3.2	1	98
11.	Federal Government Transfers to Persons	3.2	31	98
12.	State and Local Transfers to Persons	3.4	42	110
13.	Federal Government Grants-In-Aid	3.2	33	98
14.	GNP Implicit Price Deflator	7.1	1	264

The series are defined as follows:

$$Y_1 = [(X1-X8/4.-X9-X10+X11+X12+X13)/X14] \cdot 100$$

$$Y_2 = C + I$$

$$C = [(X2+X3+X4)/X14] \cdot 100$$

$$I = [(X5+X6+X7)/X14] \cdot 100$$

$$\tilde{I} = I + (X3/X14) \cdot 100$$

$$\tilde{C} = C - .9875(X3/X14) \cdot 100 + .025D_t$$

where $D_t = .975(X3/X14) \cdot 100 + .95D_{t-1}$ and $D_{47-I} = 215$.