

Federal Reserve Bank of Minneapolis  
Research Department Staff Report 234

July 1997

## **On the Size of U.S. Government: Political Economy in the Neoclassical Growth Model**

Per Krusell\*

University of Rochester, CEPR,  
and Institute for International Economic Studies

José-Víctor Ríos-Rull\*

Federal Reserve Bank of Minneapolis  
and University of Pennsylvania

### ABSTRACT

We study a dynamic version of Meltzer and Richard's median-voter analysis of the size of government. Taxes are proportional to total income, and they are used for government consumption, which is exogenous, and for lump-sum transfers, whose size is chosen by electoral vote. Votes take place sequentially over time, and each agent votes for the policy that maximizes his equilibrium utility. We calibrate the model and its income and wealth distribution to match postwar U.S. data. This allows a quantitative assessment of the equilibrium costs of redistribution, which involves distortions to the labor-leisure and consumption-savings choices, and of its benefits for the decisive voter. We find that the total size of transfers predicted by our political-economy model is quite close to the size of transfers in the data.

\*Both authors gratefully acknowledge support from the National Science Foundation. Krusell also gratefully acknowledges support from the Bank of Sweden Tercentenary Foundation. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

# 1 Introduction

Countries differ widely in their public policy choices. For example, among the OECD countries, 1983 marginal tax rates on capital income varied between  $-90\%$  and  $49.5\%$ ; the average labor income tax varied between  $24\%$  and  $63\%$ ; and the net total tax burden as measured by public expenditures varied between  $26\%$  and  $54\%$  of GNP. <sup>1</sup> Even larger differences would be revealed if one included countries on a lower level of development. Similarly, large differences in policies can also be recorded within given countries over time. To the extent one thinks actual policy outcomes affect economic performance, it seems an important task for economists to explore the origins of these wide disparities.

One way to build a positive theory of policy is to assume that policies are chosen optimally. If they are indeed, then the large observed policy differences can only depend on differences in economic primitives such as preferences and technology. In contrast, the political-economy paradigm seeks to analyze how different policy-selection procedures/collective choice mechanisms affect policy outcomes. With this approach, differences in policy outcomes depend not only on mentioned primitives and on differences in population characteristics, but they also depend on the details of the procedure by which policies are selected.

Our framework of analysis in this paper builds on Meltzer and Richard (1981)'s study of the size of government. In their analysis, the median voter has lower than average labor productivity, and thus, a proportional tax on labor used for making lump-sum subsidies implies a net transfer to this agent. Since labor supply is elastic, the political equilibrium tax rate is chosen by this agent so as to equate his marginal utility benefit from the transfer to his marginal utility loss from the tax distortion.

Our extension of Meltzer and Richard's analysis introduces both wealth heterogeneity and government consumption and makes the model dynamic. In particular, our setup has a population which is heterogeneous in two respects: labor productivity and asset wealth. One determinant of the equilibrium transfer level here is, therefore, the joint distribution of labor productivity and asset holdings. We calibrate our model so that its distribution of labor incomes and asset holdings conforms with U.S. data. This calibration, thus, indirectly determines the benefits of raising taxes from the viewpoint of the median voter. We restrict the analysis by imposing that there is a single proportional tax on all income.

---

<sup>1</sup>See McKee, Visser, and Saunders (1986) for data sources.

The quantitative assessment of the costs of redistributive taxation rests in part on assumptions about the key elasticities in the model: those assumptions governing the labor-leisure choices and those determining the consumption-savings choices. For this, we select parameters in line with studies using similar macroeconomic models. The costs of redistributive taxation also depend on the level of government consumption, which we treat as exogenous here. We divide the different items of the government budget into those which can be considered transfers and those which cannot and calibrate government consumption in the model accordingly.

Given our calibration, the political equilibrium transfer level turns out to be quite close to that in the data: measured as a percentage of GDP, the equilibrium transfer level is within a few percentage points of its observed value. Relatedly, the income tax rate is close to that observed: around 30%. For comparison, we also solve a calibrated static model, and we find that equilibrium taxes and transfers there are much too high (e.g., the tax rate is around 60%). The dynamic model is quite robust to changes in parameter values—economic and political ones—and to changes in the interpretation of what constitutes transfers in the data.

To use Meltzer and Richard's redistributive-taxation setup as a starting point for a quantitative study of the size of government requires some defense. Recent cross-country comparisons (see, e.g., Perotti (1996)) cast some doubt on our maintained hypothesis, namely, that distributional issues are key for understanding equilibrium transfer levels, since they often fail to detect a positive relationship between measures of wealth or income dispersion and transfer levels. In our view, however, the existing empirical work is far from being able to draw definite negative conclusions about the role of redistributive taxation. First, the data are often not directly comparable across countries. (Perotti's work and that of others are exemplary in attempting to remedy these problems, however.) Further, measurements for given variable definitions are often poor, implying that the number of countries in the sample may fall significantly with more stringent measurement standards. In addition, and more importantly from our perspective, cross-country comparisons always have the problem that a large number of factors which influence outcomes and which differ across countries are abstracted from. Such factors include a number of standard economic variables. For example, it may be important, as we show in our model below, whether inequality takes the form of wealth or earnings inequality, and the unidimensional measures of inequality used in the literature are unlikely to capture this fact. Perhaps even more importantly, political factors such as constitutional differences as well as social/cultural differences are likely to be important in the determination of

policy. Most of the latter are very difficult to measure, and our view is that it is at least *ex ante* worthwhile to attack the problem by focusing on one country, where many of these factors are likely to not vary much. This allows us to think more about the logic of how details of constitutions and the different elements of inequality affect outcomes. Relatedly, although we do not pursue this line in the current paper, a study of the time series of policy in a given country may also prove informative as to the role of redistributive taxation. For example, Saint-Paul (1995) argues that the observed trends toward political “conservatism” may be understood by noting that while income and wealth dispersion have increased recently, the median-to-mean ratios have narrowed. In the same spirit, but using a different set of data, Husted and Kenny (1997) argue that the median-voter framework is useful for understanding the effects of the expansion of the voting franchise on the size of government.

The extension to a dynamic framework is important for several reasons. First, given our quantitative interests, it allows us to take into account not only the labor-leisure distortion caused by an income tax but also the consumption-savings distortion implied by the taxation of capital income. As shown in the optimal taxation analysis by Chamley (1986) and followers, at least in relative terms, the capital tax has important welfare consequences. Thus, since in practice, the income tax base includes capital income, it seems important to incorporate it into the transfer-benefit vs. distortion-loss calculation by the median voter.

Second, the dynamic framework allows us to analyze several normative experiments which have important dynamic elements. Although the political framework in this paper is quite rudimentary, it admits some nontrivial constitutional experiments. In particular, we consider changes in the frequency of elections/policy reevaluations and implementation lags for policy. But it is clear that an explicitly dynamic framework is also necessary for investigating such constitutional features as a balanced budget rule, social security legislation, and so on. Although we do not study these issues here, since they would involve a different population structure, the methods we develop in this paper can be used for such analyses.

Third, one important purpose of our work is to develop tools that can be used for analyzing a broader set of dynamic models of political economy with maximizing agents. In general, such models are much harder to study than the corresponding static models, and they are also an order of magnitude more complex than standard dynamic models without political-economy elements. The main complication is introduced by dynamic voting. In particular, when agents vote, they

need to rationally predict the effects of current policy alternatives (i) on current and future prices, and (ii) on future policies. In political science theory, there are similar analyses where voters make correct forecasts and are fully rational. (See Danzau and Mackay (1981) for a two-period model, and Epple and Kadane (1990) for a multiperiod setup with uncertainty.) It has been stressed in these contexts that Condorcet cycles may occur due to a violation of single-peakedness of preferences. Our framework has the added features that the preferences over the policy are induced via the economic equilibrium and not considered primitives and that the time horizon is infinite. The requirement that voters make rational forecasts and that policy preferences be derived explicitly in an infinite-horizon equilibrium model clearly complicates the analysis, but we view these features as unavoidable if one wishes to address our types of questions in quantitatively reasonable macroeconomic settings. Our approach, therefore, is to proceed with numerical analysis.<sup>2</sup> A natural part of the paper is, therefore, our reporting throughout of how the choice of model parameters affects the results.

The outline of the rest of the paper is as follows. The first two sections are theoretical. Section 2 describes the economic framework and discusses the determination of steady states and dynamics under the assumption that policy is exogenous. Section 3 then introduces politics and shows how policy endogeneity changes the findings in the previous section. The quantitative analysis is contained in Section 4. In our calibration section (Section 4.1), we first characterize U.S. data on labor income and wealth (Section 4.1.1). We then discuss choices of constitutional characteristics (Section 4.1.2) and of preference and technology parameters (Section 4.1.3). The results from our baseline specification are then presented in Section 4.2. We start with the static version of the model—a calibrated Meltzer and Richard model—and proceed to the dynamic version. Section 5 describes comparative statics with respect to both economic and political parameters; the latter are our constitutional experiments. Our conclusions can be found in Section 6. The Appendix contains formal equilibrium definitions, our computational algorithm, and a simple example.

---

<sup>2</sup>Other recent dynamic political-economy models (for example, Glomm and Ravikumar (1992), Bertola (1993), Perotti (1993), Saint-Paul and Verdier (1993), Alesina and Rodrik (1994), and Persson and Tabellini (1994)) make one of several shortcuts: they consider intertemporal models which are not fully dynamic, *i.e.*, which do not require forward-looking, they restrict voting to a subset of the population which is not concerned about the future, or they simply assume that voters are not fully rational when predicting the effects of changes in current policy. The analysis in Krusell and Ríos-Rull (1996) is fully dynamic and analytical, but it assumes a small number of agents to limit the size of the state space. For a discussion of these issues, see Krusell, Quadrini, and Ríos-Rull (1997).

## 2 Neoclassical Growth with Income and Wealth Distribution

We consider a straightforward extension of Meltzer and Richard (1981)'s static model to a dynamic macroeconomic setup. In this section, we describe the economics of the model, and in the next section, we lay out the politics.

Agents are infinitely lived, derive utility from streams of consumption and leisure, and have access to competitive borrowing and lending markets in order to allocate resources over time. There is no uncertainty. Production takes place with constant returns to scale to labor and capital inputs, and the final output can be used one-for-one for either consumption or investment. Capital depreciates geometrically. Each agent has one unit of time available for either leisure or work, but the productivity at work may differ across agents.

Agents obtain their income from three sources: capital income, labor income, and a government transfer. Capital and labor income sources are taxed at a common, proportional rate (capital income is taxed net of depreciation), and the transfer is lump-sum and the same for all agents. We also assume that the government budget is balanced in all periods.

A typical agent, thus, faces the following maximization problem:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \text{ s.t.}$$

$$c_t + a_{t+1} = a_t(1 + r_t(1 - \tau_t)) + w_t \epsilon_t (1 - l_t)(1 - \tau_t) + T_t,$$

where  $c$  denotes consumption,  $l$  leisure,  $a$  asset holdings,  $r$  the rental rate of capital,  $w$  wage per efficiency unit of labor,  $\tau$  the income tax rate, and  $T$  the government transfer. The parameters  $\beta$  and  $\delta$  are assumed to be in  $(0, 1)$ , and  $\epsilon > 0$  is the individual's productivity parameter. We, furthermore, assume that  $u(c, l) = \lim_{\hat{\sigma} \rightarrow \sigma} ((c^\alpha l^{1-\alpha})^{1-\hat{\sigma}} - 1) / (1 - \hat{\sigma})$ . As we shall show below, this utility function gives rise to aggregation if economic policy is exogenous.

We assume that the heterogeneity in the population is of two kinds. First, agents differ in initial asset holdings, and second, they differ in labor productivity. We assume for simplicity that there is a discrete number  $I$  of types of agents with a measure  $\mu_i$  of type  $i$  agents. Each type of agent  $i$  is thus associated with an initial asset holding  $a_{i0}$  and a labor productivity level  $\epsilon_i$ . We normalize so that  $\sum_{i=1}^I \mu_i = \sum_{i=1}^I \mu_i \epsilon_i = 1$ . In the theoretical section below, we will work with 2 agents, since it is not necessary to use more than 2 agents to illustrate the main workings of the model. In the applied section, we will work with 3 types of agents, since this allows us to capture some of the key

features of the income and wealth distributions in a parsimonious way.

Given perfect competition, the rental rate and the wage rate are given by

$$r_t = F_1(K_t, N_t) - \delta \text{ and } w_t = F_2(K_t, N_t),$$

where  $F$  is the aggregate production function and  $K$  and  $N$  are the aggregate inputs of capital and labor in efficiency units, respectively.

The government budget constraint reads

$$G_t + T_t = \tau_t(K_t r_t + N_t w_t),$$

where  $G_t$  is government consumption. We take  $G_t$  as given exogenously throughout the paper, and we do not model how agents benefit from it.<sup>3</sup>

Finally, the resource constraint is

$$C_t + K_{t+1} - K_t(1 - \delta) + G_t = F(K_t, N_t),$$

where  $C_t$  is aggregate consumption.

## 2.1 Steady states and exogenous policy

One of the main reasons for studying a model with complete markets and infinitely-lived agents is that it allows us flexibility in matching the income and wealth distributions observed in U.S. data: under quite weak assumptions, this kind of framework allows any relative asset and labor income distributions as a steady state with constant policy. To see this, note that with a standard neoclassical production function, the steady-state equations can be summarized as follows.

First, the Euler equation for asset accumulation, which is the same for all agents given that they have the same discount factor and face the same rate of return on savings, reads

$$\frac{1}{\beta} = 1 + r(1 - \tau). \tag{1}$$

This equation pins down the rental rate and, therefore, the capital-labor ratio as well as the wage rate.

---

<sup>3</sup>The issue of how agents benefit from  $G_t$  would be an important one if  $G_t$  were regarded as endogenous.

Further, the labor-leisure choice is characterized by

$$u_1(c_i, l_i)w\epsilon_i(1 - \tau) = u_2(c_i, l_i)$$

for all  $i$ . The steady-state version of the budget constraint for each agent can be used to eliminate  $c_i$  for all  $i$ . This leaves  $I + 1$  equations in  $2I$  unknowns:  $(a_i, l_i)$  for all  $i$ .<sup>4</sup> Thus, the set of steady-state income and wealth distributions is typically of dimension  $I - 1$ .

Some specific examples are useful for illustration. First, if leisure is not valued, the above equations reduce to  $\sum_{i=1}^I \mu_i a_i = K^*$ , where  $K^*$  solves the rate-of-return equation. Thus, there is only one possible capital stock, and any relative asset distribution is possible. Furthermore, by choice of the individual productivity parameters, the distribution of labor income can be chosen independently of the wealth distribution.

Second, because  $u(c, l)$  is homothetic in our particular case, the equation system is linear in asset and leisure levels, and although the capital-labor ratio is pinned down, the total capital stock depends on the distribution and is generically indeterminate as well. More specifically, for any period utility function which can be written  $u(c, l) = \tilde{u}(c^\alpha l^{1-\alpha})$  for some increasing and concave  $\tilde{u}$ , the capital stock is pinned down independently of the distribution: it is given jointly with the total amount of labor effort by (i) the rate-of-return requirement on the capital-labor ratio from above and (ii) the aggregate version of the first-order condition for the labor choice:

$$\alpha(1 - N_t)F_2(K_t, N_t)(1 - \tau) = (1 - \alpha)(F(K_t, N_t) - \delta K_t - G_t). \quad (2)$$

Hence, our steady-state theory imposes no restrictions at all on the relative distributions of wealth and labor income.

For the purpose of graphical illustration, assume that there are two types of agents in the economy. Figure 1 describes the set of steady-state asset distributions.

**Figure 1 here**

The set of steady states is linear, with slope  $-\mu_1/\mu_2$  and an intercept indicating the capital stock which, along with a value for the total labor supply, yields an after tax rate of return that equals the rate of time preference (equation (1)) and the aggregate version of the labor-leisure

---

<sup>4</sup>The statement follows from replacing prices and transfers with known functions of the  $a_i$ 's and the  $l_i$ 's using market clearing in capital and labor and the government's budget.



first-order condition (equation (2)). Negative values for assets are possible, provided consumption is nonnegative for each type of agent. The figure illustrates clearly that any relative distribution of asset wealth is possible. Moreover, notice that we can select  $a_1$  and  $a_2$  independently of  $\epsilon_1$  and  $\epsilon_2$ . Therefore, any steady-state correlation between asset and labor income wealth is possible.

Changes in the exogenous tax rate will change the capital stock (capital-labor ratio) and will, hence, shift the steady-state line up or down, with unchanged slope. Changes in the exogenous tax rate, thus, do not change the set of relative wealth distributions that can arise as steady states. One of the main purposes of the ensuing analysis is to determine the set of steady-state asset distributions for this setup when taxes are endogenous.

How general is the result that there is a large set of steady states? First, agents may have differences in their period utility functions, but differences in discount factors cannot be accommodated, nor can nonadditive preferences in general. Second, the fact that the time horizon is infinite is important. Overlapping-generations structures without altruism do not give rise to the kind of steady-state Euler equation characterizing steady states in this economy.<sup>5</sup> Third, it is necessary that agents face the same return on savings. For example, if markets are incomplete, so that some agents are constrained in asset holdings, or if marginal tax rates are different for different agents, then the result does not generally apply. Fourth, individual uncertainty can be accommodated as long as market completeness is maintained.<sup>6</sup>

## 2.2 Aggregation for exogenous policy

The main focus of our analysis is on the political determination of the tax and transfer sequence. We analyze an environment where, absent the endogeneity of policy variables, an aggregation theorem applies. In particular, we will make assumptions on preferences such that aggregate outcomes do not depend on the distribution of earnings abilities nor on the initial distribution of asset holdings when policies are exogenous. This means that heterogeneity *per se* is inconsequential for aggregate outcomes, so that any effect on aggregates from changes in the distribution of wealth is solely due to politics.

---

<sup>5</sup>The same is true for overlapping-generations models where altruism is either restricted or of the nature that different generations of the same family have different preferences over the same consumption baskets.

<sup>6</sup>For examples of models with unique steady states based on (i) differences in (endogenously determined) discount factors, see Boyd III (1994); (ii) progressive taxes with differences in discount factors, see Sarte (1995); (iii) nondynastic populations, see Auerbach and Kotlikoff (1987) and Ríos-Rull (1996); and idiosyncratic, partially uninsurable risk, see Imrohoroglu (1989), Aiyagari and Gertler (1991), Diaz-Gimenez, Prescott, Alvarez, and Fitzgerald (1992), Huggett (1992), Huggett (1993), and Aiyagari (1994).

Consider, therefore, an arbitrary sequence of taxes, transfers, and government expenditures. It is easy to show that the static optimal consumption and leisure choice satisfies

$$\epsilon_i l_{it} = \frac{1 - \alpha}{\alpha} \frac{c_{it}}{w_t(1 - \tau_t)}. \quad (3)$$

That is, the supply of labor in efficiency units is  $\epsilon_i(1 - l_i)$ , and it is linear in consumption, with an intercept which is different for agents with different productivity levels and a slope coefficient which is the same for all agents. Using this condition, we can rewrite the present-value budget constraint as follows:

$$\sum_{s=t}^{\infty} \frac{p_s}{p_t} c_{is} = \alpha \left( a_{it}(1 + r_t(1 - \tau_t)) + \sum_{s=t}^{\infty} \frac{p_s}{p_t} (w_s \epsilon_i(1 - \tau_s) + T_t) \right) \equiv \alpha \omega_{it},$$

where  $p_s$  denotes the price of consumption good at  $t$  in terms of the numéraire, consumption at 0 ( $p_0 = 1$ ). The variable  $\omega_{it}$  is, thus, defined as the lifetime wealth of agent  $i$ , i.e., the present value of his current and future human wealth (which we define as the market value of the time endowment of the agent) and transfers together with the current holding of capital (nonhuman wealth).

Again employing the consumption-leisure first-order condition, we can express the intertemporal Euler equation in terms of consumption at  $t$  and  $t + 1$ :

$$\left( \frac{c_{i,t+1}}{c_{it}} \right)^\sigma = \beta \frac{p_t}{p_{t+1}} \left( \frac{w_t(1 - \tau_t)}{w_{t+1}(1 - \tau_{t+1})} \right)^{(1-\alpha)(1-\sigma)}.$$

Here,  $c_{i,t+1}$  can be solved for as a function of  $c_{it}$ : the relationship between these two variables is linear, and the slope coefficient does not depend on individual-specific variables (such as  $\epsilon_i$  or  $a_i$ ). Thus, future values of consumption can be successively substituted into the budget constraint, and consumption and leisure at date  $t$  can be solved for

$$c_{it} = e_c(z_t) \omega_{it}$$

and

$$\epsilon_i l_{it} = e_l(z_t) \omega_{it},$$

where  $z_t$  is defined as an infinite vector of current and future (as of  $t$ ) prices and taxes and the  $e$ 's

are functions thereof.<sup>7</sup> Furthermore, in our formulation,

$$p_t/p_{t+1} = 1 + (f'(k_{t+1}) - \delta)(1 - \tau_{t+1}), \quad (4)$$

where  $k$  is the capital-labor ratio and  $Nf(k) \equiv F(K, N)$ . Similarly, the wage rate equals  $f(k) - kf'(k)$ .

We know that the wage and rental rates are functions only of the capital-labor ratio. This capital-labor ratio, in turn, equals

$$k_t = \frac{K_t}{\sum_{i=1}^I \mu_i \epsilon_i (1 - l_i)} = \frac{K_t}{1 - e_l(z_t) \Omega_t}, \quad (5)$$

where  $\Omega_t$  is total wealth, i.e.,  $\sum_{i=1}^I \mu_i \omega_{it}$ . Furthermore, total wealth satisfies

$$\Omega_t = \sum_{i=1}^I \left( \mu_i a_{it} r_t (1 - \tau_t) + \sum_{s=t}^{\infty} \frac{p_s}{p_t} (f(k_t) - k_t f'(k_t)) (1 - \tau_s) \epsilon_i + T_{it} \right),$$

which allows us to write

$$\Omega_t = K_t (f'(k_t) - \delta) (1 - \tau_t) + \sum_{s=t}^{\infty} \frac{p_s}{p_t} ((f(k_t) - k_t f'(k_t)) (1 - \tau_s) + T_t), \quad (6)$$

where  $T_t = \tau_t (K_t (f'(k_t) - \delta) (1 - \tau_t) + (K_t/k_t) (f(k_t) - k_t f'(k_t))) - G_t$ . Finally, the economy's resource constraint reads

$$K_t f(k_t)/k_t - K_{t+1} + (1 - \delta) K_t - G_t = C_t = \sum_{i=1}^I \mu_i e_c(z_t) \omega_{it} = e_c(z_t) \Omega_t. \quad (7)$$

We now have a system of equations that completely determines the equilibrium evolution of aggregate variables. This system of equations consists of, for each  $t$ , (i) the aggregate resource constraint (7); (ii) the equation determining the capital-labor ratio (5); (iii) the equation determining total wealth (6); and (iv) the equation for relative prices at  $t$  and  $t + 1$  (4).<sup>8</sup> These equations embody consumer and firm optimization and market clearing.

It is clear at this point that aggregation obtains: no relative wealth or labor income variables

<sup>7</sup>More precisely,  $e_c(z_t) \equiv \alpha/\Sigma_t$  and  $e_l(z_t) \equiv (1 - \alpha)/(w_t(1 - \tau_t)\Sigma_t)$ , where  $\Sigma_t \equiv \sum_{s=0}^{\infty} \beta^{s/\sigma} [(p_t/p_{t+s})(w_t(1 - \tau_t)/(w_{t+s}(1 - \tau_{t+s})))^{1-\alpha}]^{(1-\sigma)/\sigma}$ .

<sup>8</sup>We omit transversality conditions for simplicity. For the present purpose, they can be omitted, since they are met either for no agent or for all agents.

enter into this equation system. Individual consumption, leisure, and asset levels are solved for recursively: given a solution to the above equation system for aggregate capital and the capital/labor ratio and given an initial asset position for consumer  $i$ , we can determine all we need to know about this consumer.<sup>9</sup>

In graphical terms, what the aggregation result means is indicated in Figure 2. Suppose that taxes are given by a constant sequence  $\tau_t = \tau$ , so that the steady-state line is described as in Figure 2. An initial condition with total capital  $K_0$  is a point  $(a_{10}, a_{20})$  such that  $\mu_1 a_{10} + \mu_2 a_{20} = K_0$  (the line  $t-t$  in the figure); this is a line which is parallel to the steady-state line.

**Figure 2 here**

Equilibrium implies consumption choices among agents implying a next period's capital stock  $K_1$  and an asset distribution  $(a_{11}, a_{21})$  depicted on line  $t+1-t+1$  in Figure 2. The content of the aggregation theorem is to dictate that the location of this line is *independent* of the distribution of the initial capital stock  $K_0$  among agents. An alternative initial distribution, say  $(\tilde{a}_{10}, \tilde{a}_{20})$ , which is also on line  $t-t$ , will move the economy to a new distribution  $(\tilde{a}_{11}, \tilde{a}_{21})$  with the same total capital stock as  $(a_{11}, a_{21})$ , i.e., to a point on the line  $t+1-t+1$ . Note also that initial conditions with the same total capital stock but a different asset distribution will remain different over time, and, to the extent total capital converges, the asset distributions will converge to different points on the steady-state line. In other words, there is not enough equalization of incomes and wealth in this model over time so as to eliminate the importance of initial relative wealth levels. This fact also helps us interpret the large set of steady states in this model.<sup>10</sup>

### 3 Political Equilibrium

We now introduce the political element of the model. The goal of the exercise is to describe a way in which  $\tau_t$  and, thus,  $T_t$ , the size of the government transfer program, are determined at all points in time. The mechanism of collective choice which we consider here is the same as that in Meltzer and Richard's paper: agents vote on the size of taxes, and tax outcomes coincide with those preferred by the median voter. It should be pointed out that other collective choice mechanisms have been

---

<sup>9</sup> Aggregation obtains also for a more general preference structure. For details, see Krusell and Ríos-Rull (1997).

<sup>10</sup> Chatterjee (1994) investigates the implications of different initial capital stocks on the behavior of the distribution of wealth, where wealth is interpreted in present-value terms and as a sum of human and nonhuman wealth. Caselli and Ventura (1996) explore the evolution of asset wealth and labor income in the same kind of framework.

considered in similar models. For example, a recent paper by Grossman and Helpman (1995) considers an overlapping-generations setup with two alternative choice mechanisms: one with a government with preferences over consumption of currently young and old agents and one where in addition young and old agents can lobby the government regarding its tax policy choice. We chose the median-voter setup mainly to coincide as closely as possible with Meltzer and Richard, but we agree with the view that this political mechanism only captures the political process imperfectly. It should prove fruitful to extend our analysis to alternative political contexts.

Our quantitative analysis below describes a setup in which taxes are voted on every two periods, with a period representing one year, and in which taxes are kept constant in between votes. To facilitate presentation of the political equilibrium mechanism in this section, however, we discuss a simpler framework: there is a vote every period on the income tax to be applied the next period.<sup>11</sup> Specifically, consider pairwise majority-voting contests between tax rates, so that a chosen tax rate is one which defeats all other tax rates. If the voters' preferences over tax rates are single-peaked or, less restrictively, if their preferences over tax/transfer pairs satisfy *hierarchical adherence* as defined in Roberts (1975), then the tax outcome will coincide with the median voter's preferred tax, where the median refers to a ranking of preferred tax rates in the population.

To derive the agents' preferences over tax rates, we follow Meltzer and Richard by assuming that agents think through the equilibrium effects of each tax policy and calculate their associated utility levels. This assumption is of course also simplistic; we abstract from altruistic motives, and we abstract from other reasons for differences in tax preferences such as asymmetric information (see, e.g., Piketty (1995)). We also do not consider strategic voting: by assumption, agents simply report their preferences even though no single individual agent can ever affect a voting outcome.<sup>12</sup> It is important to note that to follow Meltzer and Richard's analysis in spirit in a dynamic model is quite demanding: the agents in the model need to rationally think through all the consequences of different tax choices, and these consequences extend into the infinite future. Nevertheless, this is the route we follow.

---

<sup>11</sup>If the choice were over the current tax, capital income taxation would be nondistortionary.

<sup>12</sup>Note that what is crucial about this assumption is that the reason agents vote in real-world elections does not play any role for the way the agent votes.

### 3.1 Recursive political equilibrium

We will rely on recursive methods in defining and computing equilibria for this economy. In this section, we discuss the key elements of our equilibrium definition, and the next section discusses how the endogeneity of policy alters the equilibrium characterization compared to when taxes are exogenous. The full statement of the equilibrium definition is made in the Appendix, where we also describe the numerical method we use for computing equilibria and go through a simple example along the lines of Section 3.2.

The calculation of the indirect preference over tax rates proceeds in the following way. We restrict attention to equilibria which satisfy a Markov property: the voting outcome in period  $t$  only depends on the “minimum state variable.” By this, we mean any information that would be necessary in order to compute a dynamic equilibrium were taxes exogenously chosen.<sup>13</sup> The key state variable is the current distribution of asset wealth, i.e., the distribution of the initial capital stock. From here on, we denote this variable  $A \equiv (A_1, A_2, \dots, A_I)$ , with  $A_i$  representing the equilibrium asset holding of a type  $i$  agent. In addition, we need to include the current tax rate, since different values for the current tax rate imply different post-tax wealth distributions. Thus, our equilibrium tax rate next period,  $\tau_{t+1}$ , equals  $\Psi(A_t, \tau_t)$ . In other words, the main object of our paper is to solve for the function  $\Psi$ .

Given the equilibrium law of motion for taxes, consider an agent who contemplates a current vote. This agent needs to compare all possible tax choices  $\tau_{t+1}$ . For this, the agent needs to have a view on how both current and future events are affected by the choice of  $\tau_{t+1}$ . These events include the evolution of prices, capital, and asset holdings of the different types of agents as well as the evolution of tax rates. Our assumption here amounts to subgame-perfectness or, in macro terminology, time consistency of equilibrium: by varying the tax currently under consideration, the agent takes into account the equilibrium response of all future variables. This equilibrium response has both an economic and a political part: future asset holdings (and total capital) respond to  $\tau_{t+1}$  as dictated by (forward-looking) equilibrium savings behavior on the part of all agents, and future taxes respond to equilibrium votes as given by  $\Psi$ . In particular, when an agent thinks about a specific vote, the agent does not view it as possible to consider any future tax paths but is restricted to those which will arise as equilibrium responses to each current tax choice.

---

<sup>13</sup>Equivalently, this information is what a Ramsey planner with preferences over different types of agents would need in order to calculate an optimal plan by choosing sequences of tax rates.

To be more specific, two different tax choices  $\tau_{t+1}^1$  and  $\tau_{t+1}^2$  would lead to different current and future savings,  $\{A_{t+1}^1, A_{t+2}^1, \dots\}$  and  $\{A_{t+1}^2, A_{t+2}^2, \dots\}$ , and to different future taxes,  $\{\tau_{t+1}^1, \Psi(A_{t+1}^1, \tau_{t+1}^1), \Psi(A_{t+2}^1, \Psi(A_{t+1}^1, \tau_{t+1}^1)), \dots\}$  and  $\{\tau_{t+1}^2, \Psi(A_{t+1}^2, \tau_{t+1}^2), \Psi(A_{t+2}^2, \Psi(A_{t+1}^2, \tau_{t+1}^2)), \dots\}$ , where the evolution of the asset distribution in each case takes as given the corresponding evolution of tax rates. In particular, on the equilibrium path, the asset distribution evolves according to  $A' = H(A, \tau)$ , and on the paths where taxes are arbitrary for one period (which the voters need to contemplate to form their political preferences), the assets evolve according to  $A' = \tilde{H}(A, \tau, \tau')$  during the first period and according to  $H$  thereafter. Thus, the equilibrium evolution of taxes and assets can be completely described by applying the functions  $H$  and  $\Psi$  to any initial asset distribution and tax rate.

The forward-looking aspect of the dynamic political equilibrium makes computation nontrivial, and few analytical results can be obtained in this kind of model. Numerical characterization of equilibrium, however, is feasible.

### 3.2 Equilibrium characterization: steady states and dynamics

What are the properties of equilibria when taxes are endogenous? First, consider the problem of determining the set of steady states. In terms of equations, we need to add the condition that taxes be constant over time:

$$\tau = \Psi(A, \tau). \quad (8)$$

This equation, together with the rate-of-return equation (1), the aggregate labor supply equation (2), and asset market clearing ( $\sum_{i=1}^I \mu_i A_i = K$ ), determines the set of steady states. The unknowns are  $A$ ,  $N$ ,  $K$ , and  $\tau$ , i.e.,  $I+3$  variables, and there are 4 equations. So provided that the  $\Psi$  function exhibits nontrivial, nongeneric dependence on the  $A$  vector, the dimension of the set of steady states is the same as before (where we had one fewer equation but where  $\tau$  was exogenous).

For illustration, consider an economy with two types of agents and no differences in labor efficiency. It is easy to show for this case that equal asset wealth and  $\tau = 0$  is a steady state.<sup>14</sup> This is because there can be no transfer gain from taxation in such a situation, and any deviation from nondistortionary taxes is bad for both agents (more generally,  $\Psi(A, \tau) = 0$  has to hold whenever  $A_1 = A_2$ , including off steady state). Suppose, furthermore, that  $\mu_2 > \mu_1$ , so that agent 2 is the median voter. Consider first the set of steady states when taxes are zero exogenously, and

---

<sup>14</sup>For details, see an earlier version of this paper, Krusell and Ríos-Rull (1994).

consider a point  $(A_{10}, A_{20})$  on the steady-state line such that  $A_{20} < A_{10}$ , as depicted in Figure 3A. This point will not be a steady state, since it will benefit the median voter to impose a positive tax. A somewhat more detailed, heuristic argument for this would go as follows: imagine first that as taxes are changed, transfers do not provide net redistribution (but that they are adjusted to balance the government's budget).<sup>15</sup> Then the best possible tax rate for the median voter would be the zero tax, since it has no distortion (imposes zero costs)—the transfer benefit from changing the tax is zero by construction, and the cost is positive. Now allow a change in the tax to affect the net transfer income of the median, and it should be clear that the benefits from a tax increase must exceed the costs, since costs are of second order.

By the above arguments, the economy will leave the steady-state line associated with zero taxes and go from  $(A_{10}, A_{20})$  to a point such as  $(A_{11}, A_{21})$ , which is southwest of the steady-state line with zero taxes. The new point is to the southwest of the steady-state line with zero taxes because  $\tau_1$  has to be positive. Moreover, assuming that the economy will converge to a steady state and assuming monotonicity of the new path, we show the sequence of asset levels in Figure 3B. The steady state with endogenous taxes achieved starting from  $(A_{10}, A_{20})$  is, thus, associated with a positive tax and a lower total capital stock. A symmetric argument applies when  $A_{20} < A_{10}$ . Thus, in the case where the type 2 agent is politically pivotal, the steady-state line with endogenous taxes has a *steeper slope* than the corresponding steady-state line with exogenously set zero taxes. (Conversely, when type 1 is the median agent, the slope is flatter.)

### Figures 3A and 3B here

The issue, of course, in our quantitative section is how much steeper the steady-state line is. More precisely, and still expressed in terms of the 2-agent example, we will take as given a relative wealth ratio  $(A_2/A_1)^*$  given by the one observed in recent U.S. time series data, interpret it as a steady state, and ask what tax rate supports it. This means that we will use a “reasonably” parameterized version of our model and ask what point on the steady-state line is intersected by the line  $A_2 = (A_2/A_1)^*A_1$ . That point defines a level of the total capital stock, and we can then use the steady-state equations to find the corresponding tax rate and size of government.

---

<sup>15</sup>Of course, to be able to formally support a steady state in this model, it is necessary to study off-steady-state paths. In other words, the median voter needs to have the view that remaining in status quo is better than any alternative. Since alternatives in this kind of model necessarily involve dynamics, this means that the median voter needs to calculate through equilibrium dynamics for off-steady-state tax choices.



## 4 Quantitative Analysis of the Size of Government

In this section, we use the model developed above to analyze the transfers actually implemented in the U.S. economy. To do this, we use a version of the model discussed above. In terms of the mapping between model and data, note that the model implicitly treats households as part of integrated dynasties; i.e., they are altruistic toward their descendants. We base the choice of an infinitely-lived agent economy on the arguments in Kotlikoff and Summers (1981); they find that a very small fraction of the economy's wealth can be accounted for by life-cycle savings motives. Note also that we are using a balanced-budget assumption and that transfers are lump-sum and equal for all agents. In the U.S., a large part of transfers consist of social security payments. Given the progressivity built into this system, we find this simplifying assumption a reasonable one.<sup>16</sup>

### 4.1 Calibration

We now describe in detail how we choose a specific parameterization of the model economy so that it reproduces certain features of the U.S. economy. We begin with the properties of the joint distribution of wealth and earnings, and we then move on to the constitutional parameters and to preferences and technology parameters.

#### 4.1.1 The distribution of earnings and wealth

We decompose the population into three types of agents that make up 49%, 2%, and 49% of the population, with the group with the smallest size being the group that contains the median agent. The 49–2–49 partition is designed to make the median group small, allowing a precise identification of the political preferences of the pivotal voter.<sup>17</sup>

Households differ in both wealth and earnings, and since the model economy has fewer individuals than the actual economy, there are several ways of mapping the actual population into our three groups. We use two alternative procedures. The first of these is to sort agents according to wealth and record earnings in each group: assign the 49% of the agents with the lowest wealth to the first group, the next 2% to the middle group, and the final 49% to the third group, and then compute

---

<sup>16</sup>See Cubeddu (1995) for an analysis of the social security system.

<sup>17</sup>We could not make the median group much smaller, since that would introduce large sampling errors when measuring this group's earnings and wealth from the Survey of Consumer Finances.

average wealth and average earnings of each of these groups.<sup>18</sup> The alternative procedure is the same but ranks according to earnings. As will be seen, the two procedures do lead to different joint asset/earnings distributions, and our numerical findings will tell how these differences matter for policy outcomes. The sorting according to wealth makes wealth inequality the most extreme possible, and the sorting according to earnings makes earnings inequality the most extreme possible. Thus, the two ways of sorting should allow us to bracket our results.

The voters do not only care about the size of the transfer (net of taxes) and the magnitude of the intertemporal distortion that the policies generate, but they also care about the relative price of labor and capital. For this reason, the relative composition of income matters. As an example, suppose that the efficiency to wealth ratio of a voter is higher than the economy's average. Then this voter would like to see higher wages relative to rental rates of capital than would the average agent, and as a consequence the voter would favor policies which discourage others from working.

Table 1 shows the per capita amounts of wealth and earnings (normalized so that the middle group has wealth and earnings of 1) for the three groups of households for each of our two sorting criteria.

Household type	1	2	3
Percentiles involved	0-49	49-51	51-100
<b>Sorted by wealth</b>			
Per household wealth	0.30	1	4.78
Per household earnings	0.57	1	1.91
<b>Sorted by earnings</b>			
Per household wealth	0.55	1	2.93
Per household earnings	0.24	1	2.94

Table 1: Distributional statistics for the U.S. economy: all households.

Next, Table 2 shows the same kind of statistics for households in different age brackets. We have chosen three age groups (according to the age of the household head): up to 40 years of age, 41 to 65, and older than 65 (the earnings-based older group is excluded, since most of the households in this group are retired and have zero earnings). Note in Tables 1 and 2 that the difference in earnings and wealth dispersion between the different life-cycle groups is nontrivial. Recall that we have assumed that households are dynastic and that social security transfers can be represented

<sup>18</sup>The data come from the 1992 Survey of Consumer Finances. For details, see Díaz-Gimenez, Quadrini, and Ríos-Rull (1997).

Percentiles involved	Young: 0-40			Middle: 41-65			Older: 66+		
	0-49	49-51	51-100	0-49	49-51	51-100	0-49	49-51	51-100
<b>Sorted by wealth</b>									
Per household wealth	0.25	1	4.36	0.34	1	4.97	0.37	1	4.97
Per household earnings	0.77	1	2.14	0.60	1	2.15	0.67	1	3.07
<b>Sorted by earnings</b>									
Per household wealth	0.59	1	2.32	0.62	1	3.70			
Per household earnings	0.45	1	2.23	0.34	1	2.54			

Table 2: Distributional statistics for the U.S. economy: selected age groups.

by lump-sum transfers. This may mean that a partition of the population of all households into three groups may give the wrong picture of how different dynasties differ with respect to earnings and wealth. In the choice between the above sorting criteria, we decided to use as a baseline calibration target the data for the middle-aged group, since we believe *a priori* that it is the best representation of the dynastic household. For most of our experiments below, we will thus use the sorting according to wealth for the middle-aged group. However, we also report results for the other sortings.

We calibrate the distribution of wealth  $A$  (recall that, in this model, differences in wealth across agents are perpetuated) and the distribution of efficiency units of labor  $\epsilon$  so that equilibrium wealth and earnings of the model economy reproduce the properties of Tables 1 and 2. For the distribution of wealth, this is trivial, since the model economy's wealth distribution is an initial condition and in this sense exogenous. The earnings distribution is slightly more complicated, since it involves each agent's choice of how many hours to work.

#### 4.1.2 Constitutional characteristics

In this model, we set the length of time between elections to 2 years. This is a potentially important variable. For our baseline calculations, we take the length of time between renewals of the House of Representatives (given its budgetary powers) as the appropriate time interval. In each period, there is a vote over next period's tax rate; i.e., the policy implementation lag is one year.

#### 4.1.3 Preferences and technology

The economic period is set to one year so that it coincides with our baseline voting frequency. The choice of functional forms for preferences is an important one here, since it will determine the

elasticities that in effect determine the distortionary costs of taxation. As regards intertemporal substitution, we employ additive time separability, which simplifies the structure and allows a direct identification of the key elasticities. Given this separability, the only period utility functions which allow balanced growth are CRRA functions, which are what we employ. The elasticity with intertemporal relevance is given by the parameter  $\sigma$ , and it will be important in determining the distortions from capital taxation.

The distortions from taxing labor involve the labor supply elasticity. Furthermore, the importance of wealth effects in labor supply cannot be understated in our model: to the extent labor supply is decreasing (increasing) in wealth, a tax on labor works as an indirect subsidy (tax) to wealth. The applied macro literature has tended to use the type of period utility we employ here.<sup>19</sup> This formulation is useful for matching an important growth fact: hours worked have remained roughly constant over a long period of time. For the “macro preferences,” with consumption and real wages growing at the same rate, the choice of hours worked is constant (this fact follows directly from equation (3)). This result comes from substitution and wealth effects which cancel each other out: higher wages make leisure more expensive in relative terms, so people work more; on the other hand, higher wages make people richer, so they work less. The fact that substitution and wealth effects cancel out is also useful for matching the cross-sectional distribution of hours: hours worked are not systematically related to wages (“most” workers work about the same number of hours, independently of the wage rate the worker faces).

Within the class of period utility functions we are thus restricted to, the share on consumption,  $\alpha$ , can be used to vary the labor supply elasticity. In the steady state of the model,  $\alpha$  pins down  $l$ , the amount of chosen leisure, and given this, the ( $\lambda$ -constant) labor supply elasticity is given locally by  $l/(1-l)$ .<sup>20</sup> When we vary our labor supply elasticity in the result section below, we will, thus, vary  $\alpha$  with resulting variations in hours worked in the steady state.

Turning to technology, we choose a Cobb-Douglas production function,  $y = K^\theta N^{1-\theta}$ , for standard reasons: it allows us to obtain a balanced growth path in a growing version of the model economy with constant input shares.

The calibration of preferences and technology parameters involves the choice of the discount rate,  $\beta$ , the risk aversion parameter,  $\sigma$ , the consumption share,  $\alpha$ , the share of capital income,  $\theta$ , and

---

<sup>19</sup>An important exception is Greenwood, Hercowitz, and Huffman (1988), who use a formulation without wealth effects on labor supply.

<sup>20</sup>For a comparison of different elasticity concepts, see McLaughlin (1995).

the depreciation rate,  $\delta$ . To calibrate these 5 parameters and the size of government consumption,  $g$ , we impose 5 aggregate conditions that the model economy has to satisfy, and we use a coefficient of risk aversion of 4 (which is a commonly used value in the real-business-cycle literature). The 5 conditions that we impose on our yearly model are

1. A wealth-to-output ratio of 2.6.
2. A rate of return (before taxes) of around 6%.
3. A consumption-to-GDP ratio of 0.65 (the 1960–95 average was 0.64, and between 1990 and 1995, the ratio averaged 0.68).
4. A government expenditure-to-GDP ratio of 0.19.
5. An average time allocation to market activities of around one-third of the total time endowment.

A few of these statistics require discussion. First, the time allocation does, as argued above, imply a labor supply elasticity since it implies a value of  $\alpha$ . The elasticity obtained is, thus,  $(2/3)/(1/3)=2$ . This number is quite high compared to labor studies, which tend to find numbers less than one (see, e.g., Pencavel (1986) and Killingsworth and Heckman (1986)), but our number is standard in the macroeconomic literature. We will compute the sensitivity of the results to changes in  $\alpha$ .

Second, the government expenditure-to-GDP ratio was calculated as follows, with 1995 as an example year. In 1995 total current expenditures from the government (a total of its federal, state, and local components) amounted to 32.2% of GDP. Of this, net interest on public debt amounted to 2.5% of GDP. This leaves 29.7% of GDP, which is spent either on pure transfers or on goods and services. Since some transfers take the form of goods and services, we chose to measure  $g$  “indirectly”: we distinguish different kinds of transfers and then measure  $g$  as whatever is left after these transfers are deducted. We, thus, define transfers as consisting of two components: (i) those which are received lump-sum by all households (where, as in our theory, a household is a dynastic construct),  $T_1$ , and (ii) those which can be regarded as income security,  $T_2$ . The main components of each of these are as follows.  $T_1$  consists of Medicare and Social Security, which are largely not means-tested or income-dependent, and amounts to 6.8% of GDP;  $T_2$  consists of state and local transfers labeled public assistance—estimated as 2.7% of GDP—and federal government programs

for Medicaid and income security purposes net of grants to the states, which amounts to 1.1% of GDP. Thus, we have

$$\text{government outlays} - \text{interest on debt} = g + T_1 + T_2 = 29.7\%,$$

with  $T_1 = 6.8\%$  and  $T_2 = 2.7 + 1.1 = 3.8\%$ . This leads to an estimate for  $g$  of 19.1%.<sup>21</sup> The exclusion of  $T_2$  from  $g$  has the drawback of implying that the total exogenous tax pressure in the model is slightly too low. On the other hand, including it as part of  $g$  would be a misrepresentation. An alternative is to include  $T_2$  explicitly in the model as a transfer to the lowest income/wealth group only (it would, thus, be a lump-sum payment which depends on “type,” which is not endogenous). In this alternative, the level of  $T_2$ , or its relation to total transfers or to GDP, would be treated as exogenous. We report results for this alternative specification in Section 5.3. Finally, note that our identification of transfers makes education in the form of public schooling part of  $g$ . There are two specific arguments for this. First, schooling can be regarded as a public good more than a direct income transfer and, therefore, fits better in  $g$ . Second, funding is local and people choose where to live, which means that public schooling in many ways is close to a private good (i.e., the net transfer is small in practice and may mainly occur across households with and without schoolchildren). For comparison, however, we also report results in Section 5.3 for the case where public schooling expenditures are regarded as pure transfers.

Table 3 shows the the calibrated parameters and key steady-state relations for our baseline model.

$\beta$	$\alpha$	$\sigma$	$\delta$	$\theta$
0.96	0.429	4	0.05	0.36
$\sum_i \mu_i A_i / Y$	$r$	$C/Y$	$g/Y$	$N$
3.3	6.0%	63.8%	19.9%	0.34

Table 3: Parameters and steady-state properties for the baseline economy.

<sup>21</sup>These calculations build on the 1997 *Economic Report of the President* and the 1996 *Statistical Abstract of the United States*.

## 4.2 Results: the static vs. the dynamic model

For comparison, we start by reporting some results from a static version of the model—a calibrated Meltzer and Richard model. The static version is a one-period model with the same (intra-temporal) preference and technology structure as in the dynamic model. Furthermore, the static version has an inelastically supplied stock of capital. In the calibration of the static model, we used the following principles: (i) the parameter values for preferences and technology were set to the same values as in our calibrated dynamic model; and (ii) the capital stock was set to an amount such that the total resources available from capital income are of the same order of magnitude as capital income in the dynamic model. The consumer’s budget is, thus,

$$c_i = [w\epsilon_i(1 - l_i) + a_i\tau](1 - \tau) + T.$$

The results for this model are that politico-economic equilibrium taxes are much too high (around 60%). Given this, with the same preference parameters as in the dynamic model, hours worked are too low (around 0.2). Note here that a decreased tax rate would result if labor supply were made *more* elastic than in the standard macro formulation or if capital were elastically supplied. Given that the labor supply elasticity already is high (at least compared to findings in the labor literature), we move to elastically supplied capital: the dynamic model.

For our calibrated dynamic economy, we obtain equilibrium values for total tax rates and for transfer-to-GDP ratios as reported in Table 4. In the table, we tabulate the findings for each of the different wealth/earnings calibrations which result from the choices of how to sort agents: M refers to identifying the households with the middle-aged group in the actual economy, A to the whole population, and Y to the young; and E and W refer to the sortings by earnings and wealth, respectively. The table shows that the tax rates and transfer-to-GDP ratios implied by

	Base					
	M W	M E	Y W	Y E	A W	A E
$\tau$	31.08	31.51	32.46	27.31	27.39	33.59
$T/Y$	6.15	6.61	7.58	2.57	2.45	8.77

Table 4: Equilibrium fiscal policy in the model economies.

the dynamic Meltzer and Richard economy are of quite reasonable magnitude. For tax rates, the different sortings lead to a range of between about 27% and 33% of GDP, depending on the

income/wealth distribution employed. For transfers, the order of magnitude is between around 3% and 9%. Our most preferred sorting, that according to wealth and earnings distributions for the middle-aged group, gives transfers between 6% and 7% of GDP. Recall that our estimates of the observed U.S. transfers are 6.8% for those transfers which are not means-tested and 10.6% if means-tested transfers are included.

Compared to the results from the static version of the model, the dynamic baseline economy gives results which are remarkably close to the data. We now turn to sensitivity analysis.

## 5 What Model Features Affect the Results?

In this section, we vary some parameters of interest and examine the effects on the size of government. We have both economic and political parameters to investigate. Let us consider each in turn as perturbations of the baseline economy.

### 5.1 Distributional properties

We start by changing the distributional properties of the economy. We do this by changing the wealth and wages of the two large groups by 5% (one change at a time).

Table 5 describes the effects of changing the distributional characteristics in a variety of ways. Given that we are dealing with a median-voter model, the qualitative results are that increases in the wealth or earnings of the middle group (other groups) lower (raise) tax rates and transfers. These effects of changing the income and wealth distributions are also underlying the differences in policy outcomes in Table 4 among different sorting criteria.<sup>22</sup>

Quantitatively, what we take away from Table 5 is that (i) large changes in tax rates or transfer levels do not result from small to moderate changes in distributional characteristics and (ii) that changes in the earnings distribution seem to have somewhat larger effects than changes in the asset distribution.

### 5.2 Changes in the economic parameters

The effects of the key economic parameters are reported in Table 6. Consider first the increase in hours worked, which is brought about by an increase in  $\alpha$ . This increase implies a labor supply

---

<sup>22</sup>We performed the same experiments with the baseline based on the earnings ranking. The results are very similar.



Model economy	$\tau$	$T/Y$
Baseline	31.08	6.15
$1.05A_m, 1.05\epsilon_m$	28.99	4.84
$1.05A_R$	31.10	6.15
$1.05A_m$	31.07	6.14
$1.05A_P$	31.09	6.15
$1.05\epsilon_R$	32.59	7.71
$1.05\epsilon_m$	29.01	4.85
$1.05\epsilon_P$	31.50	6.57

Table 5: Equilibrium fiscal policies with alternative distributions.

Model economy	$\tau$	$T/Y$
Baseline	31.08	6.15
Hours worked = 0.69	50.69	24.12
$\sigma = 8$	32.21	6.84
5% reduction in $\theta$	30.92	5.20

Table 6: Equilibrium fiscal policies with alternative parameterizations.

elasticity of a little less than one-half, which is more in line with what has been found in the labor literature. With less elastically supplied labor, taxation is more severe, with the total tax rate increasing to around 50% and the transfer-to-GDP ratio to 24%.

An increase in  $\sigma$  from 4 to 8 represents a decrease in the intertemporal elasticity of substitution, and this leads to more taxation. We see from the table that a doubling of  $\sigma$  leads to a little bit more than a one percentage point increase in the income tax rate.

A decrease in  $\theta$ , the capital share, lessens the importance of capital relative to labor. Capital is distributed in a more skewed fashion—the median is much poorer relative to the mean—and this should account for the resulting drop in the size of government. The size of this effect is quite small, however.

Changes in the remaining parameters  $\delta$  and  $\beta$ , the depreciation rate and the discount rate, are closely related to changing the length of the time period. Such a change is quite nontrivial in our baseline framework. With voting every period, a change in the length of a period also means a change in the period of time over which the tax voted on is held constant. In addition, it changes the length of time before a voted policy is implemented, given that our baseline framework has a one-period implementation lag. For these reasons, it is more informative to consider the simpler

experiments of separately changing constitutional parameters such as the frequency of elections and the policy implementation lag.

### 5.3 Changes in the definition of transfers

We consider two alternative specifications: one in which public schooling expenditures are considered pure transfers and one in which we explicitly include means-tested transfers in the analysis as transfers that are made only to the poorest agent type.

Public schooling expenditures total around 5% of GDP. In the experiment, we change  $g$  so that  $g/Y$  decreases to 14.3%. With this lower value of total government consumption, we find that the equilibrium value of  $T/Y$  goes up (from 6.15% to 11.53%) and that the tax rate barely changes (it goes down from 31.1% to 30.9%). In other words, the lower value of  $g$  makes taxation less distorting, and transfers go up by about as much as  $g$  goes down! Thus, the model's predictions are still remarkably close to the data.

When we treat the two kinds of transfers separately, with  $T_1$  representing transfers which are the same for all agents and  $T_2$  being a "means-tested" transfer given only to the poorest agent type, we see an increase in the income tax rate of about one percentage point. Thus, we set  $T_2$  so that  $T_2/Y$  equals 3.8%, and we find that  $T_1/Y$  equals 4.0%. Thus, total transfers equal a little short of 8%, which should be compared with 10.6% in the data (recall that in the data,  $T_1/Y = 6.8%$ ). That is, we again find that when total exogenous expenditures/transfers change, the endogenous transfers change in an offsetting direction by a similar amount.

### 5.4 Changes in the constitutional parameters

To recall, our baseline framework is one with a one-year model period with voting every two years on taxes to be implemented one year after the vote. Table 7 shows the effects of changing the constitutional parameters. Qualitatively, Table 7 shows the following: increases in the election frequency and increases in the implementation lag increase both tax rates and transfer levels. These results are somewhat surprising in light of our earlier findings in Krusell and Ríos-Rull (1994), where the same constitutional changes have the opposite effect on taxes. The logic behind why, say, an increased implementation lag would lower taxes in the political equilibrium in that paper goes as follows. Capital responds more to changes in taxes if the taxes are levied further in the future, since capital is more elastically supplied in the future. As a result, any amount of

Model	Elect. freq. Years	Impl. lag Years	$\tau$	$T/Y$
Base	2	1	31.08	6.15
Base w. lag	2	2	31.40	6.34
Long	4	1	31.37	6.33
Long w. lag	4	4	32.33	6.92
Short	1	1	30.93	6.05

Table 7: Equilibrium fiscal policies with alternative political constitutions.

redistribution via capital taxation is more costly, so the median voter chooses to do less of it.<sup>23</sup> In the present paper, however, taxation also affects labor supply. This complicates the picture and, as is clear from the table, involves a counteracting effect which overturns the qualitative features of the economy with exogenous labor supply.

To understand the effects of timing changes on the median voter's preferred labor taxes, it is first important to note that labor taxation in this model is an indirect subsidy to wealth: higher wealth leads to fewer hours worked in our calibrated framework. As an illustration of what this implies for equilibrium tax rates, we studied a framework where agents do not differ in labor productivity and where the tax base does not include capital income. In such an economy, as expected, the political equilibrium involves a negative tax on labor provided the median voter has less than average wealth. In Tables 4–7, the earnings and wealth distribution are calibrated to match U.S. data, and the positive equilibrium tax rates seen in those tables simply reflect the fact that the median voter has both less than average labor and capital income.

Second, timing changes do not directly affect labor taxes, since the labor decision is static. However, timing changes do have an indirect effect to the extent that (i) wealth effects on labor supply are present (which, again, they are here), and that (ii) wealth differs between agents (which it does). Going back to our example with only labor income taxation, we verified that timing changes lead to changes in tax rates if and only if there are differences in wealth among agents (whether or not agents differ in earnings). Moreover, the direction of the change depends on whether the median voter is above or below the average in asset holdings. Everything else equal, the median

<sup>23</sup>This logic is similar to why capital should not be taxed in the standard neoclassical framework (see, e.g., Chamley (1986)). Also, note that the same kind of argument works for an increase in the election frequency, since in our framework, election frequency determines the period over which taxes are constant and committed to. Thus, less frequent elections mean that capital is more elastic and responds more to changes in taxes, which leads to lower equilibrium tax rates.

voter, who has less than average asset holdings, understands that a longer implementation lag will make the tax on wealth (which comes from the subsidy part of labor income in his optimal choice for the labor tax rate) more distorting. He will, thus, choose less of this tax than when the implementation lag is shorter. Since this tax takes the form of a subsidy to labor income, it means that the tax on labor income goes up.

Quantitatively, the effects of the timing aspects of the constitution are quite small. A doubling of the election period, together with a quadrupling of the policy implementation lag, increases equilibrium taxes by less than 2 percentage points.

## 6 Conclusions

We have revisited Meltzer and Richard's analysis of the size of government in the context of a dynamic model. We found that we can account for something quite close to the observed amount of transfers in the United States with a model which we consider reasonably calibrated. We compared the implications from the dynamic model to those from the corresponding static model and found that the dynamic model constitutes a substantial improvement in terms of ability to explain tax rates.

The results are sensitive mainly to the assumption regarding the elasticity of labor supply, and we find that the elasticity used in the macroeconomic literature gives more realistic tax and transfer outcomes than do the (lower) elasticities estimated in the labor literature. Whether this reflects a merit of the macroeconomic practice and of the present model or a failure of both of them is an open question.

Changes in other parameters of the model lead to less drastic effects on equilibrium tax rates, with changes in the distribution of earnings playing a more important role in relative terms. Constitutional changes in the form of changes in timing—changes in election frequency and policy implementation lags—do not lead to large effects on equilibrium tax and transfer rates.

## References

- Aiyagari, S. R. 1994. Uninsured Idiosyncratic Risk and Aggregate Saving. *Quarterly Journal of Economics*, 109, 659–684.

- Aiyagari, S. R. and Gertler, M. 1991. Asset Returns with Transactions Costs and Uninsured Individual Risk. *Journal of Monetary Economics*, 27, 311–331.
- Alesina, A. and Rodrik, D. 1994. Distributive Politics and Economic Growth. *Quarterly Journal of Economics*, 109(2), 465–490.
- Auerbach, A. and Kotlikoff, L. 1987. *Dynamic Fiscal Policy*. Cambridge University Press, New York.
- Bertola, G. 1993. Factor Shares and Savings in Endogenous Growth. *American Economic Review*, 83(5), 1184–1210.
- Boyd III, J. H. 1994. The Existence of Steady States in Multisector Capital Accumulation Models with Recursive Preferences. RCER Working Paper #392, University of Rochester.
- Caselli, F. and Ventura, J. 1996. A Representative Consumer Theory of Distribution. Mimeo.
- Chamley, C. 1986. Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica*, 54, 607–622.
- Chatterjee, S. 1994. Transitional Dynamics and the Distribution of Wealth in a Neoclassical Growth Model. *Journal of Public Economics*, 54, 97–119.
- Cooley, T. F. 1995. *Frontiers of Business Cycle Research*. Princeton University Press, Princeton, New Jersey.
- Cubeddu, L. M. 1995. *The Role of Household Composition, Formation and Dissolution in Life Cycle Economies: Theory and Evidence*. Ph.D. thesis, University of Pennsylvania.
- Danzau, A. and Mackay, R. 1981. Structure-Induced Equilibria and Perfect-Foresight Expectations. *American Journal of Political Science*, 25(4), 762–779.
- Diaz-Gimenez, J., Prescott, E., Alvarez, F., and Fitzgerald, T. 1992. Banking in Computable General Equilibrium Economies. *Journal of Economic Dynamics and Control*, 16, 533–559.
- Díaz-Gimenez, J., Quadrini, V., and Ríos-Rull, J.-V. 1997. Dimensions of Inequality: Facts on the U.S. Distribution of Earnings, Income, and Wealth. Forthcoming, *Federal Reserve Bank of Minneapolis Quarterly Review*.

- Epple, D. and Kadane, J. 1990. Sequential Voting with Endogenous Voter Forecasts. *American Political Science Review*, 84(1), 165–175.
- Glomm, G. and Ravikumar, B. 1992. Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality. *Journal of Political Economy*, 100(4), 818–834.
- Greenwood, J., Hercowitz, Z., and Huffman, G. W. 1988. Investment, Capacity Utilization and the Business Cycle. *American Economic Review*, 78(3), 402–18.
- Grossman, G. and Helpman, E. 1995. Intergenerational Redistribution. Mimeo.
- Huggett, M. 1992. The One Sector Growth Model with Idiosyncratic Risk. Mimeo.
- Huggett, M. 1993. The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies. *Journal of Economic Dynamics and Control*, 17(5/6), 953–969.
- Husted, T. A. and Kenny, L. W. 1997. The Effect of the Expansion of the Voting Franchise on the Size of Government. *Journal of Political Economy*, 105, 54–82.
- İmrohoroğlu, A. 1989. The Cost of Business Cycles with Indivisibilities and Liquidity Constraints. *Journal of Political Economy*, 97(6), 1364–1383.
- Killingsworth, M. R. and Heckman, J. J. 1986. Female Labor Supply: A Survey. In Ashenfelter, O. C. and Layard, R. (Eds.), *Handbook of Labor Economics: Volume 1*, pp. 103–204. North-Holland, Amsterdam.
- Kotlikoff, L. and Summers, L. 1981. The Role of Intergenerational Transfers in Aggregate Capital Accumulation. *Journal of Political Economy*, 89(4), 706–732.
- Krusell, P., Quadrini, V., and Ríos-Rull, J.-V. 1997. Politico-Economic Equilibrium and Economic Growth. *Journal of Economic Dynamics and Control*, 21(1), 243–272.
- Krusell, P. and Ríos-Rull, J.-V. 1994. What Constitutions Promote Capital Accumulation? A Political Economy Approach. W. Allen Wallis Institute of Political Economy Working Paper No. 1.
- Krusell, P. and Ríos-Rull, J.-V. 1996. Vested Interests in a Positive Theory of Stagnation and Growth. *Review of Economic Studies*, 63(215), 301–331.

- Krusell, P. and Ríos-Rull, J.-V. 1997. The Effect of the Wealth Distribution on Aggregate Activity. Mimeo, University of Rochester.
- McKee, M. J., Visser, J. C., and Saunders, P. G. 1986. Marginal Tax Rates on the Use of Labour and Capital in OECD Countries. *OECD Economic Studies*, 7, 45–99.
- McLaughlin, K. J. 1995. Intertemporal Substitution and  $\lambda$ -Constant Comparative Statics. *Journal of Monetary Economics*, 35(1), 193–213.
- Meltzer, A. and Richard, S. 1981. A Rational Theory of the Size of Government. *Journal of Political Economy*, 89, 914–927.
- Pencavel, J. 1986. Labor Supply of Men: A Survey. In Ashenfelter, O. C. and Layard, R. (Eds.), *Handbook of Labor Economics: Volume 1*, pp. 3–102. North-Holland, Amsterdam.
- Perotti, R. 1993. Political Equilibrium, Income Distribution, and Growth. *Review of Economic Studies*, 60(4), 755–776.
- Perotti, R. 1996. Growth, Income Distribution, and Democracy: What the Data Say. *Journal of Economic Growth*, 1, 149–188.
- Persson, T. and Tabellini, G. 1994. Is Inequality Harmful for Growth?. *American Economic Review*, 84(3), 600–621.
- Piketty, T. 1995. Social Mobility and Redistributive Politics. *Quarterly Journal of Economics*, 110(3), 551–584.
- Ríos-Rull, J.-V. 1996. Life Cycle Economies and Aggregate Fluctuations. *Review of Economic Studies*, 63(3)(215), 465–490.
- Roberts, K. 1975. Voting over Income Tax Schedules. *Journal of Public Economics*, 8, 329–340.
- Saint-Paul, G. 1995. The Dynamics of Exclusion and Fiscal Conservatism. Mimeo.
- Saint-Paul, G. and Verdier, T. 1993. Education, Democracy and Growth. *Journal of Development Economics*, 42(2), 399–407.
- Sarte, P.-D. G. 1995. Progressive Taxation and Income Inequality in Dynamic Competitive Equilibrium. Unpublished Manuscript.

## Appendix

### A A formal definition of equilibrium

We define equilibrium recursively. First, we define economic equilibrium given that policy follows the function  $\Psi$ . Second, we turn to the fixed-point problem for  $\Psi$ . As in the theoretical section, we assume here that taxes are voted on every period; for the details of how equilibrium is defined when taxes are voted on every  $n$  periods, see Krusell and Ríos-Rull (1994).

#### A.1 Economic equilibrium

In its dynamic programming version, agent  $i$ 's problem reads as follows:

$$v_i(A, \tau, a) = \max_{c, l, a'} u(c, l) + \beta v_i(A', \tau', a') \quad \text{s.t.} \quad (9)$$

$$a' = a + (ar(K/N) + w(K/N)(1-l)\epsilon_i)(1-\tau) + T - c$$

$$T = \tau(F(K, N) - \delta K)$$

$$\tau' = \Psi(A, \tau)$$

$$A' = H(A, \tau)$$

$$K = \sum_i \mu_i A_i$$

$$N = G(A, \tau).$$

Here, we let  $a$  denote the agent's own asset holding, whereas capital letters refer to economy-wide variables. Variables without primes refer to current values, and variables with primes refer to values in the next period. A solution to the dynamic-programming problem gives next period's asset holdings as a function  $a' = h_i(A, \tau, a)$  and leisure as  $l = g_i(A, \tau, a)$ .

We define recursive competitive equilibrium in the standard way, given the function  $\Psi$ , by a set of functions  $\{H, h, G, g\}$  such that

$$H_i(A, \tau) = h_i(A, \tau, A_i)$$



and

$$G(A, \tau) = \sum_i \mu_i (1 - g_i(A, \tau, A_i)) \epsilon_i$$

for all  $\tau$ ,  $A$ , and  $i$ . These conditions represent the fixed-point problem of the recursive equilibrium formulation, i.e., the conditions require that the optimal laws of motion of the individual agents reproduce the aggregate laws of motion agents perceive when solving their decision problems.<sup>24</sup>

In the above equilibrium, taxes are given by  $\Psi$  at every point in time. In order to define our political equilibrium, we need to also consider economic equilibria where taxes are set slightly differently: one-period deviations in tax policies. Consider, therefore,  $\tau'$  to be set arbitrarily but all tax rates at later dates to be given by  $\Psi$ . These are the equilibria which the voter needs to think through when contemplating a current vote. We let  $\tilde{H}$  and  $\tilde{G}$  denote the law of motion of assets and the total labor supply function, respectively, for these deviations; these functions have  $\tau'$  as an argument. Therefore, consider the following problem for a given agent of type  $i$  who has wealth  $a$ :

$$\begin{aligned} \tilde{v}_i(A, \tau, \tau', a) &= \max_{c, l, a'} u(c, l) + \beta v_i(A', \tau', a') \quad \text{s.t.} & (10) \\ a' &= a + (ar(K/N) + w(K/N)(1-l)\epsilon_i)(1-\tau) + T - c \\ T &= \tau(F(K, N) - \delta K) \\ A' &= \tilde{H}(A, \tau, \tau') \\ K &= \sum_i \mu_i A_i \\ N &= \tilde{G}(A, \tau, \tau'). \end{aligned}$$

In this problem—where next period's tax rate is given, as opposed to determined by  $\Psi$ —it is important to note that next period's value function is given by the solution to (9). The decision rules for (10) are given by  $a' = \tilde{h}(A, \tau, \tau', a)$  and  $l = \tilde{g}(A, \tau, \tau', a)$ . The equilibrium conditions for the deviation problem are  $\tilde{H}_i(A, \tau, \tau') = \tilde{h}_i(A, \tau, \tau', A_i)$  and  $\tilde{G}(A, \tau, \tau') = \sum_i \mu_i (1 - \tilde{g}_i(A, \tau, A_i)) \epsilon_i$  for all  $\tau, \tau', A$ , and  $i$ .

---

<sup>24</sup>See, for example, Cooley (1995), chapters 1–4, for an exposition and details on this concept as well as on its computation and properties.

## A.2 Politico-economic equilibrium

Turning to the determination of  $\Psi$ , suppose that an agent  $i$  contemplates the effect of different tax rates for next period  $\tau'$ , given the family of  $\tilde{H}$  equilibria, on his realized utility. By construction, the indirect utility function  $\tilde{v}_i$  can be used directly for this purpose. All the relevant effects of the tax rate  $\tau'$  are incorporated into this function: the effect on next period's transfer and, via its effect on asset accumulation and distribution, on prices, transfers, and taxes in the future. For example, the agent perceives (correctly) that future taxes are given by  $\tau'$ ,  $\tau'' = \Psi(\tilde{H}(A, \tau, \tau'), \tau')$ ,  $\tau''' = \Psi(H(\tilde{H}(A, \tau, \tau')), \Psi(\tilde{H}(A, \tau, \tau'), \tau'), \Psi(\tilde{H}(A, \tau, \tau'), \tau'))$ , and so on.

The highest utility achievable for an agent of type  $i$  then occurs for the tax rate that solves

$$\max_{\tau'} \tilde{v}_i(A, \tau, \tau', A_i). \quad (11)$$

We denote the solution to this problem as  $\psi_i(A, \tau, A_i)$ . This function returns the most preferred value for next period's tax rate of agent  $i$ , given that at all later dates, the tax policy is given by the function  $\Psi$ .<sup>25</sup>

In our numerical computations, we verify that  $\tilde{v}_i$  is single-peaked in  $\tau'$  for all  $i$ . Based on this, a median voter theorem applies, and with the three-group distributions we consider, the median agent is always the middle group: this group is large and has intermediate values both of labor efficiency and asset wealth. We refer to median agent type with an  $m$ .

The fixed-point condition determining  $\Psi$  is, thus,

$$\Psi(A, \tau) = \psi_m(A, \tau, A_m) \quad \text{for all } A, \tau. \quad (12)$$

## B Computation

Our procedure involves linear-quadratic approximations to solve for recursive equilibria for given policies ( $\Psi$  functions), and the median voter's problem is then solved given these equilibrium functions, again using linear-quadratic approximations. If the choice of the median voter coincides with the original  $\Psi$  function, an equilibrium is found; if not, we update and continue until convergence.

We are searching for an  $I - 1$ -dimensional subspace of steady states. We first choose a grid

---

<sup>25</sup>Of course, this need not be a function, but since none of the discussion below depends on (11) having a unique solution, we use the simpler notation.

on the ratio of asset holdings between the different types of agents around the point of perfect equality. For each point on this grid, the search for a steady state involves a search for a tax rate. The procedure for computing such a tax rate can be described as follows:

- (i) Let  $R_i^0(A, \tau, \tau', a, a')$  be a quadratic function that approximates the utility function in a neighborhood of the steady state (note that the budget constraint has been used here to substitute out consumption). Guess on  $\tau_0$  as a value for the tax rate, and compute the implied steady-state values of the other variables. This involves computing a value for aggregate capital with the property that the after-tax rate of return is the inverse of the discount rate.
- (ii) Fix an initial affine tax policy  $\Psi^0$ .
- (iii) Given  $\Psi_0$ , use standard methods to solve for the equilibrium elements  $h_i^0$ ,  $H^0$ ,  $g_i^0$ , and  $G^0$  as linear functions and  $v_i^0$  as a quadratic function.
- (iv) Solve for the one-period deviation equilibrium elements. Note that this is a simple static problem since we already have obtained functions  $v_i^0$ ; the key difference is that, in this case, we do not use  $\Psi^0$  as an update for next period's tax rate; instead, we leave the dependence on  $\tau'$  explicit. The application of a representative-type assumption on  $\tilde{h}_i^0$ , summing up of the  $\tilde{g}_i^0$ 's, and the matrix inversion then deliver the the equilibrium elements  $\tilde{H}^0$  and  $\tilde{G}^0$ .
- (v) Substitute the decision rules and obtained equilibrium functions into the maximand in (10) to obtain the function  $\tilde{v}_i^0$ .
- (vi) Maximize  $\tilde{v}_i^0$  with respect to  $\tau'$  to obtain a function  $\psi_i^0$  of the distribution of wealth and the own wealth of the agent. Check for the concavity of the function  $\tilde{v}_i^0$  with respect to  $\tau'$ , ensuring that the first-order conditions deliver a maximum.
- (vii) Use the representative-type condition on the median agent to obtain the function  $\Psi^1$  by letting  $\Psi^1(A, \tau) \equiv \psi_m^0(A, \tau, A_m)$ .
- (viii) Compare  $\Psi^1$  to  $\Psi^0$ . If these functions are close enough, continue to (ix). If not, redefine  $\Psi^0$  to be a linear combination of its old value and  $\Psi^1$  and go back to step (iii). This updating procedure has been used before, and it is necessary in our case for avoiding "overshooting" problems. We found that a very small step (less than 0.01 in the direction of  $\Psi^1$ ) works best.

(ix) Verify that the policy function  $\Psi$  reproduces the conjectured tax rate. In other words, the following condition has to be verified:

$$\tau^0 = \tau^1 \equiv \Psi(A, \tau^0). \quad (13)$$

If it is not, go back to step (i) and update the guess for  $\tau$ . We update using  $\tau_0 = (\tau_0 + \tau_1)/2$ .

In our experiments, we use two procedures to characterize the set of steady states. The first consists of performing steps (i) through (ix) described above. The second procedure, which is much simpler and less time-consuming, was already described in the description of the mechanics of the model. It is based on the knowledge that a zero tax and equal distribution constitute a steady state: by using the law of motion for the economy approximated around this point, one can compute the set of steady states by simply finding the set of values for  $A_1$  and  $A_2$  that are reproduced by this law of motion. Clearly, this procedure is only strictly valid locally, and it is likely to give lower accuracy further away from the point of perfect equality. Finally, note that this procedure can also be applied to extend locally the set of steady states around any steady-state point found with the first procedure.

## C A simple example

Let us illustrate the dynamics of the system by the use of linearization in a similar way to the one we use to compute equilibria numerically. For simplicity, let us also assume that leisure is not valued, i.e., that  $\alpha = 1$ . Consider a case when the type 2 group is only arbitrarily larger than the type 1 group—i.e.,  $\mu_1 = \mu_2 = 0.5$ —and where the preference parameters are the following: the discount rate,  $\beta$ , is set at  $0.96^4$  (reflecting a four-year period), and we assume a constant relative risk aversion,  $\sigma$ , equal to 2. We assume a Cobb-Douglas production function with a labor share  $\theta$  of 0.64, and we assume that the depreciation rate equals  $1 - \delta = (1 - 0.08)^4 = 0.28$ . Further, assume that the holdings of capital of each type are the same and that the total capital stock equals that of the steady state with an exogenous tax rate of zero, which is approximately equal to 0.70 for this economy.

When we compute the linearized version of  $\Psi$  for this economy, we obtain

$$\Psi(A_1, A_2, \tau) = 3.953A_1 - 3.953A_2,$$

which equals 0 as long as the two types have the same capital stock. Furthermore, note that if  $A_2 > (<)A_1$ , there will be a negative (positive) current tax: if the median is richer (poorer), savings will be subsidized (taxed).

The law of motion of the economy can be described locally by the matrix

$$H = \begin{pmatrix} 0.50 & 0.27 \\ -0.50 & 1.27 \end{pmatrix}.$$

This matrix applied to any initial deviation  $(\Delta A_1, \Delta A_2)'$  from the zero-tax, equal-wealth steady state describes the deviation implied in the next period. It is straightforward to check that this matrix has one eigenvalue equal to one and one eigenvalue positive and less than one. The eigenvalue equal to one indicates the steady-state indeterminacy we expected, and the fact that the remaining eigenvalue is less than one says that the system is nonexplosive.

It is possible to use the  $H$  matrix to find the set of steady states in the neighborhood of the zero-tax steady state. It is given by the null space of  $H$ , or equivalently, it can be calculated by substituting the obtained function  $\Psi$  into the rate-of-return equation (1) to yield

$$\frac{1}{0.85} = 1 + (0.64(0.5A_1 + 0.5A_2)^{0.36} - 0.28)(3.953A_1 - 3.953A_2).$$

Linearizing this expression and writing it in deviation form, we obtain

$$\Delta A_2 = 1.85\Delta A_1.$$

When taxes are exogenously set at zero, we know that the set of steady states is given by a line of slope  $-\mu_1/\mu_2 = -1$ . Here, in contrast, we see that the set of steady states with endogenous taxes slopes upward! The interpretation of this finding is that politics are quantitatively very powerful in this example. More to the point, the example has the property that the cost of distorting savings is low enough that the median voter finds it beneficial to use a high tax rate. This outcome depends, among other things, on the absence of a distortion of the labor-leisure choice.

It is also useful to compare the dynamics of our politico-economic equilibrium to that of the case with exogenous zero taxes. The law of motion in that case can be described by a matrix  $\bar{H}$ , which satisfies

$$\bar{H} = \begin{pmatrix} 0.89 & -0.11 \\ -0.11 & 0.89 \end{pmatrix}.$$

Clearly, the local dynamics are also very different from when taxes are exogenous. In particular, an initial increase in, say,  $A_1$  by one unit will trigger tax increases when type 2 is the median voter, and the economy will move away from the initial steady state. In the case taxes are exogenous, the dynamics following this initial change in type 1's capital are simple: the economy decumulates capital to get back to a new point on the line with slope  $-1$ , a point implying that type 1 is permanently richer than type 2 by one unit. What happens in the endogenous-tax economy in the long run? To find out, one can calculate  $H_\infty$ , which, when postmultiplied by any vector  $(\Delta A_{10}, \Delta A_{20})'$ , describes the long-run change,  $(\Delta A_{1\infty}, \Delta A_{2\infty})'$ , in the two capital stocks; i.e.,  $H_\infty = \lim_{t \rightarrow \infty} H^t$ . In our example, we obtain

$$H_\infty = \begin{pmatrix} -1.2 & 1.2 \\ -2.2 & 2.2 \end{pmatrix}.$$

The changes following initial changes in the asset distribution in this example are dramatic. If type 1 (2) is given 1 additional unit at time zero, the long-run response is for both types to decrease (increase) their capital stock by 2.2 units. The dynamic response takes us back to a new point on the positively sloped steady-state line, but we are now far away from the initial equal-capital point. The relative distribution remains intact throughout time: the type given an additional unit will remain richer indefinitely. We see that, at least in this example, the capital accumulation path fundamentally changed with a small change in the initial distribution of capital. Again, the magnitudes in this example are exaggerated due to the absence of a labor-leisure choice. Figure 4 illustrates these experiments.

**Figure 4 here**

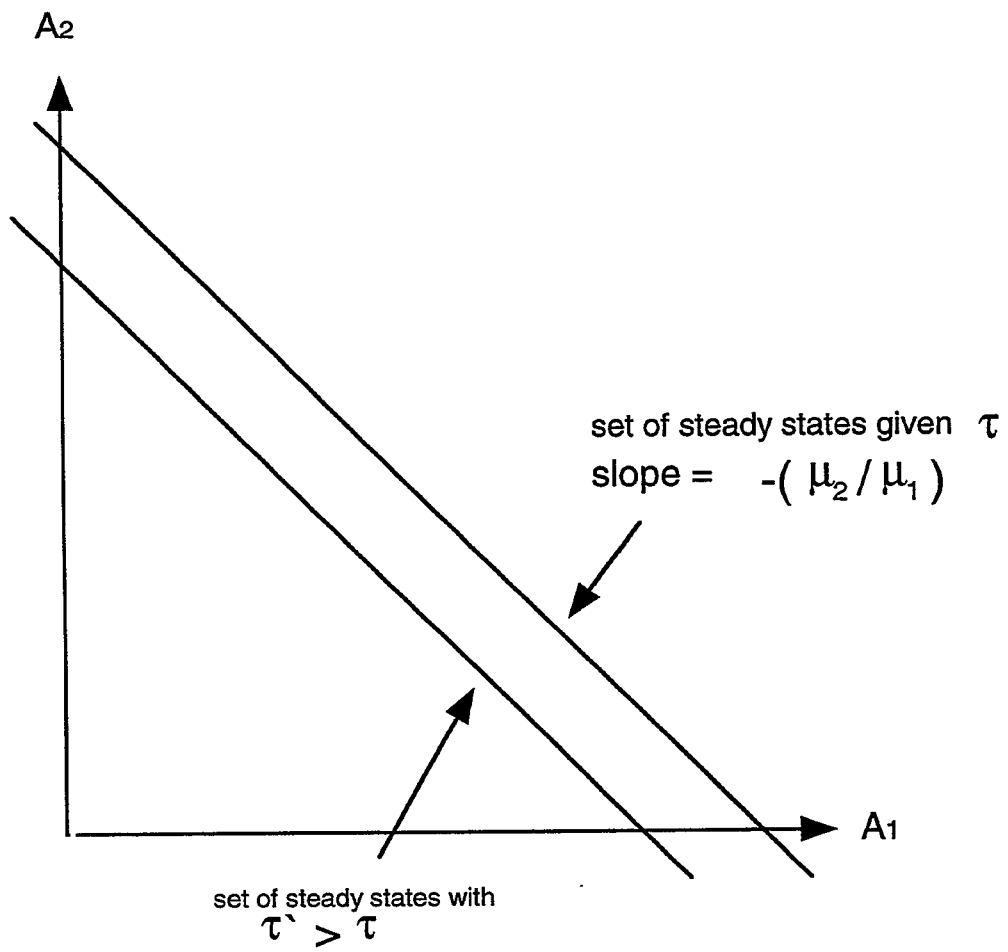


Figure 1 : exogenous taxes

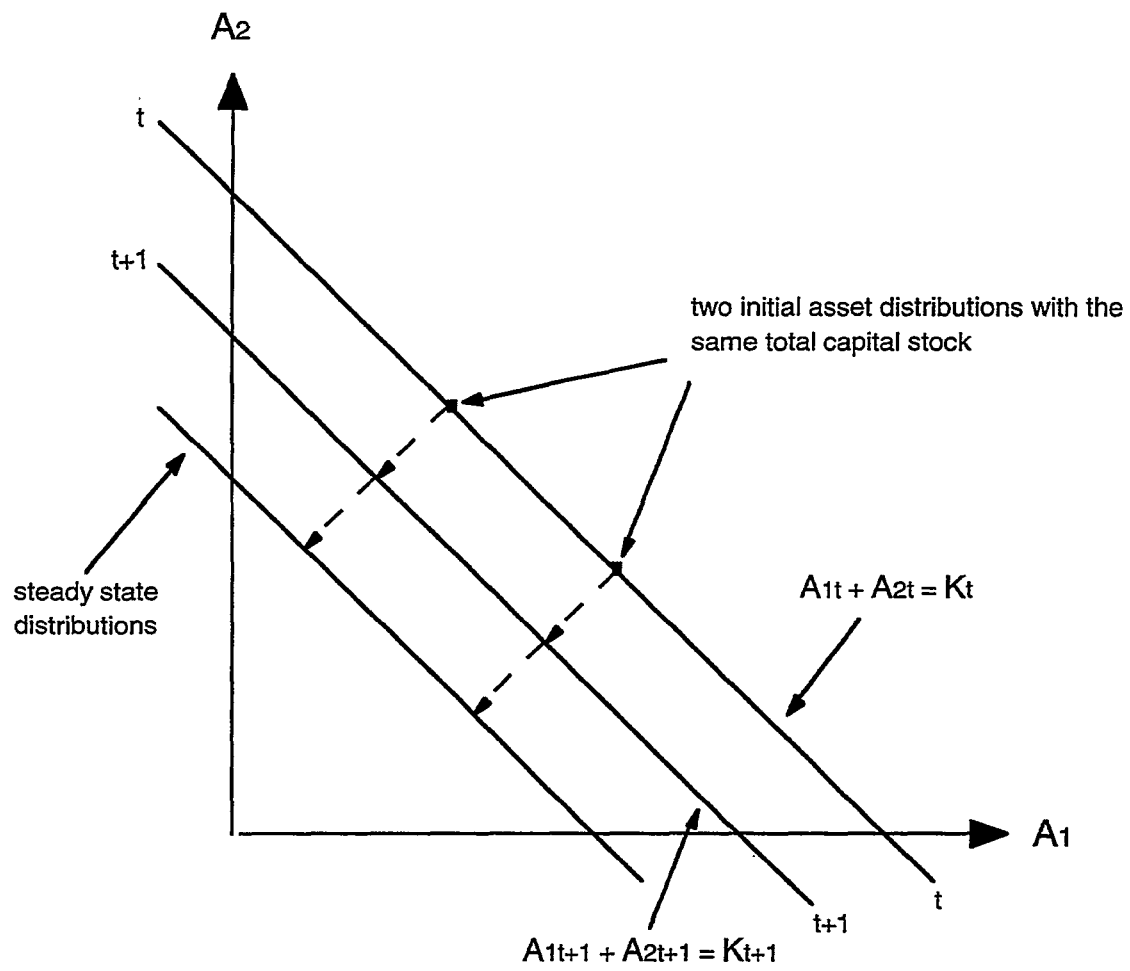


Figure 2 : aggregation with exogenous taxes



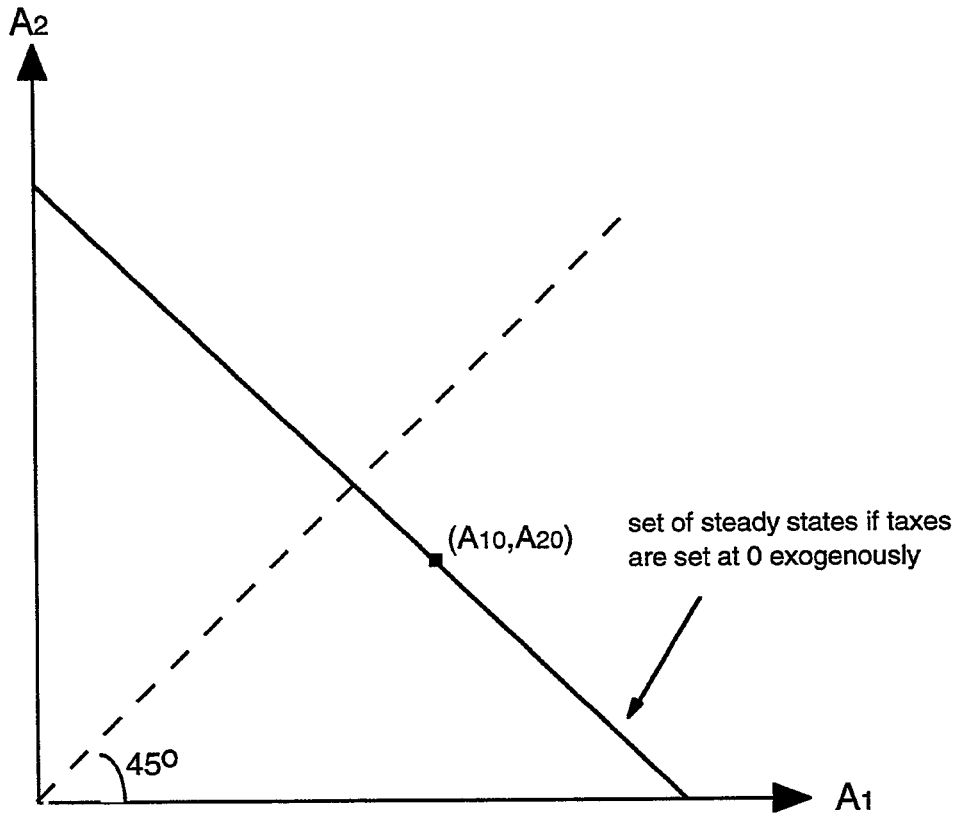


Figure 3A

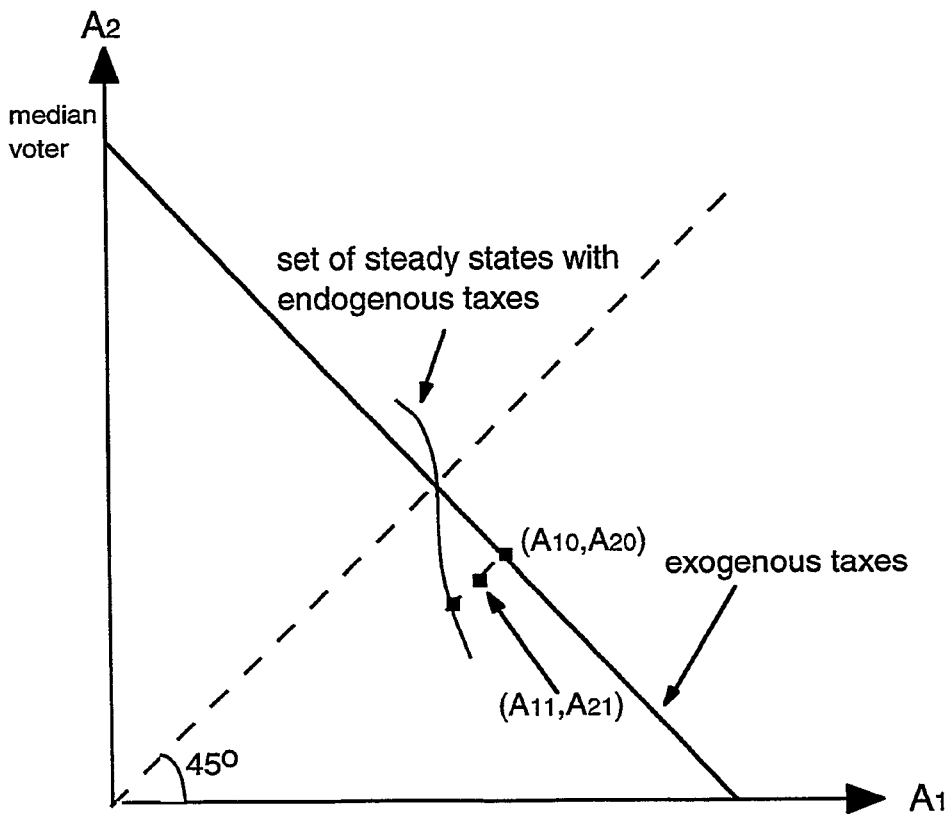


Figure 3B

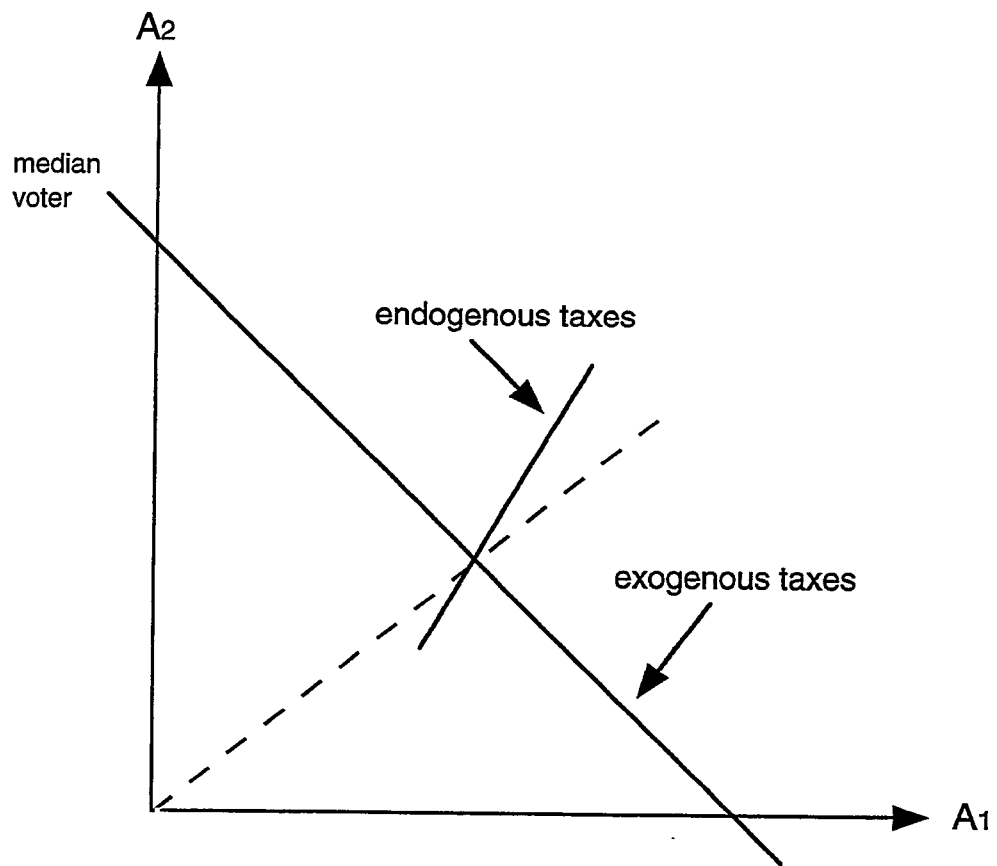


Figure 4 : illustration of the linearized set of politico-economic steady states