Federal Reserve Bank of Minneapolis
Research Department Staff Report 196

August 1995

Comments on Farmer and Guo's "The Econometrics of Indeterminacy: An Applied Study"

S. Rao Aiyagari*

Federal Reserve Bank of Minneapolis

ABSTRACT

I argue that Farmer and Guo's one-sector real business cycle model with indeterminacy and sunspots fails empirically and that its failure is inherent in the logic of the model taken together with some simple labor market facts.

Introduction

I like the spirit of the Farmer and Guo [1994] paper. The authors basically say, "We don't care how you feel about indeterminacies and sunspots. Here's a model that's got them, and either it fits the facts or it doesn't." So, I am going to treat the paper in the same spirit and talk only about the "facts." In doing so, I hope to impress upon the reader that models with indeterminacies and sunspot fluctuations are not empty boxes devoid of empirical content where anything can happen. They can be criticized on empirical grounds.

Theme and Outline

I will argue that Farmer and Guo's one-sector real business cycle (RBC) model with indeterminacy and sunspots fails empirically and that its failure is inherent in the logic of the model taken together with some simple labor market facts; hence failure was foreseeable. My discussion will proceed as follows. I will first explain how indeterminacy arises in the authors' model and the relation of indeterminacy to the labor market. I will then explain why empirical implementation of the authors' model necessarily yields strange-looking labor demand and supply curves. Finally, I will describe and criticize some empirical implications of the model.

The Labor Market and Indeterminacy

The labor market has been at the center of macroeconomics and business cycle analysis for many decades. This fact is clearly reflected in the title of Keynes' [1936] famous book and in the famous Dunlop [1938] and Tarshis [1939] criticisms of models driven by demand shocks. Dunlop and Tarshis point out that with a downward-sloping demand curve for labor, such models imply a negative correlation between labor and the real wage (or

1

productivity) that is counterfactual. The neat paper by Lucas [1970] on straight time/overtime was concerned with this problem - reconciling procyclical productivity with demand-driven cycles. Of course, the famous Phillips curve, the nonaccelerating inflationary rate of unemployment, inflation-unemployment trade-offs, and Lucas' [1972] paper all involve the labor market. The more recent RBC model with technology shocks (see Prescott 1986) also features the labor market prominently; this model has the problem that it predicts a strong positive correlation between labor and the real wage which is counterfactual, and it also predicts a volatility in labor relative to output which is too low.

Needless to say, the labor market is also at the center of Farmer and Guo [1994] and is intimately linked to indeterminacy and sunspots. In Figure 1, I have drawn all six possibilities for the labor demand and supply curves in panels a through f, where w is the real wage and z is labor.[1] The picture that most of us are used to seeing is shown in Figure 1a. The estimates in Farmer and Guo [1994] imply that the picture corresponds to Figure 1d, which is the sense in which their results are strange. I will now explain which of the pictures in Figure 1 are consistent with indeterminacy and which of the pictures are not and then explain why Farmer and Guo obtain the results they do.

In the one-sector RBC model, the position of the labor demand schedule is fixed by the capital stock and the technology shock; the position of the

---

[1] I should emphasize here that the labor supply curves I have drawn are meant to trace out the response of labor supply to a temporary change in the real wage, i.e., holding the marginal utility of consumption constant. The labor supply curve is not meant to indicate the labor supply response to a permanent change in the real wage. The latter labor supply curve may well be pretty inelastic or even have a negative slope because of strong wealth effects, but for the former one to be negatively sloped certainly is unusual. If the utility function is separable, then the marginal utility of leisure has to be increasing for the labor supply curve to be downward-sloping.

labor supply schedule is fixed by the level of consumption and the taste shocks. If, for a given capital stock and technology and taste shocks, there is a unique level of consumption, then there is a unique position of the labor supply schedule. This position determines a unique level of labor. Hence there will be a unique level of output and investment (from the resource constraint), which means a unique level of capital stock for the next period. It's clear from this that the key to indeterminacy is that there can't be a unique position of the labor supply curve, which means that there can't be a unique value of consumption. It has to be that optimistic or pessimistic expectations lead people to spend more or less on consumption, which shifts their labor supply schedule. This shift has to lead to labor, output, and investment effects that ratify the original optimistic or pessimistic expectations.

How might this happen? Presumably, current income and expectations of future income are what influence consumption most. If we assume a period utility function that is separable in consumption and leisure and is logarithmic in consumption, then we can explicitly solve out for optimal consumption from the consumer's intertemporal budget constraint and the consumption Euler equation. This yields the following solution for consumption:

(1) $$c_0 = (1-\beta)\left[\Sigma_{t=0}^{\infty} w_t z_t / \Pi_{j=1}^{t}(1+r_j) + (1+r_0)k_0\right],$$

where c_0 and k_0 are consumption and capital at time 0, β is the utility discount factor, and w_t, z_t, and r_t are the real wage, labor supply, and the net return to capital in period t. Note that r_{t+1} is also the real interest rate from t to t+1.

Thus, in order for people to consume more initially, they have to be

3

optimistic either that current and future labor incomes will be high or that current and future interest rates will be low. In the labor market of the standard RBC model with normally sloped labor demand and supply curves (as in Figure 1a), if people become optimistic and want to consume more, then their labor supply curve shifts inward (to the left), which lowers current labor. Hence current output and investment are lowered. Thereby, future capital stock and, hence, future employment, output, income, and so on, are all lowered. Further, future interest rates are raised since the capital stock is lowered. These outcomes are inconsistent with optimistic expectations.

The above argument suggests a way in which optimistic expectations may be self-fulfilling. Suppose you have an upward-sloping demand curve for labor that is steeper than the standard upward-sloping labor supply curve (as in Figure 1e).[2] In this case, optimistic expectations will shift the labor supply curve inward and will raise labor and output. By raising current output, optimistic expectations can also raise the future capital stock and possibly lower interest rates. These effects are consistent with higher initial consumption; thus the optimistic expectations can be self-fulfilling. Another possibility is a downward-sloping labor demand curve together with a downward-sloping and steeper labor supply curve (as in Figure 1f). Now, if the labor supply curve is downward-sloping, then optimistic expectations that raise consumption will shift the labor supply curve outward since the marginal utility of leisure is increasing. Since the labor supply curve is steeper than the labor demand curve, such a shift will

---

[2]This is the situation in the paper by Benhabib and Farmer [1994]. In that paper, the aggregate production function exhibits increasing returns to scale due to external economies of scale in the technology for individual firms. If the extent of increasing returns is strong enough (as assumed in that paper), then the market labor demand curve will be upward-sloping.

again raise labor, output, future capital, and so on, which will then ratify the original optimistic expectations. The same thing happens in the situation depicted in Figure 1d, where the normal slopes are simply reversed. The reader can easily verify that if the labor demand and supply curves are as depicted in Figures 1b and 1c, then optimistic expectations lead to lowered labor and output and, therefore, cannot be self-fulfilling. Thus I conclude that of the six different possibilities only three are consistent with indeterminacy. These three cases are the ones depicted in Figures 1d-f and are precisely the ones which satisfy the condition that the labor demand curve has, algebraically, a higher slope than the labor supply curve. This is precisely the condition for obtaining indeterminacy that is used in Benhabib and Farmer [1994] and in Farmer and Guo [1994].

Is there a way to narrow these possibilities some more? Looking at the empirical implications of Figures 1d-f for two key labor market facts - the productivity/labor correlation (denoted ρ) and the relative variability of labor to output (denoted σ) - provides such a way.[3] In order to derive these empirical implications, I will ignore movements in capital and assume that the real wage and productivity are proportional and interpret the vertical axes in Figures 1a-f as the log of productivity rather than the log of the real wage. Table 1 summarizes the empirical implications of Figures 1e-f for the two statistics, ρ and σ, and their empirical values.

---

[3]The following argument is adapted from Aiyagari [1994], where I use similar reasoning to calculate the contribution of technology shocks to business cycles under a variety of assumptions. These assumptions include the standard assumptions of constant returns and competitive markets as well as socially increasing returns (as in this paper), monopolistically competitive firms (as in other papers, including Benhabib and Farmer 1994), and straight time/overtime considerations.

Table 1

	ρ	σ
Figure 1e	near +1	<< 1
Figure 1f	near −1	>> 1
Empirical	0	1

In Figure 1e, since both the labor demand and supply curves are upward-sloping, the implied value of ρ will be strongly positive. This is because, regardless of where the shocks are, productivity and labor move in the same direction. However, the empirical value of ρ is about zero. Further, Figure 1e also implies a value of σ significantly less than unity. To see this, note that if the labor supply curve is perfectly elastic, then movements in the labor demand curve will not affect productivity. Consequently, output and labor will move by the same amount. However, if the labor supply curve is not perfectly elastic, then labor will have to move by a lesser amount than output. Movements in the labor supply curve will also necessarily involve smaller movements in labor than in output, because the labor demand curve is upward-sloping and both labor and productivity are moving in the same direction. Consequently, regardless of where the shocks are, the implied value of σ must be significantly less than unity. However, the empirical value of σ is about unity. Therefore, the situation depicted in Figure 1e cannot be consistent with these labor market facts.

The situation depicted in Figure 1f also cannot be consistent with the empirical values of ρ and σ, and the explanation for this inconsistency is as follows. In Figure 1f, since both the labor demand and the labor supply curves are downward-sloping, the implied value of ρ will be strongly negative. This is because, regardless of where the shocks are, productivity and labor move in opposite directions. Further, the value of σ implied by

Figure 1f must be significantly greater than unity. This is because a rightward movement in the labor supply curve will raise labor but lower productivity. Consequently, the rise in output will be less than the rise in labor. A downward shift of the labor demand curve will raise labor and lower productivity and hence raise output by less than the rise in labor. Consequently, regardless of where the shocks are, the situation in Figure 1f will imply a value of σ significantly greater than unity. Therefore, the situation depicted in Figure 1f cannot be consistent with the empirical values of ρ and σ.

It follows that the situation in Figure 1e is the only empirically plausible case (conditional on having indeterminacy) that can potentially deliver a productivity/labor correlation of about zero and a relative variability of labor to output of about unity. Why does Farmer and Guo's empirical implementation of their model yield an upward-sloping labor demand curve? Consumption is a labor supply shifter, and under the maintained hypothesis of indeterminacy and sunspots, part of consumption movements is caused by sunspots, which are uncorrelated with shocks to labor demand. This observation permits Farmer and Guo to use lagged consumption growth as a valid instrument for its current value in estimating the slope of the labor demand curve. Since, in the data, consumption is positively correlated with both labor and the real wage (or productivity), this procedure necessarily yields an upward-sloping labor demand curve. Therefore, one must necessarily also get a downward-sloping labor supply curve, because the estimation procedure is trying to reconcile the model with the roughly zero productivity/labor correlation. I think this is what is going on in Farmer and Guo [1994], and it explains why the authors estimate an upward-sloping labor demand curve and a downward-sloping labor supply curve.

So, I think I have demonstrated that the very logic of indeterminacy in

the one-sector RBC model, coupled with a few simple facts about the productivity/labor correlation, the relative variability of labor to output, and the correlation of consumption with labor and productivity, necessarily leads to an upward-sloping labor demand curve and a downward-sloping labor supply curve. Obviously, it is a pretty unorthodox conclusion – it turns our standard notion of supply and demand on its head. Nevertheless, is there any reason to try to change our thinking and entertain these new possibilities, or should we just dismiss them? Since we are all scientists, we need to have some sound scientific reasons for discarding these ideas – it's not enough to say that all textbooks have downward-sloping demand curves and upward-sloping supply curves.

In this spirit, I now consider some empirical implications of this new labor economics.

Empirical Implications: New versus Old Labor Economics

Here I will contrast the empirical implications of the new labor economics of Farmer and Guo (depicted in Figure 1d) with the old labor economics (depicted in Figure 1a). I will argue that several of the predictions of the new labor economics are inconsistent with the facts, whereas the predictions of the old labor economics are consistent with the same facts.

(i) It's generally accepted that the period after 1970 has been subject to supply shocks much more than the period 1950-70. For example, inflation was much more variable and the price level was much more countercyclical in the period after 1970, relative to the earlier period (see Wolf 1991). The old labor economics implies that the productivity/labor correlation should be less negative (or more positive) after 1970 compared to 1950-70. This is

8

because such supply shocks make the labor demand curve more volatile and, since the labor supply curve is upward-sloping, enhance the positive comovement between productivity and labor. However, the new labor economics implies exactly the opposite, because here the labor supply curve is downward-sloping, and hence supply shocks that make the labor demand curve more volatile enhance the *negative* comovement between productivity and labor. The data clearly favor the old labor economics. The productivity/labor correlation is about zero over the period 1970-88 compared to its value of -0.40 over the period 1952-69.

(ii) The old labor economics also suggests that when supply shocks are more dominant, the variability of labor input relative to output should be smaller. The reasoning behind this is as follows. Shifts in the labor demand curve change productivity and labor in the same direction. Hence output changes by more than labor input. Shifts in the labor supply curve change productivity and labor in opposite directions. Hence output changes by less than the labor input. The overall variability of labor relative to output is some weighted average of the relative variabilities due to shifts in the labor demand and the labor supply curves. It follows that if supply shocks (which shift the labor demand curve) are more dominant in a period, then the relative variability of labor input to output should be smaller. The new labor economics, however, suggests exactly the opposite. This is because, here, shifts in the labor demand curve change productivity and labor in opposite directions. Hence the change in output will be less than the change in labor. Shifts in the labor supply curve change productivity and labor in the same direction. Hence, the change in output will be greater than the change in labor. It follows that if supply shocks are more dominant in a period, then the relative variability of labor to output should be larger in

that period. The facts seem to support the old labor economics. My calculations suggest that the value of σ was 0.90 during 1970-88 compared to its value of 1.05 during 1952-69.

(iii) Studies that have tried to separate the data into periods in which demand shocks were dominant and periods in which supply shocks were dominant find that wages and productivity were much more procyclical in periods in which supply shocks were more dominant. Sumner and Silver [1989] find that during 1900-85, real wages were strongly countercyclical over the 59 periods during which the inflation rate moved procyclically and that real wages were strongly procyclical over the 21 periods during which the inflation rate moved countercyclically. This fact also favors the old labor economics over the new labor economics.

(iv) The new labor economics says that the oil price shocks of 1973 and 1979 should have raised labor and output by shifting the labor demand curve downward.[4] The shocks clearly did not. Similarly, payroll tax raises are predicted to raise labor, output, and the profits of employers. These taxes raise labor by shifting the labor demand curve to the right. This shift happens because the rise in the cost of labor raises labor demand since the labor demand curve is upward-sloping; i.e., labor demand is increasing in the real wage. Output as well as profits will rise. The Reagan tax cuts are predicted to have lowered labor and output. This is because the cut in the labor income tax raises labor supply since the labor supply curve is

---

[4]It's clear from Figure 1d that labor will rise. Whether output will rise or fall depends on the labor supply elasticity. If labor supply is highly elastic (so that productivity does not fall much), then output will also rise; if labor supply is highly inelastic (so that productivity falls a lot), then output will fall. My conclusion regarding output is based on the estimated value of the labor supply elasticity in Farmer and Guo [1994].

downward-sloping; i.e., labor supply is decreasing in the real wage. Therefore, the labor supply curve shifts to the right, and labor rises. Output also rises since productivity also rises due to the upward-sloping labor demand curve. I think these predictions are counterfactual.

(v) The article by Pencavel [1986] summarizes estimates of male labor supply elasticities. Estimates based on the static model generally yield negative labor supply elasticities, but estimates based on experimental data (such as negative income tax experiments during 1968-78) yield less negative or positive labor supply estimates. Estimates of male labor supply elasticities using the life-cycle model typically yield positive elasticities. The elasticity with respect to a permanent shift in the wage profile is also positive, but, as expected, somewhat lower (see Table 1.22, p. 85, and the discussion in section 5 of Pencavel 1986). The article by Killingsworth and Heckman [1986] summarizes estimates of female labor supply elasticities. Killingsworth and Heckman conclude that female labor supply elasticities are positive and large, both in absolute terms and relative to male labor supply elasticities, though they do note that the range of estimates is quite large (see their Table 2.26, pp. 189-92).

(vi) Estimates of labor demand elasticities from the article by Hamermesh [1986] generally indicate negative elasticities.

(vii) As Farmer and Guo note, their estimated labor supply elasticity is inconsistent with their maintained hypothesis of convex preferences. It might have been interesting if they had also conducted their estimation with the added constraint that the labor supply elasticity be nonnegative. This would presumably have produced a configuration of the labor demand and the

11

labor supply curves as in Figure 1e. This configuration cannot produce the somewhat negative productivity/labor correlation of -0.4 observed during 1952-69, but it can produce a low and close-to-zero productivity/labor correlation if the labor supply curve is highly elastic (small γ), the labor demand curve is highly inelastic (large β-1), and the shocks to the labor supply curve are much less variable than shocks to the labor demand curve. However, the implied labor supply elasticity and the extent of aggregate increasing returns seem likely to be highly unrealistic.
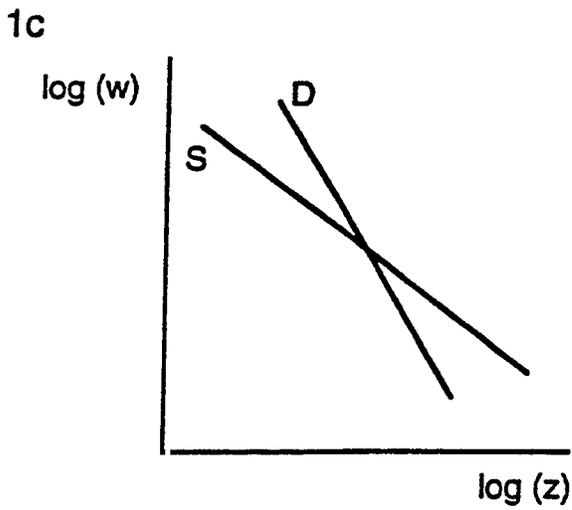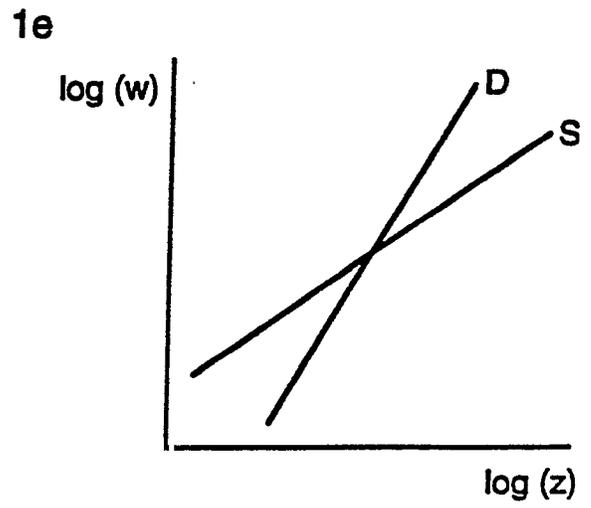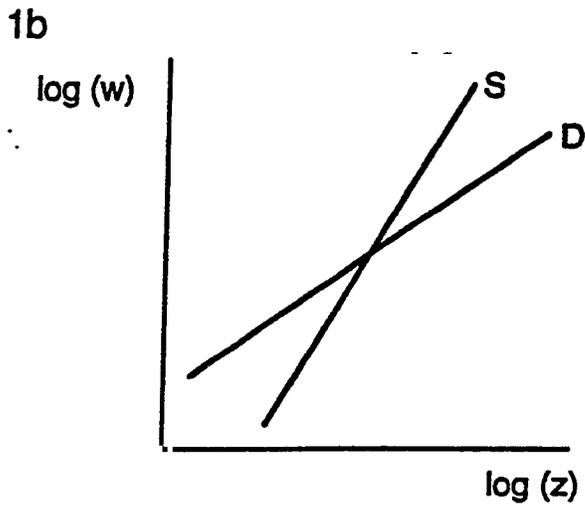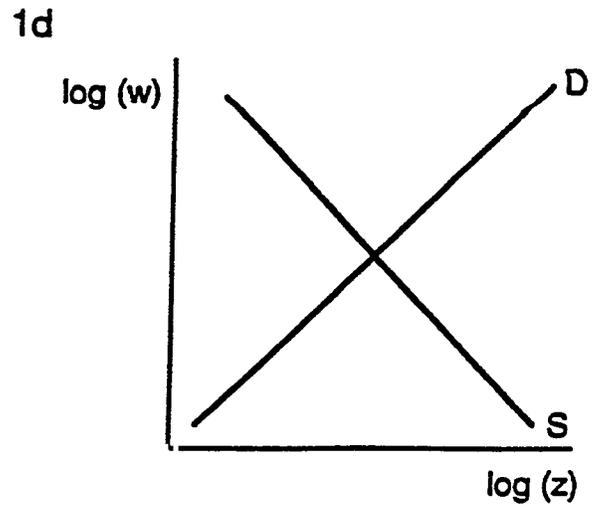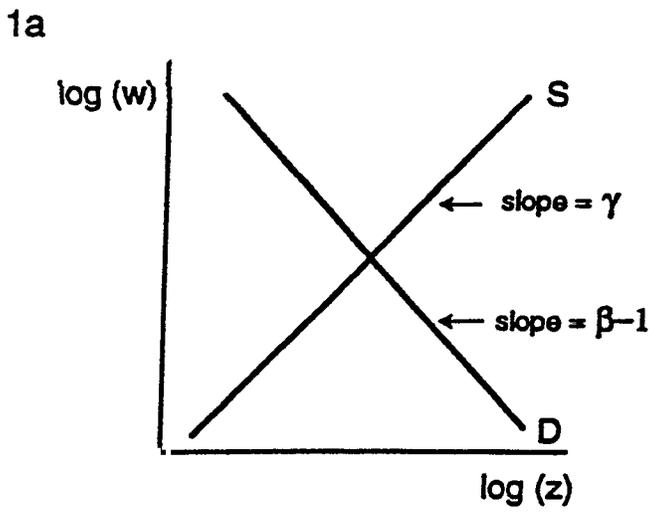
Conclusion

I conclude that the one-sector RBC models with indeterminacies and sunspots are, by virtue of their inherent logic, very unlikely to address business cycle facts satisfactorily. I have been careful to say "one-sector RBC models" and careful to avoid saying "never" since the history of science clearly demonstrates that things, like airplanes, that were once thought impossible are now facts of daily life; further, the recent history of dynamic general equilibrium theory shows that clever theorists can make the seemingly impossible happen in their models, and clever econometricians can persuade you that the seemingly impossible is, in fact, what happens in the real world.

The task of constructing empirically plausible business cycle models with animal spirits needs more work. If and when such a model is forthcoming, it will represent a major contribution to business cycle research. Farmer and Guo are to be congratulated for their part in this endeavor.

REFERENCES

Aiyagari, S. Rao. 1994. On the contribution of technology shocks to business cycles, *Federal Reserve Bank of Minneapolis Quarterly Review*, 18 (1), 22-34.

Benhabib, Jess and Roger E. A. Farmer. 1994. Indeterminacy and increasing returns, *Journal of Economic Theory*, 63 (1), 19-41.

Dunlop, John T. 1938. The movement of real and money wage rates, *Economic Journal*, XLVIII, 413-34.

Farmer, Roger E. A. and Jang-Ting Guo. 1994. The econometrics of indeterminacy: An applied study, UCLA Department of Economics Working Paper #720.

Hamermesh, Daniel S. 1986. The demand for labor in the long run, chapter 8 in *Handbook of Labor Economics*, volume 1, edited by Orley Ashenfelter and Richard Layard, North-Holland, New York.

Keynes, J. M. 1936. *The General Theory of Employment, Interest and Money*, Macmillan, London.

Killingsworth, Mark R. and James J. Heckman. 1986. Female labor supply: A survey, chapter 2 in *Handbook of Labor Economics*, volume 1, edited by Orley Ashenfelter and Richard Layard, North-Holland, New York.

Lucas, Robert E., Jr. 1970. Capacity, overtime, and empirical production functions, *American Economic Review*, 60 (2), 23-27.

————, 1972. Expectations and the neutrality of money, *Journal of Economic Theory*, 4 (2), 103-24.

Pencavel, John H. 1986. Labor supply of men: A survey, chapter 1 in *Handbook of Labor Economics*, volume 1, edited by Orley Ashenfelter and Richard Layard, North-Holland, New York.

Prescott, Edward C. 1986. Theory ahead of business cycle measurement, *Federal Reserve Bank of Minneapolis Quarterly Review*, 10 (4), 9-22. Also, 1986 in *Real business cycles, real exchange rates and actual policies*, edited by Karl Brunner and Allan H. Meltzer, Carnegie-Rochester Conference Series on Public Policy, 25 (0): 11-44. North-Holland, Amsterdam.

Sumner, Scott and Stephen Silver. 1989. Real wages, employment, and the Phillips curve, *Journal of Political Economy*, 97 (3), 706-20.

Tarshis, L. 1939. Changes in real and money wage rates, *Economic Journal*, XLIX, 150-54.

Wolf, Holger C. 1991. Procyclical prices: A demi-myth? *Federal Reserve Bank of Minneapolis Quarterly Review*, 15 (2), 25-28.

1a

log (w)

S

← slope = γ

← slope = β−1

D

log (z)

1b

log (w)

S

D

log (z)

1c

log (w)

D

S

log (z)

β −1 < γ (Determinate)

1d

log (w)

D

S

log (z)

1e

log (w)

D

S

log (z)

1f

log (w)

S

D

log (z)
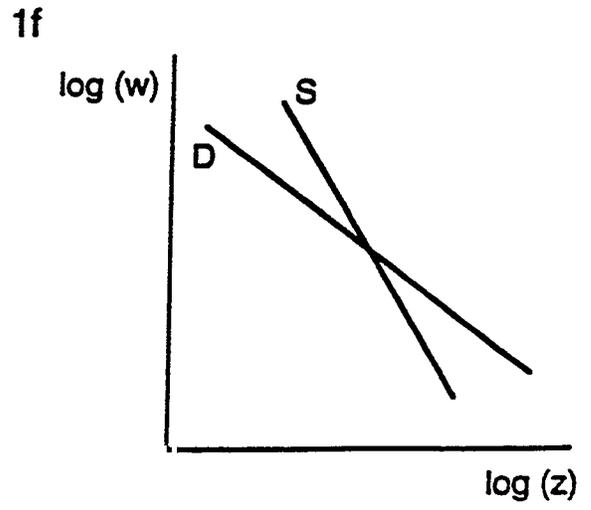
β −1 > γ (Indeterminate)

Figures 1 a − f: LABOR MARKET