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Optimal Fiscal Policy in a Business Cycle Model

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ABSTRACT

This paper develops the quantitative implications of optimal fiscal policy in a business cycle model. In a stationary equilibrium the ex ante tax rate on capital income is approximately zero. There is an equivalence class of ex post capital income tax rates and bond policies that support a given allocation. Within this class the optimal ex post capital tax rates can range from being close to i.i.d. to being close to a random walk. The tax rate on labor income fluctuates very little and inherits the persistence properties of the exogenous shocks and thus there is no presumption that optimal labor tax rates follow a random walk. The welfare gains from smoothing labor tax rates and making ex ante capital income tax rates zero are small and most of the welfare gains come from an initial period of high taxation on capital income.

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Introduction

A fundamental question in macroeconomics is how to set fiscal policy over the business cycle. Standard Keynesian models imply that fiscal policy should be countercyclical. For example, this means cutting taxes during recessions. The tax smoothing models of Barro (1979) and others imply that tax rates should change only when there are unanticipated shocks that affect the government's budget constraint. Thus when there is an unanticipated decline in output, and hence tax revenues, tax rates should be raised by enough to meet the government's expected present value budget constraint. In this paper we use standard neoclassical theory to answer this question. In particular, we use a quantitative version of the standard neoclassical growth model with distorting taxes. Using parameter values and stochastic processes for shocks similar to those in the real business cycle literature we find that after one period of transition during which labor income taxes are negative and capital income taxes are large

- tax rates on labor income should be essentially constant over the business cycle
- expected tax rates on capital income should be roughly zero in each period
- the return on debt and the ex post tax on capital income should absorb most of the shocks to the government budget constraint
- the welfare gains from smoothing labor tax rates and making ex ante capital income taxes zero are small and most of the welfare gains come from high capital income taxation in the one period of transition.

Our finding that optimal labor taxes should not respond to unanticipated shocks is quite different from the results in the tax smoothing models. In particular, such models imply that tax rates should follow a random walk regardless of the stochastic processes for the underlying shocks. In contrast, we find that optimal labor tax rates should fluctuate very little and, to the extent that they do, their serial correlation inherits the serial correlation properties of the shocks. Our finding that the ex ante tax rate on capital income is roughly zero is reminiscent of Chamley (1986) and Judd's (1985) result in the deterministic literature[1] that in a steady state the optimal capital income tax rate is zero for optimal fiscal policy.

In terms of our third finding, Lucas and Stokey (1983) showed in a model without capital that state-contingent returns on government debt can play a role in smoothing tax distortions across states of nature. In our model tax distortions across states of nature can be smoothed by state contingent taxes on capital as well as state contingent returns on debt. We find that these smoothing devices are quantitatively important: when there is an innovation in government spending about 83 percent of the resulting change in the present value of government spending is financed through the state-contingent instruments. In terms of welfare we find that, starting from a benchmark tax system which is a crude approximation to the U.S. system, switching to the Ramsey system gives welfare gains of about 1 percent of consumption. We decompose this gain into the gains from the transition period and those from smoothing labor tax rates and making ex ante capital income tax rates zero. We find that the welfare gains from smoothing labor tax rates and making expected capital income tax rates zero are about 0.2 percent of consumption while the gains from the high capital income taxation in the transitional period are about 0.8 percent of consumption.

We emphasize that our findings are quantitative in nature. In some interesting theoretical work, Zhu (1992) has shown that there is no theoretical presumption that labor tax rates should be constant or that ex ante capital income tax rates should be zero. Our contribution is to examine the quantitative significance of these features. We find that there is a quantitative presumption that labor tax rates should be constant and that ex ante capital income tax rates should be zero.

In reporting our results we have focused on three policy variables pinned down by the model. The first is the tax rate on labor income. The second is the ex ante tax rate on capital income which is defined as the ratio of the value of tax revenues across states of nature at a given date to the value of capital income across states of nature at that date. The third is the revenues from the state contingent debt and state-contingent capital income taxes. One interpretation of these revenues is that they are raised by taxing the return on debt as well as the return on capital. Since capital and debt are the assets available to private agents we call these revenues the taxes on private assets. It turns out that state-by-state capital income taxes and state-by-state returns on debt are not uniquely determined in our model. Both instruments play analogous roles in smoothing tax distortions across states of nature. Arbitrage conditions require that the returns on both type of assets weighted by the intertemporal marginal rates of substitution be equalized. There are a large variety of ways, however, of structuring the pattern of tax rates on capital and the rates of returns on bonds which meet the arbitrage conditions and raise the same revenue in each state of nature. We show there is an equivalence class of tax rates on capital income and rates of return on government debt which can be used to support the Ramsey allocations. (See Zhu 1992 for an independent derivation of this result. For some related work, see King 1990.) Indeed, from a quantitative standpoint, we find that depending on the way that policies are chosen from this equivalence class, the tax rate on capital can range from being close to i.i.d. to being close to a random walk. This finding contrasts with the results of Judd (1989) who argues that the ex post capital income tax rates should be independently and identically distributed.

1. The Economy

Consider a production economy populated by a large number of identical infinitely-lived consumers. In each period $t = 0, 1, \ldots$, the economy experiences one of finitely many events s_t. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t. The probability, as of time 0, of any particular history s^t is $\mu(s^t)$. The initial realization s_0 is given. This suggests a natural commodity space in which goods are differentiated by histories.

In each period t there are two goods: labor and a consumption-capital good. A constant returns to scale technology is available to transform labor $\ell(s^t)$ and capital $k(s^{t-1})$ into output via $F(k(s^{t-1}),\ell(s^t),s_t)$. Notice that the production function incorporates a stochastic shock. The output can be used for private consumption $c(s^t)$, government consumption $g(s^t)$, and new capital $k(s^t)$. Throughout we will take government consumption to be exogenously specified. Feasibility requires

$$(1.1)\quad c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}),\ell(s^t),s_t) + (1-\delta)k(s^{t-1}),$$

where δ is the depreciation rate on capital. The preferences of each consumer are given by

$$(1.2)\quad \sum_{t,s^t} \beta^t\mu(s^t)U(c(s^t),\ell(s^t))$$

where $0 < \beta < 1$ and U is increasing in consumption, decreasing in labor, strictly concave, and satisfies the Inada conditions.

Government consumption is financed by proportional taxes on the income from labor and capital and by debt. Let $\tau(s^t)$ and $\theta(s^t)$ denote the tax rates on the income from labor and capital. Government debt has a one-period maturity and a state-contingent return. Let $b(s^t)$ denote the number of units of debt issued at state s^t and $R_b(s^{t+1})b(s^t)$ denote the payoff at any state $s^{t+1} = (s^t,s_{t+1})$. The consumer's budget constraint is

$$(1.3)\quad c(s^t) + k(s^t) + b(s^t) \leq (1 - \tau(s^t))w(s^t)\ell(s^t) + R_b(s^t)b(s^{t-1}) + R_k(s^t)k(s^{t-1})$$

where $R_k(s^t) = [1 + (1 - \theta(s^t))(r(s^t) - \delta)]$ is the gross return on capital after taxes and depreciation and $r(s^t)$ and $w(s^t)$ are the before-tax returns on capital and labor. Competitive pricing ensures that these returns equal their marginal products, namely

$$(1.4)\quad r(s^t) = F_k(k(s^{t-1}),\ell(s^t),s_t)$$

(1.5) $w(s^t) = F_\ell(k(s^{t-1}),\ell(s^t),s_t)$.

Consumer purchases of capital are constrained to be nonnegative and the purchases of government debt are bounded above and below by some arbitrarily large constants. We let $x(s^t) = (c(s^t),\ell(s^t), k(s^t),b(s^t))$ denote an allocation for consumers at s^t and let $x = (x(s^t))$ denote an allocation for all s^t.

The government sets tax rates on labor and capital income and returns for government debt to finance the exogenous sequence of government consumption. The government's budget constraint is

(1.6) $b(s^t) = R_b(s^t)b(s^{t-1}) + g(s^t) - \tau(s^t)w(s^t)\ell(s^t) - \theta(s^t)(r(s^t) - \delta)k(s^{t-1})$.

We let $\pi(s^t) = (\tau(s^t),\theta(s^t),R_b(s^t))$ denote the government policy at s^t and let $\pi = (\pi(s^t))$ denote the policy for all s^t. The initial stock of debt, b_{-1} and the initial stock of capital, k_{-1}, are given.

Notice that for notational simplicity we have not explicitly included markets in private claims. Since all consumers are identical such claims will not be traded in equilibrium and hence their absence will not affect the equilibrium. Thus, we can always interpret the current model as having complete contingent private claims markets.

2. The Ramsey Equilibrium

Consider now the policy problem faced by the government. We suppose there is an institution or commitment technology through which the government can bind itself to a particular sequence of policies once and for all at time zero. We model this by having the government choose a policy $\pi = (\pi(s^t))$ at the beginning of time and then having consumers choose their allocations. Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices are described by rules that associate government policies with allocations. Formally, allocation rules are sequences of functions $x(\pi) = (x(s^t|\pi))$ that map policies π into allocations $x(\pi)$. Price rules are sequences of functions $w(\pi) = (w(s^t|\pi))$ and $r(\pi) = (r(s^t|\pi))$ that map policies π into price systems $w(\pi)$ and $r(\pi)$.

A Ramsey equilibrium is a policy π, an allocation rule $x(\cdot)$, and price rules $w(\cdot)$ and $r(\cdot)$ such that: (i) The policy π maximizes $\Sigma_{t,s^t}\beta^t\mu(s^t)U(c(s^t|\pi),\ell(s^t|\pi))$ subject to (1.6) with allocations and prices given by $x(\pi)$, $w(\pi)$, and $r(\pi)$, (ii) For every π', the allocation $x(\pi')$ maximizes (1.2) subject to (1.3) evaluated at the policy π' and the prices $w(\pi')$ and $r(\pi')$, and (iii) For every π', the prices satisfy

(2.1) $$w(s^t|\pi') = F_\ell(k(s^{t-1}|\pi'),\ell(s^t|\pi'),s_t)$$

and

(2.2) $$r(s^t|\pi') = F_k(k(s^{t-1}|\pi'),\ell(s^t|\pi'),s_t).$$

The allocations in a Ramsey equilibrium solve a simple programming problem called the Ramsey allocation problem. Now, it is well known that in the Ramsey equilibrium the government has an incentive to set the initial tax rate on capital income to be as large as possible. To make the problem interesting, we adopt the convention that the initial capital tax rate, $\theta(s_0)$, and the initial return on debt, $R_b(s_0)$, are fixed at some rate. We place no other restrictions on the tax rates for capital and labor income. In terms of notation, it will be convenient here and throughout the paper to let $U_c(s^t)$ and $U_\ell(s^t)$ denote the marginal utilities of consumption and leisure at state s^t and let $F_k(s^t)$ and $F_\ell(s^t)$ denote the marginal products of capital and labor at state s^t. We have, then

PROPOSITION 1 (The Ramsey Allocations). The consumption, labor, and capital allocations in a Ramsey equilibrium solve the Ramsey allocation problem

(2.3) $$\max \sum_{t,s^t} \beta^t\mu(s^t)U(c(s^t),\ell(s^t))$$

subject to

(2.4) $$c(s^t) + g(s^t) + k(s^t) = F(k(s^{t-1}),\ell(s^t),s^t) + (1-\delta)k(s^{t-1})$$

$$(2.5)\quad \sum_{t,s^t} \beta^t\mu(s^t)[U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)] = U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}].$$

Proof. In the Ramsey equilibrium the government must satisfy its budget constraint taking as given the allocation rule $x(\pi)$ and the pricing rules $w(\pi)$ and $r(\pi)$. These requirements impose restrictions on the set of allocations the government can achieve by varying its policies. We claim that these restrictions are summarized by constraints (2.4) and (2.5). We first show that the restrictions imply (2.4) and (2.5). To see this, note that we can add (1.3) and (1.6) to get (2.4) and thus feasibility is satisfied in equilibrium. Next, consider the allocation rule $x(\pi)$. For any policy π, we describe the necessary and sufficient conditions for c, ℓ, b, and k to solve the consumer's problem. Let $p(s^t)$ denote the Lagrange multiplier on constraint (1.3), then by Weitzman's (1973) theorem, these conditions are constraint (1.3) together with first order conditions for consumption and labor

$$(2.6)\quad \beta^t\mu(s^t)U_c(s^t) \leq p(s^t) \quad \text{with equality if } c(s^t) > 0$$

$$(2.7)\quad \beta^t\mu(s^t)U_\ell(s^t) \leq -p(s^t)(1 - \tau(s^t))w(s^t) \quad \text{with equality if } \ell(s^t) > 0,$$

first order conditions for capital and bonds

$$(2.8)\quad \left[p(s^t) - \sum_{s_{t+1}} p(s^{t+1})R_b(s^{t+1})\right] b(s^t) = 0$$

$$(2.9)\quad \left[p(s^t) - \sum_{s_{t+1}} p(s^{t+1})R_k(s^{t+1})\right] k(s^t) = 0,$$

and the two transversality conditions. These conditions specify that for any infinite history s^∞,

$$(2.10)\quad \lim p(s^t)b(s^t) = 0$$

$$(2.11)\quad \lim p(s^t)k(s^t) = 0$$

where the limits are taken over sequences of histories s^t contained in the infinite history s^∞.

We claim that any allocation which satisfies (1.3) and (2.6)–(2.11) must satisfy (2.5). To see this multiply (1.3) by $p(s^t)$, sum over t and s^t and use (2.8)–(2.11) to give

$$(2.12)\quad \sum_{t,s^t} p(s^t)[c(s^t) - (1 - \tau(s^t))w(s^t)\ell(s^t)] = p(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}].$$

Using (2.6) and (2.7) and noting that interiority follows from the Inada conditions, we can rewrite (2.12) as

$$(2.13)\quad \sum_{t,s^t} \beta^t\mu(s^t)[c(s^t)U_c(s^t) + \ell(s^t)U_\ell(s^t)] = U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}].$$

Thus, (2.4) and (2.5) are necessary conditions that any Ramsey equilibrium must satisfy. Next, given any allocation that satisfies (2.4) and (2.5) we can construct sequences of bond holdings and sequences of policies such that these allocations satisfy (1.3), (1.6), and (2.6)–(2.11). Therefore the restrictions on the set of allocations achievable by the government are equivalent to (2.4) and (2.5) and thus the proposition follows. □

Proposition 1 describes the consumption, labor, and capital allocations. Using these allocations we construct the bond allocation $b(s^r)$ as follows. Multiply (1.3) by $p(s^t)$ and sum over all dates and states following s^r and using (2.6)–(2.9) we can write

$$(2.14)\quad b(s^r) = \sum_{t=r+1}\sum_{s^t} \beta^{t-r}\mu(s^t|s^r)[U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)]/U_c(s^r) - k(s^r).$$

For later it will be convenient to write the Ramsey allocation problem in Lagrangian form:

$$(2.15)\quad \max \sum_{t,s^t} \beta^t\mu(s^t)W(c(s^t),\ell(s^t),\lambda) - \lambda U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]$$

subject to (2.4). The function W simply incorporates the implementability constraint into the maximand. For $t \geq 1$

$$(2.16)\quad W(c(s^t),\ell(s^t),\lambda) = U(c(s^t),\ell(s^t)) + \lambda[U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)]$$

and for $t = 0$, W equals the right side of (2.16) evaluated at s_0, minus $\lambda U_c(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}]$. Here, λ is the Lagrange multiplier on the implementability constraint, (2.5). The first order conditions for this problem imply, for $t \geq 1$

$$(2.17) \quad -\frac{W_\ell(s^t)}{W_c(s^t)} = F_\ell(s^t),$$

and

$$(2.18) \quad W_c(s^t) - \sum_{s_{t+1}} \beta\mu(s^{t+1}|s^t)W_c(s^{t+1})[1 - \delta + F_k(s^{t+1})] = 0.$$

For $t = 0$ these conditions are

$$(2.19) \quad -\frac{W_\ell(s_0) - \lambda[U_{c\ell}(s_0)(R_k(s_0)k_{-1} + R_b(s_0)b_{-1}) + U_c(s_0)(1 - \theta(s_0))F_{k\ell}(s_0)]}{W_c(s_0) - \lambda U_{cc}(R_k(s_0)k_{-1} + R_b(s_0)b_{-1})} = F_\ell(s_0)$$

and

$$(2.20) \quad W_c(s_0) - \lambda U_{cc}(R_k(s_0)k_{-1} + R_b(s_0)b_{-1}) - \sum_{s_1} \beta\mu(s^1|s_0)W_c(s^1)[1 - \delta + F_k(s^1)] = 0.$$

A useful property of the Ramsey allocations is the following. If the stochastic process on s follows a Markov process then from (2.17) and (2.18) it is clear that the allocations from date 1 onwards can be described by time invariant allocation rules $c(k,s;\lambda)$, $\ell(k,s;\lambda)$, $k'(k,s;\lambda)$, and $b(k,s;\lambda)$. The date 0 first order conditions (2.19) and (2.20) include terms related to the initial stocks of capital and bonds and are therefore different from the other first order conditions. The date 0 allocation rules are thus different from the stationary allocation rules which govern behavior from date 1 on.

3. The Ramsey Policies

Proposition 1 describes the Ramsey allocations, namely the allocations that actually occur in a Ramsey equilibrium. We are also interested in describing the set of policies and prices that may arise in a Ramsey equilibrium. That is, for some given allocations that solve the Ramsey allocation problem we construct policies and prices which decentralize it. We can pose the problem as follows. Given a Ramsey allocation c, ℓ, and k, and b given by (2.14), find the set of prices w and r, returns R_b and tax rates τ and θ that satisfy the marginal product conditions, the consumer first order conditions and the budget constraints of the consumer and the government. Now since the Ramsey allocations satisfy feasibility, any policies and prices that satisfy the consumer budget constraints must also satisfy the government budget constraints. The wage rate and the rental rate on capital are obtained from the marginal product conditions. Substituting these prices into consumer first order conditions gives an intratemporal condition

$$(3.1) \quad -\frac{U_\ell(s^t)}{U_c(s^t)} = (1 - \tau(s^t))F_\ell(s^t)$$

as well as two intertemporal conditions

$$(3.2) \quad U_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})R_b(s^{t+1})$$

$$(3.3) \quad U_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})R_k(s^{t+1})$$

where $R_k(s^{t+1}) = [1 + (1 - \theta(s^{t+1}))(F_k(s^{t+1}) - \delta)]$. The consumer's budget constraint is

$$(3.4) \quad c(s^{t+1}) + b(s^{t+1}) + k(s^{t+1}) = (1 - \tau(s^{t+1}))w(s^{t+1})\ell(s^{t+1}) + R_b(s^{t+1})b(s^t) + R_k(s^{t+1})k(s^t).$$

The tax rate on labor is determined from (3.1). Consider next the determination of bond returns R_b and the capital income tax rates θ. We will use (3.2)–(3.4) to show that these are indeterminate. Suppose that at some date t, s_{t+1} can take on N values. Then counting equations and unknowns in (3.2)–(3.4) gives 2N unknowns but only N + 2 equations. Actually, however, there is one linear dependency across these equations. To see this, multiply (3.4) by $\beta\mu(s^{t+1}|s^t)U_c(s^{t+1})$

and, summing across the states at date $t + 1$, and using (3.2), (3.3), and (2.14) yields an equation which does not depend on R_b and θ. Thus, there are $N - 1$ degrees of indeterminacy. We have

PROPOSITION 2. (The Indeterminacy of Capital Tax Rates.) If θ and R_b satisfy (3.2)–(3.4) then so do any $\bar{\theta}$ and $\bar{R}_b$ where

$$(3.5)\quad \sum_{s_{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})R_b(s^{t+1}) = \sum_{s_{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})\bar{R}_b(s^{t+1})$$

$$(3.6)\quad \sum_{s_{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})\theta(s^{t+1})(F_k(s^{t+1}) - \delta) = \sum_{s_{t+1}} \mu(s^{t+1}|s^t)U_c(s^{t+1})\bar{\theta}(s^{t+1})(F_k(s^{t+1}) - \delta)$$

and

$$(3.7)\quad \theta(s^{t+1})(F_k(s^{t+1}) - \delta)k(s^t) - R_b(s^{t+1})b(s^t) = \bar{\theta}(s^{t+1})(F_k(s^{t+1}) - \delta)k(s^t) - \bar{R}_b(s^{t+1})b(s^t).$$

To get some feel for the different possibilities we consider two extreme cases. Let us first suppose that the government is restricted to making capital taxes not contingent on the realization of the current state. That is, suppose that for each s^t

$$(3.8)\quad \theta(s^t,s_{t+1}) = \bar{\theta}(s^t) \quad \text{for all } s_{t+1}.$$

These conditions add $N - 1$ restrictions at each date and state and lead to a unique policy. The capital tax rate is pinned down by the first order condition for capital while the bond returns are then pinned down by the consumer budget constraints and the first order condition for bonds. For another extreme suppose that the government is restricted to making the returns on debt not contingent on the current state. That is, suppose for each s^t

$$(3.9)\quad R_b(s^t,s_{t+1}) = \bar{R}_b(s^t) \quad \text{for all } s_{t+1}.$$

These conditions add $N - 1$ restrictions at each date and state and lead to a unique policy. The return on bonds is pinned down by the first order condition for bonds and the capital tax rates are pinned down by the consumer budget constraints and the first order conditions for capital. More

Table 5

Percentage Welfare Gains of Alternative Tax Systems
Relative to Benchmark Economies

Benchmark Economies	Ramsey	Policy Experiments: Constant Tax Rates — Zero Capital Tax	High Capital Tax
Log Utility			
Deterministic Economy	1.00	.20	.00
Stochastic Economy: Estimated Policy	1.00	.20	.03
Stochastic Economy: Variable Policy	1.60	.80	.60
High Risk Aversion			
Deterministic Economy	1.30	−.06	.00
Stochastic Economy: Estimated Policy	1.20	−.02	.03
Stochastic Economy: Variable Policy	1.60	.33	.38
High Initial Debt			
Deterministic Economy	5.20	−.70	.00
Stochastic Economy: Estimated Policy	5.10	−.60	.05
Stochastic Economy: Variable Policy	6.10	.30	1.00

NOTES: The welfare measure is that constant percentage amount by which consumption must be increased in all dates and states in a benchmark economy, while leaving employment unchanged, so as to yield the same utility as under the policy experiment. The benchmark deterministic economy has government spending and technology shocks set to their mean values and has constant tax rates. The benchmark stochastic economy with estimated policy has shocks as in the baseline model and policies estimated from postwar U.S. data. The benchmark stochastic economy with variable policy has shocks as in the baseline model and has policies five times more responsive to technology and government spending shocks than the estimated policies. See text for details.

generally at each node s^t there are $N - 1$ degrees of freedom in determining the debt-capital tax policies. Each of the two extremes we have considered adds $N - 1$ restrictions at each node and leads to a unique policy. Of course, any other set of restrictions across capital tax rates and returns to debt that leads to $N - 1$ restrictions at each node will also lead to a unique policy.

In summary we have shown that the policies for debt and capital taxes are not uniquely determined by the Ramsey allocations. If the government has either state-contingent capital taxes or state-contingent debt it can support the Ramsey allocations. If the capital taxes are restricted to not depend on the current state the government can vary the returns to bonds in exactly the right way to support the optimal allocations. Alternatively, if the returns to debt are restricted to not depend on the current state, the government can vary the returns to capital in exactly the right way to support the same allocation. In particular, notice that restricting the government from issuing state contingent debt has no effect on either optimal allocations or on welfare. Note, however, that if the government has neither state-contingent capital taxes nor state-contingent debt there are more equations than unknowns and it cannot support the Ramsey allocations. Indeed, if the instruments available to the government are so restricted then the Ramsey problem must be modified to include extra constraints which capture the effect of these restrictions.

It will be useful for later analysis to isolate certain fiscal variables which are uniquely determined by the theory. First, as we have mentioned, the tax rate on labor income is determined. Second, while the state-by-state capital tax rates are not pinned down, (3.6) makes it clear that the value of the tax payments across states of nature is determined. To turn this value into a rate, consider the ratio of the value of tax payments across states to the value of net revenues from capital across states, namely

$$(3.10)\quad \theta^e(s^t) = \frac{\sum q(s^{t+1})\theta(s^{t+1})(F_k(s^{t+1}) - \delta)}{\sum q(s^t)(F_k(s^{t+1}) - \delta)}$$

where $q(s^{t+1}) = \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})/U_c(s^t)$ is the Arrow-Debreu price of a unit of consumption at state s^{t+1} in units of consumption at s^t. We call $\theta^e(s^t)$ the ex ante tax rate on capital income. Conceptually this rate corresponds to what Jorgenson (1963) and Hall and Jorgenson (1967) call the effective capital tax rate. The capital tax rate $\theta(s^t)$ corresponds to what Judd calls the ex post capital tax rate.

The third fiscal variable that is determined by the theory is given in (3.7), namely the revenues from capital taxation minus the value of outstanding debt. For ease of comparison with the labor tax rate and the ex-ante capital tax rate, we transform these revenues into a rate. One way of doing so is to imagine that the government achieves the desired state contingency in debt returns by promising a state-noncontingent rate of return on government debt $\bar{r}(s^t)$ which satisfies

$$(3.11)\quad \sum_{s^{t+1}} q(s^{t+1})R_b(s^{t+1}) = \sum_{s^{t+1}} q(s^{t+1})(1 + \bar{r}(s^t))$$

and by levying a state contingent tax $\nu(s^{t+1})$ on interest payments from government debt which satisfies

$$(3.12)\quad R_b(s^{t+1}) = [1 + \bar{r}(s^t)(1 - \nu(s^{t+1}))].$$

Notice that $\sum q(s^{t+1})\nu(s^{t+1}) = 0$ and thus the present value of revenues raised from taxation of interest on debt is zero. Next note from (3.5) and (3.11) that $\bar{r}(s^t)$ is pinned down and from (2.14) $b(s^t)$ is pinned down. Thus, we can think of (3.7) as pinning down the sum of the tax revenues from the capital income tax and the debt income tax given by

$$(3.13)\quad \theta(s^{t+1})(F_k(s^{t+1}) - \delta)k(s^t) + \nu(s^{t+1})\bar{r}(s^t)b(s^t).$$

We transform these revenues into a rate by dividing the income from capital and debt to obtain

$$(3.14)\quad \eta(s^{t+1}) = \frac{\theta(s^{t+1})(F_k(s^{t+1}) - \delta)k(s^t) + \nu(s^{t+1})\bar{r}(s^t)b(s^t)}{(F_k(s^{t+1}) - \delta)k(s^t) + \bar{r}(s^t)b(s^t)}.$$

A useful property of the Ramsey policies is the following. The three tax rates can be described by time invariant policy rules of the form $\tau(k,s;\lambda)$, $\theta^e(k,s;\lambda)$, and $\eta(k,s;\lambda)$ from date 1 onwards. The policy rules for date 0 are different from these time invariant rules. To see this recall from the discussion following (2.17)–(2.20) that the allocations follow time invariant rules from date 1 onwards. Inspection of (3.1), (3.10), and (3.14) makes it clear that the policy rules do also.

Notice there is a subtle asymmetry between the ex ante capital tax rate and the other two tax rates. Specifically, the tax rates on labor $\tau(s^t)$ and the tax rate on private assets $\eta(s^t)$ are levied on income received at date t while the ex ante tax rate on capital $\theta^e(s^t)$ is a weighted average of the tax rates on capital income received at date t + 1. Thus under the Ramsey policies the income from labor and private assets is taxed differently than under the stationary policies only at date 0 while the income from capital is taxed differently than under the stationary policies at date 1. Of course, the income from capital at date 0 is also taxed differently because the tax rate there is fixed at some rate.

4. A Class of Utilities

In this section we examine the nature of the Ramsey taxes for a class of utility functions. We will show that for such a class it is optimal not to distort the consumer capital accumulation decision made at date 1 or after. To motivate the result we write the consumer's first order condition for capital as

$$(4.1) \quad 1 - \sum_{s_{t+1}} q(s^{t+1})[1 - \delta + F_k(s^{t+1})] = \sum_{s_{t+1}} q(s^{t+1})\theta(s^{t+1})(F_k(s^{t+1}) - \delta).$$

Now, in an undistorted equilibrium the consumer's first order condition has the same left hand side as (4.1) but the right hand side equals zero. Thus, the right hand side of (4.1) measures the size of the wedge between the distorted and undistorted first order conditions for capital accumulation at date t. Note the right side of (4.1) is the market value at t of claims to the revenues from capital taxation at t + 1. Since the right side of (4.1) is the numerator of (3.10) the capital accumulation decision is undistorted if and only if the ex ante rate on capital income is zero.

Consider utility functions of the form

$$(4.2)\quad U(c,\ell) = c^{1-\sigma}/(1-\sigma) + V(\ell).$$

We then have

PROPOSITION 3. For utility functions of the form (4.2) it is not optimal to distort the capital accumulation decision at date 1 or after. Namely, the ex ante rate on capital income received at date t is zero for $t \geq 2$ or equivalently

$$(4.3)\quad \sum_{s_{t+1}} q(s^{t+1})\theta(s^{t+1})(F_k(s^{t+1}) - \delta) = 0 \quad \text{for } t \geq 1.$$

Proof. For $t \geq 1$ the first order conditions for the Ramsey problem imply

$$(4.4)\quad 1 = \sum_{s_{t+1}} \beta\mu(s^{t+1}|s^t)\,\frac{W_c(s^{t+1})}{W_c(s^t)}\,[1 - \delta + F_k(s^{t+1})].$$

For $t \geq 1$, consumer's first order conditions for capital imply

$$(4.5)\quad 1 = \sum_{s_{t+1}} \beta\mu(s^{t+1}|s^t)\,\frac{U_c(s^{t+1})}{U_c(s^t)}\,[1 + (1 - \theta(s^{t+1}))(F_k(s^{t+1}) - \delta)].$$

Now for any utility function of the form (4.2), it is easy to show that

$$(4.6)\quad \frac{W_c(s^{t+1})}{W_c(s^t)} = \frac{U_c(s^{t+1})}{U_c(s^t)}.$$

Substituting (4.6) into (4.4) and subtracting it from (4.5) gives the result. □

It is worth pointing out that for a deterministic version of the model Proposition 3 implies that the tax rate on capital income received at date t is zero for $t \geq 2$ and is typically different from zero at date 1. At date 0, of course, the tax rate is fixed. Now recall that in a continuous time version of the deterministic model with instantaneous preferences given by (4.2), Chamley (1986) showed the tax rate on capital income was constant for a finite length of time and zero thereafter. The reason for the difference is that Chamley imposed an exogenous upper bound on the tax rate on capital income. If we impose such an upper bound the Ramsey problem must be amended to include an extra constraint to capture the restrictions imposed by this upper bound. In the deterministic version of the model, with preferences given by (4.2), the tax rate would be constant at this upper bound for a finite number of periods, there would be one period of transition, and then it would be zero thereafter.

In the stochastic version of the model constraints of this kind can also be imposed. One can derive an upper bound endogenously. Consider the following scenario. At the end of each period t consumers can rent capital to firms for use in period t + 1 and pay taxes on the rental income from capital in period t + 1. Or, consumers can choose to hide the capital, say, in their basements. If they hide it the capital depreciates and is not available for use at t + 1. Thus if they hide it there is no capital income and consumers pay zero capital taxes. It is easy to show that this extra option leads to the following constraint on the Ramsey problem

$$(4.7) \quad U_c(s^t) \geq \sum_{s_{t+1}} \beta\mu(s^{t+1}|s^t)U_c(s^{t+1})(1-\delta).$$

One can show that for a special case of the preferences in (4.2), which we use in the baseline model in our computations, this constraint binds for a finite number of periods, then there is one period of transition and it is zero thereafter. A proof of this result is available upon request.

5. Computation and Parameterization

It turns out to be difficult to characterize theoretically the Ramsey policies for more general utility functions than those considered in Section 4. Therefore, we characterize these policies quantitatively. We are particularly interested in the quantitative properties of optimal tax rates in the class of economies similar to those studied in the business cycle literature (see, for example, Kydland and Prescott 1982). In this literature the preferences are described by utility functions of the form

(5.1) $U(c,\ell) = [c^{1-\gamma}(1-\ell)^{\gamma}]^{\psi}/\psi.$

The technology is described by a production function of the form

(5.2) $F(k,\ell,z,t) = k^{\alpha}\,[e^{\rho t+z}\ell]^{(1-\alpha)}.$

We incorporate two kinds of labor augmenting technological change into the production technology. The variable ρ captures deterministic growth in this technical change. The variable z is a zero mean technology shock which follows a symmetric two-state Markov chain. We let government consumption be given by $g_t = Ge^{\rho t+\tilde{g}}$, where G is a constant, ρ is the deterministic growth rate and the zero mean process, $\tilde{g}$, follows a symmetric two-state Markov chain. Notice that absent technological shocks the economy has a balanced growth path along which consumption, capital, and government spending grow at rate ρ and labor is constant. This formulation assumes that the economy grows over time. It is straightforward to modify the theoretical models of the previous sections to allow for exogenous growth.

We consider several parameterizations of this model. Our baseline model has $\psi = 0$ and thus has logarithmic preferences. The parameters for preferences and technology are chosen using the same procedures as in Christiano and Eichenbaum (1992) modified appropriately to take account of distorting taxes. Briefly, this procedure involves choosing parameters so that along the nonstochastic, balanced growth path of an economy with distorting taxes, the capital-output ratio, fraction of available time worked, the ratio of government spending to output and the debt-output ratio are the same as those in U.S. data. We chose the capital and labor tax rates so that their ratio matches the ratio of the mean of Barro and Sahasakul's (1983) estimate of the average marginal tax rate to the mean of Jorgenson and Sullivan's (1981) estimate of the effective corporate tax rate. Our empirical measures of the capital-output ratio, fraction of available time worked, ratio of government spending to output, and debt-output ratio are 2.71, 0.23, 0.18, 0.51, respectively. The values of the capital and labor tax rates determined by our procedure are 27.1 and 23.7 percent, respectively. We refer to these policies as the *nonstochastic benchmark policies*. It is not surprising that these tax numbers are lower than most estimates of tax rates on labor and capital since we do not have transfer payments in our model. Our procedure also determines β, γ, and G.

We choose the two parameters of the Markov chain for $\tilde{g}$ so that the autocorrelation, ρ_g, and the standard deviation, σ_g, are the same as the annualized versions of the corresponding statistics in Christiano and Eichenbaum (1992). We choose the two parameters of the Markov chain for the technology shock so that the autocorrelation, ρ_z, and the standard deviation, σ_z, are the same as the annualized versions of the corresponding statistics in Prescott (1986).

We also considered a model with high risk aversion by setting $\psi = -8$, a model with i.i.d. shocks, a model with only technology shocks and a model with only government spending shocks. In the high risk aversion model we adjusted the discount factor to keep the capital-output ratio the same as before along the balanced growth path of an economy with the benchmark nonstochastic policies. We also consider models with a range of risk aversion parameters and for each we adjusted the discount factor in a similar fashion. The initial conditions for our experiments, unless explicitly stated to the contrary, were given by the balanced growth path of a deterministic economy with the benchmark, nonstochastic policies.[2] Our parameter values are reported in Table 1.

We also considered a model with a high level of the initial debt. This model is an attempt to capture some of the consequences of including transfers into our setup. If one thinks of transfers as an obligation by the government to pay a fixed amount in present value terms then they are equivalent to government debt. In that vein we calculated the present value of transfer payments assuming that along the balanced growth path transfers were 12 percent of GNP (which is approximately their value in 1985). We then added this value to the initial government debt.

We briefly describe our computation procedure. We use the standard procedure of transforming our economy with growth into one without growth. This transformation only affects the discount factor when $\psi \neq 0$ and the depreciation rate (see Christiano and Eichenbaum 1992). Let s denote a pair of shocks (z,g) and let $\mu(s'|s)$ denote the associated transition probabilities. We begin by fixing an initial value for the Lagrange multiplier λ on the implementability constraint. Given this value of λ the solutions to the Ramsey allocation problem for $t \geq 1$ are stationary functions of (k,s). We use the resource constraint and (2.17) to express c(k,s) and ℓ(k,s) in terms of k, s, and the capital accumulation rule, k′(k,s). We use the Euler equation, (2.18), to determine k′(k,s).[3] We use the resource constraint and (2.19) to express date 0 consumption and employment in terms of the capital stock at the end-of-period capital and the Euler equation (2.20) to determine the end-of-period capital stock.

We solve for λ as follows. Since for $t \geq 1$ consumption, labor, and capital are stationary functions of (k,s) for each λ, (2.14) makes it clear that the bond allocation rule is also a stationary function of (k,s) for each λ. From (2.14) the bond allocation can be recursively written as follows

$$(5.3) \quad U_c(k,s)b(k,s) = \sum_{s'} \beta\mu(s'|s)[U_c(k',s')c(k',s') + U_\ell(k',s')\ell(k',s')U_c(k',s')b(k',s') + U_c(k',s')k'(k',s')] - U_c(k,s)k'$$

where $k' = k'(k,s)$ and $U_c(k,s)$, $U_\ell(k,s)$ are the marginal utilities of consumption and labor. Notice that (5.3) defines a linear operator mapping bond allocation rules into themselves. The stationary bond allocation rule is the fixed point of that mapping, where marginal utilities and quantities are computed using the stationary quantity allocation rules.[4] We substitute the date 0 consumption and employment rules into the marginal utility of consumption on the left-side of (5.3) and the stationary rules on the right side to derive the end of date 0 bond allocation rule.

Finally, we substitute the end of date 0 bond allocation rule together with the other date 0 allocation rules into the consumer budget constraint (1.3) evaluated at date 0. Using the equality between the marginal rate of substitution between consumption and labor and the after-tax wage rate and setting $\theta(s_0)$ to our initial rate of 27.1 percent, we calculate a value for $R_b(s_0)b_{-1}$. We iterate on λ until the initial value for $R_b(s_0)b_{-1}$ is 0.20, which is the steady state value of government obligations in a deterministic economy with the benchmark, nonstochastic policies. We compute the tax rate on labor income, the ex ante tax on capital income and the tax on private assets by substituting the allocation rules into (3.1), (3.10), and (3.14), respectively.

6. Findings

In this section we report on the statistical properties of the allocations and policies of our theoretical economies. For each setting of the parameter values we simulated our economy for 4,500 periods. As discussed in Sections 3 and 4, the optimal labor tax rate in period 0 and the optimal ex ante capital tax rate on capital income received in period 1 are different from the stationary policies. For the baseline model we found that the period 0 labor tax rate was −36 percent, and for the high risk aversion model we found that it was −17 percent. For the baseline model, the period 1 ex ante capital income tax rate was 796 percent, and for the high risk aversion model it was 1,326 percent. In terms of the properties of the stationary policies, we dropped the first 100 periods of our simulations to ensure that the allocations and policies are drawn from their stationary distributions and then computed a variety of statistics of the policies and the allocations.

Cyclical Properties

In Table 2 we report on some properties of the fiscal variables for our models. This table illustrates three of our main findings. First, in all the models the labor tax rate fluctuates very little. Second, as is to be expected from Proposition 3, in all the models with log utility the ex ante capital tax rate is identically zero. The more interesting finding is that even for the high risk aversion model, which has nonseparable utility, the ex ante capital tax rate is close to zero on average and fluctuates very little. Third, the tax rate on private assets is close to zero on average and fluctuates a great deal. In Figure 1 we plot histograms of these three tax rates for our high risk aversion model. The histograms further illustrate the three findings.

To get a feel for the sensitivity of these results we varied a number of parameters. We started by varying the risk aversion parameter ψ from 0 to -20. While adjusting the discount factor appropriately in Figures 2 and 3 we plot the means and the standard deviations of the optimal tax rates against the risk aversion parameter. These figures reinforce our basic findings. The mean labor tax rate declines as ψ becomes more negative because a lower intertemporal elasticity of substitution makes it optimal to increase the tax on capital in the transition period. This reduces revenue requirements in the steady state. The mean of the ex ante tax rate on capital is less than 1 percent even for values of the risk aversion parameter as high as 20. An interesting feature is that the standard deviation of the labor tax rate is not monotone in the risk aversion parameter. This finding is connected to a result which we discuss below, namely that the correlation between the labor tax rate and the underlying shocks changes sign near $\psi = -4$. The mean tax rate on private assets decreases with the risk aversion parameter for large negative values of ψ. This occurs because the steady state value of the debt becomes large and negative since the tax on capital in the transition phase rises as ψ falls. The standard deviation of the ex ante capital tax rate rises as ψ becomes more negative. This occurs because the lower intertemporal substitutability of consumption makes the welfare costs of varying capital tax rates over time smaller.

We also conducted a variety of experiments in which we varied other parameters of preferences, the parameters of technology and the stochastic processes for shocks. With one notable exception all the experiments confirmed our findings on the mean and variability of optimal tax rates. This exception is that when shocks are i.i.d., risk aversion is large, and initial debt is at its baseline level we found that while the mean of the ex ante capital tax rate was close to zero its standard deviation was quite different from zero. For example, when $\psi = -8$, the standard deviation was about 25 percent, and when $\psi = -20$ the mean was only 2.7 percent, but the standard deviation was about 70 percent.

Table 2 also illustrates two other features of optimal policies. The labor tax rate is highly persistent when the shocks are highly persistent and close to i.i.d. when the shocks are close to i.i.d. Thus, the labor tax rate inherits the persistence properties of the exogenous shocks. To investigate the robustness of this result we varied the autocorrelation of the technology and government spending shocks and computed the optimal policies. In Figure 4 we plot the autocorrelation of the labor tax rate against the autocorrelations of the technology and the government spending shocks where in each case we fix the other shock at its mean level. Figure 4 shows that, for both the log utility and high risk aversion models, the autocorrelation of the labor tax rate rises with the persistence of the shocks. Thus, there is no presumption that the Ramsey tax rates on labor should follow a random walk.[5]

Table 2 also illustrates that the properties of the tax rate on capital depend critically on how the Ramsey allocations are decentralized. We report the properties of the capital tax rate under two decentralizations. In one the capital tax rates are not state contingent and thus are simply the ex ante tax rates. In the other decentralization, the return on debt is not state-contingent. The statistical properties of the capital tax rates under these two decentralizations are obviously quite different. For example, for the high risk aversion model the ex ante tax rate is highly persistent while the tax rate with uncontingent debt is serially uncorrelated. Thus our model suggests that depending on the particular decentralization, the stochastic process for capital tax rates can range anywhere from i.i.d. to nearly a random walk.

Next we study the properties of optimal policies in more detail in Figures 5 and 6. In these figures we plot a segment of two simulations of the high risk aversion model, one with technology shocks only and one with government spending shocks only. Notice that all the tax rates jump when the underlying shocks change value and are relatively constant otherwise. Notice that the labor tax rate rises when there is a drop in technology and a drop in government spending. We found that, for the baseline model, these patterns are reversed, and the reversal in the patterns occurs approximately at $\psi = -4$. This finding suggests that the variance of the labor tax rate should be zero at approximately $\psi = -4$. Recall from Figure 3a that this is indeed what we found. Notice also that when there is a positive innovation to government consumption, or a negative innovation to the technology shock, there is a positive innovation in the tax on private assets. The reason is that the tax on private assets performs a shock absorber role. A positive innovation to government consumption, or a negative innovation to the technology shock, implies a negative shock to the government's budget constraint. It is efficient for these shocks to be absorbed mainly by the tax on private assets, rather than by changes in the labor tax rate.

We can get an idea of the magnitude of the shock absorber role of the tax on private assets from Figure 6. In the figure, we report government spending relative to output in the economy with the nonstochastic benchmark policies. When government spending rises from 15.2 percent to 17.5 percent of output, the tax rate on private assets rises from 0 to 300 percent. To further understand the magnitude of the shock absorber role, we regressed the innovation in the revenues from the tax on private assets on the innovations in government spending. The regression coefficient for the baseline model is 6.67 and that for the high risk aversion model is 5.49. For both economies, an increase in government spending of 1 percent of (steady state) output implies that the expected present value of government spending increases by approximately 8.06 percent. In the current period the tax on private assets finances approximately 83 percent of the innovation in this expected value for the baseline model, and finances 68 percent in the high risk aversion model.

Next we investigate the cyclical properties of the Ramsey allocations. We report on these properties for the baseline model. The results for the high risk aversion model are basically the same. We are particularly interested in how these properties compare to those in a benchmark economy in which the taxes are not optimally set. For a benchmark we constructed a crude approximation of the U.S. tax system. In doing so we had to be mindful of two issues. First, the U.S. tax system has a vast array of taxes as well as transfer payments while our model has only taxes on capital and labor and no transfer payments. Second, in the U.S. economy tax rates change in response to a variety of "shocks," while in our economy there are only two shocks. We constructed our crude approximation by considering stochastic processes for the tax rate on labor and the ex ante tax rate on capital of the form

$$\tau_t = a_0 + a_1 z_t + a_2 \tilde{g}_t \tag{6.1}$$

$$\theta^e_t = b_0 + b_1 z_t + b_2 \tilde{g}_t. \tag{6.2}$$

For the labor tax we used Barro and Sahasakul's (1983) estimate of the average marginal tax rate. For the ex ante tax rate on capital income we used Jorgenson and Sullivan's (1981) estimate of the effective corporate tax rate. For the technology shock and the government spending process we used Christiano's (1988) data. We detrended all variables using a continuous, piecewise linear trend with a single break in 1969 and obtained the coefficients (a_1,a_2,b_1,b_2) by ordinary least squares. We then set a_0 and b_0 to achieve two objectives. First, the ratio of the means of the tax rate on labor and the ex ante tax rate on capital equal those in the data. Second, in the model the tax revenues generated from the tax rate processes satisfied the government's intertemporal budget constraint with an initial debt equal to that in the deterministic economy with the benchmark, nonstochastic policies (i.e., $R_b(s_0)b_{-1} = 0.20$). (See Chari, Christiano, Kehoe 1991 for details.) We obtained: $a_1 = -0.027$, $a_2 = 0.11$, $b_1 = -0.71$, and $b_2 = 0.52$. The mean levels of the constructed labor tax rate and ex ante capital tax rate are 23.80 and 27.10 percent.

In Tables 3 and 4 we report the standard business cycle statistics for our model economy with the Ramsey policies and our version of the U.S. tax system.[6] Comparing these tables we see that the fluctuations in output, consumption, investment, and hours are smaller under the Ramsey policies. The correlation between government spending and output is higher under the Ramsey policies. The reason for these features is that labor and ex ante capital tax rates under the Ramsey system are smoother than under the estimated system and thus allocations fluctuate less. Tables 3 and 4 also show that the correlation between output and government spending is positive under the Ramsey system but negative under the estimated system. Again, the reason is that the tax rate on labor is much less responsive to shocks under the Ramsey system than under the estimated system. In fact, under the estimated system when government spending rises, the tax rate on labor rises by so much that employment, and therefore output, actually fall.

Welfare

Next we computed welfare gains from alternative tax systems relative to benchmark tax systems for a deterministic and a stochastic version of our model. Our welfare measure is that constant percentage amount by which consumption must be increased in all dates and states in the benchmark economy, while leaving employment unchanged, so as to yield the same utility as under the policy experiment. We begin by considering the balanced growth path of a benchmark deterministic economy that has government spending and the technology shocks fixed at their mean values and tax rates on labor and capital constant. The ratio of the tax rates on labor and capital in the model are chosen as in our comparison of business cycle statistics. That is, they equal the mean of those in the data and the levels are chosen so that along the balanced growth path government debt to GNP equals its average in the postwar U.S. economy, namely, 51 percent.

We conducted several experiments and report the results in Table 5. In each case, the initial conditions were given by the balanced growth path of the deterministic economy. In the first set of experiments, we computed the welfare gains from adopting the Ramsey policy for deterministic economies with log utility, high risk aversion ($\psi = -8$) and high initial debt. (Recall that the economy with high initial debt is an attempt to capture some of the consequences of introducing transfers into our setup.) For the model with log utility, the welfare gains are 1 percent of consumption, for the model with high risk aversion the gains are 1.3 percent and for the high initial debt economy they are 5.2 percent.

We decomposed the welfare gains into three sources. To motivate this decomposition recall that the Ramsey policies can be reasonably characterized as having a negative labor tax rate in period 0 followed by a constant labor tax rate in all subsequent periods together with a large positive tax rate on capital in period 1 followed by a zero tax rate on capital in all subsequent periods. The benchmark deterministic economy has constant positive tax rates on both types of income. Thus, for a deterministic economy the welfare gains come from three sources: the negative labor tax rate in period 0, the large positive capital tax rate in period 1, and the zero capital tax rate thereafter and in the steady state. We decomposed these welfare gains into these sources as follows. We computed the welfare gain relative to the benchmark policy by first computing welfare under a system where labor tax rates are constant from date zero on, keeping the capital tax rate as in the Ramsey system. The welfare gains were indistinguishable from those under the Ramsey system and so we do not report them. Thus, the negative date 0 labor tax rate plays a very minor role in the Ramsey plan. Next, we computed the welfare gains from a system under which tax rates on capital are zero in all periods from date 1 on and labor tax rates are constant. In the table we refer to this system as a constant tax system with zero capital taxes. From Table 5, we see that for our baseline model with log utility, the welfare gains are 0.2 percent. Thus, 80 percent of the welfare gains of the Ramsey system come from the large initial tax on capital income; and only 20 percent come from the subsequent and steady state elimination of capital income taxation. The results are even more dramatic for the high risk aversion and the high debt economies. Here switching to a system with zero capital taxes, in all periods including the first period of transition actually lowers welfare. Of course, from a theoretical perspective this should not be surprising since the optimal capital tax is nonstationary: a large initial tax then a zero rate thereafter. A single constant tax of zero in all periods misses the large initial tax and thus could be worse than a constant positive tax in all periods.

Next we investigate the welfare gains in stochastic economies. We consider two benchmarks. In both the policies are of the form in (6.1) and (6.2). In the estimated policy benchmark the parameters are obtained from regressions on U.S. data as described above. In the variable policy benchmark the policies were made more variable by multiplying a_1, a_2, b_1, and b_2 by a factor of five. In stochastic economies there is a source of welfare gains from the Ramsey policy, in addition to the three mentioned above. This source stems from our finding that under the Ramsey system the labor tax rates and the ex ante capital tax rates are essentially constant and not variable as in our benchmarks.

We found that, as in the deterministic economy, the negative tax rate and date 0 has insignificant welfare effects. Next we investigate the welfare effects of the high capital tax rate in period 1. Recall that under the Ramsey system, both labor tax rates and capital tax rates are essentially constant after date 1. Thus, the difference in the welfare gains between the Ramsey system and the constant tax system with zero capital tax is due to the initial capital tax. As in the deterministic economy we find this source is sizeable and accounts in the log utility case for 80 percent of the welfare gains from the Ramsey system. Finally, we investigate the welfare gains from the smoothing of the labor and ex ante capital tax rates under the Ramsey system. One way of isolating these welfare gains is to consider a system which taxes capital and labor at high average rates as in the benchmark economies but does not permit them to fluctuate. In Table 5 we refer to this system as one with constant taxes and high capital taxes. As can be seen from the table, the welfare gain from such a system is 0.03 percent for the log utility and the high risk aversion parameterization. This welfare gain is small. One reason for this small welfare gain could be that the estimated policies are not that variable to start with. To investigate this possibility, we considered benchmark economies in which labor and ex ante capital tax rates were five times as volatile as the estimated policies. For such benchmarks, we found sizeable welfare gains from eliminating fluctuations in their tax rates. For example, for the log utility case, the gain in welfare from this source accounts for almost 40 percent of the Ramsey gains.

7. Remarks on Scope

We have studied an economy in which the government uses capital and labor income taxation to raise revenues and have shown how the problem of solving the Ramsey equilibrium reduces to the simpler problem of solving for the Ramsey allocations. A wide variety of other tax systems lead to the same Ramsey allocation problem. For example, consider a tax system which includes consumption taxes as well as labor and capital income taxes. It can be shown that the Ramsey allocations can be supported by a tax system which uses any two of the three types of tax instruments. Thus, for example, the Ramsey allocations can be supported by consumption and capital income taxes only, consumption and labor income tax only, or capital and labor income taxes only.

To illustrate this point consider an economy with consumption and labor income taxes. The consumer's intratemporal first order condition is

$$(7.1) \quad -\frac{U_\ell(s^t)}{U_c(s^t)} = \frac{(1-\tau(s^t))}{(1+\nu(s^t))} F_\ell(s^t)$$

where ν is the tax rate on consumption. The consumer's first order condition for capital is

$$(7.2) \quad \sum_{s_{t+1}} q(s^{t+1}) \frac{(1+\nu(s^t))}{(1+\nu(s^{t+1}))} [1-\delta+F_k(s^{t+1})] = 1.$$

The analogue of Proposition 3 for this economy is that, for $t \geq 1$,

$$(7.3) \quad \sum_{s_{t+1}} q(s^{t+1}) \frac{(1+\nu(s^t))}{(1+\nu(s^{t+1}))} [1-\delta+F_k(s^{t+1})] = \sum_{s_{t+1}} q(s^{t+1})[1-\delta+F_k(s^{t+1})].$$

For reasons analogous to those in Proposition 2 there is an indeterminacy in the consumption tax rates and the debt policy. One way of supporting the optimal allocations is to make the consumption taxes not contingent on the current state. For such a decentralization (7.3) implies that for $t \geq 1$ all the consumption tax rates are equal. This result is a generalization of well-known results on uniform taxation (see Atkinson and Stiglitz 1972).

It should be clear from this example that the detailed implications for tax rates depend on the particulars of the tax system chosen. In contrast the theory has unambiguous implications about the relation between marginal rates of substitution and marginal rates of transformation. For example, the central implication of Proposition 3 is, for $t \geq 1$,

$$(7.4) \quad 1 - \sum_{s_{t+1}} q(s^{t+1})(1-\delta+F_k(s^{t+1})) = 0.$$

That is, it is not optimal to distort the consumer's intertemporal first order condition. In this paper we chose to focus on capital and labor income taxation to make our work comparable to the literature.

Conclusions

In this paper we investigated the quantitative properties of optimal fiscal policy in a standard business cycle model. We found that the ex ante tax on capital income is approximately zero. We found that the labor tax rate fluctuates very little and inherits the serial correlation properties of the exogenous shocks. We found that the tax on private assets fluctuates a great deal. Finally, we found that the welfare gains to optimal taxation come primarily from the transition phase of high capital income taxation.

In the model the tax on private assets plays the role of a shock absorber. To see this consider decentralizing a Ramsey allocation with state uncontingent capital taxes. In such a decentralization the fluctuations in the tax on private assets arise from the variations in the real payments on government debt. In a Ramsey equilibrium the government structures these payments in order to insure itself from having to sharply change labor tax rates when the economy is hit by shocks. In this sense state contingent debt is a form of insurance purchased by the government from consumers.

One could imagine a variety of reasons why it might be difficult to issue and enforce these types of insurance claims. One could also imagine forces which limit the state contingency of capital tax rates. As an extreme it might be useful to study economies in which both real debt and capital tax rates are restricted to be state uncontingent. We conjecture that in such economies labor tax rates will be more persistent than the underlying shocks. Another avenue of research is to explore the role of inflation in converting nominal uncontingent claims on the government into real state contingent claims. Exploring this avenue may also lead to insights into the role of optimal monetary policy. We are currently exploring both of these lines of research.

An interesting finding is that only a small fraction of the welfare gains come from smoothing tax rates and eliminating capital income taxation. Rather most of the welfare gains come from the high taxation of capital in the transition period. In this sense the temptation to renege on the previously chosen policies is large once the transition phase has passed. Thus the time inconsistency problem is quantitatively severe. Hence implementing policies of the type described here without strong safeguards against reneging in the future is likely to prove counterproductive.

Finally, our model abstracts from a variety of issues including income distribution, heterogeneity, externalities, money, and growth. (For some recent work on the last issues see Cooley and Hansen 1992 and Jones, Manuelli, and Rossi 1993.) Instead the model focuses attention on intertemporal efficiency. We think the forces driving our results will be present in more elaborate dynamic models.

Footnotes

[1]There is a voluminous literature in public finance on various aspects of optimal capital income taxes, including among others Atkinson (1971), Diamond (1973), Pestieau (1974), and Atkinson and Sandmo (1980). (See also Chapter 2 of Auerbach and Feldstein (1985) and the references cited therein.) For the most part these analyses deal with overlapping generation models while we use a model with infinitely lived agents. For analyses in an infinite-lived agents context, with human and physical capital, see Bull (1990) and Jones, Manuelli, and Rossi (1993).

[2]For our computations we used a value of $k_{-1} = 1.05$ and $R_b(s_0)b_{-1} = 0.20$.

[3]Finding a function, $k'(k,s)$ which satisfies (2.18) for all k, s is computationally infeasible. In practice, we limit ourselves to a finite-parameter class of decision rules, $k'(k,s;a) = \exp\{\Sigma_{i=0}^{n-1} a_i(s)T_i(\psi(\log(k)))\}$, where $T_i(\cdot)$ is the i^{th} Choleski polynomial (see Press, et. al., 1988), and $a_i(s)$, $i = 0, \ldots, n-1$ is a set of coefficients, for each of the four possible values of s. The 4n element vector, a, denotes these coefficients. The function $\psi(\cdot)$ maps an interval containing the ergodic set for log(k) into the interval $[-1,1]$. We chose values for a to get the expression to the left of the equality in (2.18) to be close to zero. For this, we used the following version of the Galerkin method discussed in Judd (1992). Let k_j, $j = 1, \ldots, m$ denote the values of k satisfying $T_m(\psi(\log(k))) = 0$, where $m \geq n$. Let A denote the $n \times m$ matrix with components $A_{ij} = T_{i-1}(\psi(\log(k_j)))$, $i = 1, \ldots, n$, $j = 1, \ldots, m$. Let R(s,a) denote the $m \times 1$ vector formed by evaluating the expression on the left of the equality in (2.18) using the decision rule, $k'(k,s;a)$, at the m values of k, for each s. Then, we selected the 4n parameters a so that the 4n equations, AR(s,a) all s, equal zero. For this, we used a standard nonlinear equation solver. We obtained a starting value for these calculations by finding the nonexplosive, log-linear capital decision rule that solves a version of (2.18) in which the function whose expectation is taken is log-linearized about the nonstochastic steady-state capital stock. We found that $n = 10$ and $m = 41$ works well in the

sense that larger values for these parameters resulted in no noticeable change in our results. (See Chari, Christiano, and Kehoe 1991 for further details.)

[4]It is not computationally feasible to find a function, b(k,s), that satisfies (5.3) for all k, s. Instead, we restricted the bond rule to be continuous and piecewise linear in k for each fixed s, and required that (5.3) be satisfied at a finite set of points. For each s, the nodes of our bond function occur at the values of k in the m-dimensional capital grid discussed in footnote 3. The values of the debt rule at these node points define its parameters. The requirement that (5.3) be satisfied at the 4m node points defines a linear map from the 4m-dimensional space of bond rule parameters into itself. We found the fixed point of this mapping by solving a system of 4m linear equations.

[5]This result can be established analytically for the version of our model without capital. In this case, it can be seen from the analogue of (2.17) and the resource constraint that consumption and employment, and therefore the tax rate on labor, depend only upon the current realization of the exogenous shocks. Thus, in this case, one can prove that the tax rate on labor inherits the persistence properties of the exogenous shocks. We find it interesting that in our quantitative model with capital, the labor tax rate also inherits the persistence properties of the shocks.

[6]We solved the latter model using an appropriately modified version of the method used to solve the Ramsey problem.

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Table 1

Baseline Model Parameter Values

Preferences:			
	$\gamma = .75$	$\psi = 0$	$\beta = .98$
Technology:			
	$\alpha = .34$	$\delta = .08$	$\rho = .016$
Stochastic Process for Government Consumption:			
	$G = .07$	$\rho_g = .89$	$\sigma_g = .07$
Stochastic Process for Technology Shock:			
	$\rho_z = .81$	$\sigma_z = .04$	

Table 2

Properties of Tax Rates for Model Economies

	Baseline Model	High Risk Aversion Model	Alternative Shock Processes: Only Technology	Alternative Shock Processes: Only Govt. Cons.	Alternative Shock Processes: I.I.D.
Labor Tax Rate					
Mean	23.87	20.69	23.80	23.87	23.84
Standard Deviation	.10	.04	.08	.06	.15
Autocorrelation	.80	.85	.71	.90	−.04
Correlation with Gov't Consumption	.65	−.59	NA	1.00	.10
Correlation with Technology Shock	.55	−.84	.64	NA	.95
Ex Ante Capital Tax Rate					
Mean	.00	−.06	.00	.00	.00
Standard Deviation	.00	4.06	.00	.00	.00
Autocorrelation	NA	.83	NA	NA	NA
Correlation with Gov't Consumption	NA	.33	NA	NA	NA
Correlation with Technology Shock	NA	.96	NA	NA	NA
Private Assets Tax Rate					
Mean	1.10	−.88	2.10	−.89	−.51
Standard Deviation	53.86	78.56	23.71	47.20	15.93
Autocorrelation	−.01	.02	.01	.01	−.02
Correlation with Gov't Consumption	.39	.46	NA	.45	.93
Correlation with Technology Shock	−.23	.02	−.55	NA	−.31
Capital Tax Rate With Uncontingent Debt					
Mean	.55	−.42	1.19	−.59	.23
Standard Deviation	40.93	30.35	17.67	36.22	12.03
Autocorrelation	−.01	.02	.01	.01	−.02
Correlation with Govt. Consumption	.40	.47	NA	.46	.94
Correlation with Technology Shock	−.24	−.02	−.56	NA	.33

NOTES: To compute the statistics we simulated a realization of 4,500 periods and then dropped the first 100 periods. The means and standard deviations are in percent terms. The NA signifies that the relevant statistic is not well-defined.

Table 3

Cyclical Properties of Model Economy Under the Ramsey Tax System

Variable	Standard Deviation: Percent	Standard Deviation: Relative to Output	Cross Correlation With Output at Lag k: k = −2	k = −1	k = 0	k = 1	k = 2
Output	2.72	1.00	.42	.68	1.00	.68	.42
Consumption	1.69	.62	.66	.75	.78	.45	.20
Investment	6.89	2.54	.24	.55	.96	.68	.46
Hours	1.27	.47	.00	.32	.79	.61	.45
Government Spending	3.97	1.46	.05	.09	.12	.08	.04
Productivity	1.87	.69	.60	.76	.91	.57	.30

NOTE: Statistics pertain to Hodrick-Prescott filtered data.

Table 4

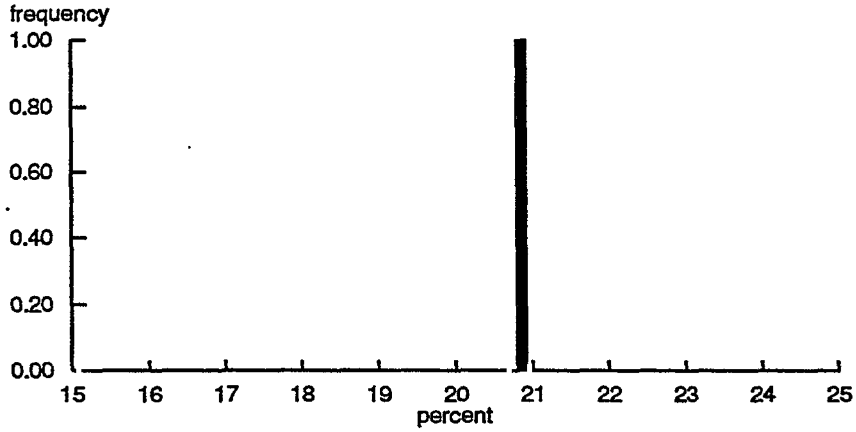
Cyclical Properties of the Baseline Model Under the Estimated Tax System

Variable	Standard Deviation: Percent	Standard Deviation: Relative to Output	Cross Correlation With Output at Lag k: k = −2	k = −1	k = 0	k = 1	k = 2
Output	3.03	1.00	.44	.69	1.00	.69	.44
Consumption	1.79	.59	.72	.79	.80	.47	.22
Investment	9.23	3.05	.26	.55	.95	.69	.48
Hours	1.57	.52	.04	.37	.85	.66	.50
Government Spending	3.97	1.31	−.03	−.06	−.10	−.09	−.08
Productivity	1.87	.62	.67	.80	.90	.56	.29

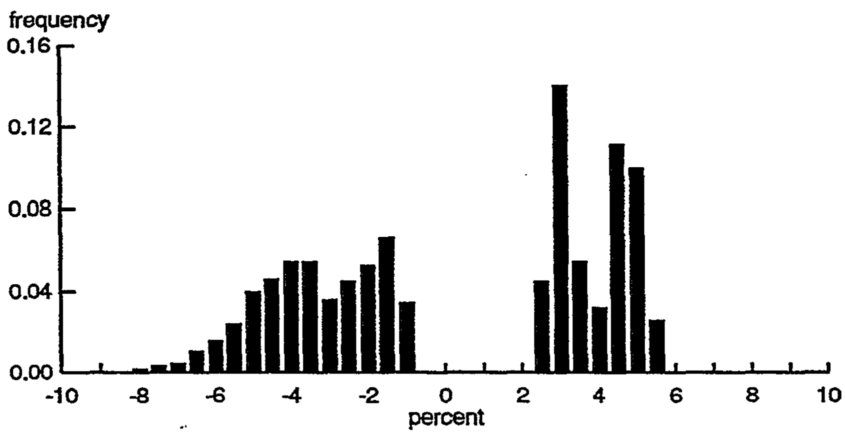
NOTE: Statistics pertain to Hodrick-Prescott filtered data.

Figure 1: Frequency distribution of tax rates*

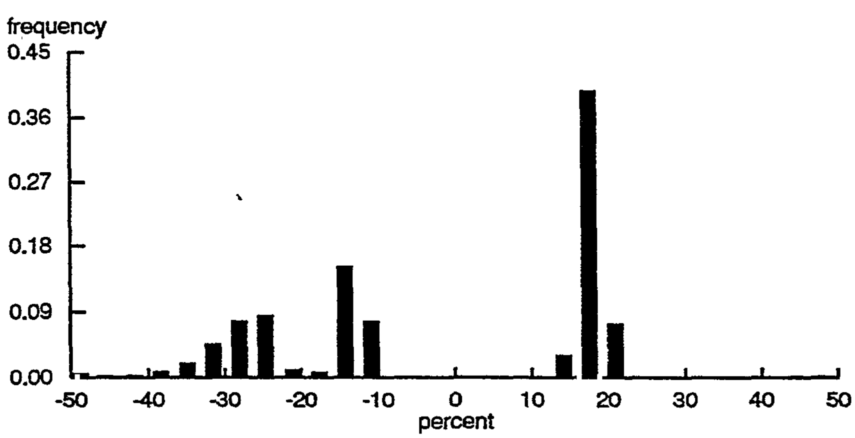
a) Labor tax rate



b) Ex ante tax rate on capital

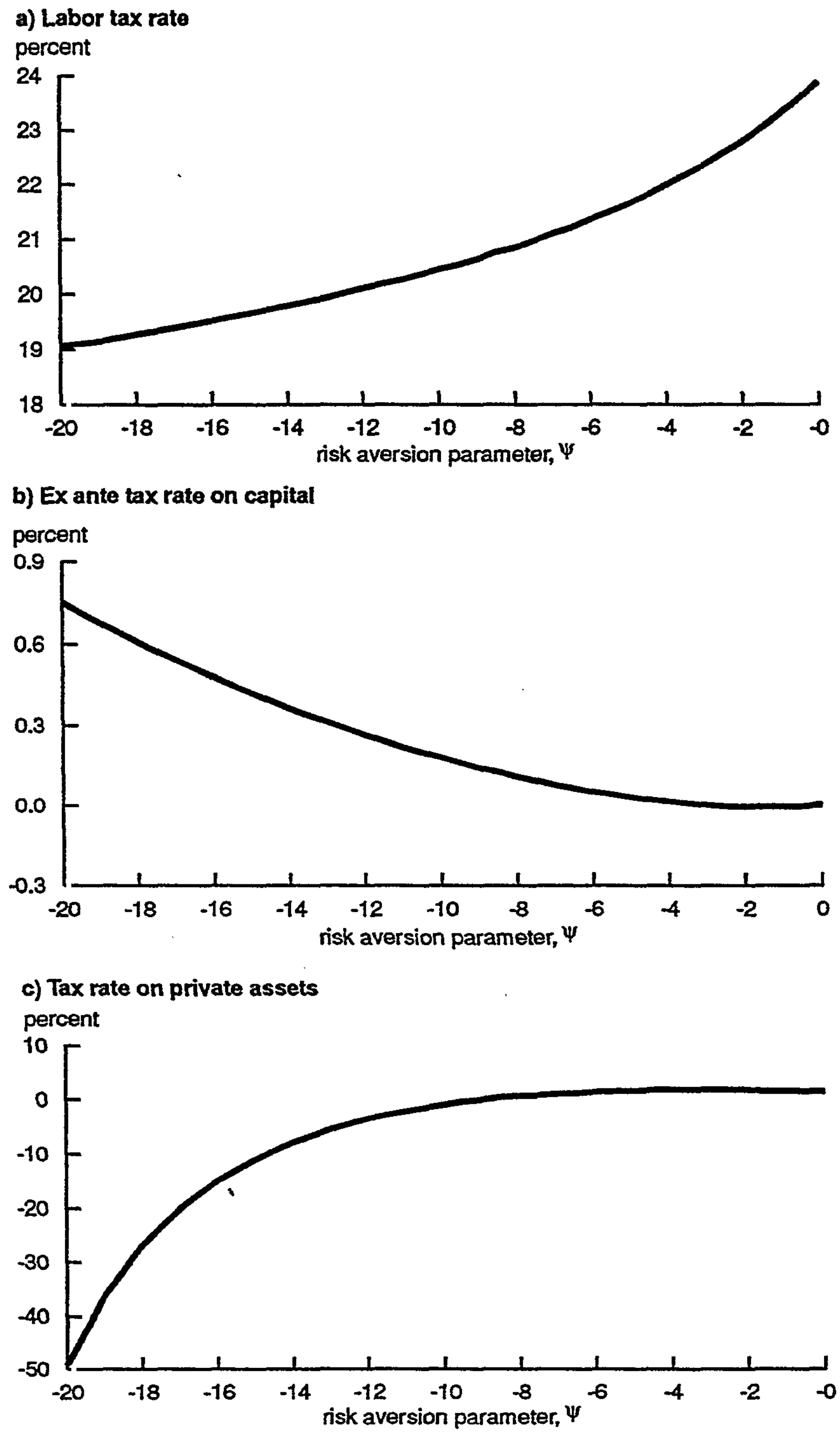


c) Tax rate on private assets**



*Frequency distribution of tax rates based on a simulation of length 4,400 from high risk aversion model.
**Frequency distribution, conditional on being in the interval, (-50,50).
The frequency of realizations lying outside this interval is 0.10.

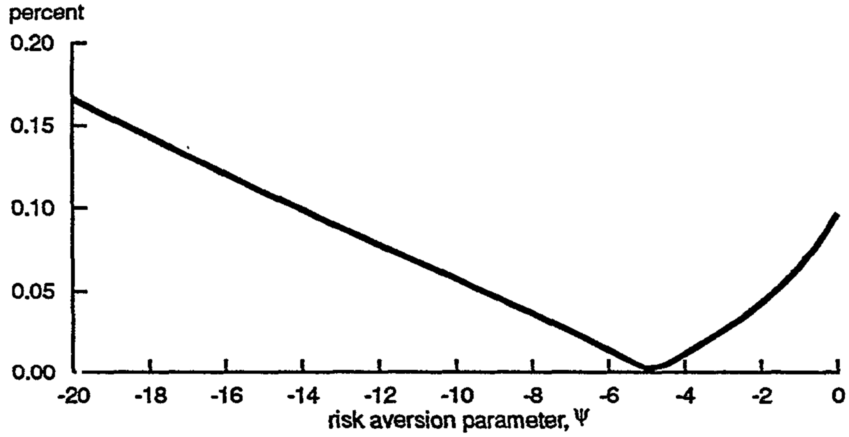
Figure 2: Mean tax rates*



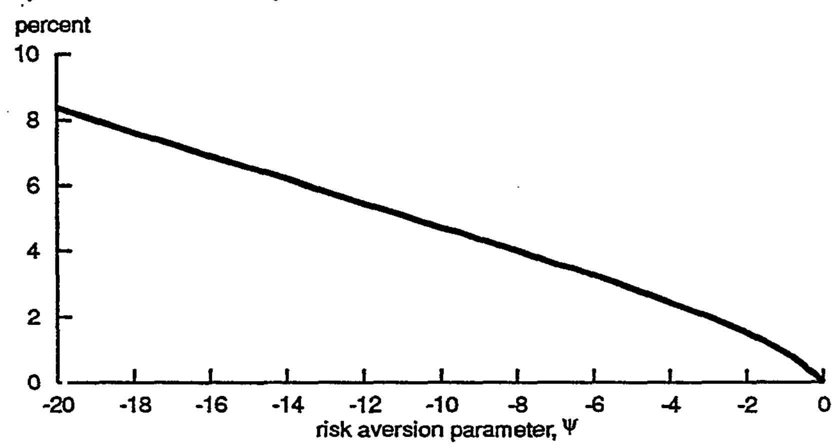
*Mean tax rates computed from simulation of length 4,400 for each value of risk aversion, ψ.

Figure 3: Standard deviation of tax rates*

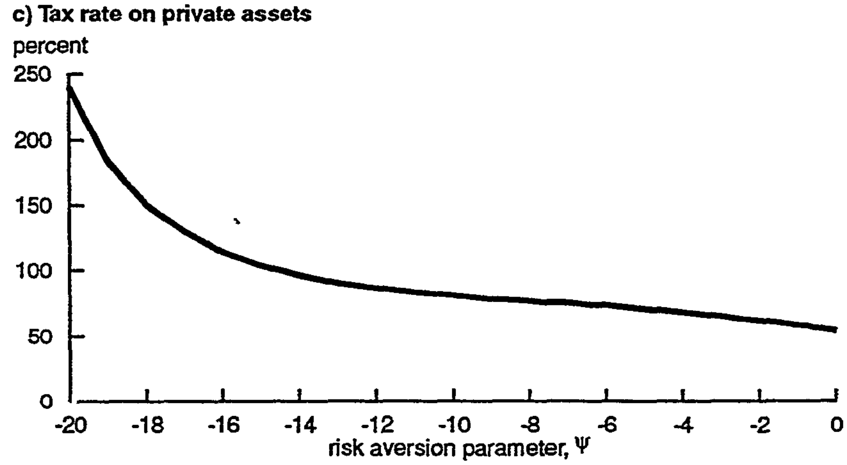
a) Labor tax rate



b) Ex ante tax rate on capital



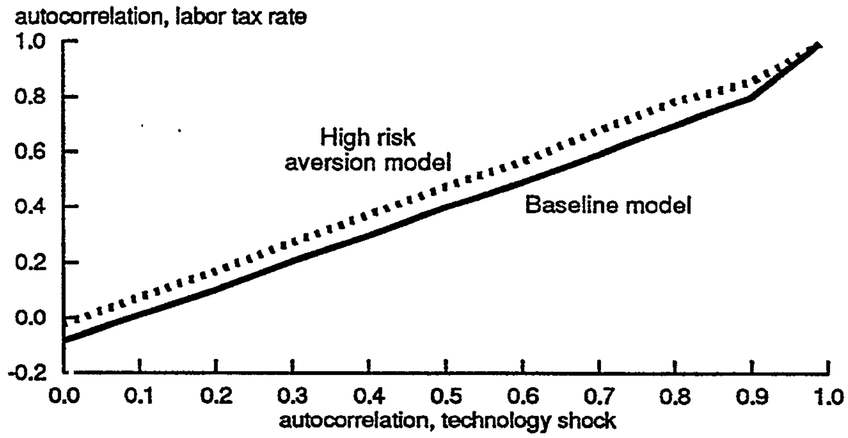
c) Tax rate on private assets



*Standard deviation, expressed in percent terms, computed from simulation of length 4,400 for each value of risk aversion, ψ.

Figure 4: Autocorrelation of labor tax rate

a) Technology shock only model



b) Government spending shock only model

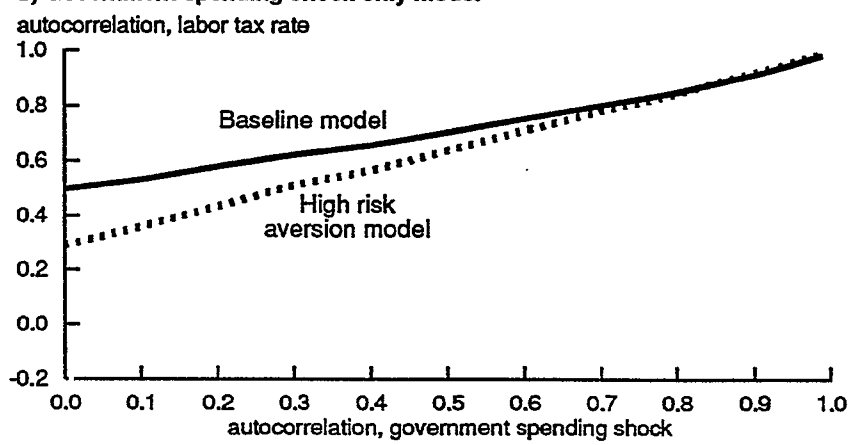
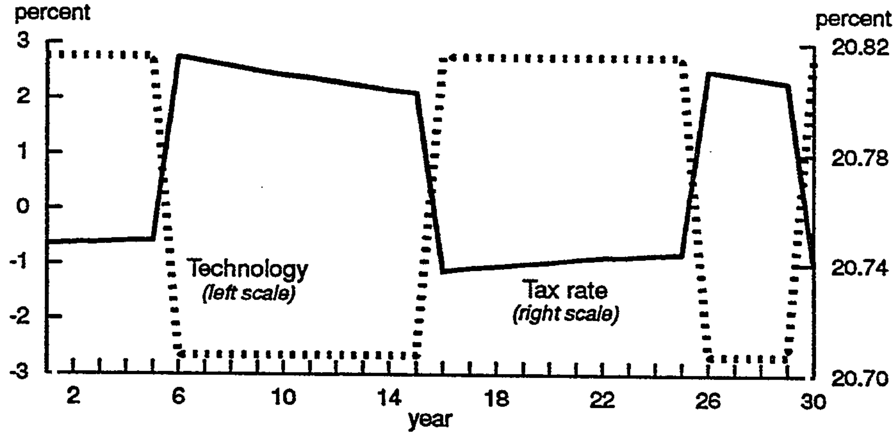
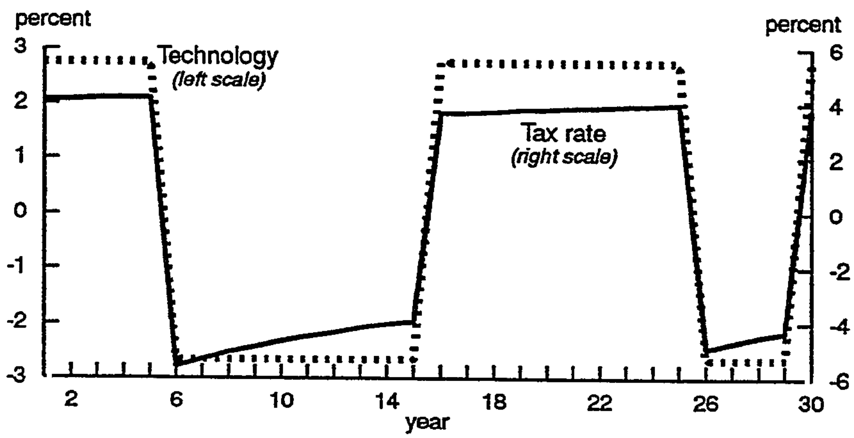


Figure 5: Technology shock model simulation*

a) Technology and labor tax rate



b) Technology and ex ante tax rate on capital



c) Technology and tax rate on private assets

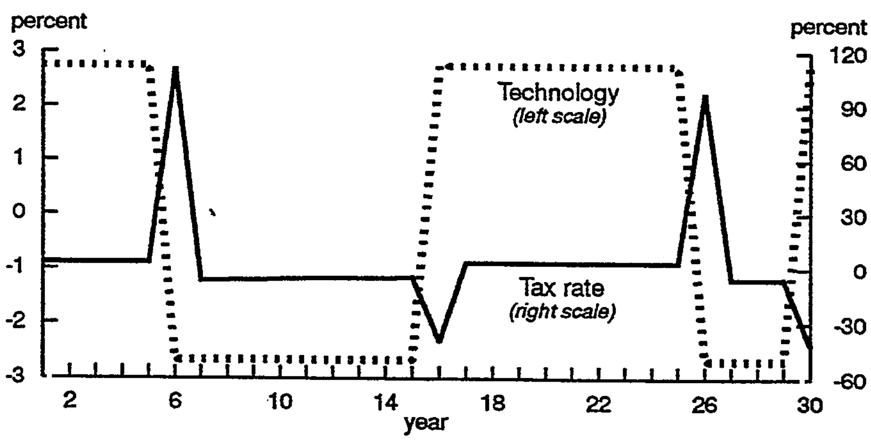


*Figure 3a-3c displays 30 simulated observations from the high risk aversion model with no government spending shocks. Tax rates are in percent terms and the technology shock is expressed as $\{\exp[(1-\alpha)z_t]-1\}\times 100$.

Figure 6: Government spending shock model simulation*

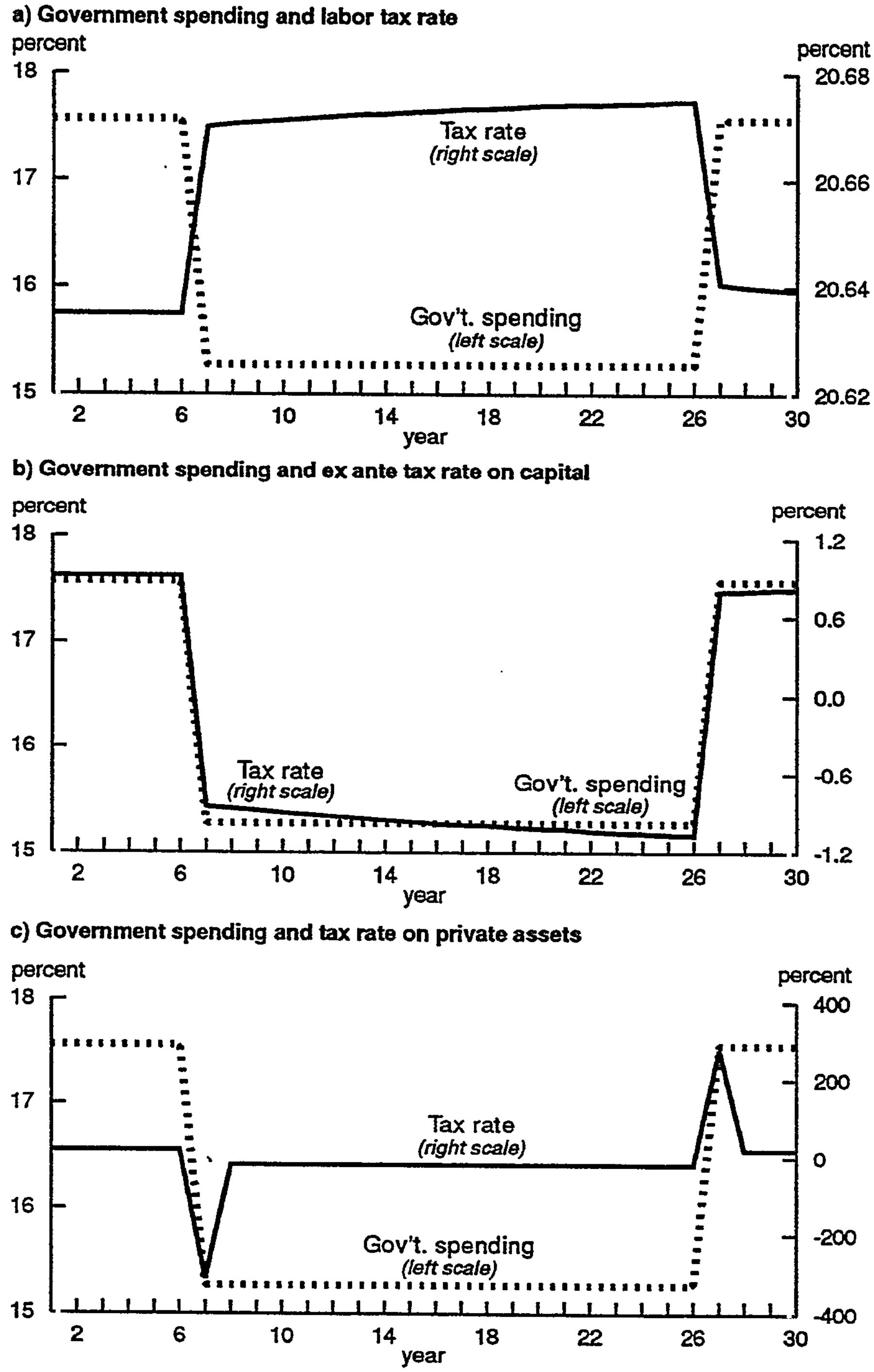


*Figure 4a-4c displays 30 simulated observations from high risk aversion model with no technology shocks. Tax rates are expressed in percent terms, and government spending is expressed as a percent of nonstochastic steady state output.