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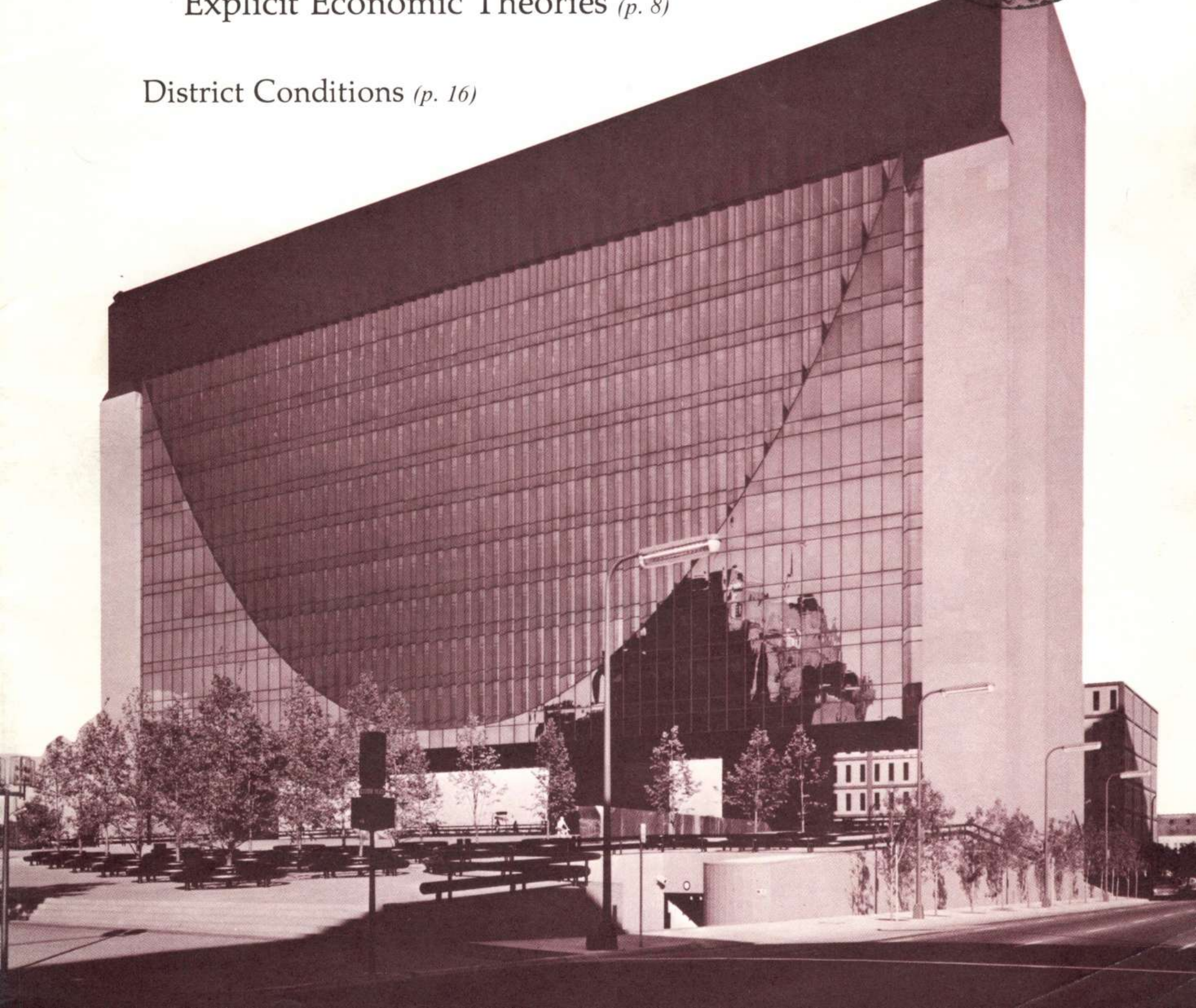

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Address requests for additional copies to the Research Department, Federal Reserve Bank, Minneapolis, Minnesota 55480.

Estimating Vector Autoregressions Using Methods Not Based on Explicit Economic Theories

Thomas J. Sargent

Adviser, Research Department
Federal Reserve Bank of Minneapolis
and
Professor of Economics
University of Minnesota

This paper describes procedures for analyzing interrelated time series[1] which are mainly intended as an alternative to using structural econometric models as forecasting devices. Alternatives to the structural models have been sought because of increasingly compelling suspicions that the a priori restrictions used in existing structural models are not implied by good dynamic economic theory and that the interpretations and policy conclusions based on those faulty a priori restrictions are worth little. The techniques described in this paper are not based on economic theory. Instead, the idea is to estimate vector autoregressions with many free parameters and to introduce restrictions not directly motivated by economic theory but rather aimed simply at forecasting better, that is, delivering estimators with small mean squared errors.

Because these techniques are not based on economic theory, they do not completely substitute for structural models. They cannot appropriately be used to analyze the range of policy interventions that structural models were designed to evaluate. The techniques are not appropriate for conditional forecasting, for predicting the behavior of the system under what may be a drastic change from the historical pattern in a feedback rule for a policy variable, for example. Instead, these techniques are designed mainly for unconditional forecasting and for compactly summarizing data. Thus, users of the statistical models described in this paper must acknowledge from the start that they are vulnerable to Lucas' (1976) criticism of econometric policy evaluation methods, and they often must restrict the domain of the questions to which answers are sought if Lucas' criticism is not to be operative.

Vector Autoregressions

For the purposes of making forecasts and displaying its operating characteristics, a linear econometric model is often represented as a particular set of random difference equations called a *vector autoregression*.[2] Thus, let z_t be an $(N \times 1)$ vector of variables, including both all of the endogenous and all of the exogenous variables in the model. Let z_t be measured in terms of deviations from means. Further, assume that z_t is a wide-sense stationary stochastic process[3] which has matrix covariogram $Ez_t z'_{t-k} = C_z(k)$. Then the M^{th}

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[1]These procedures have been developed and applied to macroeconomic data by Christopher Sims of the University of Minnesota and Robert Litterman, now of M.I.T. but until recently an analyst in the Research Department at the Federal Reserve Bank of Minneapolis. The key references are Sims 1975, 1977; Sargent and Sims 1977; and Litterman 1979. [Author names and years refer to the works listed at the end of this paper.] This paper is intended only as an introduction to the work of Sims and Litterman and makes no claim for originality of any of the ideas discussed.

[2]Good general references on the time series methods described here are Anderson 1971, Box and Jenkins 1970, Fuller 1976, Granger and Newbold 1977, and Nerlove, Grether, and Carvalho 1979. For an introduction to vector autoregressions and some of their uses in macroeconomics, see Sargent 1979, chapter XI.

[3]A vector stochastic process z_t is said to be *wide-sense stationary* if the vector of means Ez_t is a constant vector independent of time t and if the matrix covariogram $Ez_t z'_s$ depends on only the difference $(t-s)$ and not only t and s separately. Wide-sense stationarity is also referred to as *second-order stationarity* and *covariance stationarity*.

order vector autoregression for the z_t process is

$$(1) \qquad z_t = \sum_{j=1}^{M} D_j^M z_{t-j} + \eta_t^M$$

where the D_j^M's are $(N \times N)$ matrices and the $(N \times 1)$ stochastic error process η_t^M satisfies the orthogonality conditions[4]

$$(2) \qquad E\eta_t^M z_{t-k}' = 0 \qquad k = 1, \ldots, M.$$

Post-multiplying (1) by z_{t-k}' and using (2) gives the least squares normal equations (or Yule-Walker equations)

$$(3) \qquad C_z(k) - \sum_{j=1}^{M} D_j^M C_z(k-j) = 0 \qquad k = 1, \ldots, M.$$

The normal equations (3), in general, uniquely determine the matrices D_j^M in terms of the population values of the second-moment matrices $C_z(k)$, $k = 0, 1, \ldots, M$.

Under the assumptions given here, least squares estimates of the D_j^M's are known to be statistically consistent.[5] But the vector autoregressive system (1) has $(N^2 \times M)$ free parameters in the D_j^M matrices, so that for even moderate sizes of M and N, least squares estimation either is simply not feasible due to exhaustion of degrees of freedom or else is unwise due to the large sampling errors present when the number of parameters to be estimated nearly exhausts all degrees of freedom. For this reason, systems of vector autoregressions with the sizes N and M usually encountered in economics have typically been estimated by methods other than least squares.

Until recently, the most popular method of estimating vector autoregressions was to apply classical simultaneous equation estimators to the structural model that presumably underlay the vector autoregression.[6] Simultaneous equation estimators have the virtue of permitting the model builder to bring to bear a priori information of certain kinds to produce parameter estimates with smaller sampling errors of the D_j^M's than can be produced by least squares. In statistical jargon, use of this prior information produces more "efficient estimators." In the present context, these techniques can be viewed as a device for reducing the number of parameters that have to be estimated from $(N^2 \times M)$ to a much smaller number of theoretically more fundamental parameters of which the D_j^M's are functions. The argument is that the vector autoregression is a "profligately parameterized" representation[7] and that estimation proceeds much more efficiently by focusing on the structural parameters about which something is known in advance of estimation.

To make this argument more precise, we use the representation of a linear econometric model described by Lucas and Sargent (1979). The *structural equations* are

$$(4) \qquad \begin{aligned} &A_0 y_t + A_1 y_{t-1} + \ldots + A_m y_{t-m} \\ &= B_0 x_t + B_1 x_{t-1} + \ldots + B_n x_{t-n} + \epsilon_t. \end{aligned}$$

The random error generating equations are

$$(5) \qquad R_0 \epsilon_t + R_1 \epsilon_{t-1} + \ldots + R_r \epsilon_{t-r} = u_t \qquad R_0 = I.$$

Here y_t is an $(L \times 1)$ vector of *endogenous* variables, x_t is a $(K \times 1)$ vector of *exogenous* variables, and ϵ_t and u_t are each $(L \times 1)$ vectors of random disturbances. The matrices A_j are each $(L \times L)$, the B_j's are $(L \times K)$, and the R_j's are each $(L \times L)$. We assume that $L + K = N$. The $(L \times 1)$ disturbance process u_t is assumed to be serially uncorrelated with $Eu_t = 0$ and with contemporaneous covariance matrix $Eu_t u_t' = \Sigma$ and $Eu_t u_s' = 0$ for $t \neq s$.

The defining characteristic of the exogenous variables x_t is that they are uncorrelated with the ϵ's at all lags so that $Eu_t x_s'$ is an $(L \times K)$ matrix of zeros for all t and s. The x_t process is itself assumed to be generated by the vector autoregression

$$(6) \qquad x_t = C_1 x_{t-1} + \ldots + C_p x_{t-p} + a_t$$

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[4] These orthogonality conditions uniquely identify $\Sigma_{j=1}^{M} D_j^M z_{t-j}$ as the least squares projection of z_t onto the linear vector space spanned by $\{z_{t-1}, \ldots, z_{t-M}\}$.

[5] For proofs, see Ljung 1976 and Anderson and Taylor 1976.

[6] The classical simultaneous equation model and estimators are described in any good book on econometrics, for example, Goldberger 1964 or Maddala 1977.

[7] This is Sims' terminology.

where $Ea_t = 0$ and $Ea_t x_{t-j} = 0$ for $j \geq 1$.

As in Lucas and Sargent 1979, the *reduced form* of this system is

$$(7) \qquad y_t = -P_1 y_{t-1} - \ldots - P_{r+m}\, y_{t-r-m} + Q_0 x_t + \ldots + Q_{r+n}\, x_{t-n-r} + A_0^{-1} u_t$$

where

$$(8) \qquad \begin{aligned} P_s &= A_0^{-1} \sum_{j=-\infty}^{\infty} R_j A_{s-j} \\ Q_s &= A_0^{-1} \sum_{j=-\infty}^{\infty} R_j B_{s-j}. \end{aligned}$$

In these expressions for P_s and Q_s, it is to be understood that matrices not previously defined (for example, any with negative subscripts) are zero. Substituting the right side of (6) for x_t in (7) gives the component of the vector autoregression for y_t:[8]

$$\begin{aligned} (9) \qquad y_t = &-P_1 y_{t-1} - \ldots - P_{r+m}\, y_{t-r-m} \\ &+ [Q_0 C_1 + Q_1]\, x_{t-1} + [Q_0 C_2 + Q_2]\, x_{t-2} \\ &+ \ldots + [Q_0 C_p + Q_p]\, x_{t-p} + Q_{p+1}\, x_{t-p-1} \\ &+ \ldots + Q_{r+n}\, x_{t-n-r} + \{Q_0 a_t + A_0^{-1} u_t\} \end{aligned}$$

$$(6) \qquad x_t = C_1 x_{t-1} + \ldots + C_p x_{t-p} + a_t.$$

Notice that in the representation (9) and (6), y_t is written as a function of lagged y's and lagged x's, while the exogenous variables x_t only depend on lagged x_t's.

Equations (9) and (6) are the vector autoregressive representation of the structural model consisting of the structural equations (4) and (5). Equations (9) and (6) are a special case of the vector autoregression (1) with $z_t = [y_t\ x_t]'$. It is to be noted that (8) implies that the parameters of (9), the P_j's and Q_j's, are themselves complicated functions of the structural parameters, the A_j's, B_j's, and R_j's that appear in (4) and (5). Standard simultaneous equation estimators, as applied in economics, typically bring prior information to bear in the form of certain kinds of restrictions directly on the A_j's, B_j's, and R_j's. The notion is that the A_j's, B_j's, and R_j's are the parameters about which economic theory has something directly to say. Generally, the restrictions used take the form of sets of simple linear restrictions on the A_j's, B_j's, and R_j's. Most often, these assume the form simply of setting many, indeed most, of the coefficients in A_j, B_j, and R_j to zero a priori. Another set of exclusion restrictions is evident in (6), in which lagged y_j's are assumed not to appear. The asymmetrical treatment of lagged x's and y's in (6) and (9) is what distinguishes between endogenous and exogenous variables. For now, we simply note that the exclusion of lagged y's from (6) in most applications is done on an entirely a priori basis.

From the somewhat narrow viewpoint of estimating vector autoregressions, the virtue of using this body of a priori exclusion restrictions on lagged y's in (6) and on the A_j's, B_j's, and R_j's is that to the extent that the restrictions are approximately correct and numerous enough, more efficient estimates can be obtained of the parameters of the vector autoregression (6) and (9). That is, the D_j^M's of (1) can be estimated more precisely by constructing the model and introducing prior information in terms of the fundamental objects, the A_j's, B_j's, and R_j's. There is a presumption that these more efficient[9] estimates produced by a simultaneous equation estimator will lead to better predictions when the vector autoregression is used for forecasting.

Were there agreement that the a priori restrictions on the A_j's, B_j's, and R_j's described above are approximately correct, there would be no quarrel with the preceding case for using existing structural estimators as devices for estimating vector autoregressions for use in unconditional forecasting. However, over the last decade or so it has become increasingly evident that dynamic economic theories typically do not lead to prior information about the A_j's, B_j's, and R_j's of the kind described above. This argument is developed in some detail by Lucas and Sargent (1979), who argue that dynamic economic theory gives rise to restrictions of a very different form than those that

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[8] We have assumed that $p < (n+r)$. The reader can readily derive the appropriate formula where $p \geq (n+r)$.

[9] More efficient than ordinary least squares estimates.

are currently implemented or even implementable in existing computer econometric procedures. The upshot is that there is little reason from good dynamic economic theory to believe that the restrictions on the A_j's, B_j's, and R_j's imposed by existing structural macroeconometric[10] models are even approximately correct. As Sims (1977) has described the situation, the identifying restrictions used in existing macroeconometric models are "incredible."

While this argument substantially weakens the case for using structural estimators as a device ultimately to estimate vector autoregressions, it does not entirely destroy the case. For some device restricting the number of free parameters in vector autoregressions must be adopted if the estimation of systems with sizable ($N^2 \times M$) is to be practical. Sims (1977) has argued that even though the standard identifying restrictions are incredible and most likely to be false from the viewpoint of dynamic economic theory, they may still be valuable from the instrumental point of view of helping to estimate vector autoregressions by effectively reducing the dimensionality of the space of free parameters. Loosely, the idea is that even wrong prior restrictions may prove useful by permitting one to trade reduced variance of estimates for increased bias. This line of argument is Sims' defense of existing structural macroeconometric models, at least as a device for estimating vector autoregressions.[11] The argument is by imperfect analogy to the Stein paradox in statistics.[12]

This line of argument leads one to ask whether there are alternatives to the standard simultaneous equation modeling procedures that can be used to restrict the dimensionality of the free parameter space in vector autoregressions. Current research, much of it being done at the University of Minnesota and the Federal Reserve Bank of Minneapolis, is exploring several alternative lines.

One main line is much in the spirit of the classical structural or simultaneous equation procedures. The key idea underlying this work is to estimate structural models of the form (4), (5), and (6), but to use identifying restrictions on the A_j's, B_j's, C_j's, and R_j's that are motivated by dynamic economic theory. As emphasized by Lucas and Sargent (1979), these restrictions typically come in the form of complicated nonlinear restrictions across the parameters of A_j and B_j, on the one hand, and C_j, on the other. These restrictions are of a form quite different from and more complicated than the linear or exclusion restrictions implemented in standard applications of existing simultaneous equation methods. Econometric methods are currently being developed for using dynamic economic theory to impose such restrictions in estimating time series models. To the extent that these restrictions approximately reflect valid dynamic economic theory, these methods hold out the promise of being useful devices for estimating vector autoregressions.[13] From a statistical point of view, the argument is identical with the argument made above in favor of estimating at the level of the structural objects, the A_j's, B_j's, and R_j's; the disagreement is over the form taken by the prior information supplied by the appropriate dynamic theory.

While methods for implementing cross-equation restrictions delivered by dynamic theory are now being developed,[14] they are not yet readily available and certainly have not yet proved to be successful in terms of delivering good estimates of the D_j^M's for vector autoregressions of sizable dimension. Further, there remain many controversial points about what are the most appropriate assumptions for dynamic economic theories. Partly for these reasons, other alternatives to using standard simultaneous equation methods for estimating vector autoregressions are being actively explored.

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[10]Or microeconometric models, for that matter.

[11]It is hardly a defense that the model builders could welcome, since it acknowledges at the outset that those models are inappropriate for analyzing the effects on the economy of changes in feedback rules governing monetary and fiscal policy variables under the authorities' control.

[12]An instructive background to Sims' (1977) argument is the discussion of Leamer (1978) on ridge and Stein-James estimators.

[13]However, this is not the sole reason these techniques are being developed. A more important reason is that the techniques are in principle capable of isolating structural parameters (that is, parameters of preferences and technologies) that will remain invariant in the face of changes in feedback rules for policy variables. That will, in principle, overcome the objections against using econometric models as devices for evaluating monetary and fiscal policy or rules originally made by Lucas (1976) and summarized by Lucas and Sargent (1979).

[14]For example, see Hansen and Sargent 1979 and Nerlove, Grether, and Carvalho 1979.

A major alternative was initiated by Sims (1975) and is directed at introducing restrictions on vector autoregressions which are frankly admitted at the outset to have no formal basis in dynamic economic theory. The aim is to restrict the dimensionality of the free parameters of the D_j^M's while leaving room for substantial dynamic interactions across variables. Two general strategies for restricting the D_j^M's in this way have been proposed.

One method employs one of the index models described by Sargent and Sims (1977), Brillinger (1975, chapters 9, 10), and Priestly, Rao, and Tong (1974). The idea here is that the dynamic interactions among all N variables are forced to be entirely intermediated through a small number of k variables termed *indexes*; k is thought to be small, no larger than 2 or 3. Sargent and Sims (1977) describe two versions of this model which differ according to whether the index is observable or unobservable. Sargent and Sims (1977) and Litterman and Sargent (1979) describe and illustrate how these methods can be used to estimate vector autoregressions. While typically not based on a fully specified economic theory, index models do seem to faithfully represent a long-standing intuition in macroeconomics that movements in many important economic aggregates can be viewed as reflecting one underlying hidden index. This idea was present in the work of Mitchell (1951).[15] Further, a recent theory of the business cycle (see Lucas 1975) seems at least to suggest statistical models of the index form.

The restrictions on vector autoregressions implied by both observable and unobservable index models are rather complicated and involve technical intricacies in implementation. Partly for this reason, Litterman (1979) has developed procedures for introducing restrictions directly on the D_j^M's themselves. Even more so than with index models, these restrictions are admitted at the outset not to be based on dynamic economic theory. These restrictions are implemented via a version of Theil's mixed estimator,[16] a procedure for mixing data-based information about the D_j^M's with nondata-based information in the form of restrictions on the D_j^M's, which are represented as statements that known linear combinations of the parameters equal random terms with mean zero and known variance. The mixed estimation procedure has a Bayesian interpretation, but it is not really formally justified in the context used by Litterman.[17] The reason is that the implicit priors imposed by Litterman are not representations of prior beliefs about the D_j^M's that economic theorizing has led to. Instead, what is represented as prior information is being imposed simply on the hunch that by imposing it, estimators of the D_j^M's with better sampling properties can be obtained. For example, a common implicit prior used by Litterman is one with a mean which states that

$$D_1^M = I, \text{ and } D_j^M = 0 \qquad j = 2, \dots, M$$

so that the system is one with N variables, each taking a random walk and being correlated only to the extent that the contemporaneous covariance matrix $E\eta_t^M\eta_t^{M'}$ is not diagonal. In effect, Litterman's procedure selects D_j^M by moving some distance along Dickey's "curve décolletage" from the least squares estimates to the point described by the priors.[18]

Litterman has generated a variety of examples that indicate that his procedures generate forecasts outside of the estimation period that strongly outperform forecasts from least squares estimates. Further, though there are difficulties in putting things on a comparable basis, there is evidence that Litterman's procedures produce forecasts of many macroeconomic variables that are competitive with those produced outside of the estimation periods by various of the better known of the large structural macroeconometric models.[19] To the ex-

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[15]Robert E. Lucas, Jr., pointed out to me Mitchell's (1951) statement, which is quite clear on this point.

[16]See Theil 1963 or Goldberger 1964.

[17]Shiller (1973) and Leamer (1972) describe Bayesian procedures for bringing prior information to bear on distributed lags. However, as Nerlove has long emphasized (see, for example, the discussion in Nerlove, Grether, and Carvalho 1979), smoothness priors of the form imposed by Shiller and Leamer typically have no basis in dynamic economic theory. Reasoning similar to Nerlove's can be used to criticize Litterman's implicit priors as representations of information provided by dynamic economic theory.

[18]See the useful discussion of the "curve décolletage" in Leamer 1978.

[19]McNees (1975) and Fair (1978b) present evidence describing the predictive accuracy of several macroeconometric models. Fair (1978a,b) raises a number of issues involved in comparing forecasts from different models.

tent that this result holds up, it is an important one, since it suggests that good performance of a structural macroeconometric model in generating unconditional forecasts is not necessarily evidence in favor of the particular a priori theory used to overidentify and to estimate the model.

Uses of Vector Autoregressions

Let us rewrite the vector autoregression (1) in the form

$$(1') \quad z_t = \sum_{j=1}^{M} D_j z_{t-j} + \eta_t$$

where we have omitted M superscripts from the D_j's and from η_t. So-called unconditional forecasts, that is, what are really forecasts of z_{t+k}, $k \geq 1$ conditioned on $\{z_t, z_{t-1}, \ldots, z_{t-M+1}\}$, can be made as follows. Let $\hat{E}_t z_{t+k}$ be the linear least squares forecast of z_{t+k} conditioned on $\{z_t, z_{t-1}, \ldots, z_{t-M+1}\}$. Then the $\hat{E}_t z_{t+k}$'s can be generated recursively from

$$\hat{E}_t z_{t+1} = \sum_{j=1}^{M} D_j z_{t+1-j}$$

$$\hat{E}_t z_{t+2} = D_1 \hat{E}_t z_{t+1} + \sum_{j=2}^{M} D_j z_{t+2-j}$$

$$\vdots$$

The general expression is

$$(10) \quad \hat{E}_t z_{t+k} = \sum_{j=1}^{m} D_j \hat{E}_t z_{t+k-j} \qquad k \geq 1$$

where in (10) it is understood that $\hat{E}_t z_{t-j} \equiv z_{t-j}$ for $j \geq 0$. Formula (10) is called the *chain rule of forecasting*. Typically, vector autoregressions are used to forecast by substituting estimated values of the D_j's in (10).[20]

There is reason to expect that use of (10) will generate relatively good forecasts to the extent that the D_j's have been estimated with small sampling errors and to the extent that future z's will depend on current and recently past z's in the same way that current and past z's depended on previous z's during the estimation period. If there are structural changes or policy interventions that change some of the equations presumably underlying (1′)—for example, equations (4), (5), and (6)—use of (10) is likely to give poor forecasts.

One use to which the vector autoregression (1′) cannot be put is to evaluate the effects of policy interventions in the form of changes in the feedback rule governing a monetary or fiscal policy variable, say, the money supply or monetary base. Thus, suppose that one of the z_t's, say z_{it}, is the money supply. It is not appropriate to substitute a new i^{th} equation describing z_{it} under a new proposed policy regime, leave the remaining $(N-1)$ equations unchanged, and then produce forecasts using (10) in an attempt to forecast how differently the system would behave under the alternative policy rule. The reason it is not appropriate is to be found in the dynamic economic theory alluded to above and described by Lucas and Sargent (1979). That body of theory delivers a set of cross-equation restrictions which imply that when one equation of (1′) describing a policy authority's feedback rule changes, in general, all of the remaining equations will also change.

While vector autoregressions can't be used to predict the effects of changes in policy feedback rules, they can be used to characterize the response to unexpected shocks in policy and other variables. Thus, if we solve the difference equation system (1′) and ignore transient terms, we obtain the vector moving average representation

$$(11) \quad z_t = \sum_{j=0}^{\infty} H_j \eta_{t-j} \qquad H_0 = I$$

where the H_j's are $(N \times N)$ matrices.

Since $\eta_t = z_t - \hat{E}_{t-1} z_t$, equation (11) permits the analyst to trace out the likely effects of unexpected shocks to the i^{th} variable z_{it} on subsequent

[20] When the D_j's are not known with certainty but are subject to sampling error, (10) is not the correct formula for the minimum mean square error forecasts (see Chow 1973). In general, closed form formulas have not been obtained for the least squares forecasts where the D_j's are subject to sampling error. Better approximations to the least squares forecasts than are given by using estimated values as they are known in (10) can be obtained by using stochastic simulation methods (see Fair 1978b). Fair (1978b) presents evidence that in certain contexts, stochastic simulation methods do not provide much of an improvement over forecasts generated by simply using estimated values of the D_j's in (10).

values of all of the variables. By studying the H_j's together with the covariance matrix $E\eta_t\eta_t'$, the relative persistence in effects and the cross-variable effects of unexpected changes in the z_{it}'s can be characterized.

A final use of vector autoregressions is to make probabilistic statements about events in the future which depend on complicated features of sample paths. For example, one might want to know the probability of the event that, given $\{z_t, z_{t-1},\ldots\}$, a recession begins in period $t + k$, where a recession is defined as beginning at the date of a third consecutive quarter decline in the variable z_{it}, say, real GNP. The probability of this event is complicated to compute analytically. Following the proposal of Wecker (1979), Litterman (1979) uses a Monte Carlo method to generate a large number of artificial sample paths of z_{t+k}, $k > 0$, given the historical initial conditions $\{z_t, z_{t-1}, \ldots\}$. The paths are generated by drawing realizations of the sequence of disturbances $\{\eta_{t+1}, \eta_{t+2}, \ldots\}$ using a pseudorandom number generator. The resulting realizations of $\{z_{t+1}, z_{t+2}, \ldots\}$ are then recorded, and frequency distributions for various events, such as the onset of a recession at date $t + k$, are recorded. In this way, economists at the Federal Reserve Bank of Minneapolis have been using vector autoregressions to make probabilistic statements about various interesting details of sample path behavior that are inadequately summarized by the point forecasts $\hat{E}_t\, z_{t+k}$, $k \geq 1$.

Conclusions

The techniques described in this article are still in the early stages of development, so they cannot yet be regarded as having proved themselves useful in a wide variety of contexts. Further, while the techniques were developed partly in response to criticisms of standard simultaneous equation macroeconometric models, they are not intended to remedy all the defects in the standard models pointed out by critics like Lucas. Indeed, builders of statistical models constructed along the lines described in this paper admit at the outset that the models will not be capable of analyzing the range of alternative policy interventions which the standard existing macroeconometric models were designed to analyze. Users of the techniques described here must recognize that the range of uses of these models is more limited than the range of uses that would be possessed by a truly structural simultaneous equation model.

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