Quarterly Review

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Is the Stock Market Overvalued?*

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As the 20th century drew to a close, the U.S. stock market boomed. Between 1994 and 2000, the value of corporate equity relative to gross national income, or equivalently, gross national product (GNP), nearly doubled. In the first half of 2000, the value of all U.S corporate equity was close to 1.8 times GNP.[1] A ratio of 1.8 is high by historical standards. The previous post–World War II peak was 1.0, which occurred in 1968. Over the 1946–99 period, the value of corporate equity averaged only 0.67 of GNP. (See the accompanying graph.) Thus, at 1.8, the current ratio is two and a half times the ratio's average in the postwar period.

Is the current stock market value too high? Glassman and Hassett (1999) have argued that it is not. In fact, they have said that the market is undervalued by a factor of three. But others have expressed concern that the market is, indeed, overvalued. Federal Reserve Chairman Alan Greenspan (1996), for example, has suggested that the recent high value of the market may reflect "irrational exuberance" among investors. Shiller (2000) has reiterated this concern and said that a 50 percent drop in the value is plausible. General concern about an overvalued market is fueled by the experience of Japan in the 1990s. The value of Japan's corporate equity fell 60 percent in 1990, and its economy subsequently stagnated.

We use standard theory to value U.S. corporate equity and find that the current value of 1.8 times GNP is justified. An implication of the theory is that the value of cor-

porate equity should equal the value of productive assets in the corporate sector, if net indebtedness is small (as it has been recently).[2] Our basic method is to estimate the current value of corporations' productive assets and compare that value to the current value of corporate equity. This is not as easy as it may seem.

Productive assets include *tangible* assets—like factories, office buildings, and machines—and *intangible* assets—like patents, brand names, and firm-specific human capital. And a good measure of the value of these assets must include not only those used by U.S. corporations in the United States itself, but also those used outside the country, by U.S. corporations' foreign subsidiaries.

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[1] Because of data availability, our calculations are based on data for all corporations, not just those which have their shares traded in the major stock exchanges. At the end of 1999, the value of corporations traded on the New York Stock Exchange, the American Stock Exchange, and the Nasdaq Stock Market was 84 percent of the total corporate value.

Most of the data used in this study are from two sources: the U.S. Department of Commerce's national income and product accounts and the Board of Governors of the Federal Reserve System's flow of funds accounts of the United States (U.S. Commerce 2000, various dates; FR Board, various dates).

[2] Theoretically, the market value of equity plus the market value of debt liabilities should equal the market value of debt assets plus the value of productive assets. Since net indebtedness of corporations is currently small, we ignore corporate debt holdings and liabilities when modeling the U.S. economy.

The Value of U.S. Corporate Equity

Ratio of Corporate Equity to Gross National Product
Annually, 1946–51; Quarterly, 1952–2nd Quarter 2000

Ratio
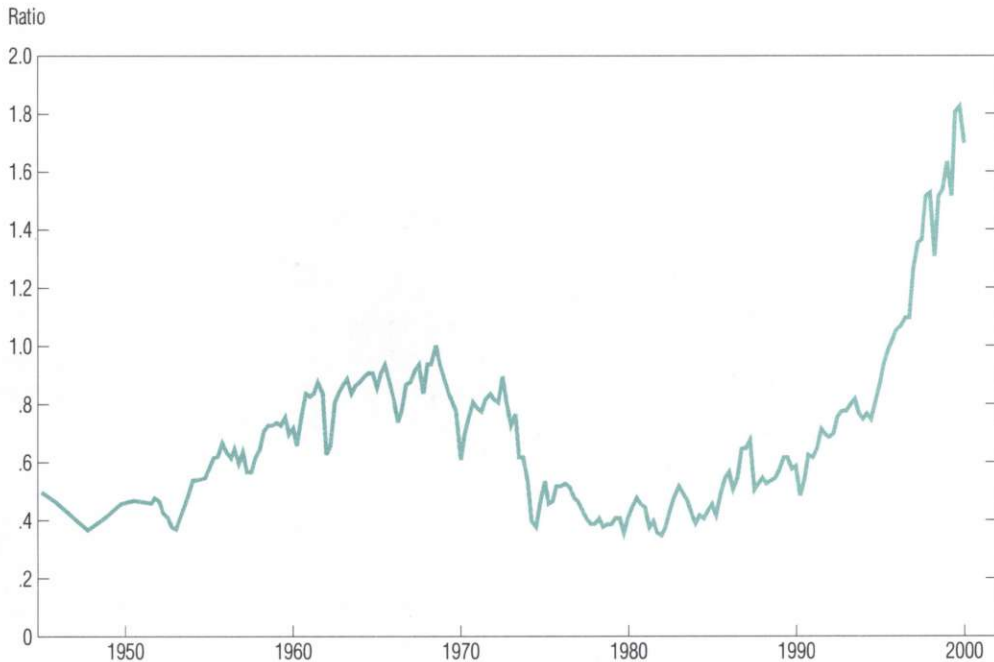
Source: U.S. Commerce, various dates

Estimates of the value of some of these assets are reported by the U.S. government. The Commerce Department's Bureau of Economic Analysis (BEA) estimates the value of tangible corporate assets located in the United States. In the 1990s, the estimate is slightly above 1.0 GNP. However, the BEA does not estimate the value of intangible assets in the corporate sector or the value of assets of U.S. corporate foreign subsidiaries. Therefore, we must construct estimates of these values ourselves.

To estimate the value of corporate intangible assets, we use data on corporate profits and tangible assets and an estimate of the return on capital used in the corporate sector. We find that corporate profits are larger than can be justified with tangible assets alone. By redoing the U.S. national income and product accounts (NIPA) with intangible assets included, we can derive formulas that allow us to residually determine the value of these assets. The key assumption is that the after-tax returns on tangible and intangible capital are equal. We find that the value of intangible capital is roughly 0.4 of GNP.

That value may seem large. We think it is reasonable in light of direct evidence. The value of high-technology companies, for example, can only be justified by their intangible capital, particularly human capital.[3] A significant fraction of the value of drug companies must be assigned to the value of the patents that they own. And as Bond and Cummins (2000) point out, brand names such as *Coca-Cola* account for much of the value of many companies.

To estimate the value of assets of U.S. corporations' foreign subsidiaries, we use profits of these subsidiaries di-

[3] In fact, Hall (2000) argues that "e-capital," which is human capital created by combining skilled labor and computers, is an important factor behind the recent rise in equity prices.

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vided by an estimate of the return on tangible capital in the United States. Our estimate of these assets is close to 0.4 of GNP.

Summing the values of corporate tangible assets located in the United States, corporate intangible assets, and assets of foreign subsidiaries gives us a total value of productive assets in the U.S. corporate sector of 1.8 times GNP—the same as the current value of corporate equity. This equality is just what economic theory predicts. According to standard economic theory, therefore, the stock market today is correctly valued.

Although our focus here is on the value of corporate equity, our work has implications for real returns on debt and equity. With our estimates of productive assets, theory predicts that returns on both debt and equity should average about 4 percent, as long as there are no important policy changes that significantly affect the pricing of financial assets. This prediction appears to be accurate so far: interest rates on U.S. Treasury inflation-protected securities with various maturities are currently around 4 percent.

Theory

Our method of estimating the value of corporate assets involves constructing a standard growth model and quantifying it.[4] The growth model we use is established aggregate economic theory and is fast becoming the textbook model in intermediate and advanced undergraduate macroeconomic courses. In this section, we derive formulas for the values of corporate equity and asset returns. In the next section, we use data from the Commerce Department and the Federal Reserve Board of Governors to derive estimates of these values for the United States.[5]

Our model economy includes two sectors, a corporate sector and a noncorporate sector. Since our focus is on the value of domestic corporations, output from the corporate sector is the gross domestic product of corporations located in the United States. Output of the noncorporate sector of our model is the remaining product of U.S. GNP. Our noncorporate sector thus includes the household business sector, the government sector, the noncorporate business sector, and the rest-of-world sector.

Willingness to Substitute
Our model economy is inhabited by infinitely lived households with preferences ordered by the expected value of

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t [(c_t l_t^\psi)^{1-\sigma}/(1-\sigma)] N_t$$

where t indexes time, c_t is per capita consumption, l_t is the fraction of productive time allocated to nonmarket activities such as leisure, and N_t is the number of household members. The fraction of productive time allocated by households to market activities is denoted by $n = 1 - l$. The size of a household is assumed to grow at the rate of population growth, η. The curvature parameter on consumption, $\sigma \geq 0$, measures how risk averse a household is. The larger this parameter's value, the more risk averse is the household. The parameter $0 < \beta < 1$ measures impatience to consume, with a smaller value implying more impatience. The parameter ψ measures the relative importance of leisure and consumption to the household. The larger ψ is, the more important is leisure.

Ability to Transform
The model economy has two intermediate good sectors— a corporate sector, denoted by 1, and a noncorporate sector, denoted by 2. These provide the inputs to produce the economy's final good.

The noncorporate production technology is simple:

$$(2) \qquad y_{2,t} \leq (k_{2,t})^\theta (z_t n_{2,t})^{1-\theta}.$$

Here y_2 is sector output, k_2 is capital services, n_2 is labor services, z is a stochastic technology parameter, and θ is the capital share parameter, $0 < \theta < 1$.

For our purposes, the corporate sector is the important sector, and it is more complicated. It has both tangible and intangible assets. U.S. corporations make large investments in such things as on-the-job training, research and development (R&D), organization building, advertising, and firm-specific learning by doing. These investments are large, and the stock of intangible assets has important consequences for the pricing of corporate assets. So we assume that production in the corporate sector requires both tangible assets, which are measured, k_{1m}, and intangible assets, which are unmeasured, k_{1u}. In addition to capital, labor services n_1 are required. The aggregate production function for the corporate sector is

$$(3) \qquad y_{1,t} \leq (k_{1m,t})^{\phi_{mt}} (k_{1u,t})^{\phi_{ut}} (z_t n_{1,t})^{1-\phi_{mt}-\phi_{ut}}$$

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[4]To justify some of the assumptions of our model, we provide evidence on U.S. household asset holdings in Appendix A. For readers unfamiliar with the basic concepts underlying standard asset pricing, we provide a primer in Appendix B.

[5]Much work in the asset pricing literature abstracts from production and stops short of matching variables in the theory with national income and product data. Notable exceptions include the work of Cochrane (1991) and Mehra (1998).

where ϕ_{mt} and ϕ_{ut} are the random capital shares for measured and unmeasured capital, respectively. In order to capture variations in profit shares over the business cycle, we make the nonstandard assumption that capital shares vary. Variations in profit shares affect the equity risk premium, which we want to estimate.

The three per capita capital stocks in this economy—corporate tangible and intangible capital and noncorporate capital—depreciate geometrically and evolve according to

(4) $\quad k_{i,t+1} = [(1-\delta_i)k_{i,t} + x_{i,t}]/(1+\eta)$

where $i = 1m$, $1u$, or 2; δ_i is the rate of depreciation for capital of type i; and $x_{i,t}$ is gross investment of type i in period t. The right side of the capital accumulation equations (4) is divided by the growth in population $(1+\eta)$ because k_i and x_i are in per capita units.

The model also has a final good sector, which combines the intermediate inputs from the corporate and noncorporate sectors to produce a composite output good that can be used for consumption and investment. This production function is

(5) $\quad c_t + g_t + x_{1m,t} + x_{1u,t} + x_{2,t}$
$$\leq y_t \equiv A[\mu(y_{1,t})^\rho + (1-\mu)(y_{2,t})^\rho]^{1/\rho}$$

where g is government consumption, $0 < \mu < 1$ is a parameter that determines the relative sizes of the corporate and noncorporate sectors, $\rho \leq 1$ is a parameter that governs the substitutability of corporate and noncorporate goods, and $A > 0$ is a scale parameter.

Government production is assumed to be included in noncorporate production. However, the government plays a special role in the economy: it taxes various activities to finance government purchases and transfers. In particular, the government taxes consumption, labor income, property, and profits. Taxes are proportional in our model economy.

Equilibrium

There are two ways to decentralize our model economy, and they lead to the same equilibrium outcome. One way is to assume that firms hire workers, make investment decisions, pay taxes directly to the government, and pay dividends to the households. Because of the investment decision, the firms' problem, in this decentralization, is dynamic. The other way to decentralize is to assume that firms rent capital and labor from households. Households

make the investment decisions and pay taxes to the government. In this decentralization, the firms' problem is simple and static. The relevant equilibrium outcomes are the same in the two decentralizations because the households effectively own the capital in both cases. Here we describe an equilibrium for the second type of economy. We find this economy easier to work with because we can consolidate all of the interesting transactions for a particular period into the household's budget constraint.

The household budget constraint in period t is

(6) $\quad (1+\tau_{c,t})c_t + x_{1m,t} + x_{1u,t} + x_{2,t}$
$$= r_{1m,t}k_{1m,t} + r_{1u,t}k_{1u,t} + r_{2,t}k_{2,t} + w_t n_t$$
$$- \tau_{1k,t}k_{1m,t} - \tau_{2k,t}k_{2,t} - \tau_{n,t}w_t n_t$$
$$- \tau_{1,t}[(r_{1m,t}-\delta_{1m,t})k_{1m,t} + r_{1u,t}k_{1u,t}$$
$$- x_{1u,t} - \tau_{1k,t}k_{1m,t}]$$
$$- \tau_{2,t}[(r_{2,t}-\delta_2)k_{2,t} - \tau_{2k,t}k_{2,t}] + \pi_t.$$

Households rent tangible and intangible capital to corporations at rental rates r_{1m} and r_{1u}, respectively. Households also rent capital to noncorporate firms at a rental rate of r_2. Wage income is wn, where $n = n_1 + n_2$ is total labor services. Taxes are paid on consumption expenditures, wage income, property, and profits. The tax rate on consumption is τ_c; that on wage income is τ_n; tax rates on property in the corporate and noncorporate sectors are τ_{1k} and τ_{2k}; and the rate on corporate profits is τ_1. Note that corporations can subtract depreciation and property taxes when they compute their corporate profits tax. Note also that unmeasured investment, for things like R&D, is untaxed. It, too, is subtracted from income when taxable income is computed. Noncorporate profits are taxed at a rate τ_2. Again, depreciation and property taxes are subtracted when taxable income is computed. Finally, transfers from the government to households are denoted by π.

Now consider equilibrium in this economy. Households maximize their expected utility (1) subject to the sequence of budget constraints (6) and the capital accumulation equations (4). Households take as given initial capital stocks as well as current and future prices and tax rates. Firms in all sectors behave competitively and solve simple, static optimization problems. The intermediate good firms choose capital and labor to maximize profits subject to the constraint on their production, namely, functions (3) or (2). Thus, wages and rental rates in the corporate and noncor-

porate sectors are equal to their marginal value products. The final good firms choose the intermediate inputs to maximize $y - p_1y_1 - p_2y_2$, where p_i is the price of the intermediate goods of sector i. Maximization is done subject to the production function (5). If households and firms choose allocations optimally, then equilibrium prices are set so that markets for goods, labor, and capital services all clear.

In this economy, the value of corporate equity is equal to the value of the end-of-period stock of capital used in the corporate sector. If we use the price of output as the unit of account, then the value is given by

$$(7) \qquad V_t = [k_{1m,t+1} + (1-\tau_{1,t})k_{1u,t+1}]N_{t+1}.$$

This follows from the facts that the cost, on margin, of a unit of measured capital is 1 and the cost, on margin, of a unit of unmeasured capital is 1 minus the corporate income tax rate. Expenditures on unmeasured investment are expensed and reduce taxable corporate income. [See the budget constraint (6).]

The return on corporate equity is given by

$$(8) \qquad r_{t,t+1}^e = (V_{t+1} + d_{t+1}N_{t+1}/V_t) - 1$$

where $\{d_t\}$ is the stream of payments to the shareholders of the corporation (that is, the households). Payments to shareholders are given by

$$(9) \qquad \begin{aligned} d_t = {} & p_{1,t}y_{1,t} - w_t n_{1,t} - \tau_{1k,t}k_{1m,t} \\ & - \tau_{1,t}[(r_{1m,t}-\delta_{1m}-\tau_{1k,t})k_{1m,t} + r_{1u,t}k_{1u,t} - x_{1u,t}] \\ & - x_{1m,t} - x_{1u,t}. \end{aligned}$$

This represents what the corporation has left over after workers have been paid, taxes on property and profits have been paid, and new investments have been made.

The return on a one-period bond, which we refer to as the *risk-free rate,* is given by

$$(10) \qquad r_{f,t} = \{\beta E_t[c_{t+1}^{-\sigma}l_{t+1}^{\psi(1-\sigma)}/(c_t^{-\sigma}l_t^{\psi(1-\sigma)})]\}^{-1} - 1$$

where $c^{-\sigma}l^{\psi(1-\sigma)}$ is the marginal utility of consumption. The value, or price, of the bond is simply the inverse of $1 + r_{f,t}$.

Findings

We can use the formulas for the asset values and returns just described to assess whether our model is consistent

with U.S. observations. It is. To demonstrate that, we first abstract from uncertainty and price corporate equity and risk-free debt using a deterministic version of the model. Without uncertainty, calculations of the relevant quantities are trivial. We then establish that, for all practical purposes, the results are the same in the deterministic and stochastic versions of the model when we introduce uncertainty consistent with the behavior of the U.S. economy.[6]

Without Uncertainty

Again, we work first with the steady state of a deterministic version of the model. We derive an estimate for the return on capital using data from the U.S. noncorporate sector. We then derive an estimate for the size of the intangible capital stock. We choose the level of intangible capital so that the returns on capital in the corporate and noncorporate sectors are equated. With the estimate for intangible capital and data on measured corporate capital and taxes paid in the corporate sector, we can estimate the value of the stock market.

□ The Return on Capital

With no uncertainty, the after-tax return on corporate equity and the after-tax return on a bond that pays 1 for sure in the following period are both equal to the after-tax interest rate, which we denote by i and define to be

$$(11) \qquad i = [(1+\gamma)^\sigma/\beta] - 1$$

where γ is the growth of the technology parameter z_t. This follows directly from the first-order conditions of the household. In fact, if there is no uncertainty, then the after-tax return on each type of capital is also given by i, and the following is true:

$$(12) \qquad \begin{aligned} i &= (1-\tau_1)(r_{1m}-\delta_{1m}-\tau_{1k}) \\ &= r_{1u} - \delta_{1u} \\ &= (1-\tau_2)(r_2-\delta_2-\tau_{2k}). \end{aligned}$$

Assuming that the U.S. economy is roughly in a steady state, we can estimate i using NIPA data. In Table 1, we report average values for income, product, and capital stocks of the United States during 1990–99. The table lists

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[6]Readers familiar with the literature on the equity premium puzzle launched by Mehra and Prescott (1985) should not be surprised by this finding. See Kocherlakota 1996 for a nice survey of the literature. For estimates of the current equity premium, see also the work of Jagannathan, McGrattan, and Scherbina (in this issue of the *Quarterly Review*).

the accounting concepts used for the NIPA data and their average values over the period 1990–99 relative to GNP. We make adjustments to these values as theory requires, in order to make the accounts consistent with our model. The table also describes and quantifies the adjustments and lists the final, adjusted averages. (In Appendix C, we provide details about the calculations made for Table 1.) In Table 2, the adjusted averages are matched up with their model counterparts.

Our estimate of the return on capital comes from noncorporate data because we observe the relevant quantities needed to infer $(1-\tau_2)(r_2-\delta_2-\tau_{2k})$. However, before we can construct an estimate of the return on capital in the noncorporate sector, we need to consider several of the adjustments made to the NIPA data. Two sets of adjustments are relevant: those to noncorporate profits and those to capital.

Consider first noncorporate profits. We make two adjustments to this item. One is to reduce the net interest payments of the sector by an estimate of the sector's purchases of intermediate financial services. We estimate that of the 0.042 of GNP of this sector's net interest payments, 0.022 should be treated as intermediate service purchases. So we reduce GNP 2.2 percent, with the reduction on the product side being in consumption of financial services and that on the income side, in imputed net interest income of households. Most of this adjustment is simply the difference in interest paid by people with home mortgages and the interest received by households who lend to the financial institutions that issue the mortgages.

The imputed net interest income that remains is 0.02 of GNP, which we see as a reasonable number. Some of this is forgone interest of people who hold currency and checking accounts that pay less than the short-term interest rate. Some of it is the reduction in insurance premiums that is possible because the insurance company earns interest on premiums for a period prior to making claims. In these cases, the household is receiving services for forgone interest, and there should be an imputation to income and product.

The other adjustment that we make to noncorporate profits is the addition of imputed capital services to government capital and to consumer durables. The BEA uses a zero percent interest rate when imputing services to government capital. We instead use the average return on capital in the noncorporate sector. So that income equals product, we add imputed services both to profits in the noncorporate sector and to government consumption. In the NIPA data, consumer durables are treated as consump-

tion. We treat them instead as investment and impute services to these durables. These imputed capital services are added to profits in the noncorporate sector and to private consumption.

We must make one addition to the capital stock of the noncorporate sector. Capital stocks reported by the BEA include only capital located in the United States. But our measure of noncorporate profits includes profits of U.S. foreign subsidiaries equal to 0.012 of GNP. To estimate the capital stock used to generate these profits, we divide 0.012 by our estimate of the return on capital i.

We are now ready to compute the after-tax return on capital in the noncorporate sector (which is equal to $(1-\tau_2)(r_2-\delta_2-\tau_{2,k})$ and to i):

(13) i = (Accounting Returns + Imputed Returns)

             $\div$ (Noncorporate Capital

                 + Capital of Foreign Subsidiaries)

(14) $= [0.064 + (0.592 + 0.287)i]/[2.153 + (0.012/i)]$

where 0.064 of GNP is noncorporate profits plus net interest less intermediate financial services; 0.592 is the net stock of government capital; 0.287 is the net stock of consumer durables; 2.153 is the sum of stocks of government capital, consumer durables, and noncorporate business; and 0.012 is net profits from foreign subsidiaries. We have assumed that τ_2 is 0 because the main categories of noncorporate income—namely, services of owner-occupied housing, government capital, and consumer durables—are untaxed. The value of i that satisfies (14) is 4.08 percent. Therefore, our estimate of the imputed services to capital is 0.036, and our estimate of the capital associated with the net profits of 1.2 percent is 0.294.

So, theory predicts that, on average, the return on capital in the noncorporate sector should be 4.08 percent. This is close to the average values of the risk-free rate on inflation-protected bonds issued by the U.S. Treasury. In the first quarter of 2000, the average return on 5-year inflation-protected bonds was 3.99 percent, and the average return on 30-year inflation-protected bonds was 4.19 percent.

□ *The Value of Corporate Equity*
We turn next to the value of domestic corporate equity. To compute our estimate, we need the value of measured tangible capital, the corporate income tax rate, and an estimate of the value of unmeasured intangible capital. [See equation (7).]

Table 1

Adjustments to the NIPA Data

Ratio of Each Item With Gross National Product, 1990–99

NIPA Concept		Average Value	Adjustment (and Its Value)	Adjusted Average Value
Income	Corporate Sector			
	Compensation	.378		.378
	Indirect Business Tax	.057	− Sales & Excise Taxes (0.037)	.020
	Capital Consumption	.069		.069
	Profits			
	After-Tax Profits	.047	+ Unmeasured Intangible Investment (0.019)	.066
	Profits Tax	.026		.026
	Net Interest	.015	− Intermediate Financial Services (0.015)	.000
	Value Added	*.592*		*.559*
	Noncorporate Sector			
	Compensation	.246		.246
	Indirect Business Tax	.022	− Sales & Excise Taxes (0.010)	.012
	Capital Consumption	.054	+ Depreciation of Consumer Durables (0.063) + Depreciation of Foreign Subsidiary Capital (0.016)	.133
	Profits	.044	+ Net Interest (0.042) − Intermediate Financial Services (0.022) + Imputed Capital Services (0.036)	.100
	Net Interest	.042	− Net Interest (0.042)	.000
	Value Added	*.408*		*.491*
	Total Income	1.000		1.050
Product	Consumption			
	Private	.588	− Sales & Excise Taxes (0.047) + Depreciation of Consumer Durables (0.063) + Imputed Capital Services (0.012) − Intermediate Financial Services (0.037) − Net Investment of Foreign Subsidiaries (0.009)	.570
	Government	.156	+ Imputed Capital Services (0.024)	.180
	Investment			
	Corporate	.100		.100
	Noncorporate	.156	+ Depreciation of Foreign Subsidiaries (0.016) + Net Investment of Foreign Subsidiaries (0.009)	.181
	Unmeasured Corporate	.000	+ Unmeasured Intangible Investment (0.019)	.019
	Total Product (GNP)	1.000		1.050
Capital Stocks*	Corporate			
	Measured	.821	+ Inventories (0.161) + Land (0.060)	1.042
	Unmeasured	.000	+ Unmeasured Capital (0.645)	.645
	Noncorporate	2.153	+ Net Capital of Foreign Subsidiaries (0.294)	2.447
	Total Capital Stocks	2.974		4.134

*Stocks are midyear.
Sources: U.S. Commerce 2000, various dates; FR Board, various dates

In Table 1, measured tangible capital as reported by the BEA (U.S. Commerce 2000) is listed as 0.821 of GNP. However, this measure does not include inventories or land. Inventories are, however, available in the NIPA data (U.S. Commerce, various dates), so we add them (0.161 of GNP). Land is not included in the NIPA data, but it is in the data collected and published by the Federal Reserve Board (FR Board, various dates). The difference between real estate values reported by the Fed and nonresidential structures reported by the BEA is 0.06 of GNP. Thus, our estimate of measured capital, with land and inventories included, is 1.042 times GNP.

In Table 1, the corporate profits tax liability is listed as 0.026 of GNP, and before-tax corporate profits are 0.073 of GNP. The tax rate is taken to be the average tax and is, therefore, equal to 0.356.

The next step is obtaining an estimate for unmeasured capital in the corporate sector. In the deterministic version of our model, the after-tax returns for the three types of capital must be equal, and this requirement ties down the size of unmeasured corporate capital. Above we computed one of these after-tax returns, namely, the return on non-corporate capital. We can use this as our estimate of both $r_{1u} - \delta_{1u}$ and $(1-\tau_1)(r_{1m}-\delta_1-\tau_{1k})$. We can also use the fact that profits in the model economy's corporate sector are equal to the NIPA value of corporate profits plus unmeasured investment. Therefore,

$$(15) \quad (r_{1m}-\delta_1-\tau_{1k})k_{1m} + r_{1u}k_{1u} = \text{NIPA Profits} + x_{1u}.$$

Replacing $r_{1m} - \delta_1 - \tau_{1k}$ by $i/(1-\tau_1)$ in (15) and rearranging, we have

$$(16) \quad i = (1-\tau_1)(\text{NIPA Profits} + x_{1u} - r_{1u}k_{1u})/k_{1m}$$
$$= (1-\tau_1)\{\text{NIPA Profits}$$
$$+ [(1+\eta)(1+\gamma) - 1]k_{1u} - ik_{1u}\}/k_{1m}$$

where we have used the fact that x_{1u} is proportional to k_{1u} on the steady-state growth path. The only unknown in equation (16) is intangible capital. Rearranging (16) and plugging into it the U.S. averages from Tables 1 and 2, we get

$$(17) \quad 0.0408 = [1 - (0.026/0.073)]$$
$$\times (0.073 + 0.03k_{1u} - 0.0408k_{1u})/1.042$$

where 0.026 of GNP is the tax paid on domestic corporate profits, 0.073 is NIPA profits, 0.03 is the growth rate of

GNP, and $0.03k_{1u}$ is the value of unmeasured net intangible investment in the steady state. The solution to this equation is $k_{1u} = 0.645$. Therefore, unmeasured intangible investment is equal to 0.019 of GNP.

With our estimate for unmeasured capital, we can now compute the model's market value of domestic corporate equity using formula (7). If the time period is not long, the total value—that is, N times the per capita value—is

$$(18) \quad V = [k_{1m} + (1-\tau_1)k_{1u}]N = 1.457N$$

where $\tau_1 = 0.356$ (which is the value of corporate income taxes divided by the value of taxable corporate income).

To compare this estimate to the data's market value of U.S. corporate equity, we need to add in the market value of U.S. foreign subsidiaries. Profits from U.S. foreign subsidiaries averaged 1.56 percent of GNP over the period 1990–99.[7] Using an interest rate of 4.08 percent, we estimate that capital of U.S. foreign subsidiaries has a value of 0.382 of GNP. Let V_{US} be the market value of U.S. corporate equity. Then,

$$(19) \quad V_{US} = V + 0.382N = 1.84N = 1.84 \text{ times GNP}.$$

We write this in terms of GNP because per capita GNP is normalized to 1, and total GNP is, therefore, N.

According to the Fed's data, the market value of domestic corporate equity at the end of the first quarter of 2000 was 1.83 times GNP in that quarter (FR Board, various dates). In the second quarter of 2000, the corporate equity market value was 1.71 times GNP. So far in 2000, therefore, the quarterly average value is 1.77. This is very close to what our model predicts (1.84).

We did not model corporate debt because it has been quite small recently. So far in 2000, it has been roughly 7 percent of GNP. This implies that the total value of U.S. corporations—equity plus debt—is 1.84 times GNP. According to our estimates, this value is equal to the value of productive assets.

Thus far, we have assumed that the premium for taking on nondiversifiable risk is small.

With Uncertainty

Now we work out the implications of a stochastic version of the model. With uncertainty, we expect that risky assets,

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[7] Above, we used net profits, which subtracts factor payments sent abroad. This is the relevant figure for computing GNP. To calculate the value of U.S. domestic corporations, we want to use gross profits from U.S. foreign subsidiaries.

like corporate equity, would be paid a risk premium. So here we quantify this premium. We find that, in fact, the premium is very small. Thus, the results of the stochastic version of the model are essentially those of the deterministic version.

☐ *Calibration*

To determine the implications of the stochastic version of the model, we must first calibrate the model. We do this in three steps. First, we compute a steady state for the model that is consistent with the adjusted accounting measures in Table 1. Second, we choose parameters for the model—including means of stochastic parameters—that are consistent with these steady-state values. Third, we choose stochastic processes for shocks in the model that lead to fluctuations in the key variables that are comparable to their U.S. counterparts. The key variables for asset pricing are output, consumption, labor, and after-tax corporate profits.

Steady State. To compute a steady state for the model we need to make some further adjustments to the NIPA data so that they are consistent with the model concepts. The adjustments that we have discussed so far are the addition of unmeasured investment; the subtraction of intermediate financial services; the imputation of consumer durable and government capital services; and adjustments to the capital stocks. The final adjustments needed are adjustments for sales and excise taxes, for depreciation of consumer durables, and adjustments for foreign subsidiary capital.

The NIPA data include sales taxes in the measure of private consumption. In our model, we treat consumption as pretax. Therefore, we must subtract sales taxes from NIPA private consumption. Consumer durables are treated as private consumption in the NIPA data and as investment in our model. Therefore, we add the depreciation of consumer durables to noncorporate depreciation and to consumption. Finally, because profits of foreign subsidiaries are included in the NIPA's national income (and therefore in noncorporate profits), we add an estimate of investment and depreciation for foreign subsidiaries. To do this, we use the same rate of depreciation as for other noncorporate capital in the United States.

The adjusted values for income, product, and capital stocks are treated as a steady state for the model. These values are reported in Table 2 along with the relevant expressions for the model.

Also in this table are values and expressions for hours worked, growth rates, and tax rates. In the United States,

hours worked per person are roughly one-quarter of discretionary time. The growth rates in the table are averages over 1990–99 of total factor productivity and population. With the exception of the labor tax rate, we use NIPA values reported in Table 1 to calculate tax rates. The corporate and noncorporate profit tax rates—which we used in earlier calculations—are set equal to 0.356 and 0, respectively. Consumption and property taxes are the two parts of indirect business taxes. Consumption taxes are 0.047 of GNP, and property taxes are 0.032 of GNP. The table's tax rate of 0.086 for consumption is found by dividing the total tax of 0.047 by the value of private consumption, which is equal to 0.544. Our tax rates on property are found by dividing total property taxes by the capital stocks in the respective sectors. For corporate property, the rate is 0.02/1.042, or 0.019. For noncorporate property, the rate is 0.012/2.447, or 0.005.

The labor tax rate is more difficult to estimate since the U.S. income tax is progressive, while taxes in our model economy are proportional. Households in the federal tax bracket of 28 percent or higher pay nearly all of the income tax. However, because of fringe benefits and before-tax contributions to retirement plans, the marginal tax rates of these households are effectively lower than 28 percent. Therefore, we choose the tax rate on labor income to be 25 percent. But our analysis is not sensitive to the exact rate used. The difference between tax revenues and government expenditures is a lump-sum transfer.

Parameters. In Table 3, we derive depreciation rates, capital shares, and parameters for the final good technology and the utility function. Most of these parameters can be pinned down by steady-state values.

There are two exceptions: the elasticity of substitution of corporate and noncorporate goods $1/(1-\rho)$ and the curvature parameter on consumption σ, which measures the degree of risk aversion. For these parameters, we experiment with different values in such a way as to get reasonable predictions for the variability of consumption relative to GNP and the variability of corporate share relative to product. Our baseline values are $\sigma = 1.5$ and $\rho = -2$.

Stochastic Shock Processes. The final choices necessary for the stochastic version of the model are the stochastic processes. We assume that the technology parameter z_t is stochastic, with the process given by

(20) $\log z_{t+1} = \log z_t + \log(1+\gamma) + \varepsilon_{zt+1}$

where ε_{zt} is an independent and identically distributed

Table 2

Steady-State Values for the Model

Ratio With GNP, Except Where Noted Otherwise

Category		Value	Formula
Income	Corporate Sector		
	Compensation	.378	wn_1
	Indirect Business Tax	.020	$\tau_{1k}k_{1m}$
	Capital Consumption	.069	$\delta_{1m}k_{1m}$
	Profits	.092	$(r_{1m}-\delta_{1m}-\tau_{1k})k_{1m} + r_{1u}k_{1u}$
	Value Added	*.559*	p_1y_1
	Noncorporate Sector		
	Compensation	.246	wn_2
	Indirect Business Tax	.012	$\tau_{2k}k_2$
	Capital Consumption	.133	$\delta_2 k_2$
	Profits	.100	$(r_2-\delta_2-\tau_{2k})k_2$
	Value Added	*.491*	p_2y_2
	Total Income	**1.050**	
Product	Consumption		
	Private*	.544	c
	Government	.180	g
	Investment		
	Corporate	.100	x_{1m}
	Noncorporate*	.207	x_2
	Unmeasured Corporate	.019	x_{1u}
	Total Product (GNP)	**1.050**	$c + x_{1m} + x_2 + x_{1u} + g$
Capital Stocks	Corporate		
	Measured	1.042	k_{1m}
	Unmeasured	.645	k_{1u}
	Noncorporate	2.447	k_2
	Total Capital Stocks	**4.134**	
Total Hours Worked (% Productive Time)†		25.0	$n_1 + n_2$
Growth Rates (%)†	Technology	2.0	γ
	Population	1.0	η
Tax Rates (%)†	Profits		
	Corporate	35.6	τ_1
	Noncorporate	0	τ_2
	Property		
	Corporate	1.9	τ_{1k}
	Noncorporate	.5	τ_{2k}
	Consumption	8.6	τ_c
	Labor	25.0	τ_n

*In a steady state of the model, gross investment is equal to depreciation plus the change in capital. To make noncorporate investment consistent with the observed stock and depreciation of the noncorporate sector, we increased it slightly (from 0.181 to 0.207). In order to leave GNP unchanged, we lowered private consumption by an equal amount (from 0.570 to 0.544).

†The values used in the model are these percentages divided by 100.

Table 3

Derivation of Parameters From the Steady State

Parameter	Derivation	Value
Depreciation Rates		
Corporate		
Measured	$\delta_{1m} = x_{1m}/k_{1m} - [(1+\gamma)(1+\eta) - 1]$	.066
Unmeasured	$\delta_{1u} = x_{1u}/k_{1u} - [(1+\gamma)(1+\eta) - 1]$	.000
Noncorporate	$\delta_2 = x_2/k_2 - [(1+\gamma)(1+\eta) - 1]$	.055
Capital Shares		
Corporate		
Measured	$\phi_m = r_{1m}k_{1m}/(p_1y_1)$	.277
Unmeasured	$\phi_u = r_{1u}k_{1u}/(p_1y_1)$	.047
Noncorporate	$\theta = r_2k_2/(p_2y_2)$	.499
Final Good Technology		
Elasticity of Substitution*	$1/(1-\rho)$	.333
Relative Weights	$\mu/(1-\mu) = p_1y_1^{1-\rho}/(p_2y_2^{1-\rho})$	.223
Scale Factor	$A = y/[\mu y_1^\rho + (1-\mu)y_2^\rho]^{1/\rho}$	1.418
Utility Function		
Risk Aversion*	σ	1.500
Discount Factor	$\beta = (1+\gamma)^\sigma/(1+i)$	.990
Weight on Leisure	$\psi = (1-\tau_n)w(1-n_1-n_2)/[(1+\tau_c)c]$	2.377

*These parameters are not pinned down by steady-state values.

(i.i.d.) normal random variable with a mean of zero. Notice that z_t grows at rate γ, as do other nonstationary variables in this economy. We choose the variance of ε_z so that the standard deviation of U.S. GNP and our model's output are roughly the same once we log the series and run them through the Hodrick-Prescott filter. The standard deviation of U.S. GNP is 1.74 percent for the postwar period.

In our baseline economy, we assume that the only shocks hitting the economy are technology shocks. We do this for two reasons. First, technology shocks in the postwar period are significant sources of aggregate fluctuations. Second, correctly identifying the shocks matters little for the size of the equity premium, provided the model has been calibrated to the steady-state observations and provided the model's variances and covariances of consumption and corporate profits match their empirical counterparts.

Table 4 summarizes the parameters for the baseline economy. One parameter included in this table that has not yet been discussed is that for the adjustment cost b. Because the cyclical variation of consumption is crucial for asset pricing, we include adjustment costs on all types of capital of the form $\varphi(x/k) = (b/2)(x/k-\hat{\delta})^2k$, where $\hat{\delta} = \delta + \gamma + \eta$.[8] We do this to ensure that the relative volatility of consumption and output in the model is approximately equal to the observed relative volatility.

□ *Simulation*

Given the parameter values, we compute an equilibrium for the economy, simulate time series, and compute asset values and returns. Following Jermann (1998), we com-

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[8]With adjustment costs, we need to modify our formula for the equity value as follows: $V = \{k_{1m}/[1 - \varphi'(x_{1m}/k_{1m})] + (1-\tau_1)k_{1u}/[1 - \varphi'(x_{1u}/k_{1u})]\}N$.

Table 4

Stochastic Model Parameter Values

Economy	Parameter	Value
Baseline With Only Technology Shocks	Preferences	$\sigma = 1.5$, $\beta = 0.99$, $\psi = 2.377$
	Technology	$\rho = -2$, $\mu = 0.182$
	Depreciation Rates	$\delta_{1m} = 0.066$, $\delta_{1u} = 0$, $\delta_2 = 0.055$
	Capital Shares	$\phi_m = 0.277$, $\phi_u = 0.047$, $\theta = 0.499$
	Growth Rates	$\gamma = 0.03$, $\eta = 0.01$
	Average Tax Rates	$\tau_1 = 0.356$, $\tau_2 = 0$
		$\tau_{1k} = 0.019$, $\tau_{2k} = 0.005$
		$\tau_c = 0.086$, $\tau_n = 0.25$
	Technology Shocks	$E\epsilon_z = 0$, $E\epsilon_z^2 = 0.013^2$
	Adjustment Cost	$b = 0.12$
With Other Shocks As Well*	Shocks to Technology and Labor Tax	$E\epsilon_z^2 = 0.01^2$
		$\rho_n = 0.95$, $E\epsilon_n^2 = 0.01^2$
		$b = 0.15$
	Corporate Capital Share	$E\epsilon_z^2 = 0.011^2$
		$\rho_\phi = 0.95$, $E\epsilon_\phi^2 = 0.006^2$
		$b = 3.1$
	Labor Tax and Corporate Capital Share	$E\epsilon_z^2 = 0.007^2$
		$\rho_n = 0.95$, $E\epsilon_n^2 = 0.01^2$
		$\rho_\phi = 0.95$, $E\epsilon_\phi^2 = 0.006^2$
		$b = 3.1$

*All innovations have a zero mean.

pute a linear approximation to the decision rules for capital. All other variables, including equity returns, can be determined in a nonlinear way once we have values for the capital stocks and the stochastic shocks. (Table 5 displays the predictions of all the versions of the model.)

Shocks Only to Technology. With no other shocks but shocks to technology, we find that the ratio of the value of corporate equity to GNP is 1.85, about what we found in the deterministic version of our model; the return on equity is 4.10; and the return on debt is 4.07. (See Table 5.) The equity risk premium in this economy is small, only 0.03 percentage point, which is close to the deterministic version's 0 equity premium.

In this economy with only technology shocks, hours of work are too smooth relative to U.S. data, and corporate earnings are too volatile. We need to get the right varia-

Table 5

Predictions of the Model

		Average Returns		
Model Version	Average Ratio of Corporate Equity to GNP	Equity (%) (1)	Debt (%) (2)	Premium (% points) (1) − (2)
Deterministic Version	1.84	4.08	4.08	0
Stochastic Versions With Shocks to				
Technology Only	1.85	4.10	4.07	.03
Technology and				
Labor Tax	1.85	4.09	4.08	.01
Corporate Capital Share	1.85	4.08	4.07	.01
Labor Tax and Corporate Capital Share	1.85	4.07	4.07	0

tions in hours as well as consumption since both are arguments of marginal utility; movements in marginal utility are what is relevant for asset pricing. We also need to get the right variation and covariation in corporate earnings since this is relevant for stock returns and the equity premium paid to stocks. Thus, we consider several variations on our baseline economy that should move the model toward greater volatility in hours and less volatility in corporate earnings. The parameters used in these variations are summarized in Table 4.

Shocks Also to Labor Taxes. To get more volatility in hours and leisure, we assume that labor tax rates are stochastic. Assume, for example, that τ_{nt} is an autoregressive process with

$$(21) \quad \tau_{nt+1} = (1-\rho_n)\bar{\tau}_n + \rho_n\tau_{nt} + \varepsilon_{nt+1}$$

where $\bar{\tau}_n$ is the mean of the process and ε_{nt} is an i.i.d. normal shock with a mean of zero. We set $\bar{\tau}_n$ equal to 0.25. In order to get a high value for the autocorrelation of hours, as is observed in U.S. data, we set ρ_n equal to 0.95. The variances of ε_{zt} and ε_{nt} are chosen to make the standard deviations of GNP and hours in the model match those in the U.S. data (which are 1.74 percent and 1.52 percent, respectively, for the postwar period). The adjustment cost parameter is set so that the relative volatility of consumption and output is roughly 0.5, as in the data.

In Table 5, we report the results of this experiment. Notice that little has changed from the economy with only technology shocks. The average ratio of the stock value to GNP is the same, and the equity and debt returns are not much different from the baseline economy's. Note also that the variation in tax rates actually leads to a fall in the premium, from 0.03 to 0.01 percentage point. This happens because the greater variation in hours reduces the correlation between consumption and earnings. But with shocks to technology and labor tax rates, the variation in corporate earnings and the correlation between consumption and earnings are still high relative to the variation in the U.S. data.

Shocks Also to Corporate Capital Share. So now we try a shock to a variable that has a significant effect on consumption and corporate earnings: the share of corporate profits in income. We assume here, as with the labor tax rate, that this variable follows an autoregressive process, with

$$(22) \quad \phi_{mt+1} = (1-\rho_\phi)\bar{\phi}_m + \rho_\phi\phi_{mt} + \varepsilon_{\phi t+1}$$

where $\bar{\phi}_m$ is the mean of the process and $\varepsilon_{\phi t}$ is i.i.d. normal with a mean of zero. If we choose ρ_ϕ and the variance of $\varepsilon_{\phi t}$ to replicate the variability in U.S. corporate shares, then the results show little difference from the benchmark economy. In fact, with shocks to both the la-

bor tax rate and the corporate profits share, we find that we are effectively back to the deterministic version of the model, with the equity premium equal to zero.

We tried some other experiments to see if we could generate a large risk premium. Introducing random corporate profit tax rates leads to counterfactually high variation in corporate earnings. With larger values of σ, we find the volatility of consumption too high and the volatility of hours too low. Different values of ρ, the parameter which affects the substitutability of corporate and noncorporate goods, change the results little.

Effects of More Rapid Growth. If we increase the growth rate of technology, we get a higher risk-free rate but a similar risk premium. The media have suggested that higher future growth justifies higher equity values. We find that this is not so. There are two consequences of higher growth for the value of the stock market. One is that with more rapid growth, future corporate payouts are larger. If market discount factors remain fixed, then these higher payouts imply higher stock market values. But higher growth also leads to greater discounting of future payouts, which reduces the current value of these future payouts. We find that these two consequences of more rapid growth for the value of corporate equity roughly offset each other. The expectation of more rapid economic growth does not justify higher equity values relative to GNP.

A change that would justify higher equity values relative to national income is an increase in the corporate after-tax profits share of income. This we see as highly unlikely because of the historic stability of this variable, once it is corrected for business cycle variation.

Conclusions

Some stock market analysts have argued that corporate equity is currently overvalued. But such an argument requires a point of reference: overvalued relative to what? In this study, we use as our reference point the predictions of the basic growth model that is the standard model used by macroeconomists today. We match up all the variables in our model with the U.S. national income and product account data.

We find that corporate equity is not overvalued. Theory predicts that if net indebtedness is small, the value of corporate equity should equal the value of productive assets. We show that it does; both values are today near 1.8 times the value of GNP. With our estimates of productive assets, theory also predicts that the real returns on debt and equity should both be near 4 percent. Therefore, barring any in-

stitutional changes, we predict a small equity premium in the future.

Appendix A
Some Financial Facts

In this appendix, we report some facts about U.S. household asset holdings that guided the selection of the model that we used in the preceding text to determine whether the U.S. stock market is currently overvalued.

We assumed that individuals in our model are not on corners with respect to their asset choices. There is some evidence that most are not. Households hold a lot of both debt and equity. In Table A1, we report the balance sheet of U.S. households in 1999 and on average for the 1946–99 period, all relative to gross national product. In 1999, households' holding of debt is 1.46 times GNP. Some of this debt is held for liquidity purposes, but the total holding is significantly above what financial planners typically recommend for emergencies and unforeseen contingencies.

In our model, we ignored transaction costs. The data suggest that these costs are quite small. Of the nonliquid assets held by households, approximately 50 percent are currently in retirement accounts. In Table A2, we report holdings in retirement accounts in 1999—by type of account and by type of asset. These pension fund assets are roughly split between debt and equity. The holdings can cheaply be shifted by pension managers or, in many cases, by individuals themselves.

Survey data find that many people do, in fact, shift between debt and equity. (See Vissing-Jørgensen 2000.) The accompanying chart captures this shifting in a graphic manner. The chart is a scatter plot of the fraction of financial assets in equity in two different years for a sample of people. Each plot depicts the positions of a person in the sample in 1989 and in 1994. The plot for a person with the same equity share in the two years falls on the 45-degree line. The large number of plots that are far from that line establishes that between these two years, many people made large changes in the share of their portfolio in equity.

We assumed that tax rates on dividends and interest were effectively zero. Corporations do pay taxes on capital income. But taxes on dividends and realized capital gains from the sale of corporate equity are not taxes on corporate capital income. People can avoid taxes on dividends and capital gains by managing their portfolios in such a way that gains are unrealized capital gains. Dividends paid to pension funds, which now own half of

Table A1

Balance Sheet of U.S. Households

	Ratio of Each Item to GNP	
Item	Average 1946–99	1999
Assets	3.96	5.29
Tangible Assets	2.10	1.99
Corporate Equity	.69	1.84
Debt	1.17	1.46
Liabilities	.46	.74
Net Worth	3.50	4.55

Source: FR Board, various dates

Table A2

Financial Assets of Pension Funds

Ratio of Each Category With GNP in 1999

Category	Ratio
Total Pension Funds†	1.47
By Type of Plan	
Defined Contribution*	.54
Defined-Benefits	.52
Public Defined-Benefits	.41
By Type of Asset**	
Equity	.63
Debt	.57

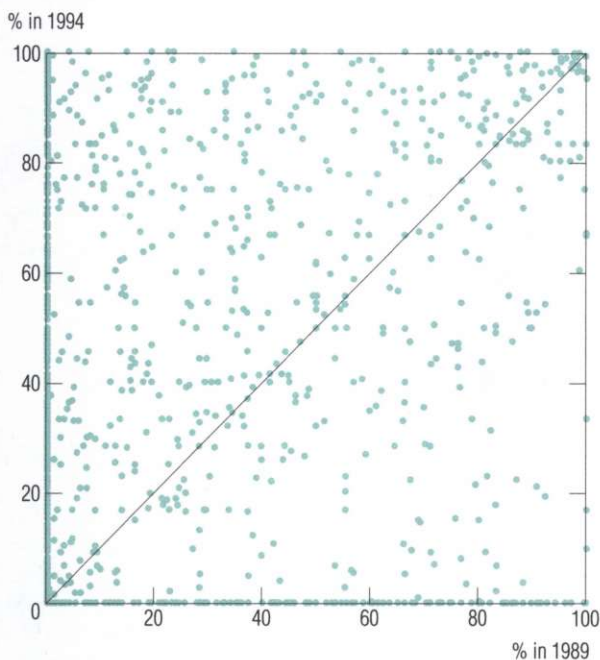† We consolidate pension fund reserves and life insurance reserves.
* This figure includes IRA and Keogh assets.
** These figures do not include IRA and Keogh assets.
Source: FR Board, various dates

Evidence of Portfolio Shifting

Percentage of Individual Financial Wealth Held as Stocks
by a Sample of U.S. Investors in 1989 and 1994

% in 1994

% in 1989

Source: Vissing-Jørgensen 2000

corporate equity, are not subject to the personal income tax. Similarly, pension funds' realized capital gains from the sale of corporate equity are not taxed. There are also tax-managed mutual funds, introduced in the mid-1990s, which are used to minimize taxes and financial fees while allowing people to hold well-diversified portfolios.*

*For an insightful discussion of taxes and how they can be avoided, see Miller 1977.

Appendix B
A Primer on Asset Pricing Under Uncertainty

Here we review the concepts that underlie standard asset pricing theory. A key idea is that consumption today and consumption in some future period are treated as different goods. Relative prices of these different goods are equal to people's willingness to substitute between these goods and businesses' ability to transform these goods into each other. In this appendix, we work through three simple examples to illustrate this point.

We begin with a simple environment with neither capital accumulation nor uncertainty (Example 1). There is only firm-specific uncertainty that averages out over the economy and consequently introduces no aggregate uncertainty. In this economy, the value of firm equity equals the present value of expected firm payouts, and all assets have the same expected return.

Next, we add economywide uncertainty that gives rise to uncertainty in consumption (Example 2). Now expected returns differ across assets. An asset that makes relatively large payments when consumption is high will have a higher expected return than one that has relatively large payouts when consumption is low.

Finally, we add capital accumulation opportunities by adding a storage technology that can transform the period t consumption good into the period $t + 1$ consumption good one-for-one (Example 3). This technology specifies the ability of people to transform goods in some period into goods in some other period. The addition of this storage technology has major consequences for the value of firm equity and for average returns. An implication of this is that when we derive the implications of theory for the pricing of assets and determine the behavior of asset returns, we must explicitly model the ability of people to substitute as well as their willingness to do so.

Willingness to Substitute

Established theory describes the willingness of people to substitute consumption goods across periods in the following way. The economy has a large number of households that maximize expected discounted utility,

(B1) $u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + ... + \beta^{t-1} u(c_t) + ...$

where u is a function determining the level of utility, c is consumption, and the parameter β is positive and less than one. The parameter β describes how impatient households are to consume. If β is small, people are highly impatient, with a strong preference for consumption now versus consumption in the future. These households live forever, which implicitly means that the

utility of parents depends on the utility of their children. In the real world, this is true for some people and not for others. However, economies with both types of people—those who care about their children's utility and those who do not—have essentially the same implications for asset prices and returns.* Thus, we use this simple abstraction to build quantitative economic intuition about what the returns on equity and debt should be. The function $u(c)$ is increasing [$u'(c) > 0$], but at an ever-decreasing rate [$u''(c) < 0$].

In empirical work, constant relative risk aversion is typically assumed. This means that if a household will accept a gamble, then that household will accept that gamble if both its wealth and the gamble amount are scaled by a positive factor. For our purposes here, we use $u(c) = \log(c)$, which empirically is not a bad representation of people's aggregate willingness to substitute. With this utility function, an individual is indifferent between a gamble that provides a 50-50 chance of either $10,000 per year consumption or $20,000 per year consumption and a certainty of consumption of $14,142. This indifference can be expressed as

(B2) $0.5\log(10,000) + 0.5\log(20,000) = \log(14,142)$.

Since the logarithmic utility function displays constant relative risk aversion, this equality holds if the three consumption levels are scaled by any factor.

Three Examples

EXAMPLE 1. *No Aggregate Uncertainty and No Ability to Transform Goods*

Assume first that the economy has one firm for every ten households. Each firm produces 100 units of output with probability 0.5 and 0 units with probability 0.5. These outcomes are randomly distributed across both firms and time. With a large number of firms, then, output per firm in every period is 50, and output per person is 5. With the assumed utility function, the wealth distribution does not matter for the pricing of assets, so for simplicity, assume that everyone owns an equal share of every firm.

Equilibrium consumption of every household is 5 units every period. Consumptions in different periods are different commodities and have different prices. In any particular period, the equilibrium price of the consumption good c_t is

(B3) $p_t = p_0\beta^t$.

Because of household impatience, consumption in the future has a lower price than consumption today.

These prices can be used to value a firm. With no aggregate uncertainty, the ex-dividend value of a firm for this economy is the present value of its expected payouts. Note that firm-specific randomness does not matter; just the expected distribution matters. If a firm has a distribution of 1,000 with probability 0.05

and a distribution of 0 with probability 0.95, then this firm has the same value as a firm with a certainty distribution of 50. The reason this is so is that households can diversify away firm risk by holding a small share of a large number of firms. Thus, the value of a firm is

(B4) $v_0 = 50p_1 + 50p_2 + 50p_3 + ... = 50\beta/(1-\beta)$.

If β is 0.95, then a firm's value is 950. The return on equity is the expected dividend per firm, 50, divided by a firm's value. Consequently, the real return on equity is 5.26 percent.

The one-period real interest rate in this economy is $r_t = p_t/p_{t+1} - 1$, or 5.26 percent. Thus, in this economy with no aggregate uncertainty, returns on debt and equity are equal.

EXAMPLE 2. *Aggregate Uncertainty and No Ability to Transform Goods*

Now assume that the economy has some aggregate uncertainty, enough to make the premium for holding equity about 5 percentage points. In order to introduce this aggregate uncertainty, assume that the probability of good times is 0.5 and so is the probability of bad times. These probabilities are independent over time. The situation is just as if each period a fair coin is tossed, and if it comes up heads, there are good times; if it comes up tails, there are bad times. In good times, the probability of a firm producing 100 units of the consumption good is two-thirds, and the probability of 0 output is one-third. In bad times, these probabilities are reversed. In good times, output per household is 6.67, and in bad times, it is 3.33. Since good and bad times are equally likely, expected output per household in future periods is 5 units, as in the previous example.

However, for this example, a richer class of commodities is needed. Consumption in period t has a different price if times are good than if times are bad. In bad times, consumption is lower, and people value an additional unit of consumption more.

So consumption must be indexed by period and by the nature of the times. Consumption in period t is c_{gt} if times are good and c_{bt} if times are bad. With prices given, the value of the firm in period t, conditional on the state $s = b$ or g, is

(B5) $v_{st} = \sum_{\tau=t+1}^{\infty} [p_{gt} (\text{Expected Payout Given } g) + p_{bt} (\text{Expected Payout Given } b)]/p_{st}$.

Now the ex-dividend value of a firm in period 0 if the state is s is

(B6) $v_{s0} = [p_{g1}66.7 + p_{b1}33.3 + p_{g2}66.7 + p_{b2}33.3 + ...]/p_{s0}$.

---

*Which environment one uses sometimes matters for the average returns. Baby boomers' saving for retirement, for example, may lower expected returns on all financial securities, but it has little effect on differences in average returns on debt and equity (Constantinides, Donaldson, and Mehra 1998).

Average Asset Returns for Examples

Type of Return	Economy With No Uncertainty (Example 1)	Economy With Uncertainty and	
		Without Storage (Example 2)	With Storage (Example 3)
Average Return (%) on			
Equity	5.26	11.67	5.28
Debt	5.26	−.76	3.62
Equity Premium (% points)	0	12.43	1.66

But what is the appropriate set of equilibrium prices? The price of consumption will be higher in bad times than in good times. With the assumed utility function, the prices are

(B7) $p_{bt} = \beta^t$ and $p_{gt} = \beta^t/2$.

These price relations are obtained by equating marginal rates of substitution to the corresponding goods' price ratio.

The ex-dividend values of a firm in terms of that period's consumption good are $v_b = 633$ and $v_g = 1{,}267$. The effect of adding aggregate uncertainty, then, is to raise the value of the firm in good times and lower it in bad times. The average return on equity is now 11.67 percent, which is more than double the return with no aggregate uncertainty.

We turn now to the return on debt. The price of a real bill if the state is s is

(B8) $q_s = \beta(p_b + p_g)/(2p_s)$.

Thus, the risk-free interest rates are

(B9) $r_s = 1/q_s - 1$.

From these equations, the risk-free interest rates are $r_{bt} = r_b = 40.35$ percent and $r_{gt} = r_g = -29.82$ percent. The average risk-free interest rate is −0.76 percent, which is far less than the average return on equity. In this economy, the average equity premium, that is, the difference between the average returns on debt and equity, is over 12 percentage points. Without aggregate uncertainty, the equity premium is 0.

EXAMPLE 3. *Aggregate Uncertainty and
the Ability to Transform Goods*

Now add to Example 2 the feature that goods can be stored. By storage, one unit of the period t good can be transformed into one unit of the period $t + 1$ good. Negative storage is not fea-

sible. The ability to intertemporally transform goods dramatically reduces the premium for holding equity.

For this economy, equilibrium values of assets and consumption depend not only on whether times are good or bad, but also on the stock of stored goods. With this complication, computing the average returns on debt and equity requires the use of a computer. But we can sketch the intuition behind the calculation.

In this economy, people save in good times and draw on savings in bad times in order to smooth consumption over time. As a result, returns on both debt and equity are lower than they would be otherwise. In fact, the average returns over long periods of time are 3.62 percent for debt and 5.28 percent for equity. For this economy with a storage technology, the average return on debt is actually higher than that for the economy without the storage technology, and the average return on equity is lower. This example establishes that any theory of debt and equity returns must model people's ability to transform consumption over time as well as people's willingness to substitute consumption over time.

The finance approach to asset pricing could be applied to this economy. Then the first step in determining the value of the stock market is to determine an appropriate list of commodities; the second step is to find payments of each of these commodities by firms; and the third is to find the prices of the commodities.

The needed list of commodities is as follows. The first three commodities—namely, contracts to deliver the period 0 consumption good, the period 1 consumption good if times are good, and the period 1 consumption good if times are bad—are the same as when the economy has no storage technology. However, in period 2, there are four, not two, event-contingent commodities. This is because people on the margin value a unit of consumption in period 2 differently if times were bad in period 1 than if they were good in period 1. This is true because the equilibrium consumption levels are different. A consequence of this fact is that the period 2 commodities must be jointly indexed by the nature of the times in period 1 and the nature of times in period 2. In general, period t commodities must be indexed by the nature of the times in periods 1 through t. Consequently, there are 2^t period commodities. With this expanded commodity space, the present value calculations work just as they did for the simpler environment considered previously.

For this set of commodities, the problem is to find the period- and event-contingent consumptions and prices for which all markets clear. The simplest way to find these quantities is to exploit the invisible hand result that the competitive equilibrium consumptions maximize welfare. We use standard computational methods to find consumption as a function of inventories x and the current state s, which is either g or b. This function is denoted $c_{t+1} = c(x,s)$. Next period's stock of inventories is, then,

(B10) $x_{t+1} = h(x_{t+1},s_t) = x_t - c(x_t,s_t) + s_t$.

If the current state of the economy is (x,s), then the interest rate is given by

(B11) $r(x,s) = \beta c(x,s)/c(h(x,s)) - 1.$

Standard computational methods can be used to find the value of the stock market as a function of the state or position of the economy, $v(x,s)$. The function v satisfies the functional equation

(B12) $v(x,s) = c(x,s)\{0.5\beta[v(x',b) + c(x',b)]/c(x',b)\}$
$+ c(x,s)\{0.5\beta[v(x',g) + c(x',g)]/c(x',g)\}$

where $x' = h(x,s)$ is next period's inventory stock.

Again, the introduction of a storage technology reduces the average return on equity from 11.67 to 5.28 percent, while its introduction increases the return on debt from −0.76 percent to 3.62 percent. (See the accompanying table for a summary of the results.) This establishes that the nature of the technology—that is, the ability to transform goods into each other—matters for valuing assets and determining their returns.

Appendix C
Adjustments to the NIPA Data

In this appendix, we describe in detail the adjustments that we made to the data from the U.S. Department of Commerce before we compared these data to our model's estimates. These adjustments are reported in Table 1.

The Data

On the left side of Table 1, we report average values for income, product, and capital stocks of the United States during 1990–99. The table first lists the accounting concepts of the national income and product account (NIPA) data. For each concept, we report average values relative to GNP. Thus, GNP is normalized to 1. Notice also that the sum of the value added for the corporate and noncorporate sectors is equal to GNP.

Corporate income is domestic income of corporations with operations in the United States. (See U.S. Commerce, various dates, NIPA Table 1.15.) *Noncorporate income* is the difference between gross national income (NIPA Table 1.14) and corporate income. Thus, noncorporate income includes income of households, the government, noncorporate business, and foreign subsidiaries. For compensation in the noncorporate sector, we include total employee compensation and 80 percent of proprietors' income. Profits of the noncorporate sector include profits of foreign subsidiaries, rental income, and 20 percent of proprietors' income.

Total product is the sum of private consumption, public consumption, and investment (NIPA Table 1.1). Investment includes fixed investment and the change in private inventories. Total investment is the sum of investment in the three types of capital—measured corporate, unmeasured corporate, and noncorporate. We include net exports in noncorporate investment since production in the rest of the world is included in our model's notion of noncorporate production.

Capital stocks are midyear stocks of corporate capital, measured and unmeasured, and noncorporate capital. (See U.S. Commerce, various dates, Fixed Asset Tables 7 and 9.) These stocks correspond to the investments listed in the product section of Table 1.

Adjustments

On the right side of Table 1, we provide descriptions and values of the adjustments that we made to the data in order to make them consistent with our theory. We now describe each adjustment.

The NIPA data include sales taxes in the measure of private consumption. In our model, we treat consumption as pretax, and

therefore, we subtract sales taxes from both the income and the product sides of the accounts. On the income side, the NIPA data include sales and excise taxes in indirect business taxes, 0.57 of GNP in corporate income and 0.022 in noncorporate income. We estimate that of the 0.079 of GNP that is total indirect business taxes, 0.047 of GNP is sales or excise taxes—0.037 in the corporate sector and 0.010 in the noncorporate sector. We attribute the remainder to property taxes. These property taxes appear in the column of adjusted average values.

The NIPA data do not include a measure of intangible investment because this type of investment is expensed. We estimate it to be 0.019 of GNP. We include an estimate of intangible investment in our notion of GNP because it raises both after-tax corporate profits and unmeasured corporate investment.

We make an adjustment to net interest in both the corporate and noncorporate sectors. We subtract the part of financial services purchased by businesses that we estimate consists of intermediate financial goods. The U.S. system of national income and product accounting treats net interest of financial intermediaries as purchases of services by the lender, typically, the household. The United Nations system of accounting treats it, instead, as purchases of services by the borrower. Thus, in the U.N. system, no entry for imputed interest is made, so imputed interest and consumption services are lower. Here, we compute lenders' (borrowers') purchases of financial services as the product of the short-term interest rate less interest received and the amount loaned (borrowed).

We assume that all of the NIPA net interest in the corporate sector, totaling 0.015 of GNP, is intermediate services, and we subtract it. We assume that only part of the net interest in the noncorporate sector is intermediate. Net interest in the noncorporate sector is equal to 0.042 of GNP. Of this value, we estimate that 0.022 of GNP is intermediate, and we subtract that from noncorporate income. The remainder of noncorporate net interest is included in noncorporate profits. Most of the 0.022 of GNP adjustment is for services implicitly purchased by homeowners with mortgages. It is the difference in interest paid by people with mortgages and the interest received by households lending to those financial institutions issuing mortgages. The adjustment that we make on the product side is to lower consumption services. We lower it by the sum of the adjustments to the corporate and noncorporate sectors on the income side (0.015 and 0.022 of GNP), which is 0.037 of GNP.

Consumer durables are treated as private consumption in the NIPA data and as investment in our model. Therefore, we add to the NIPA data the depreciation of consumer durables. For the 1990–99 period, the average depreciation of consumer durables was equal to 0.063 of GNP. We add this depreciation to noncorporate capital consumption on the income side and to private consumption services on the product side. This is the procedure used for housing services which are included in the NIPA data.

Because profits of foreign subsidiaries are part of rest-of-world profits, and therefore noncorporate profits, we add an estimate of the capital of these foreign subsidiaries to noncorporate capital. Our estimate of the capital in foreign subsidiaries is 0.294. To make the depreciation and investment of the noncorporate sector comparable to the capital stock, we add in depreciation and net investment for the foreign subsidiaries. Depreciation is added to noncorporate capital consumption on the income side and to noncorporate investment on the product side. Net investment is added to noncorporate investment and subtracted from private consumption, so that the total product does not change. Our estimate of the depreciation of foreign subsidiary capital is 0.016 of GNP. Our estimate of net investment is 0.009 of GNP. In making these estimates, we assume that depreciation rates and growth rates are the same at home and abroad.

We add to noncorporate profits our estimates of the value of imputed capital services to government capital and to consumer durables. For the NIPA data, a zero percent interest rate is used to impute services to government capital. We instead use the average return on capital in the noncorporate sector. Our estimate of this return is 4.08 percent. Thus, our estimate of imputed services is this rate times the net stock of government capital (0.592 of GNP) plus the net stock of consumer durables (0.287 of GNP). Imputed services, therefore, are equal to 0.024 of GNP for government capital and 0.012 of GNP for consumer durables, or a total of 0.036 of GNP. So that income equals product, we add the value of imputed services to government capital both to profits in the noncorporate sector and to government consumption. In the NIPA data, consumer durables are treated as consumption. We instead treat them as investment and impute services to these durables. These imputed consumer durable services are added to profits in the noncorporate sector and to private consumption.

We make several adjustments to the capital stocks. The value of measured capital is 0.821 of GNP. This value does not include the value of inventories or land. A value for inventories is, however, available in the NIPA data (Table 5.12). The value of inventories is 0.161 of GNP. The value of land is not included in the NIPA data, but it is available from the Federal Reserve Board of Governors for land owned by nonfinancial corporate businesses (FR Board, various dates). The difference between real estate values reported by the Fed and in the NIPA data is 0.060 of GNP. Thus, our estimate for the value of corporate capital, including inventories and land, is 1.042 times GNP.

We make one more adjustment to the corporate capital stock: We include an estimate of the unmeasured intangible capital. That estimate is 0.645 of GNP.

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