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Putting Home Economics Into Macroeconomics*

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Macroeconomists often divide private economic activity into two sectors, the business sector and the household sector. A lot of effort has gone into modeling the activities of businesses; much less so, into modeling the activities of households. Our purpose is to redress that imbalance. We argue that placing the household sector on an equal footing with the business sector enriches an otherwise standard real business cycle model and improves its ability to account for fluctuations in U.S. economic activity since World War II.

Considering the relatively minor role households have played in business cycle modeling to date, some may find the size of this sector surprising. This is true whether you measure its size by the amount of time spent there, the capital stock it uses, or its output. Studies such as the Michigan Time Use survey indicate that a typical married couple spend 25 percent of their discretionary time on unpaid work in the home, such as cooking, cleaning, and child care, compared to 33 percent on work for pay in the market (Hill 1984, Juster and Stafford 1991). The postwar U.S. national income and product accounts indicate that investment in *household capital* (defined as purchases of consumer durables and residential structures) actually exceeds investment in *business capital* (defined as purchases of producer durables and nonresidential structures) by about 15 percent. Finally, those who have attempted to measure the value of the household sector's output have come up with figures ranging between 20 and 50 percent

of the value of measured gross national product (Eisner 1988). Clearly, the household sector is large, and this suggests that the economics of the household (dubbed here *home economics*) is important.

The significance of home production in economic activity has long been recognized by labor economists (Becker 1965, Pollak and Wachter 1975, and Gronau 1977, 1985). But its relevance for business cycle research has only recently been investigated.¹ Those who have begun to follow the lead of labor economists have found that the costs of paying more attention to the household in real business cycle models are small compared to the benefits. All that these models require is a home production function that

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¹For instance, home production has been added to otherwise standard real business cycle models by Benhabib, Rogerson, and Wright (1991) and Greenwood and Hercowitz (1991). A dynamic general equilibrium model with home production has been developed by Rios-Rull (forthcoming) to study life cycle, business cycle, and cross-sectional wage behavior. Macroeconomic models with home production have been estimated by Fisher (1992) and McGrattan, Rogerson, and Wright (1992). And inflation's impact in a home production model has been analyzed by Fung (1992).

transforms home labor and capital into home output, just as the standard market production function transforms market labor and capital into market output. The household and business sectors simply need to be treated symmetrically.

The benefits of including home production in standard real business cycle models lie in the enriched set of choices such models produce: With home production, households must allocate their time among leisure, business work, and home work; in the standard model, their only choice is between leisure and work. Household choices are similarly expanded when it comes to allocating output. With home production, they must divide output among consumption, investment in business capital, and investment in household capital; in the standard model, their only choice is between consumption and investment. An enriched set of choices results in a model which allows more substitution into and out of market activity in response to the state of the economy.

The upshot of this greater degree of substitutability between market and nonmarket activity is that the home production model can outperform the standard real business cycle model in accounting for several basic aspects of the U.S. data. These include the volatility of market output, the volatilities of consumption and investment relative to market output, the volatility of hours worked in the market relative to either market output or productivity, the correlation between market hours and productivity, and the correlation between investment in household and business capital. Some significant deviations between the model and the data remain, but adding home production to real business cycle models appears to be a promising avenue of research.

The Basic Model

The basic real business cycle model with home production contains a large number of identical infinitely lived households. They have preferences described by this utility function:

$$(1) \quad U = \sum_{t=0}^{\infty} \beta^t [b \log(C_t) + (1-b) \log(l_t)]$$

where C_t is total consumption and l_t is leisure at date t . Total consumption is a composite of goods and services purchased in the market, c_{Mt} , and goods and services produced in the home, c_{Ht} . In particular,

$$(2) \quad C_t = [ac_{Mt}^e + (1-a)c_{Ht}^e]^{1/e}.$$

The parameter $e \leq 1$ controls the household's willingness to substitute between c_{Mt} and c_{Ht} ; the larger is e , the greater is this willingness. Leisure equals total time, which we normalize to unity, minus hours worked in the market, h_{Mt} , minus hours worked in the home, h_{Ht} . That is,

$$(3) \quad l_t = 1 - h_{Mt} - h_{Ht}.$$

Equations (1), (2), and (3) can alternatively be written as

$$(4) \quad U = \sum_{t=0}^{\infty} \beta^t u(c_{Mt}, c_{Ht}, h_{Mt}, h_{Ht})$$

where

$$(5) \quad u(c_M, c_H, h_M, h_H) = (b/e) \log[ac_M^e + (1-a)c_H^e] + (1-b) \log(1-h_M-h_H).$$

At each date, the household is subject to two types of constraints. One is the *market budget constraint* that allocates total after-tax income over its uses:

$$(6) \quad c_{Mt} + x_{Mt} + x_{Ht} = w_t(1-\tau_h)h_{Mt} + r_t(1-\tau_k)k_{Mt} + \delta_M \tau_k k_{Mt} + T_t.$$

As is shown by the left side of (6), income can be used for three purposes: the purchase of market consumption goods and services, c_{Mt} ; investment in business capital, x_{Mt} ; and investment in household capital, x_{Ht} . Here

w_t = the real wage rate in the market.

r_t = the price at which business capital can be rented to firms.

τ_h = the tax rate on labor income.

τ_k = the tax rate on capital income.

δ_M = the (tax deductible) depreciation rate on business capital.

T_t = a lump-sum transfer payment from the government.

The right side of (6) shows that the household's income derives from three sources: after-tax labor income, $w_t(1-\tau_h)h_{Mt}$; after-tax capital income, $r_t(1-\tau_k)k_{Mt} + \delta_M \tau_k k_{Mt}$; and lump-sum transfer payments from the government, T_t .

The household is also subject to the *home production constraint* at each date:

$$(7) \quad c_{Ht} = g(h_{Ht}, k_{Ht}, z_{Ht}) = k_{Ht}^\eta (z_{Ht} h_{Ht})^{1-\eta}.$$

The home production function in (7) yields consumption of the home goods and services as a function of the time spent in home work and the household capital stock brought into the period plus a shock term z_{Ht} representing technological change. Note that there are no uses for home-produced output other than consumption—it cannot be sold or transformed into capital, for example, the way that market-produced output can. This is a key asymmetry between the market and home sectors: only the former can produce capital.

An example (taken from Greenwood and Hercowitz 1991) may help to illustrate the economic environment being envisioned. A meal cooked at home combines food produced in the market using capital and time with home cooking services that use capital and time at home to create, when mixed with leisure, the end good: utility. In the spirit of Becker (1965), one can interpret the market production function $f(\cdot)$ and the home production function $g(\cdot)$ as producing intermediate goods and services, which are then used in $u(\cdot)$ with leisure to make the final product, utility.²

There is a representative firm in the economy, with a constant returns-to-scale technology described by the market production function:

$$(8) \quad y_t = f(h_{Mt}, k_{Mt}, z_{Mt}) = k_{Mt}^\theta (z_{Mt} h_{Mt})^{1-\theta}$$

where y_t is market output and z_{Mt} is a shock representing technological change in the market. For quantitative analysis, we need to be precise about the nature of technical progress. We assume here that $z_{Mt} = \lambda^t \tilde{z}_{Mt}$ and $z_{Ht} = \lambda^t \tilde{z}_{Ht}$, where λ^t is a deterministic component and \tilde{z}_{Mt} and \tilde{z}_{Ht} are stochastic processes with

$$(9) \quad \log(\tilde{z}_{Mt+1}) = \rho_M \log(\tilde{z}_{Mt}) + \varepsilon_{Mt+1}$$

$$(10) \quad \log(\tilde{z}_{Ht+1}) = \rho_H \log(\tilde{z}_{Ht}) + \varepsilon_{Ht+1}$$

The innovations ε_{Mt} and ε_{Ht} are independent and identically distributed over time, with standard deviations σ_M and σ_H and contemporaneous correlation γ . The parameters ρ_M and ρ_H govern the degree of persistence in the shocks \tilde{z}_{Mt} and \tilde{z}_{Ht} .

Investment augments the capital stock according to the law of motion:

$$(11) \quad k_{t+1} = (1-\delta_M)k_{Mt} + (1-\delta_H)k_{Ht} + x_t$$

where $x_t = x_{Mt} + x_{Ht}$ is total investment. The aggregate

capital stock can be divided between business (or market) and household capital at a point in time according to $k_t = k_{Mt} + k_{Ht}$. We assume that capital can be freely transformed between its two uses, although it may depreciate at different rates in the two sectors.³ Investments in the two capital goods are defined residually by

$$(12) \quad x_{Mt} = k_{Mt+1} - (1-\delta_M)k_{Mt}$$

$$(13) \quad x_{Ht} = k_{Ht+1} - (1-\delta_H)k_{Ht}$$

In each period, the government taxes labor and capital income, transfers T_t back to households, and consumes the surplus. Hence, government spending is given by

$$(14) \quad G_t = w_t h_{Mt} \tau_h + r_t k_{Mt} \tau_k - \tau_k \delta_M k_{Mt} - T_t$$

where, again, $\tau_k \delta_M k_{Mt}$ is the depreciation allowance. Feasibility implies that market output is allocated across market consumption, total investment, and government spending:

$$(15) \quad y_t = c_{Mt} + x_t + G_t$$

For simplicity, we assume from now on that all revenue is rebated back lump-sum to households, so that $G_t = 0$ in what follows.⁴

A competitive equilibrium for this economy is defined in the usual manner.⁵ The representative firm solves a se-

²The appropriate decision-making unit in reality is a household or family, which may, of course, consist of more than one individual. This implies that it may be possible to consume a home-cooked meal, for example, without actually cooking it. At the level of abstraction adopted here, however, the household is taken to be one single-minded decision-making unit with no internal bargaining or disagreement. This may not be particularly realistic, but it does make things simpler. Pollak and Lundberg (1991) discuss bargaining within the family and provide references to the related literature.

³Although capital is freely mobile between the home and the market at a point in time, in the experiments that we conducted it is rare that any capital physically moves between sectors, since typically gross (if not net) investment in each is positive. Hence, free mobility seems to play little role. What is important, however, is that capital does not have to be committed to either sector until the shocks have been observed. Greenwood and Hercowitz (1991) assume that capital does have to be allocated in advance, which has some advantages in terms of the results. We adopt the specification here for simplicity.

⁴More generally, G could enter the utility function, and we could assume that G in the model mimics government spending in the data (its stochastic properties or at least its average value). Note, however, that if we assume government spending is a perfect substitute for market consumption in the utility function, then a model with $G \neq 0$ generates exactly the same statistics as a model with $G = 0$, except for the fact that c_M changes one-for-one to offset changes in G .

⁵Due to the presence of distorting taxes, equilibrium allocations are not generally Pareto optimal, so we have to work with the equilibrium directly rather than the social planner's problem. The discussion here is not intended to be particularly rigorous. Greenwood and Hercowitz (1991) define more carefully a recursive competitive equilibrium for the model. The solution procedure we use here is described in detail in McGrattan, forthcoming.

quence of static problems at each date: maximize instantaneous profit, $y_t - w_t h_{Mt} - r_t k_{Mt}$, taking as given $\{w_t, r_t, z_{Mt}\}$. The household maximizes expected utility subject to the home production and market budget constraints, taking as given stochastic processes for $\{w_t, r_t, T_t, z_{Ht}\}$. Given the stochastic processes for the technology shocks and the initial capital stock, an *equilibrium* is a set of stochastic processes for the real wage, the rental rate, and transfer payments $\{w_t, r_t, T_t\}$ and quantities $\{c_{Mt}, c_{Ht}, h_{Mt}, h_{Ht}, k_{Mt}, k_{Ht}\}$ that solve both the firm's and the household's problems and satisfy the feasibility condition (15).

Calibration

The model developed above will now be *calibrated*. This involves picking values for the model's parameters either on the basis of a priori information or so that, along the model's balanced growth path, values for various endogenous variables assume their average values over the post-war U.S. period. Therefore, in order to calibrate the model, we need to derive some properties of the balanced growth path—that is, the equilibrium path to which the economy converges when $z_{Mt} = z_{Ht} = \lambda^t$ for all t . In this case, the economy converges to a path on which $h_{Mt} = h_M$ and $h_{Ht} = h_H$ are constant while all other endogenous variables grow at rate λ , so that $y_t = y\lambda^t$ for some constant y , $c_{Mt} = c_M\lambda^t$ for some constant c_M , and so on.

To describe this in more detail, substitute the market budget and home production constraints into the household's objective function and then differentiate to obtain the first-order conditions:

$$(16) \quad h_{Mt}: u_1(t)w_t(1-\tau_h) = -u_3(t)$$

$$(17) \quad h_{Ht}: u_2(t)g_1(t) = -u_4(t)$$

$$(18) \quad k_{Mt}: u_1(t)[r_t(1-\tau_k) + 1 - \delta_M + \delta_M\tau_k] = u_1(t-1)/\beta$$

$$(19) \quad k_{Ht}: u_1(t)(1-\delta_H) + u_2(t)g_2(t) = u_1(t-1)/\beta$$

where the notation $\xi(t)$ means that the function ξ is evaluated at its arguments as of date t .

Equations (16) and (17) are the efficiency conditions governing the allocation of labor to business and household production. Take equation (16), for example. The right side of this equation shows the disutility, $-u_3(t)$, that the household will realize by allocating an extra unit of time to market production. The left side shows the benefit, in terms of extra utility, that the household will earn by increasing the amount of time in market production. Specifically, after taxes, the unit of labor will be exchanged for

the equivalent of $w_t(1-\tau_h)$ units of market consumption goods generating $u_1(t)w_t(1-\tau_h)$ extra units of utility. Optimality dictates that the marginal costs and benefits from allocating time to market production be equalized.

Equations (18) and (19) are the efficiency conditions governing the accumulation of business and household capital. Consider equation (19). Suppose that the household decides to purchase an extra unit of household capital at time $t-1$ at the expense of consuming a unit of the market consumption good. This leads to a utility loss of $u_1(t-1)/\beta$, which is the right side of (19). The production of home goods and services in period t , however, increases by $g_2(t)$ units, which are worth $u_2(t)g_2(t)$ in utility terms. Also, at this time, the household can sell the undepreciated portion of this capital for $(1-\delta_H)$ units of the market consumption good, resulting in a utility gain of $u_1(t)(1-\delta_H)$. Thus, the total gain in period t utility is $u_1(t)(1-\delta_H) + u_2(t)g_2(t)$, which is the left side of (19).

The first-order conditions from the firm's problem are $f_1(t) = w_t$ and $f_2(t) = r_t$. That is, the firm hires factor services—for labor and capital—up to the point where marginal products equal factor prices. These expressions, in conjunction with the assumptions on functional forms, allow (16)–(19) to be simplified to

$$(20) \quad abc_M^{e-1}C^{-e}y(1-\theta)(1-\tau_h) = (1-b)h_M/l$$

$$(21) \quad (1-a)bc_H^e C^{-e}(1-\eta) = (1-b)h_H/l$$

$$(22) \quad \theta(1-\tau_k)y/k_M = \lambda/\beta - 1 + \delta_M(1-\tau_k)$$

$$(23) \quad \eta(1-a)c_H^e C_M^{1-e}/ak_H = \lambda/\beta - 1 + \delta_H.$$

Additionally, equations (12) and (13) imply that

$$(24) \quad x_M/k_M = \lambda - 1 + \delta_M$$

$$(25) \quad x_H/k_H = \lambda - 1 + \delta_H.$$

We now proceed to choose parameter values, setting some values based on a priori information and setting the others according to the balanced growth conditions. Since we interpret the period as one quarter, we set $\lambda = 1.005$ in order to match the quarterly growth rate of output in the U.S. data.⁶ The discount factor is set so that the annual real rate of return on assets in the model is 6 percent, which yields $\beta = 0.9898$. We set the labor income tax rate to $\tau_h =$

⁶We report exact parameter values later, in the notes to a table; in the text, we round off most parameter values to a few digits.

0.25, the average value in the series in McGrattan, Rogerson, and Wright 1992, which is based on the definitions in Joines 1981. The effective tax rate on capital income is more controversial, and there is a wide range of estimates in the literature. For example, the series in McGrattan, Rogerson, and Wright 1992 implies τ_k is about 0.50 on average, while Feldstein, Dicks-Mireaux, and Poterba (1983) estimate τ_k to be between 0.55 and 0.85 in the period 1953–79.

We use the mean of the Feldstein, Dicks-Mireaux, and Poterba estimates and set $\tau_k = 0.70$. This is higher than the numbers used in some other studies in the real business cycle literature, but for two reasons we think it is the right number for our purposes. First, given that we are trying to model both market and nonmarket investment, we want τ_k to capture all forms of government regulation, interference, or any other institutional disincentive to invest in business capital, not only direct taxation. Second, the capital share coefficient in the market production function, θ , which is calibrated below, turns out to be sensitive to the choice of the capital income tax rate. Setting $\tau_k = 0.70$ implies a value for θ that is consistent with independent evidence from the national income and product accounts. (We will return to this issue.)

We now use (20)–(25) to match the following six observations: the two capital/output ratios, the two investment/output ratios, and labor hours in the two sectors. The postwar U.S. national income and product accounts yield $k_M/y = 4$, $k_H/y = 5$, $x_M/y = 0.118$, and $x_H/y = 0.135$, on average, where household capital is measured by consumer durables plus residential structures and business capital is measured by producer durables plus nonresidential structures. Averaging data from the 1971 and 1981 time use surveys, we find $h_H = 0.25$ and $h_M = 0.33$ for a typical household, where these numbers are defined as fractions of discretionary time (24 hours per day minus personal care, which is mainly sleep). These six observations determine δ_M , δ_H , θ , η , and two of the three preference parameters a , b , and e .

The system (20)–(25) has a simple recursive structure. Equations (24) and (25) yield $\delta_M = 0.0247$ and $\delta_H = 0.0218$, which we approximate by setting the two depreciation rates to a common value of $\delta = 0.0235$. Equation (22) yields $\theta = 0.29$, and then (23) yields $\eta = 0.32$.⁷ The value $\theta = 0.29$ is also exactly what we compute from the national income and product accounts.⁸ Three preference parameters remain to be specified, a , b , and e , but we only have two equations left. In what follows, we consider sev-

eral alternative values of e , which is the parameter that determines the elasticity of substitution between c_M and c_H , and for each alternative solve for the values of a and b from (20) and (21). As e varies, a and b will change, but δ_M , δ_H , θ , and η will not.

Finally, we need to specify the parameters describing the stochastic elements of the model. As in much of the literature, we set $\rho_M = 0.95$ and set σ_M so that the innovation in $\tilde{z}_{Mt}^{1-\theta}$ has a standard deviation of 0.007. We then set $\rho_M = \rho_H$ and $\sigma_M = \sigma_H$, so that the home shock mimics the market shock. This leaves γ , which is the correlation between the innovations ε_{Mt} and ε_{Ht} . Unfortunately, there is little independent evidence to guide us in choosing this parameter. In what follows, as with the preference parameter e , we report the results of experiments with different values of γ .

To summarize, all of the parameters except e and γ have been set. The parameter e measures households' willingness, and the parameter γ measures households' incentive, to move economic activity between the home and the market. Higher values of e mean that households are more willing to substitute consumption of one sector's output for that of the other. Lower values of γ mean that the technology shocks more frequently take on different values across sectors, and this implies a greater incentive to move resources across sectors. As will be shown in the next section, changing either the willingness or the incentive to substitute between the home and the market can affect the implications of the model for business cycles.

To close this section, we return to the interaction between taxes and home production. Consider a model without taxation under the standard assumption that the entire capital stock enters into the market production function, so that k_M/y is about 9. Then, calibrating the model as we did above, we find $\theta = 0.34$, which is close to the value implied by the national income and product accounts and typically used in the real business cycle literature.⁹ However, zero taxation is clearly counterfactual. If we set $\tau_k = 0.70$,

⁷It looks as though one needs to know the parameter a in order to determine η from (23); however, a can be eliminated from (23) using the other conditions.

⁸To compute θ from the national income and product accounts, we subtract proprietor's income from total income, as is standard, and also subtract the service flow attributed to the housing stock from output since this is home and not market output. The result is $\theta = 0.29$ in our sample.

⁹Depending on details, such as how one treats proprietors' income, the national income and product accounts indicate that θ could be anywhere between 0.25 and 0.43. (See Christiano 1988, for example.) Prescott (1986) argues for $\theta = 0.36$ while, as indicated earlier, we find $\theta = 0.29$.

then in order to get $k_M/y = 9$, we need to set $\theta = 0.66$, which seems far too high. Even a more conservative tax rate of $\tau_k = 0.50$ implies that $\theta = 0.48$, which still seems far too high. Intuitively, when capital income is taxed, we must assume the marginal product of capital is big in order to get households to accumulate a stock as large as $k_M/y = 9$, and θ is the key parameter governing this marginal product. In a home production model, we do not interpret all capital as market capital; therefore, k_M/y is 4 rather than 9. This in combination with taxation implies that $\theta = 0.29$, which is just what we observe in our data.

Simulation

The model developed will now be simulated in order to assess its business cycle properties. The analysis consists of comparing a set of summary statistics characterizing the movement of variables in the model with the corresponding set describing the postwar U.S. data. The accompanying table lists some summary statistics for the U.S. economy and for several versions of the model to be described below.¹⁰ We focus on the following statistics: the standard deviation (in percent) of y ; the standard deviations relative to y of x , c_M , h_M , and w (and relative to w for h_M); the correlation between h_M and w ; and the correlation between x_M and x_H .

The variable w can be interpreted either as the real wage or, equivalently, as the average product of hours worked in the market (that is, productivity), since the wage equals the marginal product in equilibrium and the marginal product is proportional to the average product with a Cobb-Douglas technology. Investments in the two capital stocks are defined by letting business capital be producer structures plus equipment and letting household capital be residential structures plus consumer durables. Total investment is the sum. Consumption is defined to include nondurables plus services minus the service flow imputed to the housing stock. Market output is defined to be consumption plus investment and government spending. Market hours are from the household survey.

In model 1, we set $e = 0$, implying that the elasticity of substitution between c_M and c_H is unity. We also set the correlation between the shocks ε_M and ε_H to $\gamma = 2/3$, as in Benhabib, Rogerson, and Wright 1991 (although when $e = 0$ the value of γ does not matter for the results). Except for minor details, model 1 is the base model in Greenwood and Hercowitz 1991 and is designed to minimize the role of home production. This can be seen by noting that, when $e = 0$, the home production model generates

the same values for c_M , h_M , k_M , and k_H as a model without home production and a momentary utility function given by¹¹

$$(26) \quad V = a \log(c_M) + (1-a)\eta \log(k_H) \\ + [(1-a)(1-\eta) + (1-b)/b] \log(1-h_M).$$

If $\eta = 0$, this reduces to a standard utility function that ignores home production.

Hence, the home production model replicates the results of the standard model exactly when $e = \eta = 0$. Even if $\eta > 0$, when $e = 0$, the home production model generates results that are close to the standard model. As is well known, the statistics generated by the standard model differ from the data along several dimensions; therefore, so do the results generated by model 1:

- Output is less volatile in the model than in the data. Specifically, the percentage standard deviation of output is 1.36 for the model versus 1.96 for the data.
- In the model, investment fluctuates too much while consumption is too smooth. This is demonstrated by a relative standard deviation for investment of 2.82 in the model as compared with 2.61 in the data, while the standard deviation of consumption is 0.41 in the model and 0.54 in the data.
- The hours-worked series in the model is not volatile enough relative to either output (0.41 for the model vs. 0.78 for the data) or productivity (0.68 vs. 1.06).
- Hours worked and productivity are highly positively correlated in the model as reflected by a correlation coefficient of 0.96, but not in the data where that coefficient is -0.12 .
- The two investment series are positively correlated in the data (with a correlation coefficient of 0.30), but not in the model (-0.09).

See Benhabib, Rogerson, and Wright 1991 for additional discussion of these results.

¹⁰The U.S. data are quarterly and are from the 41-year period 1947:1–1987:4. Often in the literature, only data after 1955 are considered, presumably to eliminate the effect of the Korean War. Summary statistics are similar in the two periods (Hansen and Wright 1992). We take logarithms and detrend using the Hodrick-Prescott filter (as described in Prescott 1986) before computing statistics, both for the U.S. data and for data generated by the models. The notes to the table provide more details.

¹¹To prove this statement, first substitute the home production constraint into the instantaneous utility function; then maximize with respect to home work, and substitute the maximized value back into the utility function. This yields the reduced form utility function in (26). For details, see Greenwood, Rogerson, and Wright, forthcoming.

In model 2, we raise e from 0 to $2/3$. This corresponds to a situation where households are much more willing to substitute between c_M and c_H than in model 1. Notice that, between models 1 and 2,

- The volatility of output increases from 1.36 to 1.60.
- The relative volatility of investment falls from 2.82 to 2.34, and that of consumption rises from 0.41 to 0.61.
- The hours-worked series becomes more variable relative to output (from 0.41 to 0.52) and to productivity (from 0.68 to 1.00).
- The correlation between hours and productivity decreases slightly (from 0.96 to 0.86).
- The correlation between the two investment series decreases a lot (from -0.09 to -0.82).

Hence, increasing the value of e moves the model in the right direction vis-à-vis the data, except for the correlation between x_M and x_H .

Benhabib, Rogerson, and Wright (1991) set e and γ more or less arbitrarily. Another approach is to estimate the model using maximum likelihood techniques, as do McGrattan, Rogerson, and Wright (1992). This procedure yields $e = 0.4$ and $\gamma = 0$ (after rounding), which we use in model 3. These parameter values correspond to a situation where, as compared to model 2, households are less willing to substitute between the two sectors, but there is more of an incentive to do so. Notice that models 2 and 3 yield similar results. This illustrates the interaction between the assumptions that households are more willing to substitute (a higher value of e) and they have greater incentives to do so (a lower value of γ): raising e for a given γ is very similar to lowering γ for a given e .¹²

Although neither model 2 nor 3 does well in terms of the correlation between h_M and w (0.86 and 0.95, respectively, vs. -0.12 in the data), this is a statistic that can in principle be matched by introducing home production. Intuitively, the standard model with shocks only to the market technology is driven by a shifting labor demand curve, so simulations trace out in (h_M, w) space a stable upward-sloping labor supply curve and yield a correlation between the two variables close to unity. What is needed is a second shock to shift labor supply, such as a preference or home technology shock.¹³ Home technology shocks change the amount households are willing to work in the market at a given wage, shifting the labor supply curve and reducing the hours/productivity correlation. In models 2 and 3, this

effect is present but small. Increasing the standard deviation of the home technology shock can reduce the correlation between hours and productivity much more, however; see Hansen and Wright 1992 for further discussion.

We now turn to the correlation between x_M and x_H , which the above models do not capture well at all. The problem is that in times of high relative market productivity, households want to move inputs out of the home and into the market (since that is where they can build capital in order to spread the effects of a temporary productivity rise into the future). The movement of resources between the two sectors is part of what makes a home production model work: the reallocation of hours from nonmarket to market labor, rather than exclusively from leisure to labor as in the standard model, increases the volatility of h_M for a given technology shock. But it also leads to a problem: How can we make households want to invest in both business and household capital at the same time that the market and home labor inputs are moving in opposite directions over the cycle?

Greenwood and Hercowitz (1991) approach the problem by assuming a more general home production function than we have used up to now:

$$(27) \quad g(h_H, k_H, z_H) = [\eta k_H^\psi + (1-\eta)(z_H h_H)^\psi]^{1/\psi}$$

for $\psi \leq 1$. (Note that $\psi = 0$ reduces to the Cobb-Douglas case we have considered.) They also assume that the shocks z_H and z_M are highly correlated, so that when a positive technology shock hits the market, it also hits the home. When a positive shock arrives, since z_H is labor-augmenting, it is possible to move hours out of the home and into the market and still end up with more effective hours in the home. That is, $z_H h_H$ can increase while h_H de-

¹²One might think that the parameter values from McGrattan, Rogerson, and Wright 1992 would do even better than indicated by the results in the table since, after all, they were estimated by fitting the model to the aggregate time series. Several points are relevant in this regard. First, the model in that paper differs from the one here in certain respects, such as the fact that it includes stochastic taxation and government spending. Second, although we use the same e and γ , some of the other parameter values are different. Finally, the econometric technique used in that paper takes into account aspects of the time series other than the small number of second moments computed from filtered data considered in the table; for example, estimation trades off the fit at business cycle frequencies against the fit at longer run frequencies.

¹³Christiano and Eichenbaum (1992) argue for preference shocks, which they identify with changes in government spending. The idea is that, as long as government spending is less than a perfect substitute in utility for private consumption, an increase in G entails a negative wealth effect which shifts labor supply. Stochastic tax shocks, as in Braun 1990 or McGrattan 1991, can have similar effects in terms of shifting labor supply.

The Effects of Adding Home Production to a Real Business Cycle Model

	% S.D. of Market Output	Ratio of Standard Deviations				Correlation		
		Variable Relative to Market Output				Market Hours Relative to Market Wage	Market Hours and Market Wage	Investment in Business and Household Capital
		Total Investment	Market Consumption	Market Hours	Market Wage			
U.S. Time Series, 1947–87*	1.96	2.61	.54	.78	.73	1.06	-.12	.30
Models†								
1. Standard: Home Production Minimized ($e = 0, \gamma = 2/3$)	1.36	2.82	.41	.41	.60	.68	.96	-.09
2. Increased Willingness to Substitute Between Home and Market ($e = 2/3, \gamma = 2/3$)	1.60	2.34	.61	.52	.52	1.00	.86	-.82
3. Increased Incentive to Substitute Between Home and Market ($e = 0.40, \gamma = 0$)	1.59	2.44	.53	.48	.53	.91	.95	-.75
4. More General Home Production Function and Highly Correlated Technology Shocks ($e = 2/3, \gamma = 0.99, \psi = -1/2$)	1.21	2.95	.38	.39	.62	.63	.95	.50

*All data are quarterly and are divided by population, logged, and detrended using the Hodrick-Prescott filter. Nominal variables are converted into 1982 dollars. The specific time series used are gross national product minus gross housing product (for *market output*, y); fixed nonresidential private investment (for *business investment*, x_M); private residential investment plus personal consumption expenditures on durable goods (for *household investment*, x_H); personal consumption of nondurables plus services minus gross housing product (for *market consumption*, c_M); and hours worked by the employed labor force, from the household survey (for *market hours*, h_M). The *market wage* (w), or productivity, is calculated by dividing *market output* by *market hours*.

†All the models use these parameters: $\lambda = 1.004674$, $\beta = 0.9898$, $\tau_y = 0.25$, $\tau_z = 0.70$, $\delta_M = \delta_H = 0.0235$, $\theta = 0.2944$, $\eta = 0.3245$, a and b determined so that $h_M = 0.33$ and $h_H = 0.25$, $\rho_M = \rho_H = 0.95$, and $\sigma_M = \sigma_H$ determined so that the innovation in $\tilde{z}_M^{1-\theta}$ has standard deviation 0.007. Model 4 uses a CES home production function with $\psi = -0.5017$. Model statistics are sample means over 50 simulations, each the same length as the U.S. data.

Source of U.S. data: Citicorp's Citibase data bank

creases. Thus, effective hours in home production can increase during upswings in market activity, and depending on ψ , this can imply a desire to increase capital in the home.

Model 4 uses the technology in (27) with $\psi = -1/2$, $\gamma = 0.99$, and $e = 2/3$ and otherwise keeps the parameters as described above. As can be seen, this does generate a positive correlation between x_M and x_H , as demonstrated by the correlation coefficient of 0.50. (Recall that the number for the data is 0.30.) However, it requires a high correlation between the shocks, and if the two shocks are very

highly correlated, the model does not entail frequent incentives to substitute between home and market activity. Therefore, generating a positive correlation between x_M and x_H involves sacrificing at least part of the other improvements that can be achieved by introducing home production. It is not obvious how to resolve this tension. Additionally, the U.S. data display a clear phase shift, with investment in household capital leading investment in business capital. Building a model that better accounts for these phenomena remains an open project.

Let us summarize the findings from these experiments.

With $e = 0$, the model generates second moments that are similar to those of a standard model without home production. By increasing e for a given γ , we can affect the volatility of output, investment, consumption, and hours in the right direction. A similar effect can be obtained by decreasing γ for a given e . These results do not require a large home shock, and in fact, the model performs about as well if the home technology is nonstochastic.¹⁴ However, the larger the home shock, the better the resulting correlation between hours and productivity implied by the model. The correlation between investments in the two sectors can also be improved by considering a more general home technology, although this tends to reduce the impact of home production along other dimensions.

Conclusion

Home production is empirically sizable, and we have suggested that there may be interesting interactions between the home and market sectors. We have shown how to incorporate home production into an otherwise standard real business cycle model. We then calibrated the resulting model. With reasonable parameter values, this model can replicate long-run properties of the U.S. data, including the observed allocation of capital and time to both market and home production. Finally, by simulating the model, we analyzed its business cycle properties. Adding home production to a typical real business cycle model improves its ability to account for the standard features of observed business cycles. There do remain deviations between the theory and data, such as some aspects of the behavior of the two investment series. We have demonstrated how the results depend on households' willingness and incentive to substitute between the home and market sectors and on the functional form of the home technology. There is unfortunately not a lot of independent evidence on the parameters dictating these features of the model, and it seems worthwhile for future research to investigate this in more detail.

¹⁴This is because even if the home technology is nonstochastic, shocks to the market production function obviously still induce relative productivity differentials between the sectors.

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