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How Should Taxes Be Set?

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Are current U.S. budget deficits too high? Is a balanced budget amendment to the U.S. Constitution a good idea? After all, most state governments are required to balance their budgets over a two-year period. Why not insist that the federal budget also be balanced over some suitably short period of time, if not year by year?[1] Should exceptions be made in the event of a war? For instance, would the United States have been better off raising taxes immediately to pay for the Vietnam War instead of financing it by running up the federal debt (as was done)? How should tax policy respond if the government anticipates entering a medium-term war in the near future?

All of these questions have a common feature: They force us to confront the issue of how taxes should be set given the usually fluctuating, and often unpredictable, requirements for government expenditures.[2] The purpose of this article is to explain the principle of tax setting and budget management and use this principle in a specific model to try to answer the above questions. The point of view I will adopt here is that of a benevolent federal government which has the welfare of its citizens as its prime consideration and which properly takes account of the impact of taxes on incentives and welfare. I will also assume that the path of government expenditures over time is given exogenously. That is, the choice of the level and composition of government expenditures is not of concern here.

The general tax-setting principle that I will explain and illustrate is the following: The path of tax rates over time should be such as to produce a constant path of the marginal welfare cost of taxation, that is, to equalize the marginal welfare cost of taxation at different times. The *marginal welfare cost of taxation* is the loss in consumer welfare due to an extra dollar's worth of taxes over and above the extra taxes raised. Such a loss is always incurred when taxes are not lump sum.[3] The above principle emerges as the best way to minimize the present value of the welfare cost of taxation or, equivalently, to maximize consumer welfare.

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[1]State governments maintain separate accounts for current and capital spending and are required to balance the budget on current account only; they are permitted to borrow for capital spending. The federal government does not have separate current and capital accounts.

[2]By *government expenditures* I mean (throughout) net of interest expenditures, that is, government purchases of goods and services plus transfer payments.

[3]A *lump-sum tax* is a tax that is not related to any economic decision. It is a head tax that specifies the total amount of tax to be paid regardless of what the individual does. Those familiar with the literature on market imperfections may be surprised to read that non–lump-sum taxes lead to losses in consumer welfare; for often ad valorem taxes and subsidies are recommended to correct such losses, due to externalities (pollution, for example) and monopolies. My concern here is with raising tax revenues to meet exogenously specified government expenditures and not with using taxes and subsidies to correct for market imperfections. I will, therefore, assume that there are no market imperfections other than those caused by the government's need to raise revenues.

A Preview

Applying the general principle leads, in some instances, to two more specific conclusions.

One is that taxes at any date should depend on the permanent level of government expenditures plus interest on government debt from that date onward. The *permanent level* of government expenditures is defined as that constant level of expenditures (from now till forever) which has the same present value as the actual stream of expenditures (from now till forever). It represents an average value of the stream of current and future government expenditures and is equivalent to an annuity value. This concept is borrowed from Milton Friedman's (1957) concept of *permanent income* which he used to explain the relationship between consumption and income.[4] In particular, this conclusion implies that, even if actual government expenditures fluctuate, taxes should not change as long as the permanent level of government expenditures plus interest on debt does not change. This is analogous to Friedman's theory that consumption is proportional to permanent income.

The other conclusion that arises is that, in some instances, the time path of taxes should fluctuate less (that is, be smoother) than that of government expenditures.[5] This results from the simple fact that the stream of annuity values associated with a given stream of government expenditures will generally fluctuate much less than the expenditure stream. As an example, suppose that government expenditures are alternating between \$100 and \$200 year after year while the interest rate is constant at 10 percent. Then the permanent value of government expenditures will alternate between \$147.62 and \$152.38 year after year.[6] The reasoning behind this conclusion is again similar to that behind Friedman's theory of consumption. Friedman argued that consumers typically prefer to avoid highly fluctuating patterns of consumption (feasts followed by fasts); they want to maintain as smooth a pattern of consumption as possible. Therefore, consumers will typically borrow (or use up savings) when income is low and repay the debt (or replenish savings) when income is high.

This second conclusion implies that it may be entirely appropriate to finance unusually high (that is, higher than average) expenditures by issuing debt (borrowing) rather than raising tax rates and to use the surpluses (savings) in periods of below-average expenditures to retire some of the debt. In this way, high expenditures would be allowed to result in deficits and debt accumulation which would be offset by surpluses and debt retirement in periods of low expenditures so that tax rates could be held steady.

It follows that a constitutional amendment to balance the budget over short horizons may not be such a good idea. This would be the equivalent of a consumer feasting when income is high and fasting when it is low.

The question of whether current deficits are too high cannot be answered definitively since it depends on a judgement regarding future government expenditures. Current government expenditures may or may not be judged as being unusually higher than average future government expenditures. If so, current deficits may not be too high and could be offset by future surpluses when expenditures dip below average. But if not, current deficits may be too high and present tax policy inappropriate. Different readers may be inclined toward different conclusions.

We can push the analogy between the tax-setting problem and the permanent income theory of consumption a little further. Friedman's theory also implies that if consumers experience an unexpected windfall in income (for example, a lottery win), then they will not consume the entire windfall immediately. Instead, they will only consume the annuity value of the windfall and will save the rest for the future. Thus, consumption increases by the amount of the increase in permanent income, but by much less than the actual windfall in income. Similarly, if current government expenditures are much higher than expected (with no change in future government expenditures), tax rates should probably be raised so that revenues increase roughly by

[4] Robert Barro's text (1984, chap. 4, p. 92) provides a simple exposition of the concept. Chaipat Sahasakul (1986) uses this concept for an empirical study of U.S. taxation.

[5] The government budget constraint which will be developed later requires that the discounted present value of tax revenues be sufficient to finance the discounted present value of expenditures plus interest payments on government debt. In this present value sense, the government budget is always balanced. Further, the average level of tax revenues must be equal to the average level of expenditures plus interest on debt. The question of how tax rates should be set is about the appropriate time path of tax rates given the present value or, equivalently, the average level of tax revenues. The pioneering work in this area is by Frank Ramsey (1927).

[6] Readers may verify that a constant stream of expenditures of \$147.62 has the same present value as an alternating stream of \$100 and \$200. Similarly, a constant stream of \$152.38 has the same present value as an alternating stream of \$200 and \$100. If r is the interest rate and g_t is government expenditures in period t, then the formula for the *present value* at time t (PV_t) is

$$PV_t = g_t + [g_{t+1}/(1+r)] + [g_{t+2}/(1+r)^2] + [g_{t+3}/(1+r)^3] + \dots.$$

Then the *permanent value* of government expenditures at time t ($\bar{g}_t$) is

$$\bar{g}_t = [r/(1+r)](PV_t).$$

the amount of the increase in permanent government expenditures, but by much less than the actual increase in government expenditures. The permanent income theory of consumption also implies that if consumers expect a permanent increase in income (that is, an increase in every period) of, say, $1, then permanent income increases $1 and, hence, so does consumption. Similarly, if government expenditures are expected to increase permanently, then tax rates should probably increase, too, so that tax revenues increase by about the same amount.

I will illustrate the general principle of tax setting, as well as the above more specific conclusions, using a relatively simple model of tax determination and debt policy developed by Robert Barro in 1979. First I will describe Barro's model. Then I will explain how and why the model leads to the conclusions outlined above. And then I will note some of the model's limitations and discuss how they qualify its conclusions.

A Simple Model of Tax Setting

The Barro model treats the private sector as consisting of a single, representative, infinitely lived consumer/worker. Models of this type have been found to be very useful to study a variety of issues in economics, including consumption theory (Christiano 1987), business fluctuations (Kydland and Prescott 1982, Prescott 1986), investment theory (Sargent 1986), and long-run growth (Romer 1986). While this type of model seems highly abstract, under certain conditions it can be shown to have properties identical to those of models which seem more realistic, that is, have a variety of consumers constantly being born and dying (Aiyagari 1987).

As for taxation, I will assume that the government cannot levy lump-sum taxes. This certainly seems very realistic. Because of this assumption, taxes will generally be related to the level at which people choose to undertake various economic activities and so will affect their incentives in making consumption and work decisions. Consider the effects of an income tax, for instance. An income tax will generally affect people's incentives to work (since the after-tax income from a second job may not be worth the loss in leisure, for example) and to save (since interest income is also taxed). There can also be more subtle intertemporal effects. If people know, for example, that the tax rate will be much higher next year than it is this year, then they will have a great incentive to increase work this year (and postpone the unpaid vacation to next year) and to decrease saving. The converse will be true if the tax rate is expected to be much lower next year.

To get a feel for the importance of the incentive effects of a non–lump-sum tax, consider the following scenario. Suppose that government expenditures are fluctuating in a regular and predictable way. What would be the effects of raising and lowering the income tax rate in step with expenditures in order to maintain a balanced budget? Clearly, this would create incentives for people to work less and therefore produce less in periods when expenditures are high. We will see that, on average, this leads to a lower level of output and, hence, private consumption (total output less government expenditures less investment). A policy of maintaining the tax rate roughly constant clearly would not create similar incentives to shift work intertemporally and so would lead to a higher average level of private consumption. Thus, the intertemporal incentive effects of fluctuating tax rates can imply a smooth time path of tax rates (relative to government expenditures) as being best from a social point of view.

I now describe the model more fully:

- This economy has one infinitely lived agent who works and produces a good which may be either consumed or stored for future consumption. Work (measured in, say, hours per week) involves an opportunity cost measured in units of foregone consumption, and the agent cares for *net consumption,* that is, consumption net of the opportunity cost of working. Storage of the good yields a constant net return equal to r.
- The path of government expenditures over time is given exogenously, and taxes are proportional to labor income.[7] The government may also issue debt to meet expenditures.
- The individual agent maximizes welfare given by the discounted sum of the utility of net consumption by choosing the allocation of work, net consumption, and saving over time and taking the time path of tax rates as given.
- The government chooses the time path of tax rates to maximize the agent's welfare subject to its own budget constraint, taking account of the effects of changing tax rates on the agent's behavior. The

[7]The assumptions that taxes are proportional (rather than progressive or regressive) and levied only on labor income, not on capital income, simplifies the exposition considerably. Permitting capital taxation leads to some interesting complications which I will touch on later. Note that implicit in my assumption is another, that interest income on government debt is not taxed either.

government also treats the time path of expenditures exogenously.

The Consumer/Worker

Let $C(t)$, $l(t)$ be consumption and work (labor), respectively, in period t, where t takes values $0, 1, 2, \ldots$. Let $H(l)$ denote the opportunity cost of work in units of foregone consumption, and let

$$(1) \qquad c(t) = C(t) - H(l(t))$$

denote net consumption. A fairly typical opportunity cost function, $H(\cdot)$, is shown in Figure 1. The *marginal opportunity cost of work* is defined as the increase in opportunity cost resulting from a one-unit increase in work and corresponds to the slope of the curve $H(\cdot)$ in Figure 1. Assume that both the opportunity cost and the marginal opportunity cost are zero at zero work and that both are increasing as the level of work increases. This means that both the level of work and the marginal unit of work are costless at zero work and both become increasingly unpleasant as work increases. Next assume that each unit of work results in w units of output,[8] and denote by $y(t)$ *net labor income,* that is, labor income net of the opportunity cost of work as well as taxes. This is given by the following, where $\theta(t)$ is the tax rate on labor income in period t:

$$(2) \qquad y(t) = [1-\theta(t)]wl(t) - H(l(t)).$$

Tax revenues are denoted by $T(t)$ and are obviously given by

$$(3) \qquad T(t) = \theta(t)wl(t).$$

Let $W(t)$ be the total wealth of the individual measured in units of consumption at the beginning of period t and consisting of $K(t)$ units of capital and $D(t)$ units of government debt. The individual earns interest on both of these at the constant rate r. We can now write the individual's intertemporal budget constraint this way:

$$(4) \qquad c(t) + [W(t+1)/(1+r)] = y(t) + W(t).$$

Equation (4) says that net labor income plus wealth is either spent on net consumption or accumulated as future wealth. Note that since $W(t+1)$ is wealth at the beginning of period $t+1$ in units of $t+1$ consumption, its value in units of period t consumption is only $W(t+1)/(1+r)$. Now assume that the individual maximizes welfare as of period 0, denoted by $V(0)$, which is given by

$$(5) \qquad V(0) = \textstyle\sum_{t=0}^{\infty}\beta^t U(c(t)).$$

In this expression, $U(\cdot)$ measures the utility derived in period t and depends on period t net consumption,[9] while β is the discount factor assumed to be positive but less than one. This implies that a unit of utility derived tomorrow is less valuable (by the factor β) than a unit of utility derived today; that is, as a consumer, the individual is impatient with regard to the future.

In order to analyze the consumer's welfare maximization problem, it will be convenient to rewrite the budget constraint (4) in present value form. To do this, assume that r is positive and wealth is bounded below; that is, wealth is always greater than some (possibly negative) number. Under these conditions, the consumer's present value budget constraint can be written this way:[10]

$$(6) \qquad \textstyle\sum_{t=0}^{\infty} c(t)/(1+r)^t = \sum_{t=0}^{\infty}[y(t)/(1+r)^t] + W(0).$$

The consumer maximizes $V(0)$ given by (5) subject to (6) by choosing the time paths of net consumption and net income. Note that from equation (2) net income is determined by the choice of work and will depend on the tax rate, $\theta(t)$.

The solution to the problem of choosing the time path of work simply amounts to choosing $l(t)$ in each

[8]For now, take labor productivity to be constant over time in order to focus on the relationship between the time paths of government expenditures and tax rates. Note that with labor productivity fixed, the time paths of total tax revenues and tax rates will be similar. This need not be true when labor productivity also fluctuates over time. I will comment later on the effect this may have on tax setting.

[9]This formulation of preferences is equivalent to one in which utility depends both on consumption, $C(t)$, and on work, $l(t)$, in the following special way: Utility $= U[C(t) - H(l(t))]$. This specification implies that the income effect on work is zero, which simplifies the exposition considerably. I am also implicitly assuming that government purchases of goods and services do not enter the consumer's welfare. This is only a simplification and makes no difference to the subsequent analysis since the path of government purchases is treated as exogenous.

[10]Since wealth consists of capital (which is nonnegative) plus government debt (which may be negative; that is, the agent may be borrowing from the government), the restriction that wealth be bounded below amounts to prohibiting the agent from engaging in Ponzi games in which the agent borrows to finance consumption and keeps borrowing more and more to pay off previous debt without ever redeeming any debt. The present value budget constraint may be obtained by solving equation (4) for $W(0)$ by repeatedly substituting for future values of wealth. I am implicitly assuming that transfer payments from the government are zero; otherwise they would have to be entered on the right side of (4). However, this makes no difference to the subsequent analysis since, again, transfer payments are treated as exogenous.

period to maximize net income, $y(t)$. As is clear from equation (6), this results in the maximum possible present value of net income and, hence, also of welfare, $V(0)$. This happens because any net consumption path that is feasible for the individual for a particular value of total wealth [$W(0)$ plus the present value of net income] is also feasible for a higher value of total wealth. That is, the maximum welfare that the consumer can attain depends only on total wealth and is always increasing with it. Therefore, regardless of the particular form of the utility function, maximizing consumer welfare is equivalent to maximizing the present value of net income.[11]

The choice of $l(t)$ is illustrated in Figure 1. In this figure, the straight line OB with slope w represents the relationship between before-tax income and work; the straight line OA with slope $w[1-\theta(t)]$ represents the relationship between after-tax income and work; and, recall, the curve $H(l)$ represents the opportunity cost of work. The *marginal after-tax income* is defined as the extra income after taxes that the individual gets by working an extra unit, and it corresponds to the slope of OA. For any value of $l(t)$, then, the vertical distance between OB and OA gives the tax revenues and the vertical distance between OA and $H(l)$ gives net income [equations (3) and (2)]. The maximum value of net income occurs when the marginal after-tax income, $w[1-\theta(t)]$, equals the marginal opportunity cost of work, the slope of the curve $H(l)$. In Figure 1, maximum net income occurs at $l(t)$. This completes the description of the consumer's behavior.

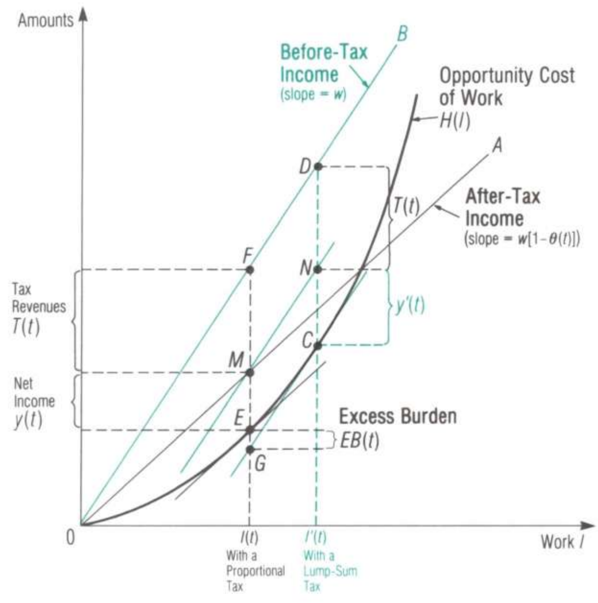
The Welfare Cost of Taxation

Now we can define the concept of *excess burden*, which is a measure of the welfare cost of taxation on the consumer.[12] In Figure 1, the tax rate $\theta(t)$ results in a net income of $y(t)$ and tax revenues of $T(t)$ as shown. If, however, the government could raise the same amount of tax revenues by levying a lump-sum tax, then net income would be higher and so would consumer welfare.[13] This difference in net income when the tax is proportional and when it is lump sum (when both yield the same tax revenues) is said to be the *excess burden* of taxation on the consumer.

Let us show that net income would indeed be higher under a lump-sum tax that raises the same tax revenues as a proportional labor income tax. Under a lump-sum tax, the marginal after-tax income is w since the amount of the tax is independent of work. In Figure 1, the line MN represents after-tax income under a lump-sum tax and has the same slope as OB. As before, the

Figure 1

The Consumer's Choice: How Much to Work



consumer chooses the amount of work so as to equate the marginal opportunity cost of work [the slope of $H(l)$] to the marginal after-tax income, which is equal to the slope of MN under a lump-sum tax. In Figure 1, the choice of work under a lump-sum tax is $l'(t)$. Let $y'(t)$ and $EB(t)$ be the net income and the excess burden, respectively, under a lump-sum tax. As can be seen from the figure, $l'(t)$ is greater than $l(t)$ since the marginal opportunity cost of work is increasing. Further, $y'(t)$ is greater than $y(t)$ since NC equals MG ($MNCG$ is a parallelogram). The difference between

[11] This conclusion depends on the facts that the interest rate r is given by the return on capital independently of tax policy and that the tax rate (or the level of tax revenues) does not enter the utility function $U(\cdot)$. What is critical in generating the latter feature is the fact that the consumer cares for consumption net of the opportunity cost of work; or, equivalently, the income effect on work is zero. If consumption and the opportunity cost of work enter the utility function in some other fashion, this will not be true.

[12] Barro's (1979) analysis was in terms of this concept. See also that of Christophe Chamley (1985).

[13] Then why doesn't the government use lump-sum taxes? One reason, of course, is that we have assumed it can't (except conceptually). More generally, though, this requires a deeper look at taxation than is possible here.

$y(t)$ and $y'(t)$, EG, represents the excess burden. We then have

$$(7)\quad EB(t) = y'(t) - y(t) = [wl'(t) - H(l'(t)) - T(t)] - y(t).$$

The Government

I will now describe the government's budget constraint and maximization problem. Let $g(t)$ be government expenditures in period t, and recall that $D(t)$ is the face value of government debt outstanding at the beginning of period t. Then the period t government budget constraint is given by

$$(8)\quad g(t) + D(t) = T(t) + [D(t+1)/(1+r)].$$

Equation (8) says that the government's expenditures and debt must be paid off by tax revenues and additional borrowing. Note that since $D(t+1)$ is the face value of debt at the beginning of period $t+1$, its market value as of period t is only $D(t+1)/(1+r)$.

Now we can develop the resource constraint for this economy. First, using equation (3), substitute $T(t)$ for $\theta(t)wl(t)$ in (2) and then substitute the resulting expression for $y(t)$ in (4). In the resulting equation, substitute for $c(t)$ from (1) and for $T(t)$ from (8), and then use the fact that $W(t) = K(t) + D(t)$. This yields the following resource constraint:

$$(9)\quad C(t) + g(t) + [K(t+1)/(1+r)] = wl(t) + K(t).$$

Equation (9) can also be interpreted as the national income identity ($C + G + I \equiv Y$). Since $K(t)/(1+r)$ is the capital at the end of period $t-1$, we may interpret $rK(t)/(1+r)$ as the return on capital in period t and $[K(t+1)-K(t)]/(1+r)$ as gross investment, $I(t)$, in period t. By subtracting $K(t)/(1+r)$ from both sides of equation (9), we can see that (9) is equivalent to the national income identity (where gross income Y is the sum of wage income and the return on capital).

Just as we rewrote the consumer's budget constraint in present value form [equation (6)], we can do the same for the government's. Assume that $D(t)$ has some upper bound, so that, just as the individual is, the government is prohibited from running a Ponzi game, or perpetually rolling over debt (note 10). Then rewrite equation (8) by repeatedly substituting for $D(t+1)$ in terms of $D(t+2)$ and so on:

$$(10)\quad \Sigma_{t=0}^{\infty}[g(t)/(1+r)^t] + D(0) = \Sigma_{t=0}^{\infty}T(t)/(1+t)^t.$$

Equation (10) says that the present value of tax revenues must be sufficient to pay for the present value of expenditures plus the debt outstanding at date 0. Another way to write this equation is this:

$$(11)\quad \Sigma_{t=0}^{\infty}\{g(t)+[rD(0)/(1+r)]-T(t)\}/(1+r)^t = 0.$$

The term $rD(0)/(1+r)$ represents interest payments on debt. Therefore, equation (11) says that, in present value terms, the government's budget is always balanced: The present value of deficits (total expenditures + interest on debt − tax revenues) must be zero. Therefore, the government cannot run deficits forever. Indeed, it cannot run surpluses forever, either. Thus, periods of deficits must be followed by periods of surpluses, and in a present value sense they must cancel each other out.

In fact, on average, the government deficit must be zero if the *average* levels of tax revenues and government expenditures are defined as follows:

$$(12)\quad \bar{T} = [r/(1+r)]\ \Sigma_{t=0}^{\infty}T(t)/(1+r)^t$$

$$(13)\quad \bar{g} = [r/(1+r)]\ \Sigma_{t=0}^{\infty}g(t)/(1+r)^t$$

$$(14)\quad \bar{T} = \bar{g} + [rD(0)/(1+r)].$$

Here $\bar{T}$ and $\bar{g}$ are the geometrically weighted averages of the time paths of tax revenues and government expenditures, respectively. The result that the average deficit is zero is a simple consequence of the government budget constraint (11). $\bar{T}$ and $\bar{g}$ are also the *permanent* levels of tax revenues and expenditures, respectively (as defined in note 6). Thus, in this weighted average sense, the budget is always balanced, even though during any period t, tax revenues $T(t)$ may exceed or fall short of expenditures plus interest on debt. The average level of tax revenues is, thus, determined by the average level of government expenditures and the initial level of debt.

The Best Time Path for Taxes

The question that remains to be answered is, What is the best time path of tax revenues given the average value? Should tax revenues be fairly smooth over time, staying close to their average value? Or should they vary highly, up and down, perhaps in step with government expenditures?

Assume again, that the government chooses the time path of tax rates, $\theta(t)$, subject to its budget constraint and taking account of its effect on the consumer's behavior in such a way as to maximize the consumer's welfare, $V(0)$. As we have seen, this is equivalent to maximizing the present value of net income. It is also equivalent to minimizing the present value of the excess burden, $EB(t)$, which, recall, measures the welfare cost of taxation on the consumer. To see this, rearrange equation (7) and rewrite it in present value terms:

$$(15) \quad \Sigma_{t=0}^{\infty} y(t)/(1+r)^t = \Sigma_{t=0}^{\infty}[wl'(t) - H(l'(t))]/(1+r)^t - \Sigma_{t=0}^{\infty} T(t)/(1+r)^t - \Sigma_{t=0}^{\infty} EB(t)/(1+r)^t.$$

Note that the choice of $l'(t)$ is, in fact, independent of tax revenues. [It depends only on labor productivity, w, and the opportunity cost function $H(\cdot)$.] Further, the present value of tax revenues is independent of the time path of tax revenues [from the government budget constraint (10)]. So maximizing the present value of net income by choosing the time path of tax rates is equivalent to minimizing the present value of the excess burden. Thus, the government's problem can be stated as one of minimizing the present value of the consumer's welfare cost of taxation that results because taxes are proportional rather than lump sum.

To analyze this problem, it is convenient to let the government choose tax revenues $T(t)$ directly and the tax rate $\theta(t)$ indirectly, since the government's budget constraint is written in terms of $T(t)$. That is, the government picks the level of tax revenues and lets the tax rate be whatever it must be to raise that level. Now we obviously need to determine the relationship between tax revenues $T(t)$, the tax rate $\theta(t)$, and net income $y(t)$. This can be obtained from Figure 1 by varying the tax rate $\theta(t)$ between zero and one, determining the consumer's choice of work at each tax rate, and using those values to calculate the amount of tax revenues and the net income.

The result of this exercise is shown in Figure 2. The relationship between the tax rate $\theta(t)$ and tax revenues $T(t)$ has an inverted U-shape because, when the tax rate is zero, tax revenues will also be zero and, when the tax rate is one, the consumer will choose zero work, so tax revenues will again be zero.[14] Net income $y(t)$, however, will always decrease as the tax rate rises. We may as well restrict attention to tax rates below θ^*, the rate that produces the maximum amount of tax revenues, because any revenue that can be raised by a rate above θ^* can also be raised by a rate below it and leave the consumer with more income. Therefore, the government will never choose a tax rate above θ^*.

Given this restriction, we see that (not surprisingly) there is a one-to-one, increasing relationship between tax revenues and the tax rate and a one-to-one, decreasing relationship between tax revenues and net income. Therefore, we may as well let the government specify tax revenues $T(t)$ and then calculate the corresponding value of net income $y(t)$ from Figure 2. This relationship is illustrated in Figure 3.

Figure 3 also shows the relationship between tax revenues $T(t)$ and the consumer's excess burden $EB(t)$. From equation (7), we can see that, since $l'(t)$ is independent of tax revenues, $y'(t)$ decreases one-to-one with $T(t)$. When tax revenues are zero, $y'(t)$ equals $y(t)$ because the tax rate is also zero. Therefore, $y'(t)$ always lies above $y(t)$, and the vertical distance between the two at a given level of tax revenues measures the excess burden. $EB(t)$ is always increasing in tax revenues because when tax revenues go up by \$1, net income falls by at least that much. The slope of $EB(t)$, then, is the *marginal excess burden,* or the increase in the excess burden on the consumer due to a \$1 increase in tax revenues. Under reasonable assumptions, we can show that the marginal excess burden is always increasing in tax revenues, is zero at zero tax revenues, and is infinitely high at the value T^*, which is the maximum possible tax revenues. The shape of the $EB(t)$ curve in Figure 3 reflects this description.

Now the government chooses the time path of tax revenues $T(t)$ to minimize

$$(16) \quad \Sigma_{t=0}^{\infty} EB(T(t))/(1+r)^t$$

subject to the constraint

$$(17) \quad \Sigma_{t=0}^{\infty} T(t)/(1+r)^t = [\bar{g}(1+r)/r] + D(0).$$

The expression in (16) explicitly shows the dependence of the excess burden on tax revenues that we saw in Figure 3. Equation (17) is simply equation (14) rewritten using (12). Since the time path of expenditures $g(t)$ is given and the amount of government debt outstanding in period 0 is also given (by past budget policies), the right side of (17) is independent of tax rates. This emphasizes one of the conclusions we noted earlier, that taxes should depend on the permanent level

[14]This is the famous *Laffer curve* relationship between the tax rate and tax revenues.

Figures 2 and 3

The Government's Choices:

Figure 2 **How Much to Tax . . .**

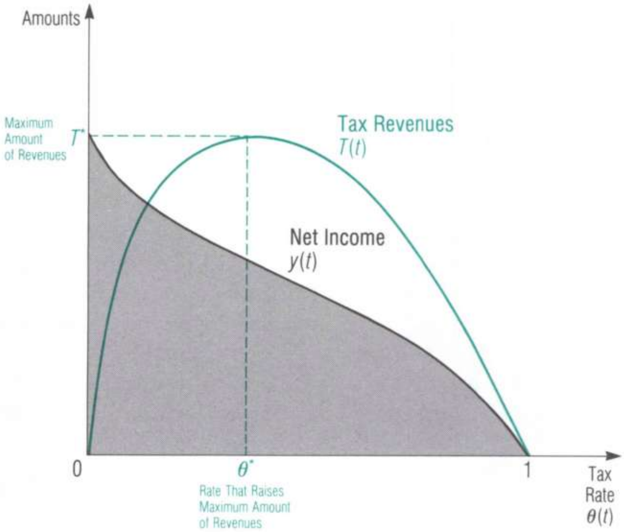
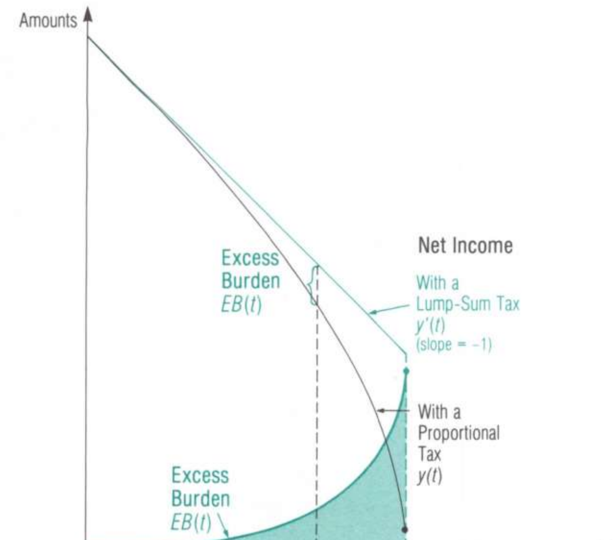


Figure 3 **. . . And Burden the Consumer**



of government expenditures plus interest payments on government debt.

The solution to problem (16)–(17) is quite simple: Keep tax revenues constant at the level $\bar{T}$ forever. From (14), this means that tax rates should be such as to generate revenues which equal the permanent level of government expenditures plus the interest on government debt.

The explanation for this remarkable conclusion is also fairly simple. First we must see that the marginal excess burden must be the same in any two successive periods. For suppose the contrary, that the marginal excess burden in period t (say, 2 units) is greater than that in period $t+1$ (say, 1 unit). If the government reduces tax revenues in period t by 1 unit and increases them by $1+r$ units in period $t+1$, then the government budget constraint (17) will still be satisfied. But the excess burden will go down by 2 units in period t and will go up by $1+r$ units in period $t+1$. Hence, the present value of the excess burden will go down by 1 unit. A similar argument can be made if the marginal excess burden in period t is less than that in period $t+1$. This proves that, unless the marginal excess burden is the same in every period, the present value of the excess burden cannot be at its lowest possible value.

The conclusion that tax revenues $T(t)$ must be the same in every period follows because, recall, the slope of the $EB(t)$ curve in Figure 3 is, by assumption, always increasing in tax revenues. If tax revenues differ in any two periods, therefore, the marginal excess burden cannot be the same in those two periods. From (12), if tax revenues are constant over time, then that constant level must be $\bar{T}$. It also follows that the tax rate must be the same in every period. This is because, from Figure 2, the relationship between tax rates and tax revenues is fixed over time.[15]

Applying the Model

Now I will describe the implications of the above analysis for some of my opening questions.

A Balanced Budget Amendment

First, is a balanced budget amendment to the U.S. Constitution a good idea? The Barro model suggests that the answer is no. According to this model, tax rates

[15] I should emphasize here that the conclusion that tax revenues must be the same in every period depends on the assumption that labor productivity, w, is constant over time. If w is, instead, changing over time, then the relationship between $y(t)$ and $T(t)$ shown in Figure 3 will also be changing over time, as will the relationship between $EB(t)$ and $T(t)$. Thus, a constant path of the marginal excess burden will lead to a fluctuating path of tax revenues over time. In general, the tax rate will also be changing over time.

should be determined by the permanent level of government expenditures and should be chosen so as to equalize the marginal welfare cost of taxation over time. Only by coincidence would this imply that tax revenues equal government expenditures plus interest on debt in every period. Therefore, it would not be a good idea to raise and lower tax rates in step with expenditures so as to maintain a balanced budget. If that were done, the incentive to vary work over time in response to the changing tax rates would lower consumer welfare by raising the present value of the excess burden of taxation. Therefore, it is better to let the deficit rise in periods of above-average expenditures, by issuing more government debt, and pay off the debt with surpluses in periods of below-average expenditures.

Responses to Spending Increases

How should tax policy respond if the government faces increases in expenditures, such as those required for a war or, perhaps, for cleanup of toxic nuclear wastes? To simplify this discussion, I will assume that initially the level of government expenditures is constant and that tax rates are chosen as Barro's model says they should be. Initially, therefore, expenditures plus interest on debt will also be constant and equal to tax revenues, and the budget will have been in balance in every period. Let's consider several alternative types of spending increases.

□ *An Immediate Permanent Increase*

Suppose that, starting in period 0, the level of government expenditures increases permanently and uniformly by, say, one unit. From (13), we can see at once that permanent expenditures also increase by one unit. Therefore, tax revenues should be raised immediately and permanently by one unit. Note that this conclusion is independent of the assumption that the initial path of expenditures was constant and that, initially, the budget was always in balance. Therefore, what remains unaffected under such a tax response is the time path of the deficit.

□ *An Immediate Temporary Increase*

Now suppose that, starting in period 0, the level of expenditures increases temporarily (for a certain number of periods) by, say, one unit. From (13) again, we can see that permanent expenditures increase by less than one unit, and so tax revenues should increase immediately and permanently by the amount of the rise in permanent expenditures. This implies that the government will be running deficits during the time that expenditures are higher than usual and, hence, more and more debt will be issued. Once expenditures return to normal, tax revenues will exceed expenditures by just enough to meet interest payments on the higher level of debt. That is, the budget will be in balance and stay that way.

□ *An Expected Temporary Increase*

Finally, suppose that, starting in some future period, expenditures are expected to increase for a certain number of periods before returning to the initial level. How should tax policy respond?

A balanced budget rule says to do nothing until the periods in which expenditures actually increase and then to raise tax rates by the required amount to keep the budget in balance. Barro's model recommends a different policy. This is to raise tax rates immediately to a higher constant level to match the increase in permanent expenditures, thereby running budget surpluses until the period in which expenditures actually increase. The surpluses should be used to make loans to the consumer and build up credit. Then the interest income from these loans plus tax revenues should be used to partly offset the higher level of expenditures, the rest being made up by issuing more debt. When expenditures return to normal, the level of debt will be higher than initially, but the budget will be in balance and stay that way. This happens because tax revenues go up uniformly in all periods and, hence, will be higher than the initial level of expenditures plus interest.

As you can see, the appropriate pattern of tax response to expected government spending increases is pretty much the same regardless of how large the increase is or how long it is expected to last or how soon it occurs. What differs is the magnitude of the immediate (and permanent) increase in tax revenues. This happens simply because the magnitude of the rise in permanent expenditures increases with the magnitude of the rise in expenditures, the number of periods for which it lasts, and the proximity of the rise in expenditures. This is a simple result of the definition of *permanent expenditures* in (13). However, note that the magnitude of the increase in tax revenues is always less than that of an expected increase in expenditures.

Qualifying the Conclusions

I have described a very simple model of tax determination and debt management and obtained some interesting conclusions. Now I will highlight my simplifying assumptions and consider whether the model's results are affected if these assumptions are relaxed.

Robustness of Result

My principal conclusion is that tax rates should depend on permanent government expenditures plus interest on debt. In particular, tax rates and tax revenues should move with current government expenditures only to the extent that such movements imply changes in permanent government expenditures. The particular model that I used to illustrate these ideas led to a much stronger result—that tax rates and tax revenues should be constant over time. However, this result depends on a lot of special features in this model and will not necessarily hold for all others. These special features include, for example, the assumptions that labor productivity is constant, that utility depends only on the difference between consumption and the opportunity cost of work, and that the rate of return on capital is independent of the amount of work.

Commitment and Time Consistency

The tax-setting problem has been formulated here as one in which the government chooses and announces at date 0 the entire infinite sequence of tax rates $\{\theta(t), t \geq 0\}$. But what if, once period 0 passes and period 1 arrives, the government does not remain committed to the announced time path of tax rates? What if it is allowed to choose a different time path from period 1 onward $\{\theta'(t), t \geq 1\}$? Would the model's conclusions be different? If the government is not committed to follow through with whatever tax rates it announces for future periods, how is the consumer supposed to form beliefs about future tax rates?

Whenever the best choice of tax rates made in period $t+1$ for period $t+1$ and beyond differs from the best choice made in period t for period $t+1$ and beyond, there is said to be a problem of *time consistency*. Discussing this problem is beyond the scope of this article. Interested readers can consult papers by V. V. Chari (1988) and Chari, Patrick Kehoe, and Edward Prescott (1988).

A lack of commitment by the government also leads to other problems. For instance, the government may default on its debt, having promised not to. It may promise not to tax capital, thereby encouraging saving and capital formation, and then levy a tax. Similarly, it may promise to pursue a low inflation policy, encouraging people to accumulate nominally denominated assets, and then engineer an inflation to tax away the real value of nominal assets. Chari and Kehoe (1988a,b) have analyzed the first two problems. In Aiyagari 1989, I discuss these problems, at an elementary level, in more detail than I do here.

In Conclusion

In this article, I have considered the issue of how best to choose the time paths of tax rates and revenues and, hence, the time paths of government deficits and debt as well. This study was motivated by several real-world questions. To study this issue, I have presented a simple model of tax determination which was first analyzed by Barro in 1979. This model illustrates the general principle that tax rates should be chosen to equalize the marginal welfare cost of taxation on the consumer across time periods. The model also suggests that tax rates and revenues should depend on the permanent level of government expenditures plus interest on debt, rather than fluctuating with the current level of government expenditures, and that the paths of tax rates and revenues over time should be smoother than that of government expenditures.

While the Barro model is conceptually useful, the applicability of its specific conclusions is subject to several qualifications. The development and analysis of more sophisticated models capable of studying how taxes should be set is today an active area of research in macroeconomics and public finance.

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