A Currency Premium Puzzle*

Tarek Hassan,[†] Thomas M. Mertens,[‡] and Jingye Wang[§]

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Abstract

Canonical long-run risk and habit models reconcile high equity premia with smooth risk-free rates by inducing an inverse functional relationship between the variance and the mean of the stochastic discount factor. We show this highly successful resolution to closed-economy asset pricing puzzles is fundamentally problematic when applied to open economies with complete markets: It requires that differences in currency returns arise almost exclusively from predictable appreciations, not from interest rate differentials. In the data, by contrast, exchange rates are largely unpredictable and currency returns differ because interest rates differ widely across currencies. We show that no complete-markets model with canonical long-run risk and habit preferences can match this fact. We argue this tension between canonical asset pricing and international macroeconomic models is a key reason why researchers have struggled to reconcile the observed behavior of exchange rates, interest rates, and capital flows across countries. The lack of such a unifying model is a major impediment to understanding the effect of risk premia on international markets.

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†**Boston University**, 270 Bay State Rd, Boston, MA 02215; thassan@bu.edu.

‡**Federal Reserve Bank of San Francisco**, 101 Market Street, Mailstop 1130, San Francisco, CA 94105; Thomas.Mertens@sf.frb.org.

§**Renmin University of China**, 59 Zhongguancun Road, Haidian District, Beijing 100872, China; jingyewang@ruc.edu.cn.

1 Introduction

Recent years have seen a surge in interest in the role of risk premia in shaping international capital markets. A wealth of studies has examined the ramifications of shocks to both global and country-specific risk premia, investigating their impact on exchange rates, interest rates, capital flows, and financial stability. This research addresses critical phenomena such as the violation of uncovered interest parity (di Giovanni et al., 2022), contagion (Forbes, 2013), the global financial cycle (Miranda-Agrippino and Rey, 2020),[1] and events like flight to safety, capital retrenchments (Forbes and Warnock, 2021; Chari, Dilts Stedman, and Lundblad, 2020), and sudden stops (Mendoza, 2010), which are of paramount concern for policymakers.

Despite the intense interest in these topics, it has proven difficult to construct quantitative models that can reconcile the observed behavior of exchange rates with large and persistent differences in interest rates across countries. For example, to our knowledge, no quantitative model has been able to reproduce the fact that New Zealand on average has had a risk-free interest rate five percentage points above that of Japan for multiple decades, or that the cost of borrowing in the Euro Area is persistently lower than that in Canada. The lack of such a unifying model is a major impediment to understanding the effect of risk premia on the allocation of capital across countries.

In this paper, we show these challenges are rooted in a fundamental tension between the canonical asset pricing models that have been highly successful in addressing closed-economy asset pricing puzzles (Bansal and Yaron, 2004; Campbell and Cochrane, 1999), and the empirically observed behavior of exchange rates: strong forces in these models require that any large differences in risk premia across countries manifest themselves not as persistent differences in interest rates, but as highly predictable exchange rates. In the data, we see large and persistent differences in interest rates across countries and exchange rates that are notoriously hard to predict (Meese and Rogoff, 1983), putting this class of model at odds with the data. We term this tension a 'currency premium puzzle' and argue it is a fundamental reason the literature has struggled to construct a quantitative model that can match both asset prices and macroeconomic quantities across countries.

The classical quantitative challenge in asset pricing is to reconcile a high equity premium (Mehra and Prescott, 1985) with a low and stable risk-free rate (Weil, 1989). The former requires the stochastic discount factor (SDF) to be highly volatile, while the latter requires the sum of the mean and variance of the log SDF to be low and stable. Canonical long-run risk and habit models resolve these puzzles jointly by introducing

[1] Also see Rey (2015), Bai et al. (2023), Morelli, Ottonello, and Perez (2022), for example.

a negative, functional relationship between the mean and the variance of the log SDF, so that whatever increases the SDF's variance also decreases its mean. This negative functional relationship keeps the risk-free rate low and stable while allowing for a large equity premium.

This "trick" – hard-wiring a functional relationship between the mean and the variance of the log SDF – has been highly successful in closed economy settings, allowing the construction of representative agent models that match both asset prices and macroeconomic quantities, such as aggregate investment and consumption. *We argue the same trick is also the fundamental reason why these models fail in an open economy setting.*

Specifically, in an open economy with complete markets and log-normal SDFs, the means of log SDFs govern expected changes in exchange rates, while the variances of the log SDFs govern the expected return on the currency (Backus, Foresi, and Telmer, 2001). Data on exchange rates and currency premia thus put bounds on how these moments should differ across countries. In particular, a large literature documents that (1) exchange rates are largely unpredictable, which requires the means of log SDFs to be similar across countries ; and (2) currency premia are quantitatively large, which requires the variances of log SDFs to differ significantly across countries (Hassan and Mano, 2019). Taken together, this means that countries' log SDFs should differ widely in their variances, but not in their means. This restriction from the international data, however, is impossible to fit with canonical long-run risk and habit preferences, which hard-wire an almost one-for-one negative functional relationship between the two moments.

In this sense, large currency risk premia pose a fundamentally different quantitative challenge to these models than the classical closed-economy asset pricing puzzles; and this challenge has not been widely recognized in the literature.

To illustrate this point, we consider a conventional long-run risk model with complete markets, two representative agents (one in the home country and one in the foreign country), and Epstein-Zin preferences with parameters equal to those in Bansal and Shaliastovich (2013). In this model, 94% of *any* difference in currency returns between the two countries must result from a predictable depreciation of the exchange rate, with the remaining 6% coming from interest rate differentials. Because of the hard-wired functional relationship between the mean and variance of the log SDFs, this 94:6 split is fully determined by the parameters of the Epstein-Zin utility function and independent of the features of the economic environment.

In other words, we demonstrate a theorem showing that no complete-markets model with these canonical preferences (risk aversion around 10 and elasticity of intertemporal substitution around one) can fit the international data.

The same holds for canonical habit preferences (Campbell and Cochrane, 1999), where again the link between the mean and variance of the log SDF forces the vast majority of cross-country differences in currency returns to manifest themselves as predictable depreciations, not persistent interest rate differentials.

This finding is all the more problematic because leading theories of why risk premia and interest rates might differ persistently across countries typically point to differences in the economic environment, such as differences in country size (Martin, 2011; Hassan, 2013), their role in global trade (Richmond, 2019), resource endowments (Ready, Roussanov, and Ward, 2017), their level of indebtedness (Wiriadinata, 2021), or the volatility of shocks (Menkhoff et al., 2012; Colacito et al., 2018a). All of these features of the environment are irrelevant for the results above: With complete markets, canonical long-run risk and habit preferences force all of these features of the real economy to manifest as large predictable depreciations, and not persistent interest rate differentials.

To corroborate our theoretical results, we simulate seven widely used versions of long-run risk and habit models, and show the currency premium puzzle manifests in each of them: Specifically, for long-run risk models, we consider a heterogeneous country general equilibrium model (Colacito et al., 2018a), a symmetric country general equilibrium model with production (Colacito et al., 2018b), a symmetric country partial equilibrium model with stochastic volatility (Bansal and Shaliastovich, 2013), and a symmetric country general equilibrium model with endowments (Colacito and Croce, 2013). For habit models, we consider a discrete-time habit model (Verdelhan, 2010), a continuous-time habit model (Stathopoulos, 2017), and a deep habit model (Heyerdahl-Larsen, 2014). We obtain moments on exchange rates and currency premia numerically and show that their contradictions with the data are consistent with our theoretical predictions.

We show the same issue also arises in models that combine disaster risk with Epstein and Zin (1989) preferences (Gourio, 2012; Gourio, Siemer, and Verdelhan, 2013), and in other canonical applications of the rare disaster paradigm (Farhi and Gabaix, 2016).

In sum, the currency premium puzzle we highlight in this paper applies to the most advanced quantitative models that have attempted a synthesis between international asset prices and quantities.

After establishing these core results of the paper, we explore a number of avenues through which the currency premium puzzle might be addressed. Our analytical results show that stochastic volatility as in Bansal and Yaron (2004) and departures from log-normality do not substantially alleviate the problem. At a deep level, any complete-markets model in which representative agents in two countries have identical habit or Epstein-Zin preferences with a preference for early resolution of uncertainty appears to feature the currency premium puzzle.

Of course, this leaves open the possibility that frictions induced by market incompleteness may take a form that solves the tension between large interest rate differentials and unpredictable exchange rates. Because the vast majority of the literature in this area relies on the complete markets assumption, it is difficult to make an assessment to what extent this might be possible in general. However, we are able to study one specific type of incompleteness that arises under incomplete spanning (Lustig and Verdelhan, 2019), when the representative agent in one country cannot trade all assets available in other countries.

Here again we find no easy resolution to the currency premium puzzle: Although incomplete spanning loosens the relationship between marginal utility and exchange rates, it does so in a limited way. In particular, we show that every percentage point incomplete spanning deducts from exchange rate predictability is also eliminated from that currency's expected return. That is, while it is conceivably possible to construct a pattern of incomplete spanning that eliminates exchange rate predictability from the models mentioned above, that same friction will then again compress cross-sectional differences in currency returns towards zero, and thus to levels similar to those one would obtain with conventional (constant relative risk aversion) preferences. As a result, incomplete spanning may help in eliminating exchange rate predictability, but those models would then again fail to generate the large differences in interest rates and currency returns we see in the data.[2]

Of course, the possibility remains that the patterns we observe in the data can be rationalized with systematic differences in preferences across countries. However, in the absence of direct evidence of such differences—such as systematic demonstrations of higher risk aversion among households in New Zealand versus Japan, or in Canada relative to the EU—we are unable to evaluate this possibility. Ultimately, any observed economic behavior can be explained by appealing to sufficiently heterogeneous preferences, which is why economists are traditionally skeptical of purely preference-based explanations, favoring instead interpretations grounded in empirical evidence.

Our paper contributes to a large and growing literature at the intersection of international macroeconomics and asset pricing. One important strand of this literature studies in reduced-form the effects of time variation in global and country-specific risk premia, where shocks to global and local risk-bearing capacity motivate variation in global and local borrowing costs, retrenchments, capital flight, and the global financial cycle

[2]This result dove-tails with several recent papers (Lustig and Verdelhan, 2019; Jiang, 2023; Jiang et al., 2022; Jiang, Krishnamurthy, and Lustig, 2023; Sandulescu, Trojani, and Vedolin, 2021; Chernov, Haddad, and Itskhoki, 2023), which have consistently found that various forms of incomplete markets do not necessarily help in resolving many of the well-known international financial puzzles, including the volatility puzzle of Brandt, Cochrane, and Santa-Clara (2006), the Backus and Smith (1993) puzzle, and the exchange rate disconnect puzzle (Meese and Rogoff, 1983).

(Mendoza, 2010; Forbes, 2013; Rey, 2015; Miranda-Agrippino and Rey, 2020; Chari, Dilts Stedman, and Lundblad, 2020; di Giovanni et al., 2022; Forbes and Warnock, 2021; Morelli, Ottonello, and Perez, 2022; Bai et al., 2023; Colacito et al., 2018b).

Several important papers have attempted to microfound such time variation in global risk premia by combining conventional international macro models with long-run risk, habits, and other advances from the asset pricing literature (e.g., Colacito and Croce, 2011, 2013; Colacito et al., 2018a; Verdelhan, 2010; Heyerdahl-Larsen, 2014; Stathopoulos, 2017; Gourio, Siemer, and Verdelhan, 2013). While most of these models are designed to address a variety of quantitative puzzles, ranging from the forward premium puzzle to exchange rate disconnect, none of them are able to match the empirical fact that currency premia are large while exchange rates are largely unpredictable. We contribute to this literature by highlighting this tension as a major obstacle to its further development.

Our paper is also tightly linked to the classic approaches to resolving the equity premium and risk-free rate puzzles. In particular, we show that models that are highly successful in resolving these well-known puzzles (Campbell and Cochrane, 1999; Bansal and Yaron, 2004), face new challenges when confronted with international data.

Our paper also speaks to the growing international macroeconomics literature that has documented large and persistent differences in currency risk, currency returns, and interest rates across countries (Lustig and Verdelhan, 2007; Lustig, Roussanov, and Verdelhan, 2011; Hassan and Mano, 2019).[3] Various papers have related these persistent differences in currency risk to features of the economic environment, such as differences in country size (Hassan, 2013; Martin, 2011), trade centrality (Richmond, 2019), commodity trade (Ready, Roussanov, and Ward, 2017), indebtedness (Wiriadinata, 2021), and others. We contribute to this literature by showing that all of these forces, when evaluated through the lens of a quantitative model with long-run risk or habit preferences will result in large predicted depreciations, but not the large interest rate differentials we observe in the data.

Lastly, our paper also relates to a growing body of papers that highlights the importance of incompleteness of international asset markets. Since the seminal work of Backus, Foresi, and Telmer (2001), complete markets have been a popular assumption in the international finance literature for its simplicity. But recent works by Sandulescu, Trojani, and Vedolin (2021), Lustig and Verdelhan (2019), Jiang (2023), Jiang et al. (2022), Jiang, Krishnamurthy, and Lustig (2023), and Chernov, Haddad, and Itskhoki (2023), to highlight a few prominent

[3]See Hassan and Zhang (2021) for a survey of this literature.

contributions, have established in a reduced form way that market incompleteness, or even segmentations and Euler equation wedges, help in matching salient features of currency markets. We show that incomplete spanning is also subject to the currency premium puzzle. While other forms of market incompleteness might help in resolving the puzzle, we are not aware of any work that has successfully done so.

The remainder of this paper is organized as follows. Section 2 lays down the basic concepts and framework for analysis. Section 3 discusses the long-run risk model and Section 4 discusses the habit model. Section 5 and Section 6 expands the argument by allowing for departures from log-normal distributions, and incomplete spanning. Section 7 concludes.

2 Basic framework and data

This section connects the closed and open economy asset pricing frameworks and introduces a simple graphical representation of the currency premium puzzle. We show how data on exchange rates and currency returns put restrictions on the choice of stochastic discount factors across countries. For simplicity, we first focus the discussion on log-normally distributed stochastic discount factors and discuss more general distributions in Sections 5 and 6.

2.1 Closed Economy Asset Pricing Puzzles

Our starting point is a complete-markets setup where we assume a log-normal distribution for the stochastic discount factor M_{t+1}. Absence of arbitrage requires

$$\mathbb{E}_t(M_{t+1}R_{t+1}) = 1,$$

where $\mathbb{E}_t$ denotes the mathematical expectation conditional on the information set at time t and R_{t+1} the return on an arbitrary asset. Applied to the risk-free rate with log return $r_{f,t}$, this equation relates the mean and variance of the stochastic discount factor via

$$\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) - r_{f,t}. \tag{1}$$

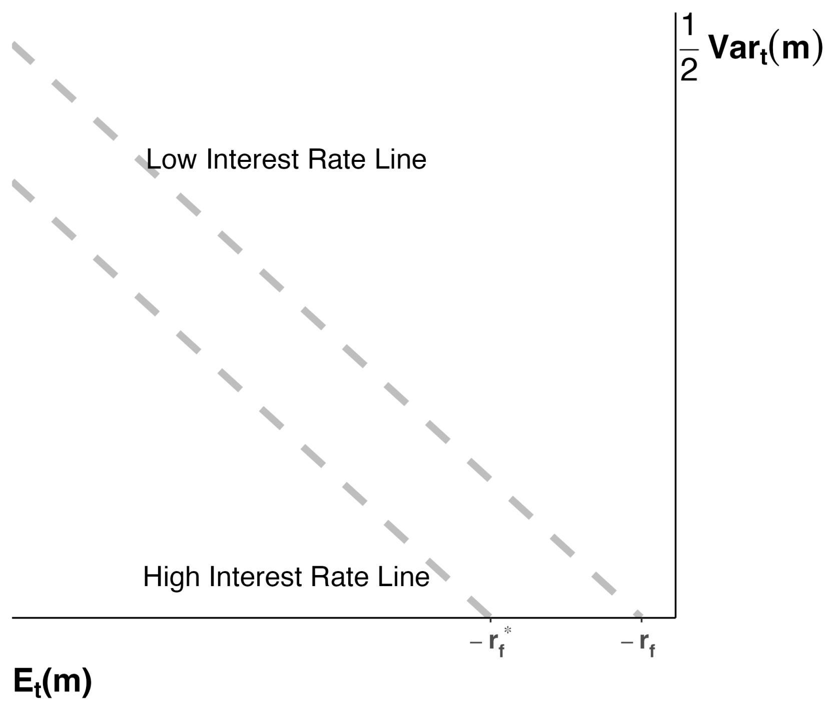
Throughout the paper, lowercase letters denote the logarithm of a variable such that $x = \log(X)$.

Resolving the equity premium puzzle of Mehra and Prescott (1985) requires the variance of the logarithm of the stochastic discount factor (log SDF) $\mathrm{var}_t(m_{t+1})$ to be large (Hansen and Jagannathan, 1991). Equation (1) shows that this restriction has an immediate implication for the risk-free rate. In the data, risk-free interest rates tend to be low and stable, as is known from the risk-free rate puzzle of Weil (1989).

The joint resolution of the risk-free rate and equity premium puzzles thus requires a mean of the log SDF that moves to accommodates a high, and potentially changing, variance of the SDF to maintain low and stable risk-free rates. As we show in later sections, standard resolutions of the puzzles such as long-run risks and habit utility achieve this feature by imposing a negative functional relationship between the two moments of the log SDF.

Figure 1 represents equation (1) graphically. It plots the (conditional) mean of the log SDF on the horizontal axis and the scaled conditional variance on the vertical axis. Each point in this "SDF space" determines a log stochastic discount factor represented by its first two moments. By equation (1), the first two moments of each stochastic discount factor pin down the risk-free interest rate. In fact, equation (1) implies that any stochastic discount factor that lies on the same negatively-sloping 45-degree line produces the same risk-free rate. We refer to these lines as “iso-risk-free rate (or iso-rf) lines".

Figure 1: The SDF Space and Iso-rf Lines



This figure plots two iso-rf lines (grey dashed line) in the SDF space. The lower line represents a higher risk-free rate. The intercept of each line with the x-axis represents the negative of the corresponding risk-free rate.

Figure 1 shows two examples of iso-rf lines. The intercept of the line with the x-axis represents the (negative of the) risk-free rate, and lower lines represent higher risk-free rates. We thus have a graphical representation of (1) for different risk-free rates. The equity premium puzzle and the risk-free rate puzzle require a high variance with $(\mathbb{E}_t(m_{t+1}), \frac{1}{2}\,\mathrm{var}_t(m_{t+1}))$ pairs on or around a particular iso-rf line to maintain a stable risk-free rate.

2.2 Exchange Rates and the SDF

Next, we show that the logic of the previous section puts additional restrictions on the log SDFs when applied to open economies. Under complete markets, Backus, Foresi, and Telmer (2001) show that the expected change in exchange rates, quoted as units of foreign currency per home currency, is given by[4]

$$\mathbb{E}(\Delta s_{t+1}) = \mathbb{E}(m_{t+1}) - \mathbb{E}(m^\star_{t+1}), \tag{2}$$

where Δs_{t+1} is the change in log exchange rate (depreciation rate of the foreign currency) and we use $\star$ to denote the foreign country.

From equation (3), the difference in risk-free rates between the two countries is given by

$$\begin{aligned}\mathbb{E}(r^\star_t - r_t) = {} & \mathbb{E}(m_{t+1}) - \mathbb{E}(m^\star_{t+1}) \\ & - \frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(m^\star_{t+1}) - \mathrm{var}_t(m_{t+1})).\end{aligned} \tag{3}$$

Analogous to the equity premium, which is defined as the unconditional difference between returns on equity and the risk-free rate, we define the currency premium as the expected return a home investor would earn if she invests in a foreign bond versus a home bond.[5] She earns the difference in interest rates net of the depreciation rate of the foreign currency. Using (2) and (3), we obtain the currency premium as

$$\mathbb{E}(rx_{t+1}) = \mathbb{E}(r^\star_t - r_t) - \mathbb{E}(\Delta s_{t+1}) = \frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(m_{t+1}) - \mathrm{var}_t(m^\star_{t+1})). \tag{4}$$

where rx_{t+1} denotes the currency return earned between t and $t+1$. For ease of exposition, we assume that the home country has a lower risk-free rate and that the investor is short the home country and long the

[4]All equations in this subsection also hold conditionally. We show unconditional equations here because we can only estimate the unconditional moments given data on exchange rates; and we emphasize the cross-country variations of unconditional moments.

[5]In the literature, this return is sometimes referred to as "currency risk premia" or simply "currency returns". We call it currency premia in analogy to equity premium.

foreign country unless specified otherwise.

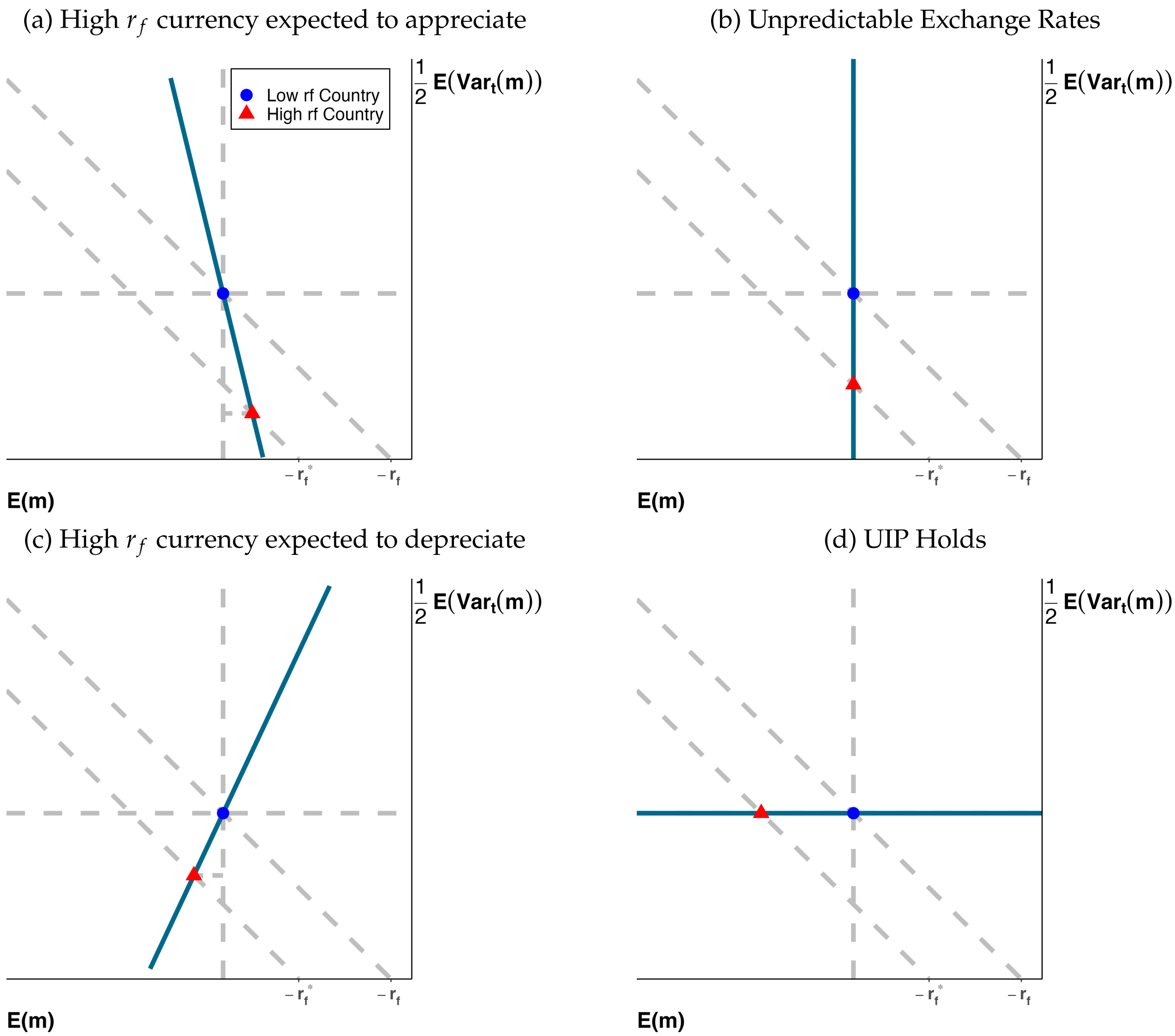
Note that we can decompose currency premia into an interest rate differential and an expected change in the exchange rate. The relative importance of these two terms is key to our analysis. To facilitate later discussion, we define the share of the expected change in the exchange rate in the currency premium, $\frac{-\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})}$, and label it "FX-share". The FX-share measures the fraction of the currency premium that comes from predicted changes in the exchange rate, rather than the difference in interest rates. In particular, a negative FX-share means investors lose money from expected depreciations, i.e., the high interest rate currency depreciates on average.

Equations (2) and (4) show that exchange rates and currency premia are tightly linked to means and variances of log SDFs. Each country's stochastic discount factor is characterized by one mean-variance pair $(\mathbb{E}(m), \frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})))$, which we represent as a point in the (unconditional) SDF space. Data on exchange rates and currency premia between any two countries provide information on the relative position of their corresponding points. In particular, expected depreciations of the exchange rate determine the horizontal differences (equation (2)) and currency premia govern vertical differences (equation (3)).

Figure 2 visualizes properties of exchange rates and currency premia in SDF space under various configurations. Panel (a) shows the high-interest rate currency depreciating on average relative to the low-interest currency. The high-interest rate foreign currency (red triangle) lies on a lower iso-rf line and is expected to appreciate ($\mathbb{E}_t(m_{t+1}) < \mathbb{E}_t(m^\star_{t+1})$) relative to the low-interest rate home currency (blue dot). The blue line connecting the two dots has a negative slope. In Panel (b), exchange rates are unpredictable, i.e., $\mathbb{E}(\Delta s) = 0$ and the means of the log SDFs are consequently equalized across countries, so that the blue line is vertical. Panel (c) shows the situation where the high-interest rate currency is expected to depreciate – the blue line has a positive slope. If uncovered interest rate parity holds, interest rate differences are exactly offset by expected exchange rates and currency premia (differences in variances) vanish, so that the blue line is horizontal (Panel (d)).

The main take-away from Figure 2 is that the slope of the solid blue line that connects the two countries provides a useful statistic for their relative position in the SDF space. We call this statistic the "FX-slope".

Figure 2: Currency Premium in the SDF Space



This figure plots four scenarios of currency premia in the SDF space. In each panel, grey dashed lines represent iso-rf lines and vertical/horizontal lines. The blue dot represents the low-interest rate country, and the red triangle represents the high-interest rate country. Panel (a) represents the case when the high-interest rate currency is expected to appreciate; Panel (b) represents the case when exchange rates are unpredictable; Panel (c) represents the case when the high-interest currency is expected to depreciate; and Panel (d) represents the case when uncovered interest rate parity holds.

Note that the FX-slope is equal to the negative reciprocal of the FX-share

$$\begin{aligned}\text{FX-slope} &= \frac{\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1}) - \frac{1}{2}\,\text{var}_t(m^{\star}_{t+1}))}{\mathbb{E}(m_{t+1} - m^{\star}_{t+1})} \\ &= \frac{\mathbb{E}(rx)}{\mathbb{E}(\Delta s)} = -\frac{1}{\text{FX-share}}. \qquad (5)\end{aligned}$$

This FX-slope characterizes the composition of the currency premium: It pins down the split between expected depreciations and interest rate differentials. As we discuss below, the currency premium puzzle states that long-run risk and habit models generate an FX-slope that is at odds with the data, or a currency premium that is of the wrong composition (FX-share).

To discipline the moments of log-SDFs, we discuss critical features of the data on exchange rates and currency premia. To this end, we use the dataset constructed by Hassan and Mano (2019).[6] Instead of looking at individual currencies or currency pairs, we form portfolios of currencies to control for currency-specific idiosyncrasies (see Lustig and Verdelhan (2007) and Lustig, Roussanov, and Verdelhan (2011)). The rationale for this approach is that portfolios provide a better proxy for a representative country since they reduce the idiosyncratic risk of an individual currency. Our empirical results are robust to alternative ways of portfolio construction, and the general pattern that we present is confirmed in many related works (e.g., Lustig, Roussanov, and Verdelhan (2011)). We defer the details of portfolio construction to Appendix C.

Table 1: Static Trade Returns in the Data

	Return (%)	Change in FX (%)	Interest Rate Diff (%)	FX-share	FX-slope
	$\mathbb{E}(rx)$	$-\mathbb{E}(\Delta s)$	$\mathbb{E}(r^\star - r)$	$\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)}$	$-\frac{1}{\text{FX-share}}$
Static Trade	3.46	-1.30	4.76	-0.37	2.67
	[1.18,5.54]	[-3.82,0.60]	[1.30,8.46]	[-1.16,0.23]	$[0.86, \infty) \cup (-\infty, -4.42]$

We use the 1-rebalance sample of Hassan and Mano (2019). All moments are annual. Static trade returns are calculated by first sorting the unconditional forward premia (interest rates), and then always shorting a weighted portfolio of the currencies with below average unconditional forward premia, and longing a weighted portfolio of the rest. Details on the construction of portfolios can be found in Appendix C. Confidence intervals are reported in brackets and are obtained by bootstrapping over countries. The confidence interval of the FX-slope is obtained by choosing FX-slopes between which 95% of the bootstrapped points lie. Alternative regression-based estimations can be found in Appendix C and in Hassan and Mano (2019).

Table 1 summarizes the empirical regularities with point estimates and confidence intervals. Static trade refers to fixed portfolios of high and low interest rate currencies that do not rebalance over time. The resulting currency premium between these two portfolios is 3.46% per year, with a confidence interval of [1.18,5.54]. The high-interest rate portfolio depreciates on average ($\mathbb{E}(\Delta s) > 0$), but since the confidence interval includes zero, we cannot rule out the possibility that exchange rates are unpredictable ($\mathbb{E}(\Delta s) = 0$).

On average, investing in high-interest-rate currencies makes money on the interest rate differential and loses money on the exchange rate. The share of the expected change in the exchange rate within currency

[6] The data ranges from 1983-2010. Our results are robust to including newer data. We keep our sample identical to Hassan and Mano (2019) to facilitate comparison. We use their 1 Rebalance sample, which consists of 16 countries: Australia, Canada, Switzerland, Denmark, Hong Kong (China), Japan, Kuwait, Malaysia, Norway, New Zealand, Saudi Arabia, Sweden, Singapore, United Kingdom, South Africa, and USA.

premium, i.e., the FX-share, is negative (-37%). Consequently, the FX-slope is positive, as in Panel (c) in Figure 2), with a confidence interval that includes a vertical relationship (unpredictable exchange rates as in Panel (b). However, the data clearly exclude the possibility that investors on average expect high-interest rate currencies to appreciate (Panel (a)) or that UIP might hold (Panel (d)).

In sum, a complete-markets model that rationalizes the large and persistent differences in interest rates we see in the data must satisfy two key properties:

Property 1. *A large difference in the variances of log SDFs*[7]

$$\mathbb{E}(var_t(m) - var_t(m^\star)) \geq 0.07$$

Property 2. *The FX-slope is positive or vertical*

$$\frac{\frac{1}{2}\,\mathbb{E}(var_t(m) - var_t(m^\star))}{\mathbb{E}(m_{t+1} - m^\star_{t+1})} \geq 0$$

Equivalently, the FX-share is negative, $\frac{-\mathbb{E}(\Delta s)}{\mathbb{E}(rx)} \leq 0$*, and the high interest rate currency depreciates on average.*

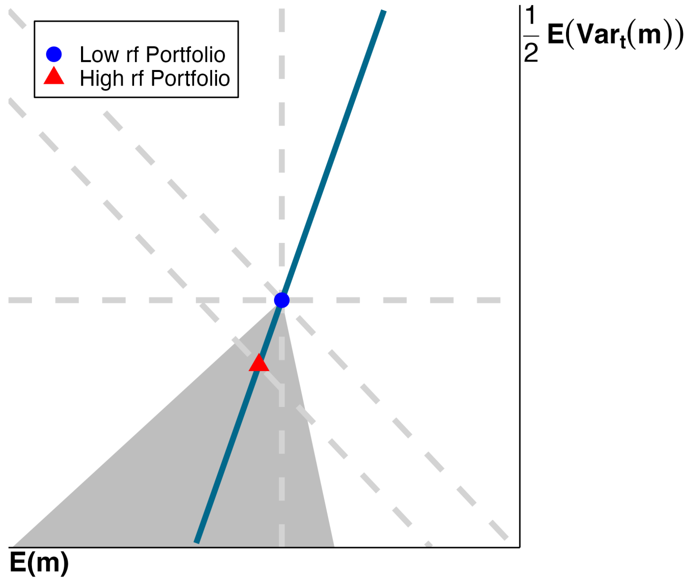
We incorporate these empirical results in the graphical representation of stochastic discount factors in Figure 3. In SDF space, the data require a weakly positive FX-slope.

These features of the data are not only of importance for our understanding of financial markets. They affect the relative marginal product of capital across countries and direct capital flows. They thereby have first-order implications for the real economy, as we discuss further below (see Hassan, Mertens, and Zhang (2016, 2021) and Richers (2021)).

In the following sections, we show that canonical long-run risk and habit models cannot satisfy both properties simultaneously. They either generate unpredictable exchange rates with small currency premia (Property 2 but not Property 1) or generate large currency premia through predictable changes in exchange rates (Property 1 but not Property 2), i.e. both models are incompatible with large interest rate differentials that are the primary drivers of currency returns. We show theoretically that the fundamental reason that these models fail to account for the two properties is that they introduce a functional negative relationship between means and variances of log SDFs.

[7]To put things into perspective, the Hansen and Jagannathan (1991) bound implies $var_t(m)$.0.25.

Figure 3: Static Trade in the SDF Space



This figure plots the relative position of the low-interest rate portfolio and the high-interest rate portfolio implied by the static trade returns in the data, as well as the confidence intervals of the FX-slope. The position of the low-interest rate portfolio is arbitrarily chosen because only the relative positions of these dots matter for our discussion. The position of the high-interest rate portfolio is inferred from the data using equations (2) and (4). The shaded area represents confidence intervals for the FX-slope.

3 Long-Run Risk Models

In this section, we set up and analyze a canonical long-run risk model under complete markets. We derive in closed form the relationship between the first two moments of the log SDFs and argue that such a relationship is at odds with data from currency markets. We then simulate four representative long-run risk models and show the currency premium puzzle quantitatively.

A representative agent derives utility according to

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{6}$$

U_t denotes utility at time t and C_t denotes consumption. δ is the subjective time discount factor, ψ governs the elasticity of intertemporal substitution, and γ scales risk aversion. Following the long-run risk approach, we further assume that consumption follows[8]

$$\Delta c_{t+1} = \mu + z_t + \sigma \varepsilon_{t+1}$$
$$z_t = \rho z_{t-1} + \sigma_{LR} \varepsilon_{LR,t},$$

where μ is the mean growth rate of consumption. z_t is a long-run process that moves the mean of consumption growth, and ρ denotes its persistence. σ governs the volatility of short-run shocks and σ_{LR} the volatility of long-run shocks. ε_{t+1} and $\varepsilon_{LR,t}$ are short-run and long-run shocks, respectively. For simplicity, we set $\sigma = 0$ for our derivation in the main text.(See Appendix A.1.2 for a version including short-run shocks).

Under these preferences and with complete markets, the log SDF is given by

$$\begin{aligned} m_{t+1} &= \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} \\ &+ \left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \frac{1}{1-\gamma}\log\left(\mathbb{E}_t[\exp((1-\gamma)u_{t+1}])\right)\right). \end{aligned} \tag{7}$$

Assuming that u_{t+1} is normally distributed, we solve for the first and second moment of the log SDF as

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)\mathbb{E}(\text{var}_t(u_{t+1})) \tag{8}$$
$$\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\text{var}_t(u_{t+1}))$$

These two equations show that the first and second moments of the log SDF are tightly linked. In fact, when we substitute out $\mathbb{E}(\text{var}_t(u_{t+1}))$ from these conditions, we get a functional relationship between the mean and variance

$$\frac{1}{2}\,\mathbb{E}(\text{var}_t(m_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\,\mathbb{E}(m_{t+1}) + \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\left(\log(\delta) - \frac{1}{\psi}\mu\right). \tag{9}$$

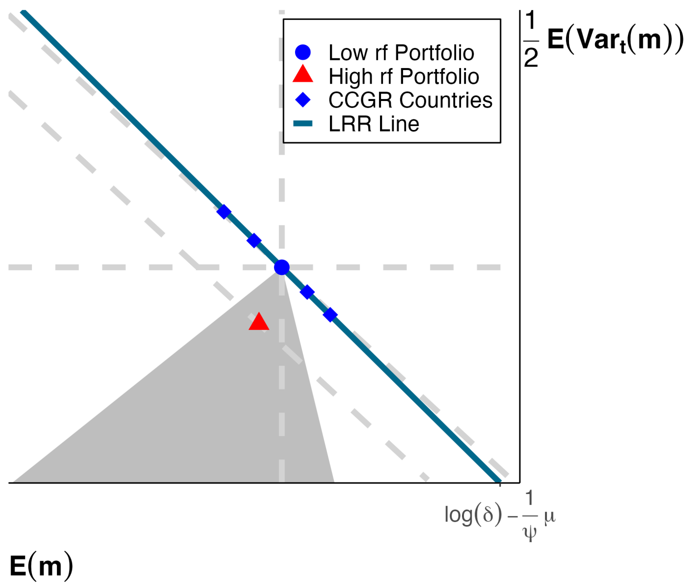
Standard calibrations use $\gamma > 1$ and $\gamma > \frac{1}{\psi}$ and thus imply a negative coefficient on the mean log SDF. As a result, increases in the variance lower the mean log SDF.

Importantly, the relationship in equation (9) represents a line in SDF space. We add this "long-run risk line" to Figure 3 as the blue line with a common calibration of $\gamma = 6.5$ and $\psi = 1.6$ (see Figure 4). Note that

[8] For simplicity, we do not consider time-varying volatility here. In Appendix A.2, we show that our results hold up under stochastic volatility.

the long-run risk line has the slope -1.07 and thus has a negative slope very similar to that of the iso ris-free lines (-1). This feature of the long-run risk model helps in resolving both the equity premium puzzle and the risk-free rate puzzle since it ensures a stable risk-free rate even when the variance of the SDF is large or changes over time.

Figure 4: The LRR Line in the SDF Space



This figure plots the long-run risk line with $\gamma = 6.5$ and $\psi = 1.6$. The red triangle represents the high-interest rate portfolio in the data. The blue dot represents the low-interest rate portfolio in the data. The shaded area represents confidence intervals inferred from the data. Blue squares correspond to countries in the heterogenous country general equilibrium model of Colacito et al. (2018a). We obtain the relative position of these points by solving their model in closed form using a risk-adjusted affine approximation.

However, if all countries share the same preference parameters, all stochastic discount factors have to lie on the same blue line. Consequently, no specification of the model exists that would lead to large difference in interest rates without generating exchange rate predictability. Specifically, the model cannot generate a high interest rate portfolio, such as the red dot in the figure, which are consistent with the data on exchange rates. To illustrate this point, we plot the five countries (blue squares) in the calibrated long-run risk model of Colacito et al. (2018a) (a heterogeneous country general equilibrium model, hereafter CCGR) in Figure 4.[9] These dots confirm the theoretical finding that there is a linear relationship between the mean and the variance of the log SDF. They also confirm that none of the currencies match the empirical pattern established in Section 2.2 (the red triangle).

[9]The relative positions of these points are obtained by solving their model using risk-adjusted-affine approximation (log-linearization with risk-adjustments. See, e.g, Chen and Palomino (2019), Malkhozov (2014) and Lopez, Lopez-Salido, and Vazquez-Grande (2015)). The Mathematica file for the solution is available upon request.

The long-run risk model thus pins down the slope of the relationship between means and variances of log SDFs via equation (9) whenever $\psi \neq 1/\gamma$.

Proposition 1. *Let γ, ψ, μ and δ be identical across countries and assume absence of short-run risk ($\sigma = 0$). Further assume that u_{t+1} is normally distributed. Then, for any two countries, the long-run risk model implies*

$$FX\text{-}slope = \frac{\mathbb{E}(rx_{t+1})}{\mathbb{E}(\Delta ex_{t+1})} = -\frac{\frac{1}{\psi} - \gamma}{1 - \gamma} \tag{10}$$

If agents prefer early resolution of uncertainty so that $\gamma > 1/\psi$, and we assume $\gamma > 1$, the FX-slope is negative and the model cannot satisfy Property 2. In particular, if $\gamma > 2 - \frac{1}{\psi}$, FX-share $= \frac{-\mathbb{E}(\Delta ex_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{1-\gamma}{\frac{1}{\psi}-\gamma} > \frac{1}{2}$. As a result, the expected change in the exchange rate $\mathbb{E}(\Delta ex_{t+1})$ accounts for more than 50% of the currency premium.

Proof. Implied by equations (2), (4) and (9). □

Equation (10) shows that the FX-slope is determined purely by preference parameters. It is thus deeply connected with the assumption of recursive preferences. Equivalently, the FX-share, and thus the composition of currency premia, directly follows from the choice of preferences. As we show in Appendix A.2 and Section 5, this relationship is robust to adding volatility shocks and relaxations of the log-normality assumption.

To show the implications of the theoretical analysis, we simulate five state-of-the-art long-run risk models in the literature and summarize the results in Table 2. These models are replications of the seminal works in Colacito et al. (2018a), Colacito et al. (2018b), Bansal and Shaliastovich (2013), and Colacito and Croce (2013), as well as the original long-run risk model in Bansal and Yaron (2004).[10] Of the four international finance models, only the heterogeneous-country general equilibrium model (Colacito et al. (2018a)) is able to generate sizable currency premium thus matching our empirical Property 1. However, consistent with the currency premium puzzle, more than 90% of this currency premium arises from predictable appreciation of the high-interest rate currency, implying an FX-slope close to -1 and contradicting our empirical Property 2. The symmetric models do not generate unconditional differences in returns consistent with the data thus fail to match Property 1. The FX-slope in each of these symmetric models is very close to -1 (the slope of iso-rf line), suggesting that FX-shares would make up well over 90% even if country heterogeneities were introduced. Similarly, the FX-slope for the canonical closed-economy long-run risk model of Bansal and Yaron (2004) is close to negative one. This feature, while being useful in keeping the risk-free rate low and stable, leads to a currency premium with a counterfactual composition of interested rate differences and

[10]We thank Ric Colacito, Max Croce, Federico Gavazzoni and Robert Ready for providing their code.

Table 2: Static Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{st})$	Change in FX (%) $-\mathbb{E}(\Delta s^{st})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,st} - r^{st})$	FX-share $\frac{-\mathbb{E}(\Delta s^{st})}{\mathbb{E}(rx^{st})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	3.46 [1.18,5.54]	-1.30 [-3.82,0.60]	4.76 [1.30,8.46]	-0.37 [-1.16,0.23]	2.67 $[0.86, \infty) \cup (-\infty, -4.42]$	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	7.10	5.98	1.12	0.93	-1.07	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	0.00	0.00	0.00	0.98	-1.02	No	No
Bansal and Shaliastovich (2013) RFS	0.00	0.00	0.00	0.96	-1.04	No	No
Colacito and Croce (2013) JF	0.00	0.00	0.00	0.95	-1.05	No	No
Bansal and Yaron (2004) JF	-	-	-	0.94	-1.06	No	No

This table shows the simulation results of four long-run risk models. For Colacito et al. (2018a), we simulate the model in their Section II, with calibrations in their Table II (exactly the same model as the heterogeneous EZ model in their Table III). For Colacito et al. (2018b), we simualte the model in their Section II, with calibrations in their Table 2 (exactly the same model as EZ-BKK in their Table 3). For Bansal and Shaliastovich (2013), we simulate the model in their Section 2, with calibrations in their Table 4 and Section 4.4 (exactly the same model as Table 7), we report the simulation results for the real model to be consistent with other models. For Colacito and Croce (2013), we simulate the model in their Sections II and III, with calibrations in their Table II, Panel A (exactly the same model as model (1) in their Table II, Panel B). For Colacito et al. (2018a), we follow their approach and simulate 100 economies, each with 320 periods and 100 burn-in periods (discarded). For all the other models, we simulate 100 economies of 10000 periods. Static trade returns are computed using the same method in our empirical analysis. All moments are averaged across periods and simulations. Because Colacito et al. (2018b), Bansal and Shaliastovich (2013) and Colacito and Croce (2013) feature symmetric countries and currency premia are close to 0, we show the theoretical FX-slopes for these models using Proposition 1 instead. We also show the theoretical FX-slope for the calibration of Bansal and Yaron (2004) for comparison (with $\gamma = 10, \psi = 1.5$). The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively.

predictable changes in exchange rates. To be clear, the composition of currency premia was not a target of any of the papers. However, our analysis reveals that the standard long-run risk setup will not be able to match this additional empirical fact.

An alternative popular trading strategy, the carry trade, aims at exploiting differences in interest rates across countries over time. The carry trade goes long high interest rate currencies and short low interest rate currencies, just like the static trade. It differs, however, in that the carry trade compares risk-free rates period-by-period and re-balances the portfolio accordingly. Because of period-by-period re-balancing, carry trade returns also contain conditional information, which is missing in our analysis of unconditional moments. To account for that, we report carry trade returns in Table 3.

Table 3 displays a similar pattern to the static trade. The data again suggest large currency premia which

Table 3: Carry Trade Returns under LRR Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 $[0.90, \infty) \cup (-\infty, -6.56]$	-	-
Colacito, Croce, Gavazzoni and Ready (2018) JF	4.47	2.76	1.71	0.62	-1.62	Yes	No
Colacito, Croce, Ho and Howard (2018) AER	-0.09	-0.52	0.44	6.11	-0.16	No	No
Bansal and Shaliastovich (2013) RFS	-0.03	-0.26	0.23	9.54	-0.10	No	No
Colacito and Croce (2013) JF	0.05	-0.35	0.41	-7.02	0.14	No	No

This table shows the simulation results for the four long-run risk models of Table 2. Instead of the static trade, this table focuses on the carry trade return. The last two columns summarize whether the simulated results can match our empirical Properties 1 and 2, respectively.

are mostly accounted for by risk-free rate differences.[11] Symmetric country models with long-run risk fail to generate the size of currency premia. And all four models imply counterfactually large predictability in exchange rates.

To summarize, in this section, we derived a closed-form functional relationship between the mean and the variance of the log SDFs under long-run risk models with EZ preference. We show that under standard calibrations where agents prefer early resolution of uncertainty, the model can only generate a negative FX-slope and cannot satisfy Property 2. Most of the generated currency premia are accounted for by an appreciation of the high interest rate currency, in contrast with the data.

4 External habit models

In this section, we show that external habit models display similar difficulties in matching the data as long-run risk models. We illustrate these challenges using the pioneering work in Verdelhan (2010). This model has the advantage that it is a direct extension of the classical habit model in Campbell and Cochrane (1999)

[11]The FX-slope now measures the relative position of the portfolio that we long each period and the portfolio that we short each period.

and can be solved in closed form. Agents feature habit utilities of the form

$$\mathbb{E}\sum_{t=0}^{\infty}\delta^t \frac{(C_t - H_t)^{1-\gamma} - 1}{1-\gamma},$$

where H_t is an externally given habit level. With the surplus consumption ratio

$$X_t \equiv \frac{C_t - H_t}{C_t},$$

the pricing kernel becomes

$$M_{t+1} = \delta\left(\frac{X_{t+1}}{X_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}.$$

Log consumption follows a random walk with drift given by

$$\Delta c_{t+1} = \mu + \sigma\varepsilon_{t+1}.$$

Following Campbell and Cochrane (1999), instead of specifying an exogenous process for H_t, we directly assume the following process for the log surplus consumption ratio

$$x_{t+1} = (1-\phi)\bar{x} + \phi x_t + \lambda(x_t)(\Delta c_{t+1} - \mu).$$

Following the literature, we use the sensitivity function $\lambda(x_t)$

$$\lambda(x_t) = \begin{cases} \frac{1}{\bar{X}}\sqrt{1-2(x_t-\bar{x})} - 1 & \text{when } x < x_{max} \\ 0 & \text{elsewhere} \end{cases}$$

where the logarithm of the upper bound x_{max} is given by

$$x_{max} = \bar{x} + \frac{1-\left(\bar{X}\right)^2}{2}$$

and the steady-state of the surplus consumption ratio by

$$\bar{X} = \sigma\sqrt{\frac{\gamma}{1-\phi-B/\gamma}}.$$

Note that $\gamma(1-\phi) - B > 0$ by construction.[12] Using the process for the log surplus consumption ratio x_t, we derive the log SDF as

$$m_{t+1} = \log(\delta) - \gamma(\Delta c_{t+1} + \Delta x_{t+1}).$$

This specification implies the first two moments of the log SDF as

$$\mathbb{E}_t(m_{t+1}) = \log(\delta) - \gamma\mu + \gamma(1-\phi)(x_t - \bar{x}) \tag{11}$$

$$\begin{aligned} \frac{1}{2}\text{var}_t(m_{t+1}) &= \frac{1}{2}\gamma^2(1+\lambda(x_t))^2\sigma^2 \\ &= \frac{1}{2}(\gamma(1-\phi) - B) - (\gamma(1-\phi) - B)(x_t - \bar{x}). \end{aligned} \tag{12}$$

Both the mean and the variance of the log SDF depend on the term $x_t - \bar{x}$. Substituting it out unveils a strong relationship between the first two moments, which are conditioned on time t information:

$$\begin{aligned} \frac{1}{2}\text{var}_t(m_{t+1}) = &-\frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}\,\mathbb{E}_t(m_{t+1}) \\ &+ \frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}(\log(\delta) - \gamma\mu) + \frac{1}{2}(\gamma(1-\phi) - B). \end{aligned} \tag{13}$$

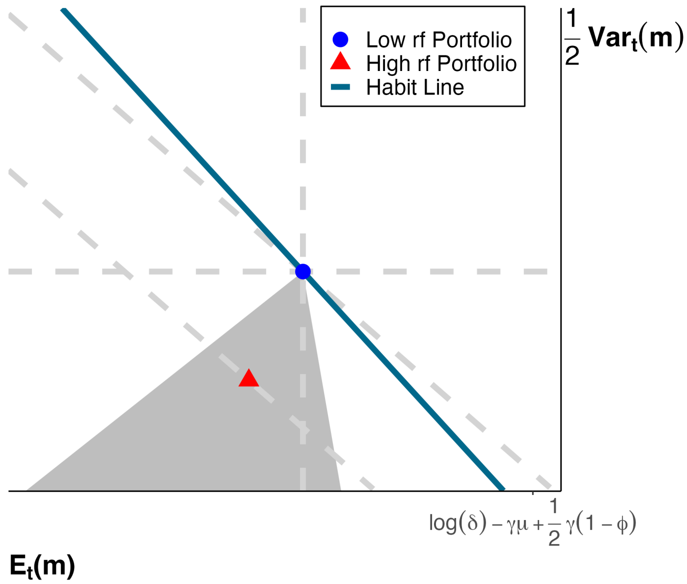
Equation (13) shows that, similar to long-run risk models, habit models also imply a negative functional relationship between the first two moments of log SDFs. Here, again, this feature is useful for resolving closed-economy asset pricing puzzles.

Equation (13) again implies a line in the, now conditional, SDF space. We plot this habit line in Figure 5 with a calibration of $\gamma = 2$, $\phi = 0.01$ and $B = -0.01$, which are taken from Verdelhan (2010).

Figure 5 shows that all stochastic discount factors implied by this external habit model (blue line) lie close to the iso-rf line. As a result, the currency premium puzzle applies to the habit model when taken to international asset pricing data. Similar to the above discussion of the long-run risk model, the ability of the habit model to match the data can be characterized by the FX-slope. We summarize this slope, as well as the predictions of the habit model in terms of the two properties that the data calls for, in the following proposition.

[12] The specification of the $\lambda()$ function strictly follows Campbell and Cochrane (1999). As they have pointed out in their paper, the specific functional form is designed to keep the risk-free rates stable. Parameter B nests different SDFs from the literature. $B < 0$ in Verdelhan (2010), $B = 0$ in Campbell and Cochrane (1999), and $B > 0$ in Wachter (2006).

Figure 5: The Habit Line in the SDF Space



This figure plots the habit line with $\gamma = 2$, $\phi = 0.01$, and $B = -0.01$. The red triangle represents the high-interest rate portfolio in the data. The blue dot represents the low-interest rate portfolio in the data. The shaded area shows the confidence intervals inferred from the data.

Proposition 2. *If preferences are symmetric across countries, the FX-slope is given by*

$$\textit{FX-slope} = \frac{\mathbb{E}_t(rx_{t+1})}{\mathbb{E}_t(\Delta s_{t+1})} = -\frac{\gamma(1-\phi) - B}{\gamma(1-\phi)}.$$

Because $\gamma(1-\phi) - B > 0$ is required by stationarity, the FX-slope is always negative and the model cannot satisfy Property 2. Furthermore, if $\gamma(1-\phi) > -B$, FX-share $= \frac{-\mathbb{E}(\Delta s_{t+1})}{\mathbb{E}(rx_{t+1})} = \frac{\gamma(1-\phi)}{\gamma(1-\phi)-B} > \frac{1}{2}$. This means an appreciation of the high interest currency accounts for more than 50% of the currency premium.

Proof. Implied by equations (2), (4) and (13). □

We focus our numerical comparison of various models with the data on carry trade returns. An analysis of static trades would not be suitable since symmetric habit models in the literature provide conditional moments and unconditional returns from the static trade are zero. To show that the currency premium puzzle applies to a wider set of habit models, we simulate several prominent and state-of-the-art models in the literature.

We summarize our results in Table 4. The habit model of Verdelhan (2010) closely matches the carry trade returns in the data (Property 1). Almost half of these returns, however, arise from expected changes in exchange rates as our theory predicts, contradicting Property 2. The FX-slope is strongly negative at -2.13 in

Table 4: Carry Trade Returns under Habit Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 $[0.90, \infty) \cup (-\infty, -6.56]$	-	-
Verdelhan (2010) JF	4.54	2.19	2.35	0.48	-2.07	Yes	No
Stathopoulos (2017) RFS	-1.23	-2.40	1.17	1.95	-0.51	No	No
Heyerdahl-Larsen (2014) RFS	3.48	3.05	0.43	0.88	-1.14	Yes	No
Campbell and Cochrane (1999) JPE	-	-	-	1.00	-1.00	No	No

This table shows the simulation results of three habit models. For Verdelhan (2010), we simulate the model in Section I, with calibrations in Table II. For Stathopoulos (2017), we simulate the model in Section 1, with calibrations in Table 1. For Verdelhan (2010) and Heyerdahl-Larsen (2014), we simulate 100 economies of 10000 periods. All moments are averaged across periods and simulations. Results for Heyerdahl-Larsen (2014) are obtained by using exactly the same simulated samples that generated his Table 10. Details on carry trade return construction can be found in Appendix B and the same method is used for both the empirical and the simulation results. We also show the theoretical FX-slope for the calibration of Campbell and Cochrane (1999) for comparison. The last two columns summarize whether the simulated results can match our empirical properties 1 and 2, respectively.

this model while it is positive in the data (2.30). In the continuous-time habit model of Stathopoulos (2017), the high-interest rate currency does depreciate on average.[13] But it depreciates so much that the expected depreciation exceeds the interest rate differential and the resulting currency premium is negative, at odds with the data and contradicting both our empirical properties. The deep habit model of Heyerdahl-Larsen (2014) features an FX-share of 89.19% and and FX-slope of -1.16 and also fails to match Property 2.[14] We further report the theoretical FX-slope for Campbell and Cochrane (1999). The FX-slope for each of the habit models is very close to -1. As for the literature on long-run risks, none of these models were designed to fit the composition of currency premia. Our analysis shows that the persistent differences in interest rates combined with unpredictable exchange rates pose a tough challenge for any model in international finance.

To summarize, in this section we derived a closed-form functional relationship between the conditional mean and variance of the log SDFs under external habit models. We showed that under standard calibrations, standard models in this literature generate a negative FX-slope and cannot satisfy Property 2, thereby running into the currency premium puzzle.

[13]We thank Andreas Stathopoulos for providing us with his code.
[14]We thank Christian Heyerdahl-Larsen for providing us with his simulated samples.

5 Relaxing log-normality

While log-normality is a common assumption in the international finance literature, there are reasons, such as the skewness of exchange rates, to explicitly model higher moments. In this section, we explore whether relaxing the log-normality assumption resolves the currency premium puzzle. We therefore first generalize the standard framework to a non-normal distribution and show that the tension between the equity premium puzzle, the risk-free rate puzzle, and the currency premium puzzle exists regardless of log-normality. We then examine two existing prominent international asset pricing models that feature disaster risk (Gourio, Siemer, and Verdelhan (2013) and Farhi and Gabaix (2016)) and thus focus on higher moments of the distribution. We find that these models are also subject to the currency premium puzzle.

For general distributions, the risk-free rate can be written as (see Backus, Foresi, and Telmer (2001))

$$\begin{aligned} r_{f,t} &= -\log(\mathbb{E}_t\, M_{t+1}) = -\,\mathbb{E}_t(m_{t+1}) - \left[\log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})\right] \\ &= -\,\mathbb{E}_t(m_{t+1}) - \Xi_t(m_{t+1}), \end{aligned}$$

where $\Xi_t(m_{t+1}) = \log(\mathbb{E}_t(M_{t+1})) - \mathbb{E}_t(m_{t+1})$ denotes the entropy of the SDF.

We investigate whether the two empirical regularities about the composition of currency premia also put restrictions on models with higher moments in the distribution. In the generalized case, the expected change in exchange rates and currency premia are now given by

$$\begin{aligned} \mathbb{E}_t(\Delta s_{t+1}) &= \mathbb{E}_t(m_{t+1}) - \mathbb{E}_t(m^{\star}_{t+1}) \\ \mathbb{E}_t(rx_{t+1}) &= \Xi_t(m_{t+1}) - \Xi_t(m^{\star}_{t+1}). \end{aligned}$$

The expressions are identical to the ones in Section 2 except that the entropy $\Xi_t(m_{t+1})$ takes the place of what used to be the variance of the log SDF, $\frac{1}{2}\,\mathrm{var}_t(m_{t+1})$. The relationship between the risk-free rates, the exchange rates, the currency premia, and the mean, and now the entropy of the SDFs is, however, preserved. As a result, the discussion in Section 2 still applies. The log-normal model emerges as a special case and the tension between a high equity premium, a low and stable risk-free rate, and the composition of currency premium extends to the generalalized setup.

As an illustration, we consider the framework of Gourio, Siemer, and Verdelhan (2013), which focuses on disaster risk and, thus, higher moments of the distribution of the SDF. Agents feature Epstein and Zin

(1989) preference as in (6), and their log SDF is given by (7). To obtain the entropy, we assume absence of normalally-distributed shocks and consider the special case when the probability of disasters is small.[15] In this case, the mean and the entropy of the SDF are given by

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\,\mathbb{E}(\Delta c_{t+1}) + \frac{\frac{1}{\psi} - \gamma}{1-\gamma}\,\mathbb{E}\left[((1-\gamma)u_{t+1}) - \log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)\right] \tag{14}$$

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\,\mathbb{E}\left[\left(\frac{1}{\psi} - \gamma\right)u_{t+1} - \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right)\right]. \tag{15}$$

We defer the detailed derivation to Appendix B. An immediate observation is that the mean and the entropy are again tightly linked. We start with a discussion of the special case where the elasticity of intertemporal substitution ψ is equal to one. Now we obtain a functional link between the mean of the log stochastic discount factor and the entropy.

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\,\mathbb{E}(m_{t+1}) + \log(\delta) - \mathbb{E}(\Delta c_{t+1}).$$

This linear functional relationship has a slope of negative one in a generalized SDF space, in line with the results for the lognormal case where the y-axis is entropy instead of variance of SDF. That is to say, countries with identical preference parameters and consumption growth end up on the same iso-rf line, and there is no cross-country variation in risk-free rates in the model. The FX-share is one implying that all currency premia arise from predictable variation in exchange rates. It thus fails to satisfy the two empirical properties required by the data.

This failure of this model to match the composition of currency premia is rooted in the choice of preferences. As we saw for long-run risk models, Epstein-Zin preferences help in keeping the risk-free rate stable. But this feature is precisely the reason why it is hard to generate differences in interest rates across countries.

Looking beyond the case of a unit elasticity of intertemporal substitution, we investigate whether differences in exposure to downside risks across countries would resolve the currency premium puzzle. Specifically, we capture the key ingredient in the disaster risk literature by allowing for country-specific skewness in the distribution of consumption. To highlight the role of higher moments, we exploit the recursivity of preferences to recover the relationship between the first moment and the entropy of the SDF. From equations

[15] This is similar to the "no-short-run-shocks" assumption in the long-run risk framework (Section 3). As a result, consumption growth Δc_{t+1} is known at time t and disaster risk affects the model through continuation utility. For a detailed discussion, see Appendix B.

(14) and (15), using cumulant generating functions (Backus, Foresi, and Telmer (2001)), we obtain

$$\mathbb{E}(m_{t+1}) = -\left(\log(\delta) - \frac{1}{\psi}\,\mathbb{E}(\Delta c_{t+1})\right) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{2,t}(u_{t+1})) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots,$$

where $\kappa_{i,t}(u_{t+1})$ is the ith cumulant of u_{t+1}.[16] A similar expansion applies to the entropy

$$\mathbb{E}(\Xi_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2\mathbb{E}(\kappa_{2,t}(u_{t+1})) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3\mathbb{E}(\kappa_{3,t}(u_{t+1})) + \ldots$$

With skewness ($\kappa_{3,t}(u_{t+1})$) differing across countries and setting all other cumulants to be identical, we get

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)^2\mathbb{E}(m_{t+1}) + \text{constant}. \tag{16}$$

We again see a tight relationship between the entropy and the first moment. In particular, (16) implies

$$\text{FX-slope} = -\frac{1}{\text{FX-share}} = -\left(\frac{\frac{1}{\psi}-\gamma}{1-\gamma}\right)^2$$

which is always negative and close to -1 under standard calibrations ($\gamma > \frac{1}{\psi}$, $\gamma > 1$ and $\psi > 1$). For example, with $\gamma = 8.5$ and $\psi = 2$, we have a FX-slope $= -1.13$.

We show quantitatively that the currency premium puzzle applies to disaster models in the existing literature. We therefore consider two highly successful contributions from the international finance literature, as well as a special case of Gourio (2012), and compare them to the data.[17] Closely related to our theoretical results, Gourio, Siemer, and Verdelhan (2013) features a model with both Epstein and Zin (1989) utility and heterogeneous exposures to global disaster risk. Farhi and Gabaix (2016) build a general disaster model with constant relative risk aversion and tradable and nontradable goods. We list the unconditional results (UN) as well as the results conditional on no disaster happening in the sample (ND).

As Table 5 shows, the currency premium puzzle is also evident in these disaster models. The conclusions are the same as for long-run risk and habit models: With FX-shares above 50% and FX-slopes close to -1, the model cannot match the composition of currency premia.

To summarize, the inability of standard models to simultaneously generate a high equity premium, a

[16] Cumulants are functions of the usual central moments.

[17] Barro (2009) is another closed-economy disaster model with Epstein and Zin (1989) preference, which is also subject to our puzzle.

Table 5: Carry Trade Returns under Disaster Models

	Return (%) $\mathbb{E}(rx^{ct})$	Change in FX (%) $-\mathbb{E}(\Delta s^{ct})$	Interest Rate Diff (%) $\mathbb{E}(r^{\star,ct} - r^{ct})$	FX-share $\frac{-\mathbb{E}(\Delta s^{ct})}{\mathbb{E}(rx^{ct})}$	FX-slope $-\frac{1}{\text{FX-share}}$	P1	P2
Data	4.95 [1.50,8.34]	-2.15 [-4.98,0.49]	7.11 [2.22,13.22]	-0.43 [-1.10,0.15]	2.30 [0.90, ∞)$\cup$ ($-\infty$, -6.56]	-	-
Gourio, Siemer and Verdelhan (2013) JIE	2.36	1.81	0.55	0.77	-1.31	Yes	No
Farhi and Gabaix (2016) QJE (UN)	4.9	3.39	1.51	0.69	-1.44	Yes	No
Farhi and Gabaix (2016) QJE (ND)	-	-	-	0.75	-1.33	Yes	No
Gourio (2012) AER	-	-	-	1.00	-1.00	No	No

The return, FX-share and FX-slope for Gourio, Siemer and Verdelhan (2013) are calculated from their tables 2 and 4; return for Farhi and Gabaix (2016) is from their table III, FX-shares and FX-slopes are calculated using their calibrations in Tables I and II, and their equations (24) and (25). UN stands for unconditional moments, ND stands for moments conditional on no disaster in the sample. We also show the theoretical FX-share and FX-slope for a special case of the closed-economy model of Gourio (2012) for comparison, assuming an IES of 1 ($\psi = 1$). Alternatively, if we use Gourio's preferred calibration ($\psi = 2, \gamma = 3.8$) and use the approximation in equation (16), we end up with FX-share = 0.72 and FX-slope = -1.39. The last two columns summarize if the simulated results can match our empirical properties 1 and 2, respectively.

low and stable risk-free rate, as well as a weakly positive FX-slope extends beyond the case of log-normality. In particular, existing disaster models are also subject to the currency premium puzzle.[18] However, we acknowledge that allowing multiple higher cumulants or moments to differ across countries at the same time does offer more degree of freedom and could potentially be helpful in resolving the puzzle. How such a model should be constructed and what would drive these differences is not obvious and would require further research.

6 Incomplete Spanning

The presence of complete markets underlies the analysis in Sections 3 and 4. Thus a natural question arises: Can market incompleteness help in resolving the currency premium puzzle? In this section, we relax the assumption of complete markets and explore incomplete spanning as a potential explanation of the empirical patterns in exchange rates. We find that it does not resolve the currency premium puzzle.

We consider the scenario when agents do not have full access to foreign financial markets. Specifically, we assume that agents can freely only trade their domestic risk-free assets so that the link between the risk-free rate and the mean and variance of the log SDF, as in equation (1), holds within each country. But if investors

[18] In fact, Jurek (2014) and Farhi et al. (2009) find that when disaster risk are hedged using options, carry trade returns are still large, suggesting disasters are not the whole story anyway.

cannot freely trade all assets, the expected change in the exchange rate might no longer be determined by the ratio of expected SDFs across countries. In other words, (2) no longer holds. Following Lustig and Verdelhan (2019), we summarize incomplete spanning in international financial markets by a wedge, η_{t+1}, between changes in exchange rates and log SDFs so that[19]

$$\begin{aligned}\mathbb{E}(\Delta s_{t+1}) &= \mathbb{E}(m_{t+1}) - \mathbb{E}(m^{*}_{t+1}) - \mathbb{E}(\eta_{t+1}) \\ &= \mathbb{E}(m_{t+1}) - \mathbb{E}(m^{im,*}_{t+1}) \end{aligned} \tag{17}$$

Again, all unconditional equations in this section also hold conditionally.

To facilitate discussion, we define $\mathbb{E}(m^{im,*}_{t+1}) = \mathbb{E}(m^{\star}_{t+1}) + \mathbb{E}(\eta_{t+1})$, the expectation of a foreign log SDF adjusted for the incomplete-market wedge, in a slight abuse of notation. Similarly, we compute its expected variance $\frac{1}{2}\mathbb{E}(\text{var}_t(m^{im,*}_{t+1})) = \frac{1}{2}\mathbb{E}(\text{var}_t(m^{*}_{t+1})) - \mathbb{E}(\eta_{t+1})$. Plugging (1) and (17) into the first part of equation (4) yields the currency premium

$$\begin{aligned}\mathbb{E}(rx_{t+1}) &= \mathbb{E}(\eta_{t+1}) + \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m^{*}_{t+1})) \\ &= \frac{1}{2}\mathbb{E}(\text{var}_t(m_{t+1}) - \text{var}_t(m^{im,*}_{t+1})). \end{aligned} \tag{18}$$

Equations (17) and (18) show that the expected incomplete market wedge, $\mathbb{E}(\eta_{t+1})$ provides an extra degree of freedom. The question is whether this degree of freedom can help in resolving the currency premium puzzle. To this end, a properly chosen wedge can render the exchange rate unpredictable, i.e., $\mathbb{E}(\Delta s_{t+1}) = 0$, which helps the model to match our Property 2. In other words, we can choose $\mathbb{E}(\eta_{t+1})$ so that $\mathbb{E}(m^{im,*}_{t+1}) = \mathbb{E}(m_{t+1})$. However, this wedge also moves the scaled variance of the foreign log SDF and thus has direct implications for the magnitude of currency premium (equation (18)). We illustrate how the wedge can move the foreign points in SDF space in Figure 6.

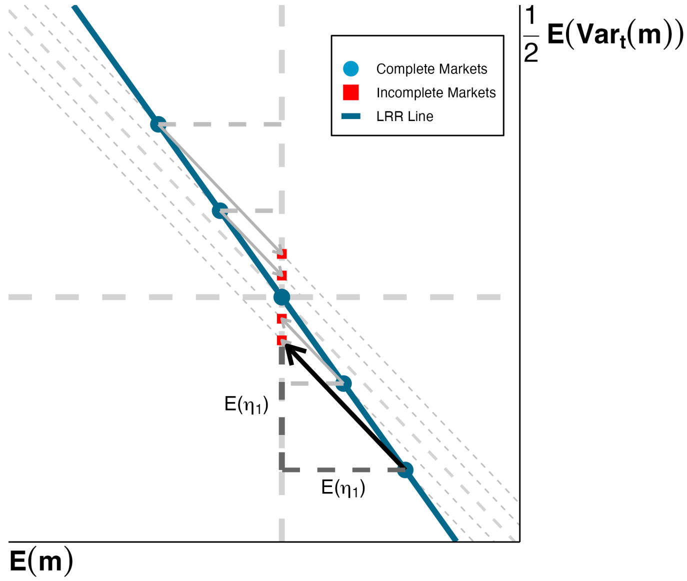
Figure 6 plots a line for the long-run risk model (LRR line) and five countries (indexed by i) that differ in their variances of log SDFs to illustrate the effects of incomplete spanning.[20] Country 1 is assumed to have the highest interest rate and country 5 the lowest.[21] When markets are complete, all stochastic discount factors consistent with the model lie on the same long-run risk line (blue dots on the blue line), as we showed

[19] Lustig and Verdelhan (2019) assume that agents can trade all the risk-free assets internationally. We only need the weaker assumption that each agent have access to her domestic risk-free asset.

[20] Habit models would result in a line like the blue line as well.

[21] These differences could be driven by heterogeneous loadings on a global shock (Colacito et al. (2018a)), country sizes (Hassan (2013), Martin (2011)), trade centrality (Richmond (2019)), or any other form of heterogeneity in economic environments.

Figure 6: Incomplete Markets vs Complete Markets



This figure plots a long-run risk line (blue line) with five countries. The blue dots represents the countries under complete market, and the red square incomplete markets, assuming that the incomplete market wedge removes the predictability of exchange rates so that change in exchange rates are 0 for all country pairs.

in Section 3. We assume that for each country, we choose a wedge $\mathbb{E}(\eta^i_{t+1})$ such that exchange rates are unpredictable for all country pairs. We plot the log SDF for each country after adjusting it for the incomplete market wedge, $(\mathbb{E}(m^{im,i}_{t+1}), \frac{1}{2}\,\mathbb{E}(\text{var}_t(m^{im,i}_{t+1})))$, as red squares. By construction, all five countries lie on the same vertical line indicating unpredictable exchange rates. As a result, the FX-slope is vertical and FX-shares are zero, consistent with our empirical Property 2.

Two observations from this analysis stand out. First, as shown in Figure 6, the incomplete market wedges $\mathbb{E}(\eta^i)$ have to differ significantly across countries to achieve exchange rate unpredictability across country pairs. It is unclear what type of mechanism would bring about such a constellation of wedges so as to render exchange rates unpredictable.

Second, currency premia, i.e., the vertical differences of these points, shrink considerately relative to the complete markets benchmark. In fact, the expected wedge lowers exchange rate predictability and currency premia by the same amount (as implied by equations (17) and (18)). Consequently, incomplete spanning makes it harder for any model to satisfy empirical Property 1, which states that currency premia need to be large. In particular, as shown in the figure, the incomplete market wedge moves the corresponding country point along the corresponding iso-rf line

$$
\begin{aligned}
\mathbb{E}(r^i_{f,t}) &= -\,\mathbb{E}(m^i_{t+1}) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m^i_{t+1})) \\
&= -(\mathbb{E}(m^i_{t+1}) + \mathbb{E}(\eta^i_{t+1})) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m^i_{t+1}) - \mathbb{E}(\eta^i_{t+1})) \qquad (19) \\
&= -\,\mathbb{E}(m^{im,i}_{t+1}) - \frac{1}{2}\,\mathbb{E}(\text{var}_t(m^{im,i}_{t+1})). \qquad (20)
\end{aligned}
$$

Consequently, the entire currency premium arises from interest rate differentials. This feature poses a challenge for standard models. When agents have symmetric preferences, face identical growth rates, and have access to their own risk-free assets, their $(\mathbb{E}(m), \mathbb{E}(\frac{1}{2}\,\text{var}_t(m_{t+1}))$ pairs lie on the same long-run risk line in SDF space (see (9)), whose slope is close to -1. Consequently, differences in interest rates across countries are small and the model will likely fail empirical Property 1.

Table 6: Implied Wedges For CCGR Countries

Country	Return (%)		Change in FX (%)		Interest Rate Diff (%)	Implied Wedge
	Complete	Incomplete	Complete	Incomplete		
1	2.91	0.16	-2.75	0.00	0.16	-2.75
2	1.52	0.08	-1.44	0.00	0.08	-1.44
3	0.00	0.00	0.00		0.00	0.00
4	-1.64	-0.07	1.57	0.00	-0.07	1.57
5	-3.41	-0.12	3.28	0.00	-0.12	3.28

This table shows currency premia, expected depreciation of the high-interest rate currency, and interest rate differences for the five countries in the model of Colacito et al. (2018a) under both complete and incomplete markets. The currency of the middle, country 3, is used as the base currency. The implied incomplete market wedge is set to equal differences in means of log SDFs to match the fact that exchange rates are unpredictable.

To illustrate these two observations quantitatively, Table 6 shows currency premia and exchange rate appreciations for five countries in the model of Colacito et al. (2018a) under both complete and incomplete markets, using country 3 (the middle country) as the base currency. The currency premium puzzle is again evident across all country pairs, with counterfactually small interest rate differences and large currency premia driven mostly by expected appreciation. As in Figure 6, the incomplete market wedge can be utilized to achieve unpredictable exchange rates as in the data. In particular, by setting $\mathbb{E}(\Delta s_{t+1})$ in (17) to zero, we can back out the required wedges to eliminate predictability from exchange rates for each of the country pairs. We report these implied wedges in the last column. Setting the incomplete wedge to these values improves the model's performance in matching the composition of currency premia (Property 2), since all currency premia arise entirely from interest rate differences. The implied wedges strongly differ across country pairs.

As a direct implication, the entire currency premia arise from risk-free rate differences, which are substantially smaller than the ones observed in the data. As an example, country 1 produces a currency premia of 2.91% relative to country 3 under complete markets. If we set the incomplete market wedge to -2.75% so that exchange rates are unpredictable, we end up with a currency premia of merely 0.16%.[22] In other words, while variation in the wedge across countries can ensure that Property 2 holds, Property 1, which says currency premia are large, will fail in those cases.[23]

This argument applies to all models considered in this paper. Recall that in all models, changes in exchange rates account for more than 50%, in many cases for more than 90%, of currency premia. Fixing the composition of currency premia in these models using the incomplete spanning wedge implies a significant decrease in currency premia, weakening their ability to match the empirically large currency premia. This feature is embedded in preferences chosen to keep risk-free rate stable by imposing a tight functional relationship between means and variances of log-SDFs.

7 Conclusion

In this paper, we highlight a fundamental tension between canonical asset pricing models, which have been notably successful in explaining closed-economy puzzles, and the empirical observations in open economies. This tension, which we term the 'currency premium puzzle,' manifests in the inability of these models to reconcile large and persistent interest rate differentials with the unpredictability of exchange rates.

We have demonstrated that in the context of an open economy with complete markets, canonical long-run risk and habit models require that differences in currency returns should predominantly stem from predictable exchange rate movements, rather than from interest rate differentials. This theoretical prediction sharply contrasts with empirical observations, where exchange rates are notoriously difficult to predict, and interest rate differentials are the primary source of differences in currency returns.

This failure is not mitigated by introducing stochastic volatility, by deviating from log-normal distribu-

[22]Similarly, consider our simulation results in Table 2, the simulated difference in variances of log SDFs is 7.10%, while the difference in means is -5.98%. If we believe that exchange rates are unpredictable as the data suggests and set $\mathbb{E}(\eta_{t+1}) = 5.98\%$, we can indeed generate a $\mathbb{E}(\Delta s_{t+1}) = 0$ and $\mathbb{E}(rx_{t+1}) = 1.12\%$. Now, all currency premia are accounted for by differences in risk-free rates: incomplete market helps the long-run risk model of Colacito et al. (2018a) to match Property 2, i.e., to get the right composition of currency risk premia. But it does so by shrinking the currency premia by more than 90%, significantly weakening the model's ability to match Property 1.

[23]In principle, one could design a long-run risk (or habit) model with even larger differences in variance of SDFs and the right incomplete market wedge to match the data, but how such extreme heterogeneity can be justified, and how such strong market frictions can prevent exchange rates to move in such models, remain unclear.

tions, or by modifying the economic environment in other ways. Instead, it is inherent to the preference structures in these models: The same preference structures that reconcile large equity premia with low and stable risk-free rates in the context of a closed economy also require that the vast majority of any differences in currency returns across countries must transmit themselves through predicted depreciations of the exchange rate – as long as markets are complete.

This leaves loosening the complete markets paradigm as an important avenue to explore. Predominantly, the existing literature in this area presupposes the completeness of financial markets, so that more research is needed to assess to what extent deviations from this standard could contribute to resolving the currency premium puzzle in general.

Our own exploration has focused on one specific type of market incompleteness – incomplete spanning. We showed this form of market imperfection does not offer a straightforward solution. Incomplete spanning tends to simultaneously diminish the predictability of exchange rates and the magnitude of currency returns, thus falling short in explaining the significant interest rate differentials observed in the data. This finding underscores that while moving away from the assumption of complete markets is a logical direction for future research, it is far from a straight-forward solution.

The implications of our findings are two-fold. Firstly, they underscore a significant limitation of the current generation of asset pricing models when applied to open economies.

Secondly, the currency premium puzzle highlights the necessity for new models that can simultaneously account for large interest rate differentials and the unpredictability of exchange rates. This need is not merely academic; it is a crucial step in building models that can assess the effect of variation in global and local risk premia on capital flows and the allocation of capital across countries.

By pinpointing a crucial inconsistency between canonical asset pricing and international macroeconomic models, we hope to spur more work on the broader challenge of integrating these two areas. In our view, this integration is essential for developing a more comprehensive understanding of critical phenomena, including the violation of uncovered interest parity, contagion, the global financial cycle, flights to safety, capital retrenchments, and sudden stops. All these phenomena ultimately result from the interplay of international financial markets, risk premia, and allocations. They are critical to understand. The currency premium puzzle, as we have defined it, calls for innovative approaches to address this challenge.

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A Details on the LRR models

A.1 Derivation of Moments of log SDF

Start from EZ preference.

$$U_t = \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A1}$$

SDF is given by

$$\begin{aligned} M_{t+1} &= \frac{\frac{\partial U_t}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} \\ &= \frac{\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial C_{t+1}}}{\frac{\partial U_t}{\partial C_t}} \end{aligned}$$

Note that

$$\begin{aligned} \frac{\partial U_t}{\partial U_{t+1}} &= \delta \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\ &= \delta U_t^{\frac{1}{\psi}} \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}} U_{t+1}^{-\gamma} \\ \frac{\partial U_t}{\partial C_t} &= (1-\delta) \left(1 - \frac{1}{\psi} \right) \left((1-\delta) C_t^{1-\frac{1}{\psi}} + \delta \left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}} C_t^{-\frac{1}{\psi}} \\ &= (1-\delta) \left(1 - \frac{1}{\psi} \right) U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}} \end{aligned}$$

We can then easily see that

$$M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{ \mathbb{E}_t \left[U_{t+1}^{1-\gamma} \right] \right\}^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}$$

Taking logs on both side, we have the log SDF as

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(u_{t+1} - \frac{1}{1-\gamma}\log(\mathbb{E}_t\left[\exp((1-\gamma)u_{t+1})\right])\right)$$

A.1.1 Model without Short-run shocks

Now assume

$$\Delta c_{t+1} = \mu + z_t \tag{A2}$$

$$z_t = \rho z_{t-1} + \sigma \varepsilon_t \tag{A3}$$

Assumption 1. *Assume that u_{t+1} is normal.*

We can easily see that now

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\,\mathrm{var}_t(u_{t+1}) \\
\frac{1}{2}\,\mathrm{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathrm{var}_t(u_{t+1})
\end{aligned}$$

In particular, if we look at unconditional moments:

$$\mathbb{E}(m_{t+1}) = \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\,\mathbb{E}(\mathrm{var}_t(u_{t+1})) \tag{A4}$$

$$\frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(m_{t+1})) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 \mathbb{E}(\mathrm{var}_t(u_{t+1})) \tag{A5}$$

A.1.2 Model with Short-run shocks

If we instead assume

$$\begin{aligned}
\Delta c_{t+1} &= \mu + z_t + \varepsilon_{SR,t+1} \\
z_t &= \rho z_{t-1} + \sigma \varepsilon_t
\end{aligned}$$

Now calculating the second moment is a bit tricky. To simplify things, we define $V_t = \frac{U_t}{C_t}$. Now we have

$$
\begin{aligned}
V_t &= \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(\frac{U_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}} \\
&= \left((1-\delta) + \delta \left\{ \mathbb{E}_t \left[\left(V_{t+1} \frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}
\end{aligned}
$$

The nice thing about this transformation is that v_{t+1} is now independent of Δc_{t+1}. To see this, suppose after some terminal date T, all shocks are 0 and $z_T = 0$. Then we have

$$
\begin{aligned}
v_T &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma) \left(v_{T+1} + \Delta c_{T+1} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\
&= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_T \left[\exp(1-\gamma) \left(v_{T+1} + \mu \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)
\end{aligned}
$$

Obviously $v_T = v_{T+1}$, so we can solve for v_T as a constant. At $T-1$, we have

$$
\begin{aligned}
v_{T-1} &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma) \left(v_T + \Delta c_T \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\
&= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-1} \left[\exp(1-\gamma) \left(v_T + \mu + z_{T-1} + \varepsilon_{SR,T} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)
\end{aligned}
$$

Note that $\mathbb{E}_{T-1} \left[\exp(1-\gamma) \left(v_T + \mu + z_{T-1} + \varepsilon_{SR,T} \right) \right]$ is a function of z_{T-1} only. So v_{T-1} is a function of z_{T-1} only. Similarly

$$
\begin{aligned}
v_{T-2} &= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma) \left(v_{T-1} + \Delta c_{T-1} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right) \\
&= \frac{1}{1-\frac{1}{\psi}} \log \left((1-\delta) + \delta \left\{ \mathbb{E}_{T-2} \left[\exp(1-\gamma) \left(v_{T-1} + \mu + z_{T-2} + \varepsilon_{SR,T-1} \right) \right] \right\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)
\end{aligned}
$$

One can see that v_{T-2} is also a function of z_{T-2} only. Using backward induction, we can conclude that for any t, v_{t+1} is a function of z_{t+1} only, and thus

$$\text{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0.$$

The reason is simple, v_{t+1} depends on $z_{t+1} = \rho z_t + \sigma \varepsilon_{t+1}$ only and $\Delta c_{t+1} = \mu + z_t + \varepsilon_{SR,t+1}$. Given z_t, since $\text{cov}_t(\varepsilon_{t+1}, \varepsilon_{SR,t+1}) = 0$, obviously $\text{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0$.

Now let's re-visit our SDF. Now our SDF has the form:

$$\begin{aligned}
M_{t+1} &= \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma} \\
&= \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}\frac{C_{t+1}}{C_t}}{\left\{\mathbb{E}_t\left[\left(V_{t+1}\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}
\end{aligned}$$

Take logs, we have

$$m_{t+1} = \log(\delta) - \frac{1}{\psi}\Delta c_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(v_{t+1} + \Delta c_{t+1} - \frac{1}{1-\gamma}\log \mathbb{E}_t\left[\exp((1-\gamma)(v_{t+1} + \Delta c_{t+1}))\right]\right)$$

Now assume that v_{t+1} is log-normal, we have

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) = \log(\Delta) &- \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) \\
&+ \left(\frac{1}{\psi} - \gamma\right)\left(\mathbb{E}_t(v_{t+1}) + \mathbb{E}_t(\Delta c_{t+1}) - \mathbb{E}_t(v_{t+1}) - \mathbb{E}_t(\Delta c_{t+1}) - \frac{1}{2}(1-\gamma)\,\text{var}_t(v_{t+1} + \Delta c_{t+1})\right) \\
&= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi} - \gamma\right)(\text{var}_t(v_{t+1} + \Delta c_{t+1}))
\end{aligned}$$

And the conditional variance of the log SDF is given by

$$\begin{aligned}\frac{1}{2}\operatorname{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \operatorname{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \operatorname{var}_t(v_{t+1}+\Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right) cov_t(\Delta c_{t+1}, v_{t+1}+\Delta c_{t+1}) \\ &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \operatorname{var}_t(\Delta c_{t+1}) + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \operatorname{var}_t(v_{t+1}+\Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right) \operatorname{var}_t(\Delta c_{t+1}).\end{aligned}$$

The second equality uses the fact that $\operatorname{cov}_t(v_{t+1}, \Delta c_{t+1}) = 0$. Now substitute in our processes, we have

$$\begin{aligned}\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\mu + z_t) - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)(\operatorname{var}_t(v_{t+1}+\Delta c_{t+1})) \\ \frac{1}{2}\operatorname{var}_t(m_{t+1}) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \operatorname{var}_t(v_{t+1}+\Delta c_{t+1}) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right)\sigma_{SR}^2.\end{aligned}$$

Unconditionally[24]

$$\begin{aligned}\mathbb{E}(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\mu - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\mathbb{E}\left(\operatorname{var}_t(v_{t+1}+\Delta c_{t+1})\right) \\ \frac{1}{2}\mathbb{E}(\operatorname{var}_t(m_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}\right)^2 \sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \mathbb{E}(\operatorname{var}_t(v_{t+1}+\Delta c_{t+1})) - \frac{1}{\psi}\left(\frac{1}{\psi}-\gamma\right)\sigma_{SR}^2.\end{aligned}$$

All short-run shocks do is that it adds a constant. If we don't allow short-run shock volatilities to differ across countries, then nothing changes; if we do allow them to differ across countries, the currency premia they can generate is given by

$$\frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi}+\gamma\right)(\sigma_{SR}^2 - (\sigma_{SR}^\star)^2)$$

Consumption growth volatility is low, so is the difference across countries, so this term is quantitatively not important.

A.2 Long-run risk model with time-varying volatility

A.2.1 Main Results

In this section, we explore if adding time-variation in the second moments might help in mitigating our currency premium puzzle. We abstracted from this mechanism in our discussion in Section 3. Following

[24] Note that I have left $\operatorname{var}_t(v_{t+1}+\Delta c_{t+1})$ as a whole while further simplification could be made. The reason is to highlight the fact that this term as a whole is the same in the mean and the variance of the EZ SDFs and thus can be cancelled when calculating the EZ line.

Bansal and Yaron (2004), long-run risk models often feature stochastic volatility to generate time-varying risk premia. We find that the puzzle is deeply linked to the structure of Epstein-Zin preference and changing the endowment process to incorporate second moments shocks matters little for the FX-share or the FX-slope.

With time-varying volatility, the endowment process for the long-run risk model is given by

$$\begin{aligned}\Delta c_{t+1} &= \mu + z_t + \sigma \varepsilon_{SR,t+1} \\ z_t &= \rho z_{t-1} + w_{t-1}\varepsilon_{LR,t} \\ w_t^2 &= (1-\phi)w_0^2 + \phi w_{t-1}^2 + \sigma_w \varepsilon_{w,t}\end{aligned}$$

where w_t^2 is the time varying volatility and ϕ governs its persistence and σ_w its volatility. The remaining setup is identical to the model in Section 3. That is, we continue to abstract from short-run shocks so that $\sigma = 0$.

When including stochastic volatility, a closed-form solution is only available for approximations of the model. Using a log-linearization with risk adjustments to solve the model, we show that[25]

$$\mathbb{E}(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)(A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2) \tag{A6}$$

$$\frac{1}{2}\operatorname{var}_t(\hat{m}_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)^2 (A_{vw}^2\sigma_w^2 + A_{vz}^2 w_0^2) \tag{A7}$$

The various coefficients labeled A are constants and the variables with a hat denote deviations from the deterministic steady state. Substituting out the right-hand side again shows the tight link between first and second moments of the log SDFs

$$\frac{1}{2}\mathbb{E}(\operatorname{var}_t(\hat{m}_{t+1})) = -\frac{\frac{1}{\psi} - \gamma}{1-\gamma}\mathbb{E}(\hat{m}_{t+1})$$

Adding stochastic volatility to the model does not help in resolving the currency premium puzzle. In fact, this link between the moments is identical to the case without time-varying volatility (see equation (9)). The reason is that the relationship between the first and second moments of the log SDF is deeply embedded in the EZ preferences and determined only by preference parameters. Changing the structure of the endowment process then matters little in terms of breaking the link.

[25]See a detailed proof in Section A.2.2

A.2.2 Detailed Derivation

Now let's solve the model using risk-adjusted affine approximation.

Divide both side of the EZ preference by C_t and define $V_t = U_t/C_t$, we have

$$\begin{aligned}
\exp\left(\left(1-\frac{1}{\psi}\right)v_t\right) &= 1-\delta+\delta\exp\left(\left(1-\frac{1}{\psi}\right)q_t\right)\\
q_t &= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(v_{t+1}+\Delta c_{t+1})\right)^{1-\gamma}\right]\right\}\\
m_{t+1} &= \log(\delta)-\frac{1}{\psi}\Delta c_{t+1}+\left(\frac{1}{\psi}-\gamma\right)\left(v_{t+1}-q_t+\Delta c_{t+1}\right)
\end{aligned}$$

Log-linearize the system around the deterministic steady state, we have

$$\begin{aligned}
\Delta\hat{c}_{t+1} &= z_t+\sigma_{SR}\varepsilon_{SR,t+1}\\
z_t &= \rho z_{t-1}+w_{t-1}\varepsilon_{LR,t}\\
w_t^2 &= (1-\phi)w_0^2+\phi w_{t-1}^2+\sigma_w\varepsilon_{w,t}\\
\hat{v}_t &= \delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)\hat{q}_t\\
\hat{m}_{t+1} &= -\frac{1}{\psi}\Delta\hat{c}_{t+1}+\left(\frac{1}{\psi}-\gamma\right)\left(\hat{v}_{t+1}-\hat{q}_t+\Delta\hat{c}_{t+1}\right)
\end{aligned}$$

We leave the equation with an expectation operator as it is

$$\hat{q}_t = \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(\hat{v}_{t+1}+\Delta\hat{c}_{t+1})\right)^{1-\gamma}\right]\right\}$$

Now guess

$$\hat{v}_t = A_{v0}+A_{vz}z_t+A_{vw}w_t^2$$

We plug it into the equation immediately above, and obtain:

$$\begin{aligned}
\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}\left(A_{v0}+A_{vz}z_t+A_{vw}\hat{w}_t^2\right) &= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0}+A_{vz}z_{t+1}+A_{vw}\hat{w}_{t+1}^2+z_t+\sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\} \\
&= \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{v0}+A_{vz}\rho z_t+A_{vz}\hat{w}_t\varepsilon_{LR,t+1})\right)^{1-\gamma}\right]\right\} \\
&+ \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(A_{vw}(1-\phi)w_0^2+A_{vw}\phi w_t^2+A_{vw}\sigma_w\varepsilon_{w,t+1})\right)^{1-\gamma}\right]\right\} \\
&+ \frac{1}{1-\gamma}\log\left\{\mathbb{E}_t\left[\left(\exp(z_t+\sigma_{SR}\varepsilon_{SR,t+1})\right)^{1-\gamma}\right]\right\} \\
&= A_{v0}+A_{vz}\rho z_t+\frac{1}{2}(1-\gamma)A_{vz}^2w_t^2 \\
&+ A_{vw}(1-\phi)w_0^2+A_{vw}\phi w_t^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 \\
&+ z_t+\frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
&= A_{v0}+A_{vw}(1-\phi)w_0^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
&+ (1+A_{vz}\rho)z_t+\left(\frac{1}{2}(1-\gamma)A_{vz}^2+A_{vw}\phi\right)w_t^2
\end{aligned}$$

Matching coefficients, we have

$$\begin{aligned}
\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{v0} &= A_{v0}+A_{vw}(1-\phi)w_0^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2 \\
\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{vz} &= 1+A_{vz}\rho \\
\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{vw} &= \frac{1}{2}(1-\gamma)A_{vz}^2+A_{vw}\phi
\end{aligned}$$

We can easily solve for all three coefficients.

$$\begin{aligned}
A_{vz} &= \frac{1}{\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}-\rho} \\
A_{vw} &= \frac{\frac{1}{2}(1-\gamma)A_{vz}^2}{\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}-\phi} \\
A_{v0} &= \frac{A_{vw}(1-\phi)w_0^2+\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2+\frac{1}{2}(1-\gamma)\sigma_{SR}^2}{\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}-1}
\end{aligned}$$

We can then solve for the pricing kernel

$$
\begin{aligned}
\hat{m}_{t+1} &= -\frac{1}{\psi}\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t + \Delta\hat{c}_{t+1}\right) \\
&= -\gamma\Delta\hat{c}_{t+1} + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \hat{q}_t\right) \\
&= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(\hat{v}_{t+1} - \frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}\hat{v}_t\right) \\
&= -\gamma(z_t + \sigma_{SR}\varepsilon_{SR,t+1}) + \left(\frac{1}{\psi} - \gamma\right)\left(A_{v0} + A_{vz}(\rho z_t + w_t\varepsilon_{LR,t+1}) + A_{vw}((1-\phi)w_0^2 + \phi w_t^2 + \sigma_w\varepsilon_{w,t+1})\right) \\
&- \left(\frac{1}{\psi} - \gamma\right)\left(\frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}(A_{v0} + A_{vz}z_t + A_{vw}w_t^2)\right) \\
&= \left(\frac{1}{\psi} - \gamma\right)\left(A_{v0} + A_{vw}(1-\phi)w_0^2 - \frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{v0}\right) \\
&+ \left(-\gamma + \left(\frac{1}{\psi} - \gamma\right)\left(A_{vz}\rho - \frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{vz}\right)\right)z_t \\
&+ \left(\frac{1}{\psi} - \gamma\right)\left(A_{vw}\phi - \frac{1}{\delta\exp\left(\left(1-\frac{1}{\psi}\right)\mu\right)}A_{vw}\right)w_t^2 + \left(\frac{1}{\psi} - \gamma\right)A_{vz}w_t\varepsilon_{LR,t+1} \\
&+ \left(-\gamma\sigma_{SR}\varepsilon_{SR,t+1} + \left(\frac{1}{\psi} - \gamma\right)(A_{vw}\sigma_w\varepsilon_{w,t+1})\right) \\
&= -\left(\frac{1}{\psi} - \gamma\right)\left(\frac{1}{2}(1-\gamma)A_{vw}^2\sigma_w^2 + \frac{1}{2}(1-\gamma)\sigma_{SR}^2\right) \\
&- \frac{1}{\psi}z_t + \left(\frac{1}{\psi} - \gamma\right)\left(-\frac{1}{2}(1-\gamma)A_{vz}^2\right)w_t^2 + \left(\frac{1}{\psi} - \gamma\right)A_{vz}w_t\varepsilon_{LR,t+1} \\
&+ \left(-\gamma\sigma_{SR}\varepsilon_{SR,t+1} + \left(\frac{1}{\psi} - \gamma\right)(A_{vw}\sigma_w\varepsilon_{w,t+1})\right)
\end{aligned}
$$

And we have the conditional mean of the log SDF given by

$$
\mathbb{E}_t(\hat{m}_{t+1}) = -\frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2 - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)\sigma_{SR}^2 - \frac{1}{\psi}z_t - \frac{1}{2}\left(\frac{1}{\psi} - \gamma\right)(1-\gamma)A_{vz}^2 w_t^2
$$

And the conditional variance of the log SDF given by

$$\frac{1}{2}\,\mathrm{var}_t(\hat{m}_{t+1}) = \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 A_{vz}^2 w_t^2 + \frac{1}{2}\gamma^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 A_{vw}^2\sigma_w^2$$

Unconditionally

$$\begin{aligned}
\mathbb{E}(\hat{m}_{t+1}) &= -\frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)A_{vw}^2\sigma_w^2 - \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)\sigma_{SR}^2 - \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)(1-\gamma)A_{vz}^2 w_0^2 \\
\frac{1}{2}\,\mathbb{E}(\mathrm{var}_t(\hat{m}_{t+1})) &= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 A_{vz}^2 w_0^2 + \frac{1}{2}\gamma^2\sigma_{SR}^2 + \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 A_{vw}^2\sigma_w^2 \\
&= \frac{1}{\psi}\left(-\frac{1}{2}\frac{1}{\psi}+\gamma\right)\sigma_{SR}^2 - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\,\mathbb{E}(\hat{m}_{t+1})
\end{aligned}$$

Setting $\sigma_{SR} = 0$ yields (A6) and (A7).

B Details on the Disaster Models

The Gourio, Siemer, and Verdelhan (2013) model features EZ preference and thus the SDF is given by

$$M_{t+1} = \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}}{\left\{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$$

We can easily get:

$$\begin{aligned}
\mathbb{E}_t(m_{t+1}) &= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) \\
&\quad + \mathbb{E}_t\left(\left(\frac{1}{\psi}-\gamma\right)u_{t+1}\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right) \\
&= \log(\delta) - \frac{1}{\psi}\,\mathbb{E}_t(\Delta c_{t+1}) \\
&\quad + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\,\mathbb{E}_t\left((1-\gamma)u_{t+1}\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right)
\end{aligned} \tag{A8}$$

To derive the entropy $\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1})$, we need $\log \mathbb{E}_t(M_{t+1})$. Notice that

$$
\begin{aligned}
\log \mathbb{E}_t(M_{t+1}) &= \log \mathbb{E}_t\left(\delta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right) \\
&= \log(\delta) + \log\left(\mathbb{E}_t\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \mathbb{E}_t\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} + \operatorname{cov}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}, \left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right]\right)
\end{aligned}
$$

To make things simpler, we consider the special case where $\frac{C_{t+1}}{C_t}$ is known at time t. While this assumption seems strong at first sight, it has a similar effect to the "no-short-run-shocks" assumption in the long-run risk framework. Intuitively, it says that although agents anticipate a small probability of disaster, the chances that it happens in the next period are minimal, and the effect of disaster mainly materializes in the expected utility U_{t+1} rather than consumption growth. We confirm the harmlessness of this assumption in our simulation results in Table 5. With this simplification, we have

$$
\begin{aligned}
\log \mathbb{E}_t(M_{t+1}) &= \log(\delta) + \log \mathbb{E}_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\right) + \log \mathbb{E}_t\left(\left(\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}\right) \\
&= \log(\delta) - \frac{1}{\psi}(\Delta c_{t+1}) + \left[\log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\log\left(\mathbb{E}_t\left[U_{t+1}^{1-\gamma}\right]\right)\right]
\end{aligned}
$$

Combine with (A8), we have the entropy

$$
\Xi_t(m_{t+1}) = \log \mathbb{E}_t(M_{t+1}) - \mathbb{E}_t(m_{t+1}) = -\mathbb{E}_t\left(\left(\frac{1}{\psi} - \gamma\right) u_{t+1}\right) + \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) \tag{A9}
$$

By comparing the red terms in (A8) and (A9), we can already see a tight link between the two. In particular, if $\psi = 1$, we have

$$
\Xi_t(m_{t+1}) = -\mathbb{E}_t(m_{t+1}) + \log(\delta) - \mathbb{E}_t(\Delta c_{t+1})
$$

Taking unconditional expectations, we have

$$\mathbb{E}(\Xi_t(m_{t+1})) = -\mathbb{E}(m_{t+1}) + \log(\delta) - \mathbb{E}(\Delta c_{t+1})$$

This is a line with a slope of -1 in the generalized SDF-space with entropy on the y-axis. This suggests as long as all countries feature the same preference and consumption growth rate, they all lie on this "EZ-entropy line", which actually coincides with an iso-rf line. That is to say, when $\psi = 1$, all countries would feature exactly the same risk-free rates!

In standard calibrations, ψ is typically close to but larger than 1. In this case, we can use cumulant generating functions and obtain:

$$\begin{aligned}
\Xi_t(m_{t+1}) &= \log\left(\mathbb{E}_t\left(U_{t+1}^{\frac{1}{\psi}-\gamma}\right)\right) - \mathbb{E}_t\left(\left(\frac{1}{\psi}-\gamma\right)u_{t+1}\right) \\
&= \frac{1}{2}\left(\frac{1}{\psi}-\gamma\right)^2 \kappa_{2,t}(u_{t+1}) + \frac{1}{6}\left(\frac{1}{\psi}-\gamma\right)^3 \kappa_{3,t}(u_{t+1}) + \frac{1}{24}\left(\frac{1}{\psi}-\gamma\right)^4 \kappa_{4,t}(u_{t+1})\dots \\
\mathbb{E}_t(m_{t+1}) &= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) - \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left(\log\left(\mathbb{E}_t[U_{t+1}^{1-\gamma}]\right) - \mathbb{E}_t\left((1-\gamma)u_{t+1}\right)\right) \\
&= \left(\log(\delta) - \frac{1}{\psi}\mathbb{E}_t(\Delta c_{t+1})\right) \\
&\quad - \frac{1}{2}(1-\gamma)\left(\frac{1}{\psi}-\gamma\right)\kappa_{2,t}(u_{t+1}) - \frac{1}{6}(1-\gamma)^2\left(\frac{1}{\psi}-\gamma\right)\kappa_{3,t}(u_{t+1}) - \frac{1}{24}(1-\gamma)^3\left(\frac{1}{\psi}-\gamma\right)\kappa_{4,t}(u_{t+1}) + \dots
\end{aligned}$$

C Portfolio Construction and Alternative Estimates

In this section we discuss how our static trade and carry trade portfolios are constructed, as well as alternative estimates of the FX-slope. We follow the procedure and estimates in Hassan and Mano (2019). Our results are robust to alternative portfolio construction methods, for example the equal weighted method utilized in Lustig, Roussanov, and Verdelhan (2011).[26] We use the Hassan and Mano (2019) portfolios because of their close connection to regression based results, which we would base our alternative estimates of the FX-slope on in this section. [27]

Static trade and carry trade returns are constructed by forming a portfolio of currencies weighted by their forward premium relative to the US (equivalently, their risk-free rates). Let $f_{i,t}$ be the log one-period forward

[26]The results are available upon request. The broad pattern that carry traders lose money on the exchange rate is also evident in Table 1 of Lustig, Roussanov, and Verdelhan (2011).

[27]We only provide essential information in this section. Interested readers should refer to Hassan and Mano (2019) for details

exchange rate of currency i at time t, $s_{i,t}$ be the spot rate, both quoted in units of currency i per US dollar, by covered interest rate parity, we have

$$r_{i,t} - r_{US,t} = f_{i,t} - s_{i,t} = fp_{i,t}$$

Sorting currencies by their forward premia ($fp_{i,t}$) is thus equivalent to sorting currencies on their risk-free rates. Let $rx_{i,t+1} = f_{i,t} - s_{i,t+1} = r_{i,t} - r_{US,t} - \Delta s_{i,t+1}$ be the currency premium of currency i relative to the US dollar, our static trade return is then given by

$$\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]$$

where $\overline{fp}_i^e$ denotes the estimated[28] forward premium of currency i over time and $\overline{fp}^e = \frac{1}{N}\Sigma_i \overline{fp}_i$. Intuitively, investors conducting the static trade would weight the currencies using their long-term forward premium, longing high interest rate currencies and shorting low interest ones. They fix their portfolio (the weights, $\overline{fp}_i^e - \overline{fp}^e$, do not change over time), thus conducting a "static" carry trade.

Our carry trade return is given by

$$\Sigma_{i,t}[rx_{i,t+1}(fp_{i,t} - \overline{fp}_t)]$$

where $\overline{fp}_t = \frac{1}{N}\Sigma_i fp_{i,t}$. The only difference from static trade is that the weights now change over time. In each period, investors would weight currencies based on their forward premium at that period.

For both of these trades, we can view the currency portfolio that investors long as one representative country, and currency portfolio that investors short another representative country. Using static trade portfolio as an example, mathematically, the currency return for the representative high-interest rate country relative to the US is given by

$$\Sigma_{i\in\{\forall i \text{ s.t. } \overline{fp}_i - \overline{fp} > 0\},t}[rx_{i,t+1}(\overline{fp}_i - \overline{fp})]$$

The risk-free rate (or forward premium) and exchange rate of this representative country are defined in a similar manner, simply replacing $rx_{i,t+1}$ with $fp_{i,t+1}$ and $-\Delta s_{i,t+1}$, respectively. Using the unconditional

[28]From an investor's perspective, a currency's forward premium over the whole sample period is unknown. We assume investors simply expect $\overline{fp}_i$ to be equal to the mean of $\overline{fp}_{i,t}$ across all available data prior to the investment period. We do exactly the same thing when running our all our simulations in the paper.

Table 7: Estimation of FX-slope

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Horizons (months)	1	1	6	12	1	1	6	12
Sample	1 Reblance				3 Reblance			
Static T: β^{stat}	0.47	0.37	0.56	0.60	0.26	0.18	0.26	0.25
	[0.31, 0.63]	[0.19, 0.55]	[0.36, 0.76]	[0.40, 0.80]	[0.16, 0.36]	[0.08, 0.28]	[0.18, 0.34]	[0.13, 0.37]
Static T: FX-slope	0.89	0.59	1.27	1.50	0.35	0.22	0.35	0.33
	[0.46, 1.68]	[0.24, 1.20]	[0.57, 3.10]	[0.68, 3.90]	[0.19, 0.56]	[0.09, 0.39]	[0.22, 0.51]	[0.15, 0.58]
Carry T: β^{ct}	0.68	0.55	0.62	0.71	0.57	0.45	0.42	0.43
	[0.15, 1.21]	[0.04, 1.06]	[0.05, 1.19]	[0.20, 1.22]	[0.20, 0.94]	[0.10, 0.80]	[0.01, 0.83]	[0.06, 0.80]
Carry T: FX-slope	2.13	1.22	1.63	2.45	1.33	0.82	0.72	0.75
	[0.18,+∞)∪(−∞,-5.78]	[0.04,+∞)∪(−∞,-17.78]	[0.05,+∞)∪(−∞,-6.31]	[0.25,+∞)∪(−∞,-5.55]	[0.25, 16.36]	[0.11, 4.07]	[0.01, 4.94]	[0.06, 4.06]
Sample	6 Reblance				12 Reblance			
Static T: β^{stat}	0.23	0.15	0.25	0.24	0.34	0.23	0.31	0.30
	[0.13, 0.33]	[0.05, 0.25]	[0.17, 0.33]	[0.14, 0.34]	[0.18, 0.50]	[0.05, 0.41]	[0.15, 0.47]	[0.14, 0.46]
Static T: FX-slope	0.30	0.18	0.33	0.32	0.52	0.30	0.45	0.43
	[0.15, 0.49]	[0.05, 0.33]	[0.21, 0.49]	[0.17, 0.51]	[0.22, 0.99]	[0.06, 0.68]	[0.18, 0.88]	[0.17, 0.84]
Carry T: β^{ct}	0.56	0.45	0.45	0.11	0.67	0.52	0.57	0.22
	[0.21, 0.91]	[0.12, 0.78]	[0.08, 0.82]	[-0.16, 0.38]	[0.36, 0.98]	[0.21, 0.83]	[0.26, 0.88]	[-0.11, 0.55]
Carry T: FX-slope	1.27	0.82	0.82	0.12	2.03	1.08	1.33	0.28
	[0.26,10.47]	[0.13,3.61]	[0.08,4.63]	[-0.14,0.62]	[0.55, 59.98]	[0.26, 5.01]	[0.34, 7.59]	[-0.10, 1.24]

Point estimates are taken from Table III in Hassan and Mano (2019). Confidence intervals are calculated using the corresponding standard errors.

moments of these returns, as well as their composition, we could infer FX-slopes for these representative countries as in the main text of this paper.

As shown in Hassan and Mano (2019), one major advantage of forming portfolios in this way is one can easily relate these portfolios to regression results. For example, the static trade is closely related to β^{static} in the following regression:

$$rx_{i,t+1} - \overline{rx}_{t+1} = \beta^{static}(\overline{fp}_i^e - \overline{fp}^e) + \epsilon_{i,t+1}^{static}$$

More importantly, estimates of β^{static} is closely related to FX-slope.

$$\text{FX-slope} = \frac{\hat{\beta}^{static}}{1 - \hat{\beta}^{static}}$$

To see this, note that $\hat{\beta}^{static} = \frac{\Sigma_{i,t}[(rx_{i,t+1} - \overline{rx}_{t+1})(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]} = \frac{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[(\overline{fp}_i^e - \overline{fp}^e)^2]}$, so we have

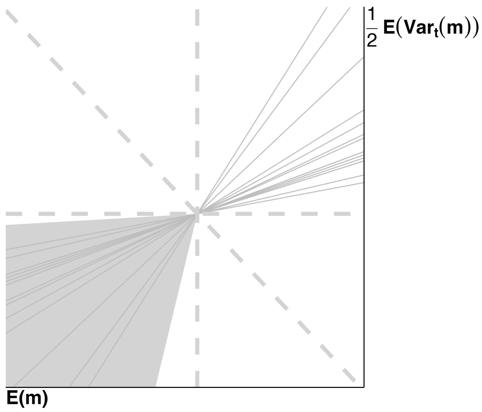
$$\frac{\hat{\beta}^{static}}{1 - \hat{\beta}^{static}} = \frac{\Sigma_{i,t}[rx_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]}{\Sigma_{i,t}[\Delta s_{i,t+1}(\overline{fp}_i^e - \overline{fp}^e)]} = -\frac{1}{\text{FX-share}} = \text{FX-slope}$$

A similar result can be obtained for carry trade returns. We then use the estimates of β^{static} and β^{ct} in Hassan and Mano (2019) to construct alternative estimates of FX-slope.

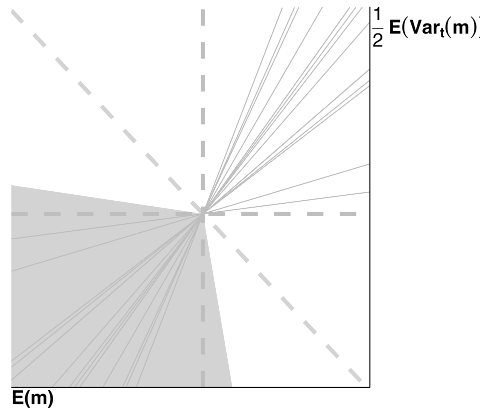
Table 7 lists all the estimates using different samples. We illustrate all these estimates in our SDF space in Figure 7. Each of the lines represent an estimate of the FX-slope, with the shaded area showing the widest confidence interval. One can clearly see the results are consistent with our bootstrap-based estimats: high interest rate currency tend to depreciate, and FX-slope tend to be positive or vertical at best.

Figure 7: Alternative Estimations of the FX-slope

(a) Static trade



(b) Carry trade



This figure plots the point estimates and confidence intervals of the FX-slopes inferred from estimates of β^{static} and β^{carry}. Solid grey lines represent point estimates across different samples. The shaded grey area represents the widest confidence interval.